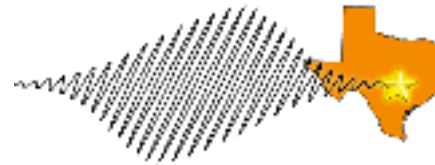


Criticality and nongaussian moments in heavy ion collisions

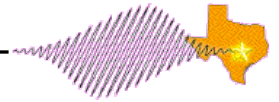
In collaboration with Jiunn-Wei Chen, Jian Deng and Hiroaki Kohyama

Lance Labun

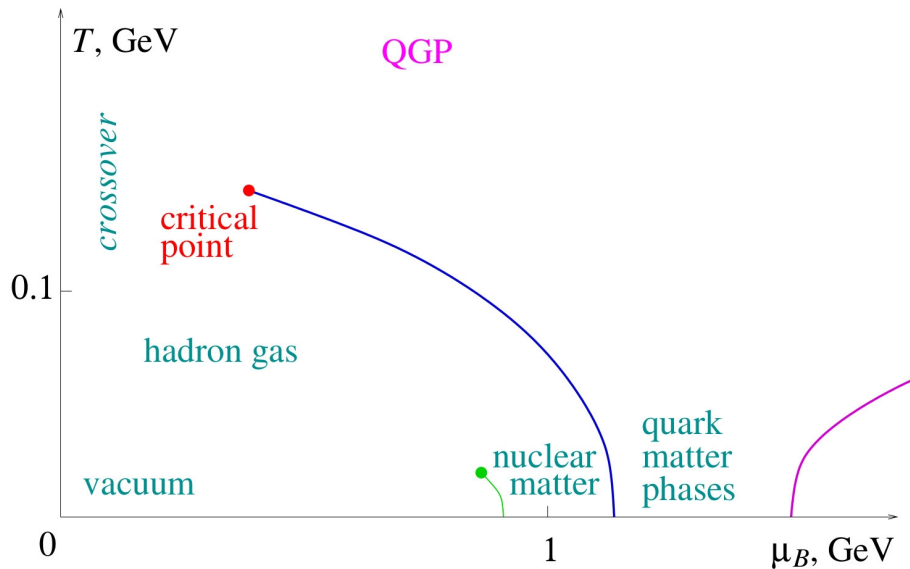
**Center for High Energy Density Science
Dept. of Physics
University of Texas
labun@utexas.edu**



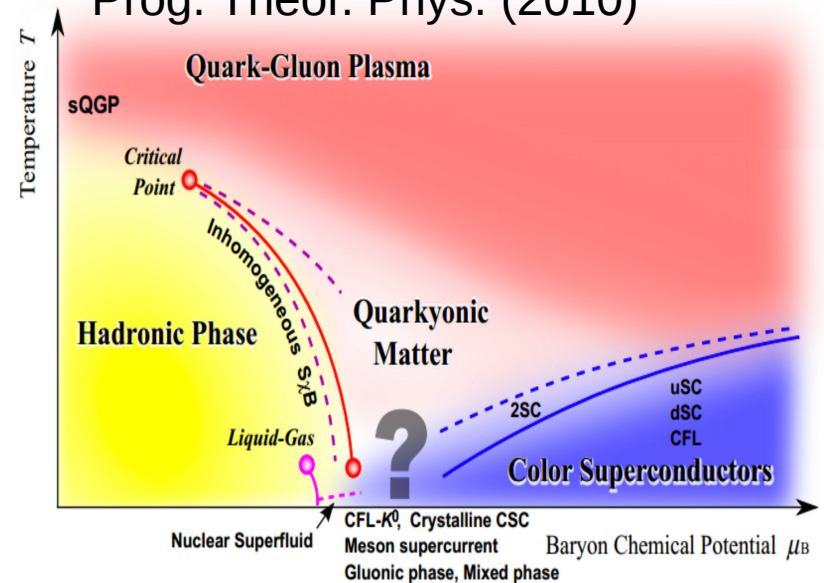
Searching for a Critical End Point



Hypothetical phase diagrams:
Stephanov, PoS LAT2006



Fukushima & Hatsuda,
Prog. Theor. Phys. (2010)

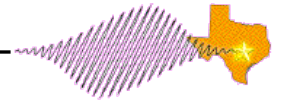


- Models of chiral symmetry breaking suggest First Order Line at high μ_B
- Collider experiments plus lattice establish crossover at $\mu_B \rightarrow 0$

[Gupta et al., Science (2011), MILC lattice, Phys. Rev. D (2009) and many others]

>> Critical end point at moderate μ_B ? Accessible to heavy-ion collisions?
Experimental evidence needed! Important input on QCD phase diagram

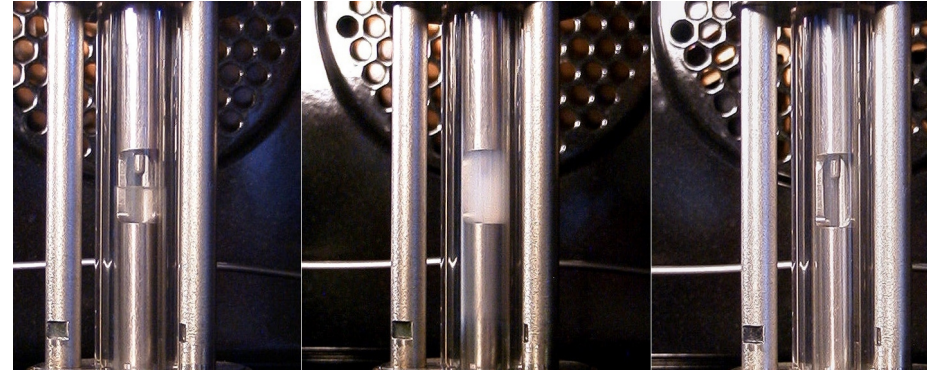
Basic signature of critical phenomena is diverging correlation length



CEP is boundary between 1st order line and crossover
 >> phase transition is 2nd order on CEP

At 2nd order transition,
 Mass of order parameter field --> 0

$$\kappa_2 = \langle \sigma^2 \rangle = \frac{T}{V} \frac{1}{m_\sigma^2} = \frac{T}{V} \xi^2 \rightarrow \infty$$



At $m \rightarrow 0$ there is **no scale** in the system
 >> fluctuations the same at all length scales (critical opalescence)

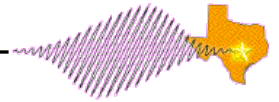
$T < T_c$ $T \sim T_c$ $T > T_c$

Baryon number couples to order parameter:

$$\langle \delta N^2 \rangle \sim \kappa_2 \quad \delta N = N - \langle N \rangle$$

** Requires (quasi)-equilibrium to achieve large correlation length
 (correlations can only grow as fast as velocity of sound)

