Heavy Quark Diffusion and Jet Quenching Parameter in Strong Magnetic Field at Weak Coupling

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Strangeness Quark Matter
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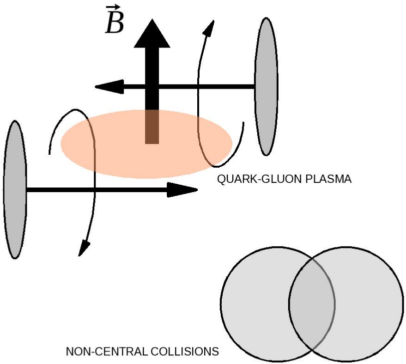
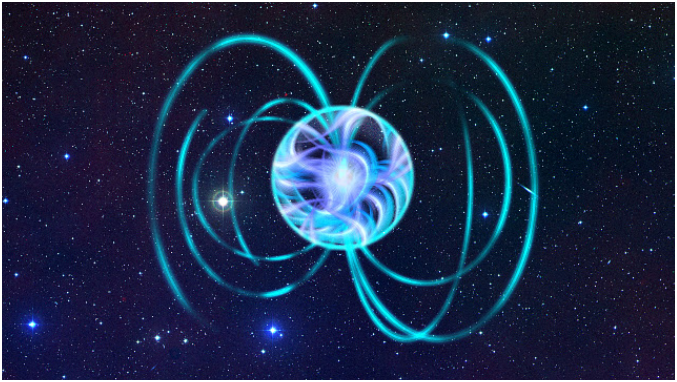
arXiv:1512.03689 (PRD 93, 074028, K. Fukushima, K. Hattori, HUY, Y. Yin)
and arXiv:1605.00188 (S. Li, K. Mamo, HUY)

In heavy-ion collisions, two magnetic fields from the projectiles overlap along the same direction out of reaction plane



$eB \sim (300\,\mathrm{MeV})^2 \sim T^2$,
but the life time may be short $\tau \lesssim 1$ fm/c

Astrophysical eB of magnetars is about 10^{15-16} $G \sim (30\,\mathrm{MeV})^2$, but $T^2 \ll eB$

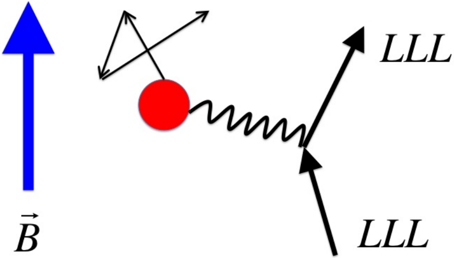


We study how a heavy-quark and a jet transverse momentum diffuse in a thermal Quark-Gluon Plasma (QGP) in the presence of strong magnetic field in complete leading order (leading log + constant under the log) of perturbative QCD,

assuming the hierarchy $\alpha_s eB \ll T^2 \ll eB$

A nice consistent Hard Thermal Loop (HTL) power counting scheme emerges:
soft scale $m_D^2 \sim \alpha_s eB$ from HTL self-energy,
hard scale $\sim T$, and UV cutoff $\Lambda_{UV}^2 \sim eB$

Heavy-quark Diffusion
(small velocity $v \ll 1$)



Coulomb scatterings with thermally excited quasi-particles give random kicks and drag force

A Langevin dynamics

$$\frac{dp_{\parallel}}{dt} = -\eta_{\parallel} p_{\parallel} + \xi_{\parallel}\,, \qquad \frac{d\boldsymbol{p}_{\perp}}{dt} = -\eta_{\perp}\boldsymbol{p}_{\perp} + \boldsymbol{\xi}_{\perp}$$

$$\langle \xi_{\parallel}(t)\xi_{\parallel}(t')\rangle = \kappa_{\parallel}\delta(t-t')\,, \qquad \langle \boldsymbol{\xi}^{i}_{\perp}(t)\boldsymbol{\xi}^{j}_{\perp}(t')\rangle = \kappa_{\perp}\delta^{ij}\delta(t-t')$$

Heavy-quark diffusion constants $\kappa_{\parallel}$ and $\kappa_{\perp}$ in leading order

$$\kappa_{\parallel} \equiv \frac{d}{dt}\langle(\Delta p_{\parallel})^2\rangle = \int d^3\boldsymbol{q}\,\frac{d\Gamma(\boldsymbol{q})}{d^3\boldsymbol{q}}\,q_{\parallel}^2$$

$$\kappa_{\perp} \equiv \frac{d}{dt}\langle(\Delta p_{\perp})^2\rangle = \frac{1}{2}\int d^3\boldsymbol{q}\,\frac{d\Gamma(\boldsymbol{q})}{d^3\boldsymbol{q}}\,|\boldsymbol{q}_{\perp}|^2$$