In real system correlation length does not diverge



Fireball in heavy ion collisions is **expanding and finite size**

Want a hierarchy $\frac{1}{V^{1/3}}, \frac{1}{t_{\text{exp}}} \ll m_{\sigma} \ll T$

Ensure **correlation length is largest dynamical length scale** $\frac{1}{m_{\sigma}} \sim \xi \rightarrow \infty$

And **approximately infinite system** (finite size corrections small)

Equivalent to dropping derivatives in σ effective potential

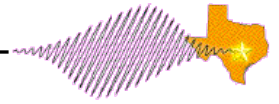
Can relax the slow expansion condition by taking into account memory effects [Mukherjee et al Phys. Rev. C (2015), arXiv:1605.09341]

For finite correlation length, investigate **observables that diverge with higher power of ξ**

$$\langle \delta N^3 \rangle \sim \kappa_3 = \langle \sigma^3 \rangle \sim \xi^6 \quad \langle \delta N^4 \rangle \sim \kappa_4 = \langle \sigma^4 \rangle - 3 \langle \sigma^2 \rangle^2 \sim \xi^8$$

[Stephanov Phys. Rev. Lett (2009), (2011)]

Universality: Long-wavelength physics of correlation length the same within universality class (effective field theory)



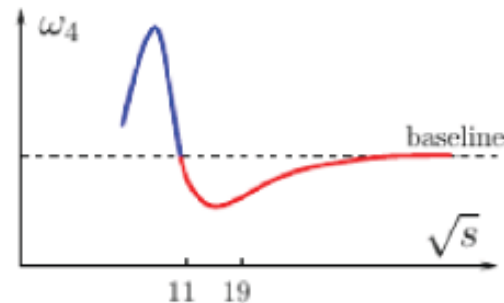
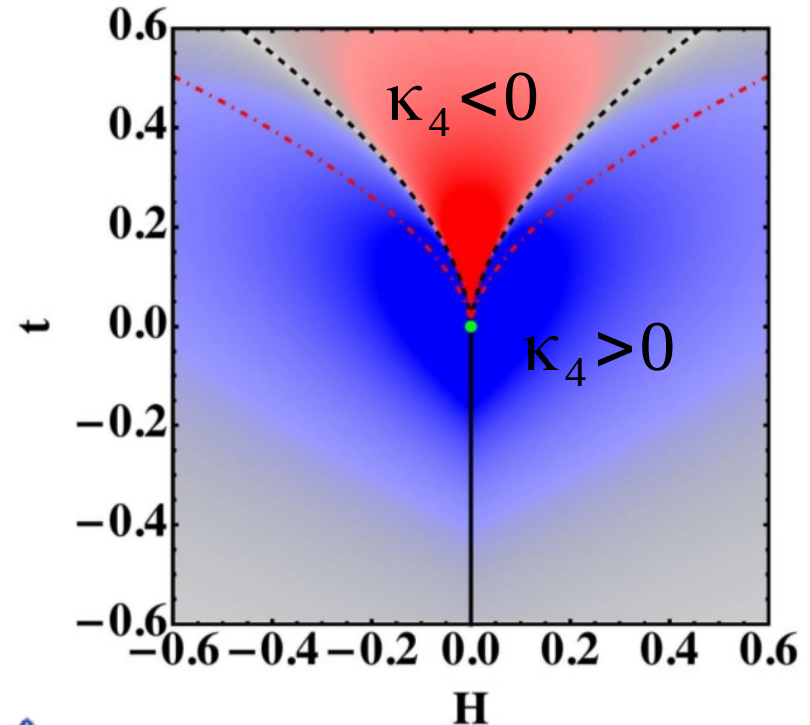
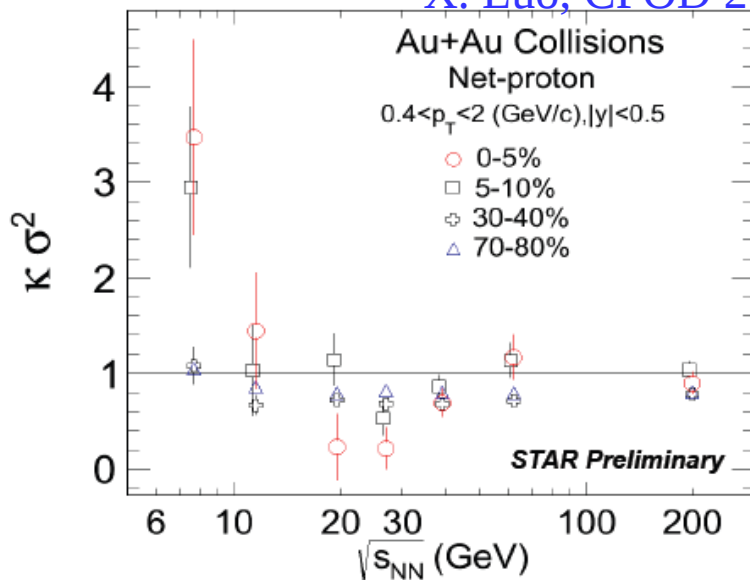
QCD near the CEP belongs to Ising-model class

Mapping from QCD coordinates to Ising coordinates not known

$$(\mu_B, T) \rightarrow (H, t)$$

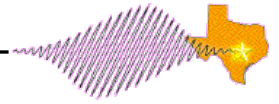
Can compare qualitative behaviour to observables

X. Luo, CPOD 2014



[Stephanov Phys. Rev. Lett (2011)]

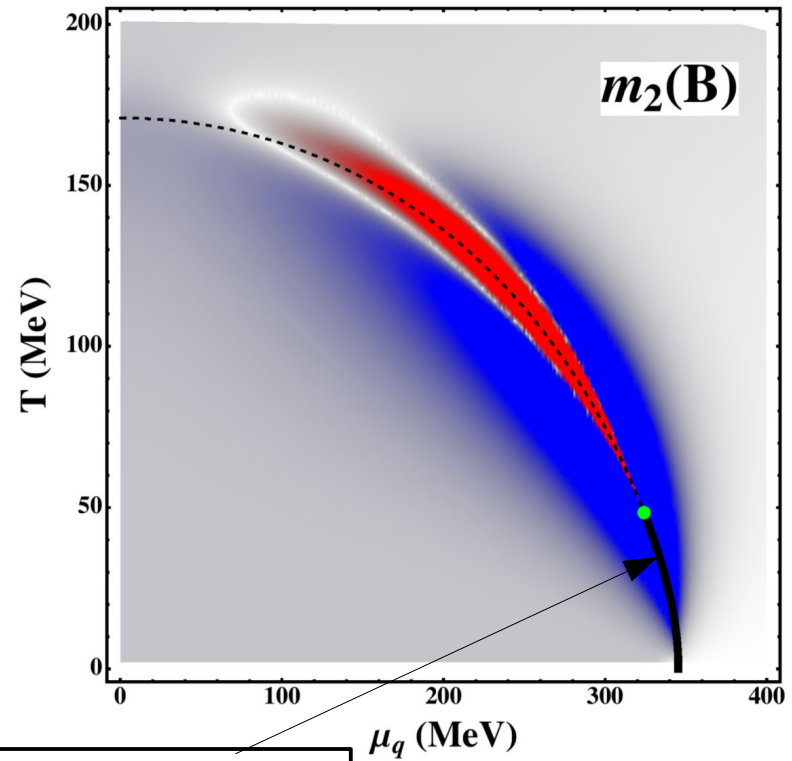
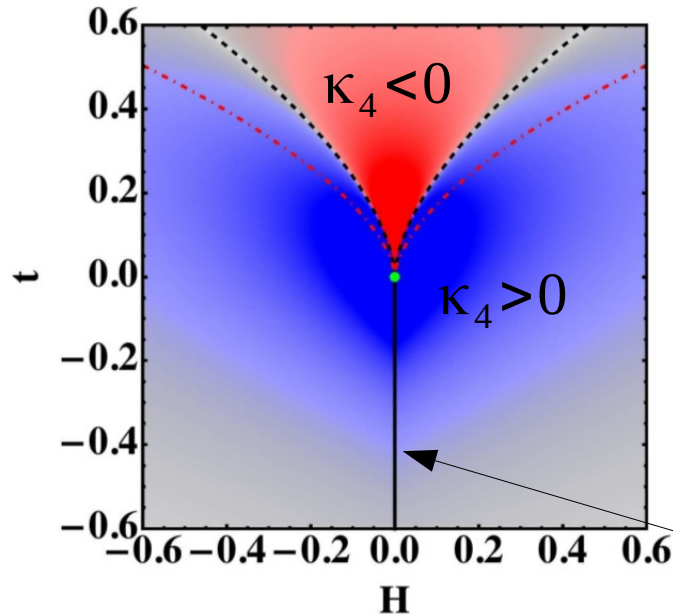
Universality: use NJL model also in Ising/QCD class



NJL near the CEP also belongs to Ising-model class

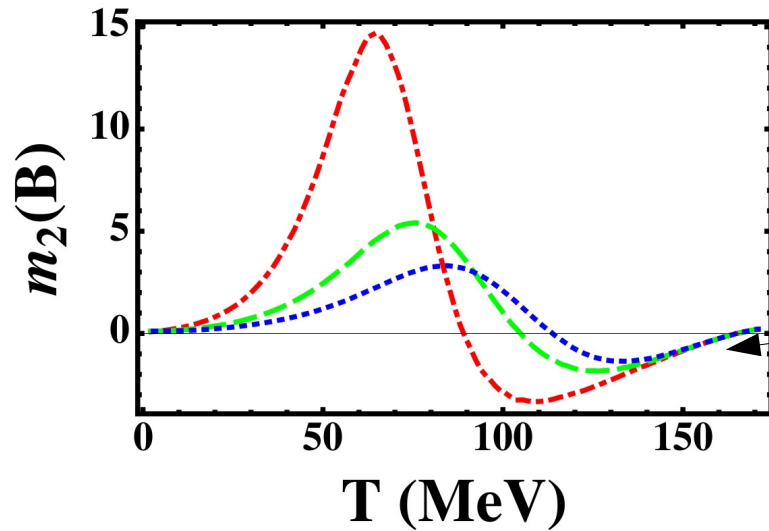
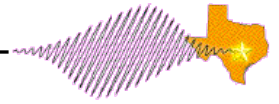
Compute experimental observables in NJL for more systematic study how baryon fluctuations connect to order parameter fluctuations

$$\chi_n = \frac{\partial^n \ln Z}{\partial \mu_B^n} \quad m_2 = \frac{T^2 \chi_4}{\chi_2}$$

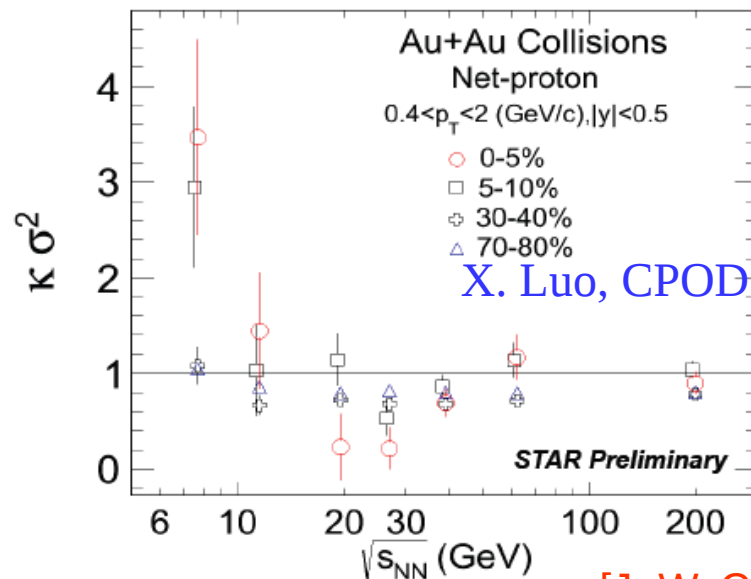
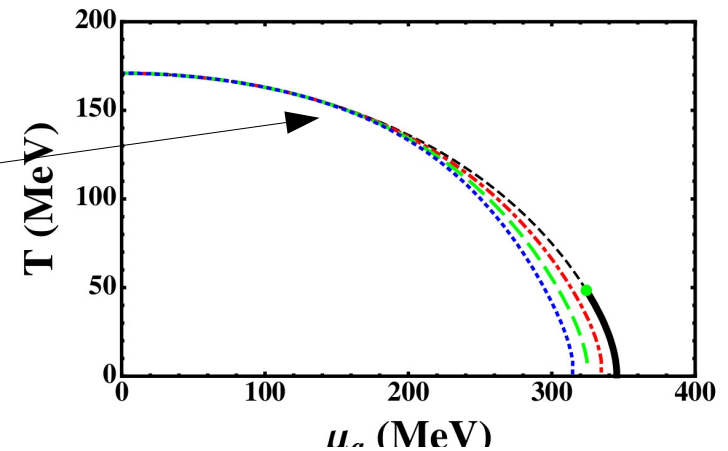


1st order line

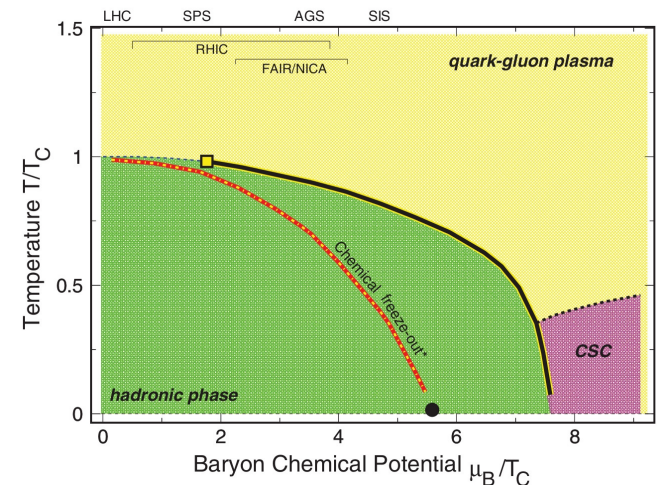
Robust characteristics from NJL model investigation