$\frac{d\Gamma(\boldsymbol{q})}{d^3\boldsymbol{q}}$ **= Coulomb scattering rate with momentum transfer $\boldsymbol{q}$**

Jet Quenching Parameter $\hat{q}$ (energetic jet $v \sim 1$)

BDMPS-Z, GLVW

$$\hat{q} \equiv \frac{d}{dz}\langle \boldsymbol{p}_\perp^2 \rangle = \frac{1}{v}\int d^3\boldsymbol{q}\,\frac{d\Gamma}{d^3\boldsymbol{q}}\boldsymbol{q}_\perp^2$$

=transverse momentum diffusion constant of an energetic particle

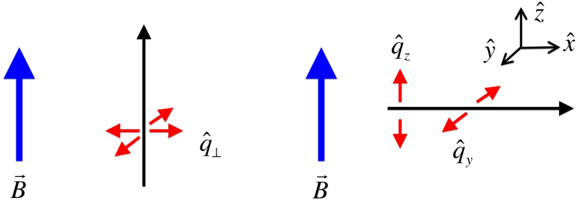
A gluon of energy ω emitted from a jet has a formation time $t_F \sim \omega/k_\perp^2 \sim \omega/(\hat{q}t_F) \sim \sqrt{\frac{\omega}{\hat{q}}}$, and the energy loss is

$$\frac{dI}{d\omega} \sim L\frac{\alpha_s}{t_F} \sim \alpha_s L\sqrt{\frac{\hat{q}}{\omega}}$$

with $\omega_{\rm max} = \hat{q}L^2$. The total energy loss is

$$E_{loss} = \int^{\omega_{max}} \frac{dI}{d\omega} \sim \alpha_s \hat{q} L^2$$

Asymmetric momentum diffusion when a jet is perpendicular to the magnetic field

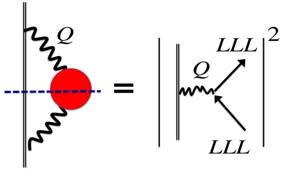


$$\hat{q}_z = \int d^3\boldsymbol{q} \frac{d\Gamma}{d^3\boldsymbol{q}}(q^z)^2 \,, \quad \hat{q}_y = \int d^3\boldsymbol{q} \frac{d\Gamma}{d^3\boldsymbol{q}}(q^y)^2$$

The BDMPS-Z emission vertex evolution becomes an asymmetric imaginary harmonic potential

$$i\frac{\partial F(\boldsymbol{b})}{\partial t} = -\frac{1}{2\omega}\nabla^2_{\boldsymbol{b}} F(\boldsymbol{b}) + \frac{i}{4}\left(\hat{q}_z \boldsymbol{b}_z^2 + \hat{q}_y \boldsymbol{b}_y^2\right) F(\boldsymbol{b})$$

Scattering Rates $\frac{d\Gamma}{d^3\boldsymbol{q}}$ when $eB \gg T^2$



The density of states of the lowest Landau levels (LLL) is $\sim (eB)T$ while that of gluons is $\sim T^3$

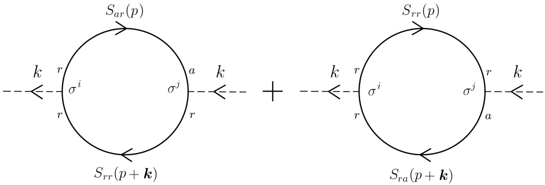
The dominant source of scatterings is the thermally populated LLL quarks

Similarly, the dominant Debye screening is provided by LLL with $m_D^2 \sim \alpha_s eB \gg \alpha_s T^2$

It is enough to consider the HTL from the hard thermal LLL states

The retarded HTL gluon self-energy

$$\Pi_R^{\mu\nu}(Q) = i g_s^2 T_R \langle J_r^\mu(Q) J_a^\nu(-Q) \rangle$$



Then the gluon spectral density

$$\rho^{\rm g}_{\mu\nu}(Q) = -2\ {\rm Im}[\langle A_\mu(Q) A_\nu(-Q) \rangle_R]$$

and the scattering rate

$$\int d^3\boldsymbol{q} \frac{d\Gamma}{d^3\boldsymbol{q}} = \frac{g^2}{2} C_2^J \int \frac{d^4 Q}{(2\pi)^4} \left(n_B(q^0) + n_F(p^0+q^0)\right)$$
$$\times\ (2\pi)\delta(p^0 + q^0 - \sqrt{(\boldsymbol{p}+\boldsymbol{q})^2 + M^2})\rho^{\rm g}_{\alpha\beta}(Q)$$
$$\times\ {\rm Tr}\left[\gamma^\beta \gamma^0 \mathcal{P}(\boldsymbol{p}+\boldsymbol{q}) \gamma^\alpha \gamma^0 \mathcal{P}(\boldsymbol{p})\right]$$

The LLL quark propagators factorize as

$$S_{r/a,r/a}(p) = e^{-\frac{\boldsymbol{p}_\perp^2}{eB}}\ \mathcal{P}\ S^{1+1D}_{r/a,r/a}(p^0, p_\parallel)$$

where $\mathcal{P}$ is the projection onto 1+1 D Dirac spinors, and S^{1+1D} are the real-time propagators of 1+1 D Dirac fermions.