Compare qualitative behaviour along hypothetical freeze-out lines and **find robust characteristics**

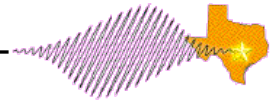


X. Luo, CPOD 2014

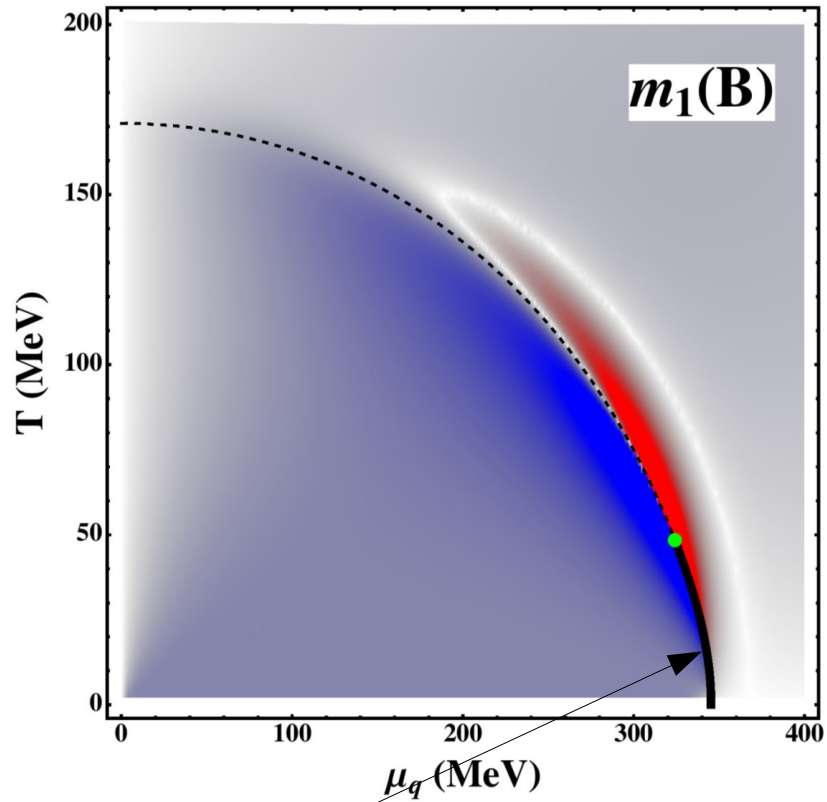
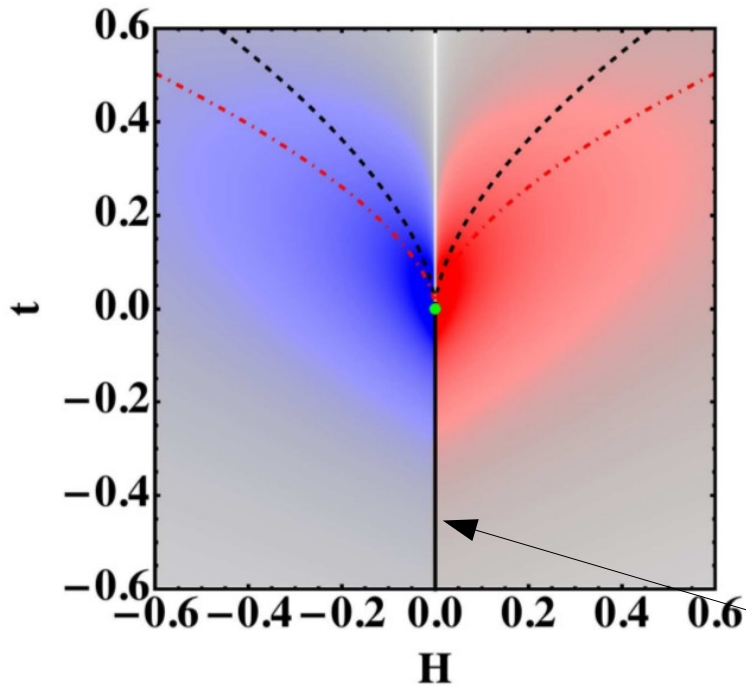


[J.-W. Chen, J. Deng, H. Koyama, and LL PRD (2016)]

Compare multiple observables

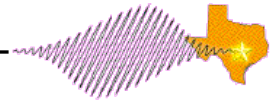


$$\chi_n = \frac{\partial^n \ln Z}{\partial \mu_B^n} \quad m_1 = \frac{T \chi_3}{\chi_2}$$

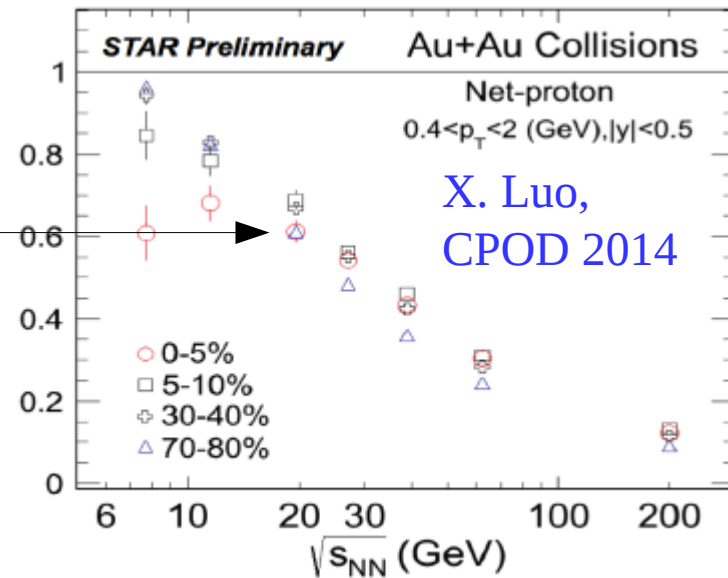
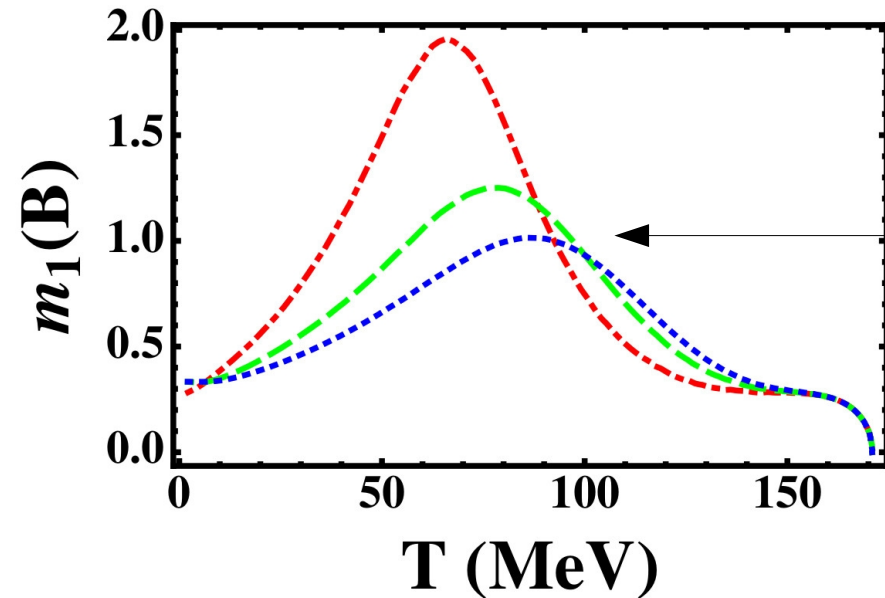
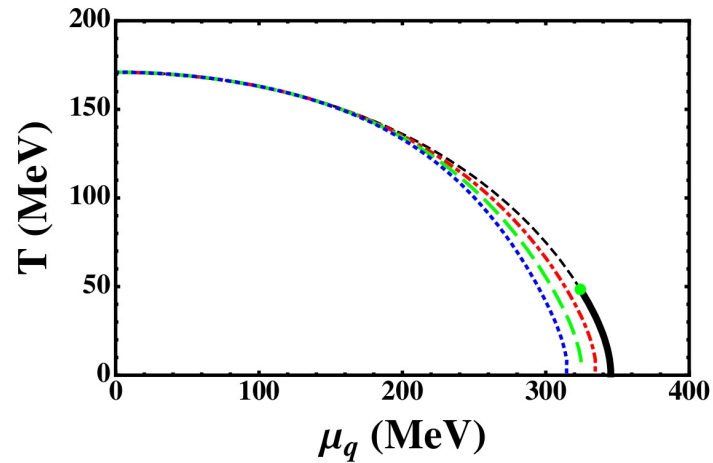


1st order line

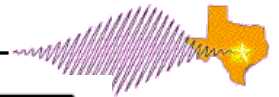
Qualitative agreement with data continues...



$$m_1 = \frac{T \chi_3}{\chi_2} \sim S \sigma(\text{exp})$$

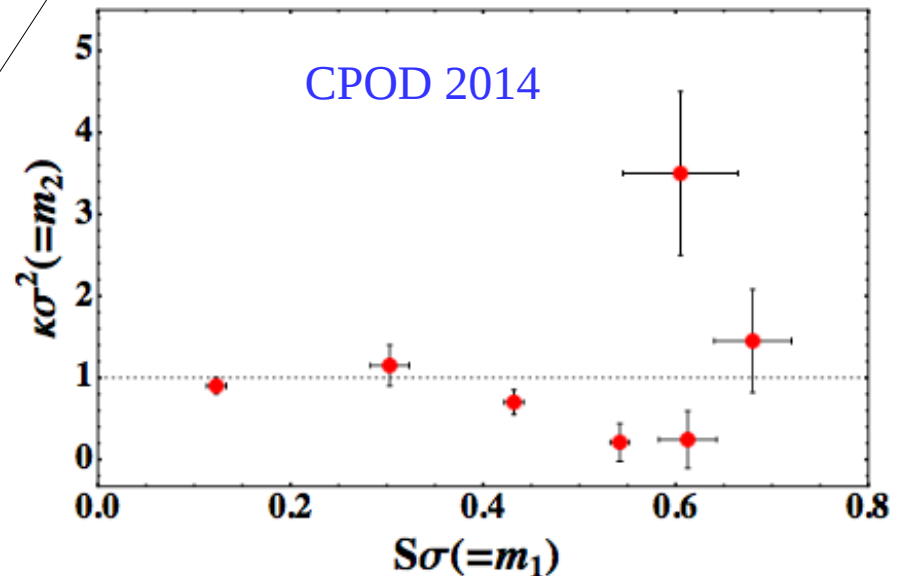
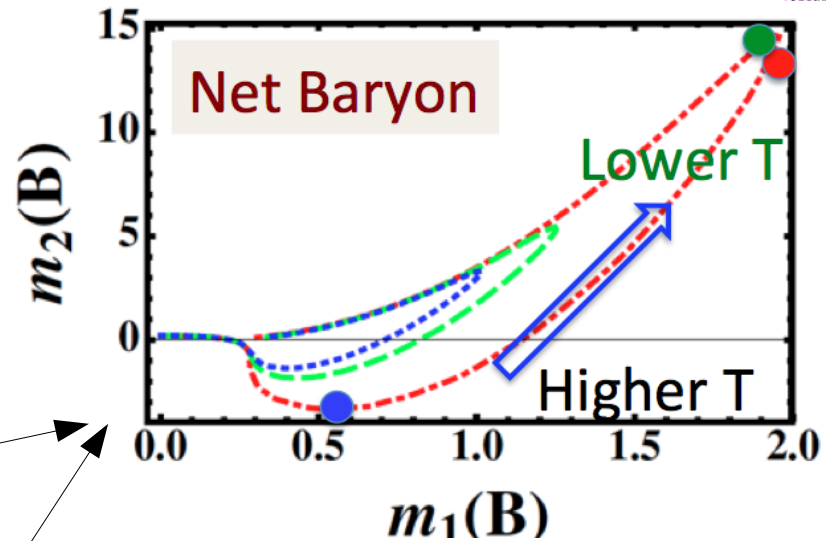
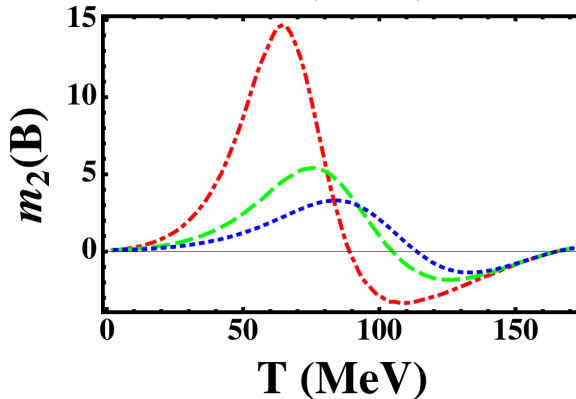
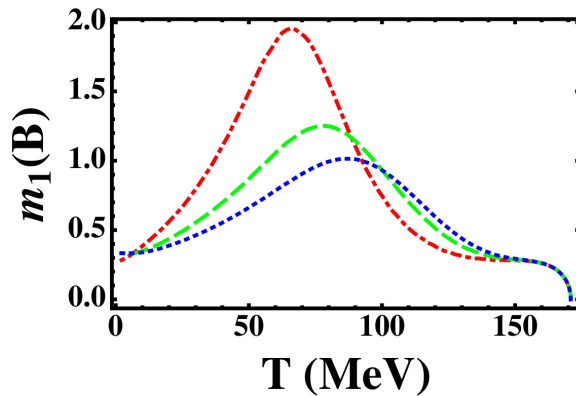