(N.B. Schwinger phase cancels for a 1-loop diagram)

Transverse momentum integral in 1-loop factorizes

$$4\int \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\ e^{-\frac{\boldsymbol{p}_\perp^2}{eB}} e^{-\frac{(\boldsymbol{p}_\perp+\boldsymbol{q}_\perp)^2}{eB}} = \frac{eB}{2\pi} e^{-\frac{1}{2}\frac{\boldsymbol{q}_\perp^2}{eB}}$$

where $eB/(2\pi)$ is simply the transverse space density of states of the LLL

The remaining part is simply the one for 1+1 D fermions!

1+1 D massless Dirac fermion is equivalent to a real scalar field (bosonization) by the mapping

$$J^\mu = \sqrt{\frac{1}{\pi}}\epsilon^{\mu\nu}\partial_\nu\phi\,, \quad J_A^\mu = \sqrt{\frac{1}{\pi}}\partial_\mu\phi$$

Using this, it is easy to compute the gluon spectral density from the hard LLL states

$$\rho^{\rm g}_{\alpha\beta}(Q) = \frac{(2\pi)Q_{\parallel\alpha}Q_{\parallel\beta}\frac{g^2}{\pi}T_R N_F\left(\frac{eB}{2\pi}\right)e^{-\frac{\boldsymbol{q}_\perp^2}{2eB}}{\rm sgn}(q^0)\delta(Q_\parallel^2)}{\left(\boldsymbol{q}_\perp^2 + \frac{g^2}{\pi}T_R N_F\left(\frac{eB}{2\pi}\right)e^{-\frac{\boldsymbol{q}_\perp^2}{2eB}}\right)^2}$$

Note that it is independent of the temperature

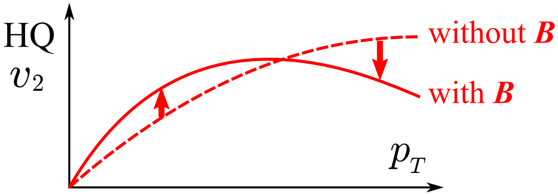
Results in Complete Leading Order

Heavy-quark Diffusion Constants

$$\kappa_\perp = 2\alpha_s^2 T C_R T_R \left(\frac{eB}{2\pi}\right)\left[\log(1/\alpha_s) - \log(T_R/\pi) - \gamma_E - 1\right]$$

$$\kappa_\parallel = 0 + \frac{4\pi\alpha_s^2}{9} N_c C_R T^3 \left[\log\left(\frac{T^2}{\alpha_s eB}\right) - \log(T_R/2\pi) - 1.294\right]$$

Note $\kappa_\perp \gg \kappa_\parallel$ which implies



Jet Quenching Parameter

When a jet is parallel to the magnetic field

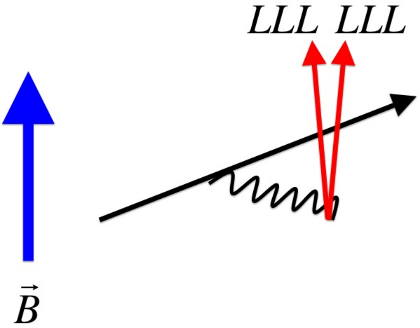
$$\hat{q} = \frac{2}{\pi} C_R T_R N_F \, \alpha_s^2 (eB) T \left(\log\left(1/\alpha_s\right) - 1 - \gamma_E - \log\left(T_R N_F/\pi\right) \right)$$

When a jet is perpendicular to the magnetic field

$$\begin{aligned}
\hat{q}_z &= \frac{16}{3(2\pi)^{3/2}} C_R T_R N_F \alpha_s^2 (eB)^{3/2} \\
&+ \frac{1}{\pi} C_R T_R N_F \alpha_s^2 (eB) T \left(\log\left(\frac{T^2}{\alpha_s T_R N_F \left(\frac{eB}{2\pi}\right)} \right) - 2 \right) \\
\hat{q}_y &= \frac{8}{3(2\pi)^{3/2}} C_R T_R N_F \alpha_s^2 (eB)^{3/2} \\
&+ \frac{1}{\pi} C_R T_R N_F \alpha_s^2 (eB) T \left(\log\left(\frac{T^2}{\alpha_s T_R N_F \left(\frac{eB}{2\pi}\right)} \right) + 0 \right)
\end{aligned}$$

The first part is the vacuum contribution !

Physics of Vacuum Contribution:
Pair Creation of LLL Quark-Antiquark



Thank You !!!