Consistency check: compare observables to each other

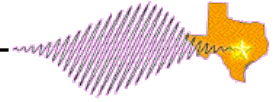


$$\chi_4 = \frac{\partial \chi_3}{\partial \mu_B}$$

Should be a relation, may be robust up to non-thermal effects

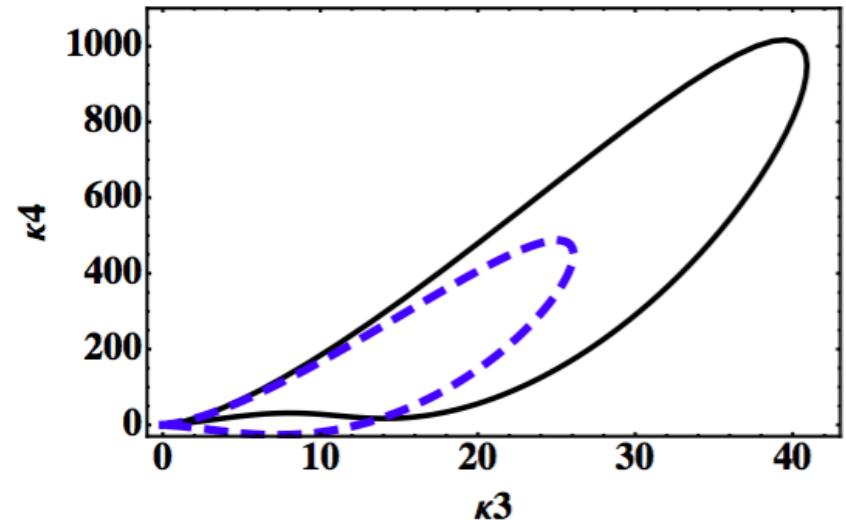
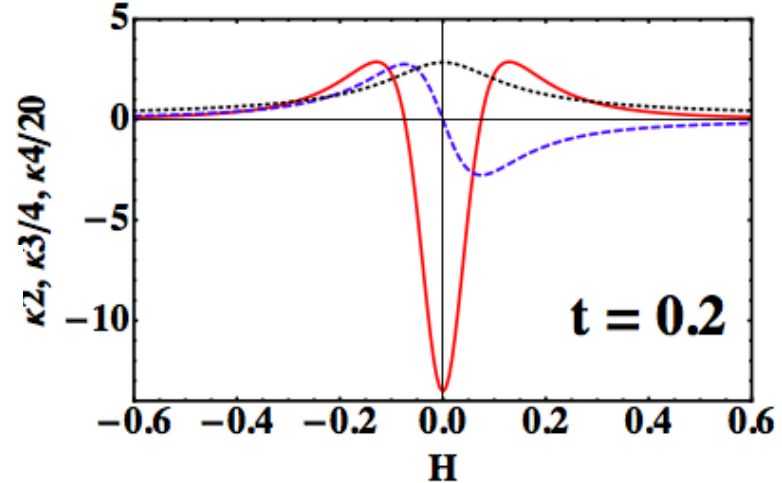
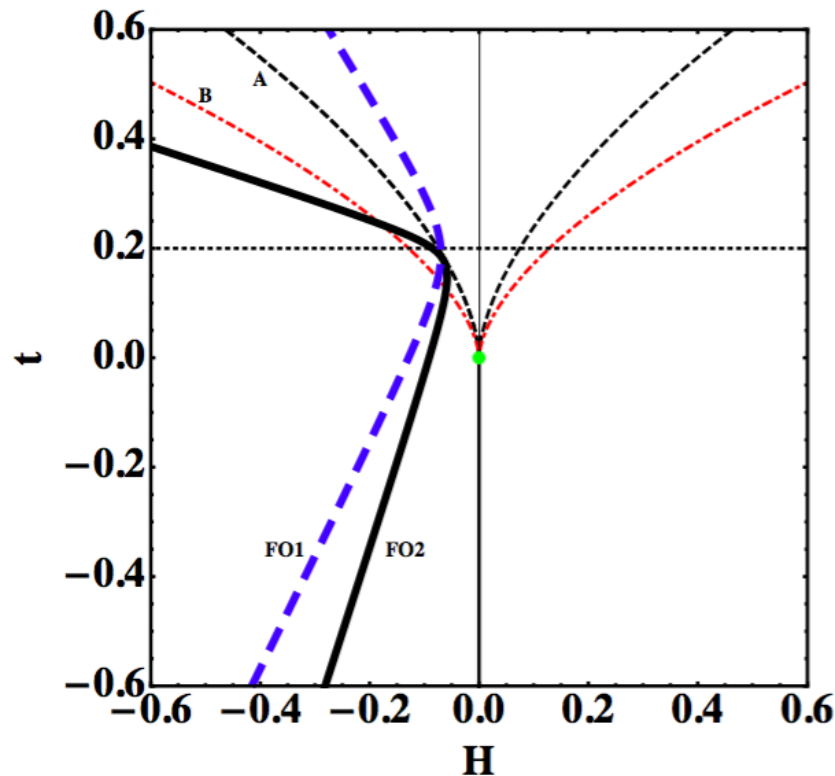


Banana shape may be robust: general argument for ordering of features

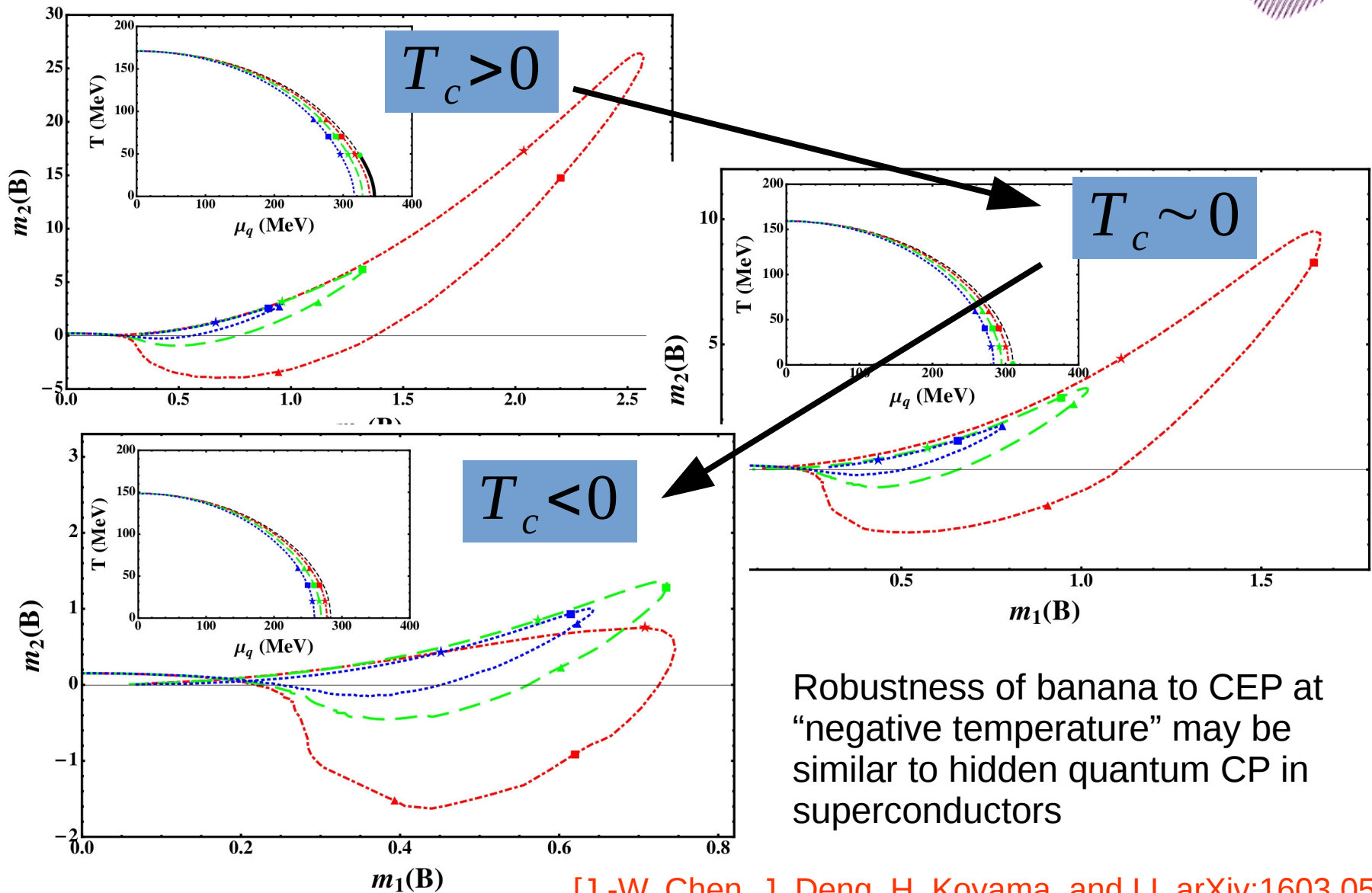
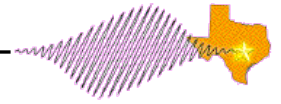


$$T_{min, m_2} > T_{max, m_1} > T_{max, m_2} > T_{CEP}$$

Found also in Gross-Neveu model
 [J.W. Chen, J. Deng and LL, Phys. Rev. D (2015)]
 And **Ising model**



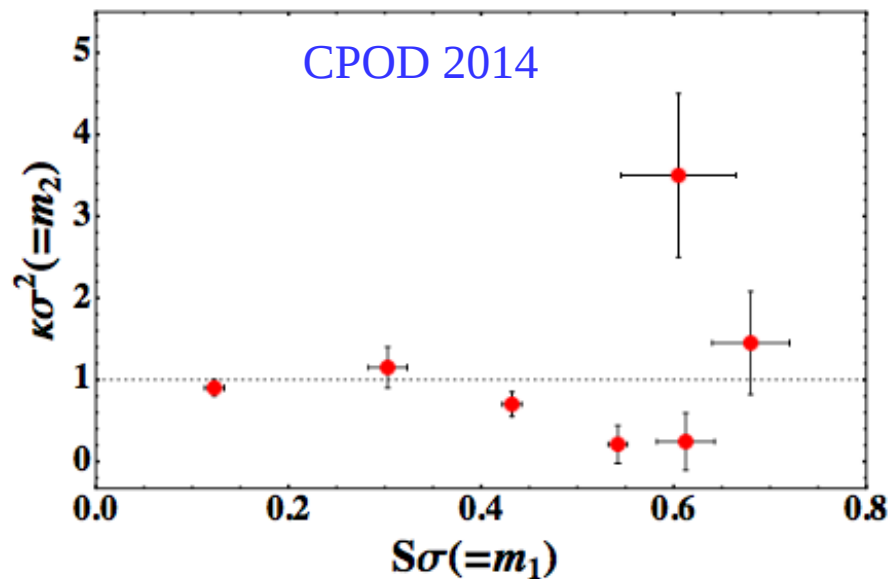
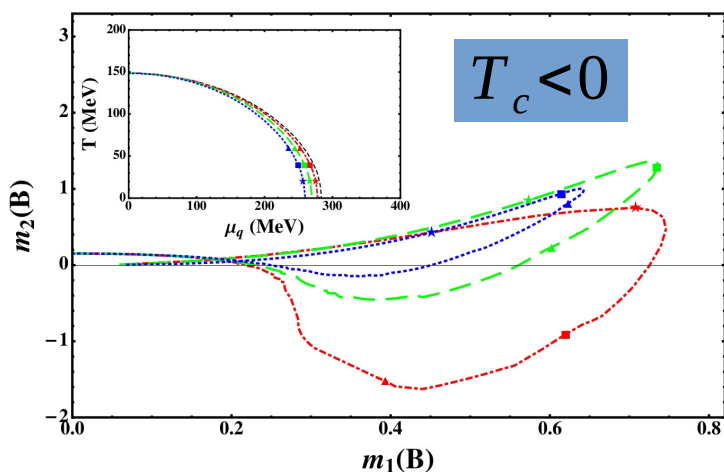
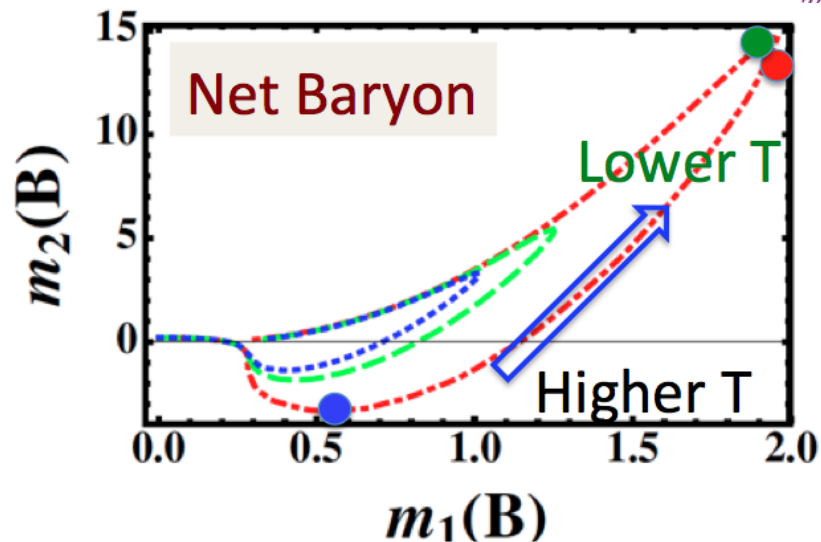
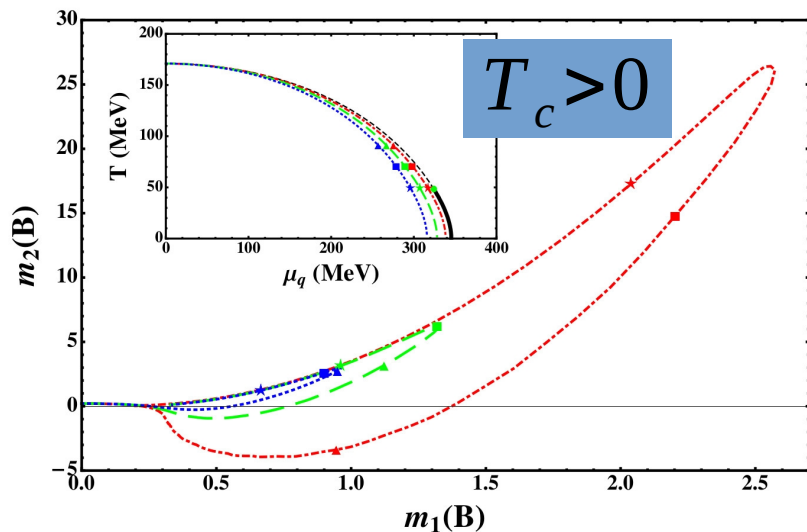
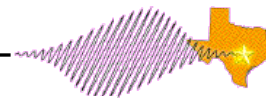
Banana shape is a necessary condition, but not sufficient!



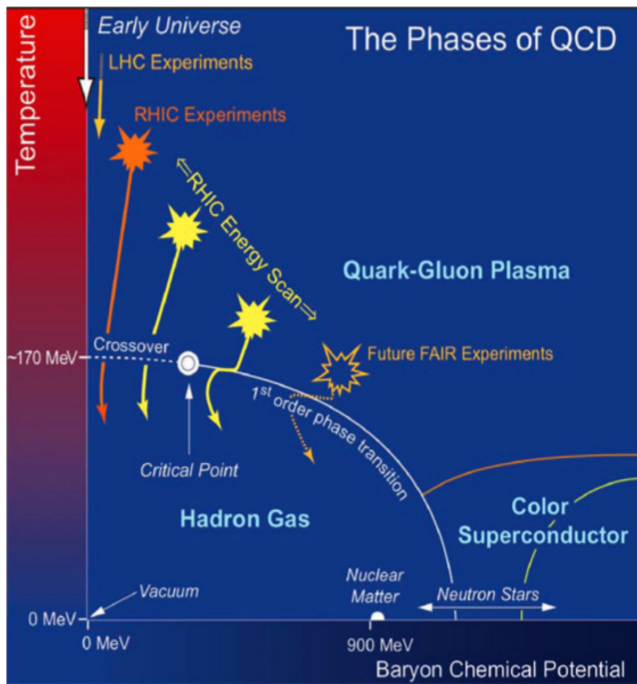
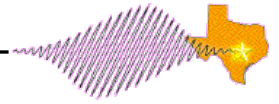
Robustness of banana to CEP at “negative temperature” may be similar to hidden quantum CP in superconductors

Conclusions:

Try to grow a banana but don't bet the farm



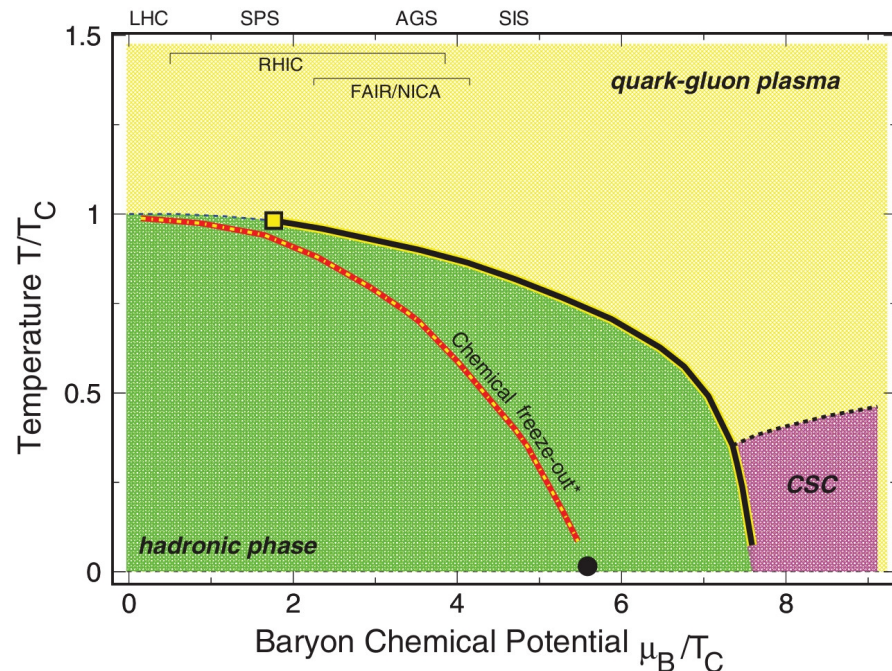
Experimental challenges of the search



STAR White paper 2014

In ideal world:

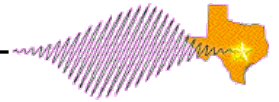
Simply scan beam energy to create fireballs at different densities and temperatures
 >> probe phase diagram along freeze-out line



In real world:

- Systematic changes in observables unrelated to equilibrium phase diagram
- Identification of (μ_B, T) relies on well-motivated models of hadronization, but model-dependence difficult to relate to other models and theory (see below)

Our goal: multiple consistent observables to strengthen interpretation of data



In experimental search for theorized object,
can **only predict necessary conditions on observables** (consequences of theory)
but **not sufficient conditions** (there may be many ways to produce similar results)

Best second option:
set of observables over-constrained by theory

Example: Show the 125 GeV boson is THE Higgs

EW Theory

Higgs potential has 2 parameters:

$$\mu^2 < 0, \lambda$$

Solving the potential relates the 3 measurable properties to 2 parameters of theory
>> over-constrained

Collider experiment

Higgs particle involves several measurable quantities:

$$m_H^2 > 0, v, g_3$$

VEV and trilinear coupling