Symmetric Qumulants and event-plane correlations

with Giuliano Giacalone, Li Yan, Jaki Noronha-Hostler,
based on arXiv:1605.08303

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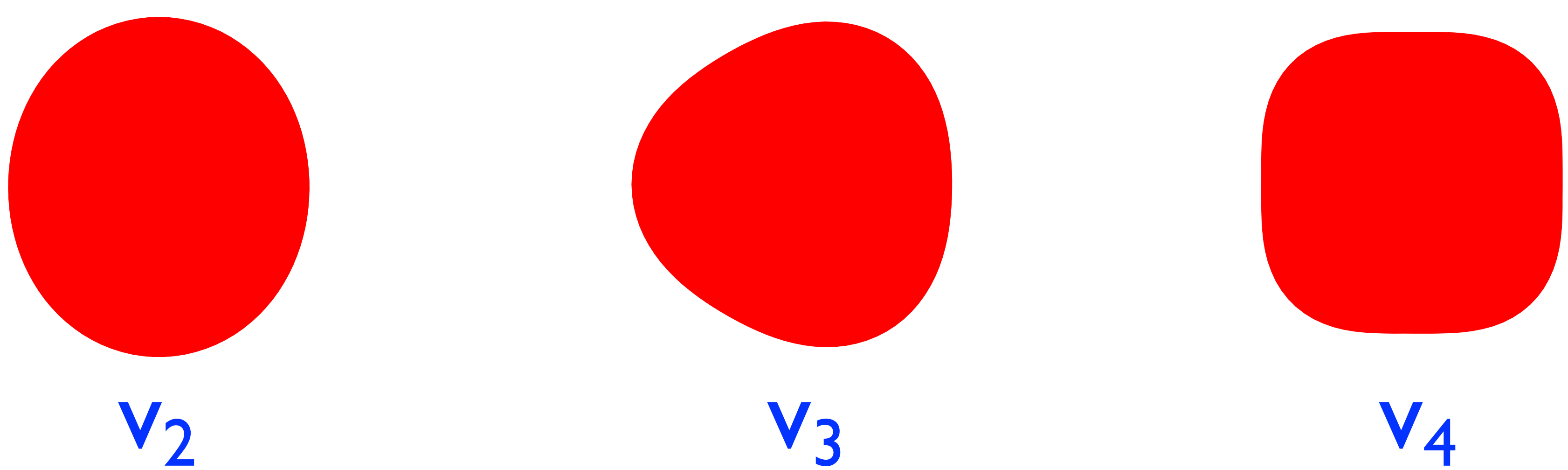
The *flow paradigm*

- Particles are emitted independently, with an *underlying probability distribution* P(φ) that is not isotropic in φ, and is different in every event.
- Fourier decomposition : $P(\varphi)=\sum_n V_n\, e^{-in\varphi}$
- $v_n \equiv |V_n|$=anisotropic flow
  [phase of $V_n \equiv \psi_n$=event plane]



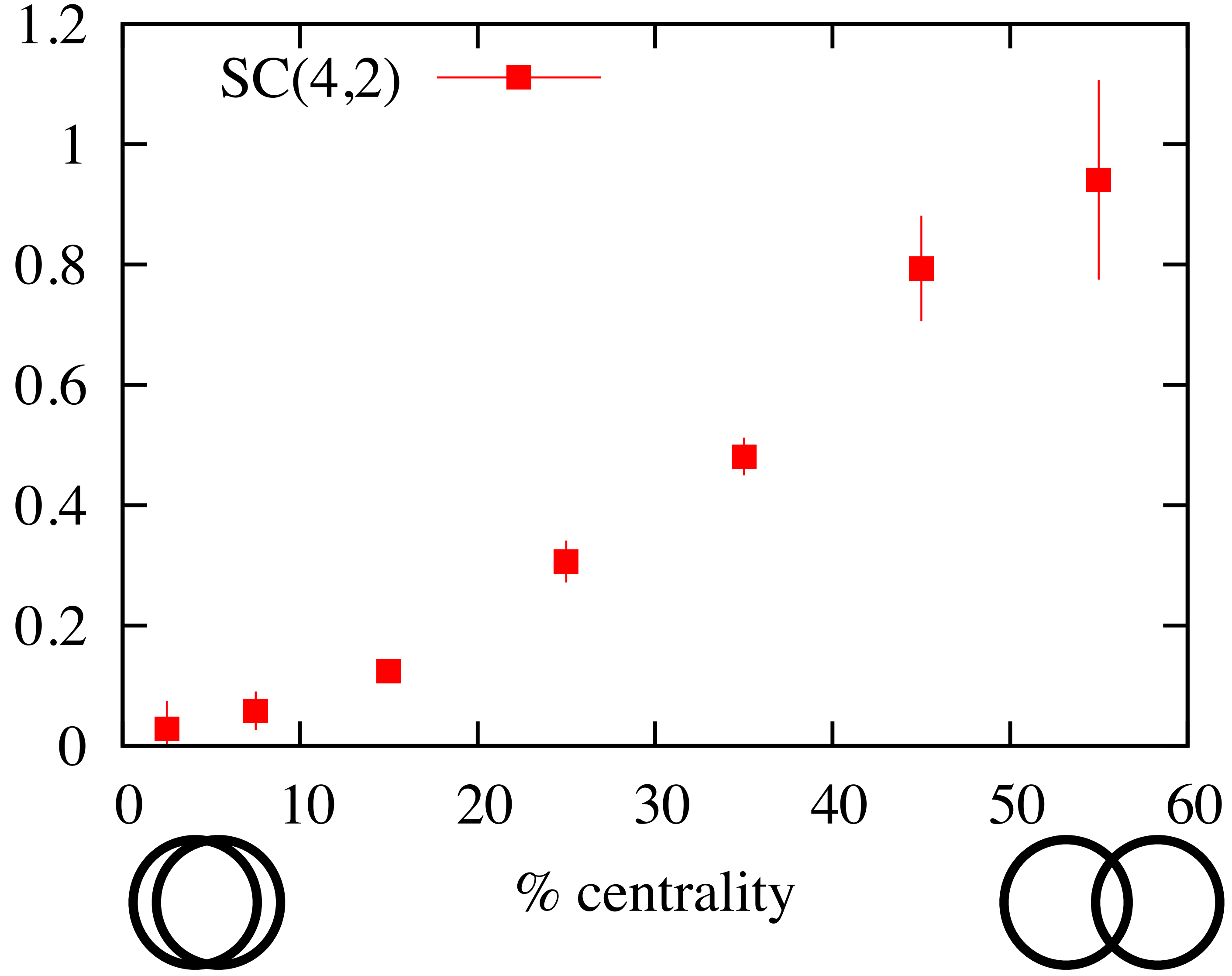
New data from ALICE

- Correlation between the magnitudes of v_4 and v_2: *Symmetric cumulant* defined as

$$SC(4,2) \equiv \frac{\langle v_4^2 v_2^2 \rangle - \langle v_4^2 \rangle \langle v_2^2 \rangle}{\langle v_4^2 \rangle \langle v_2^2 \rangle}$$

- ALICE has recently measured SC(4,2) as a function of centrality (also SC(3,2), not covered in this talk)

New data from ALICE



ALICE Collaboration, arXiv:1604.07663

ATLAS « event-plane correlation »

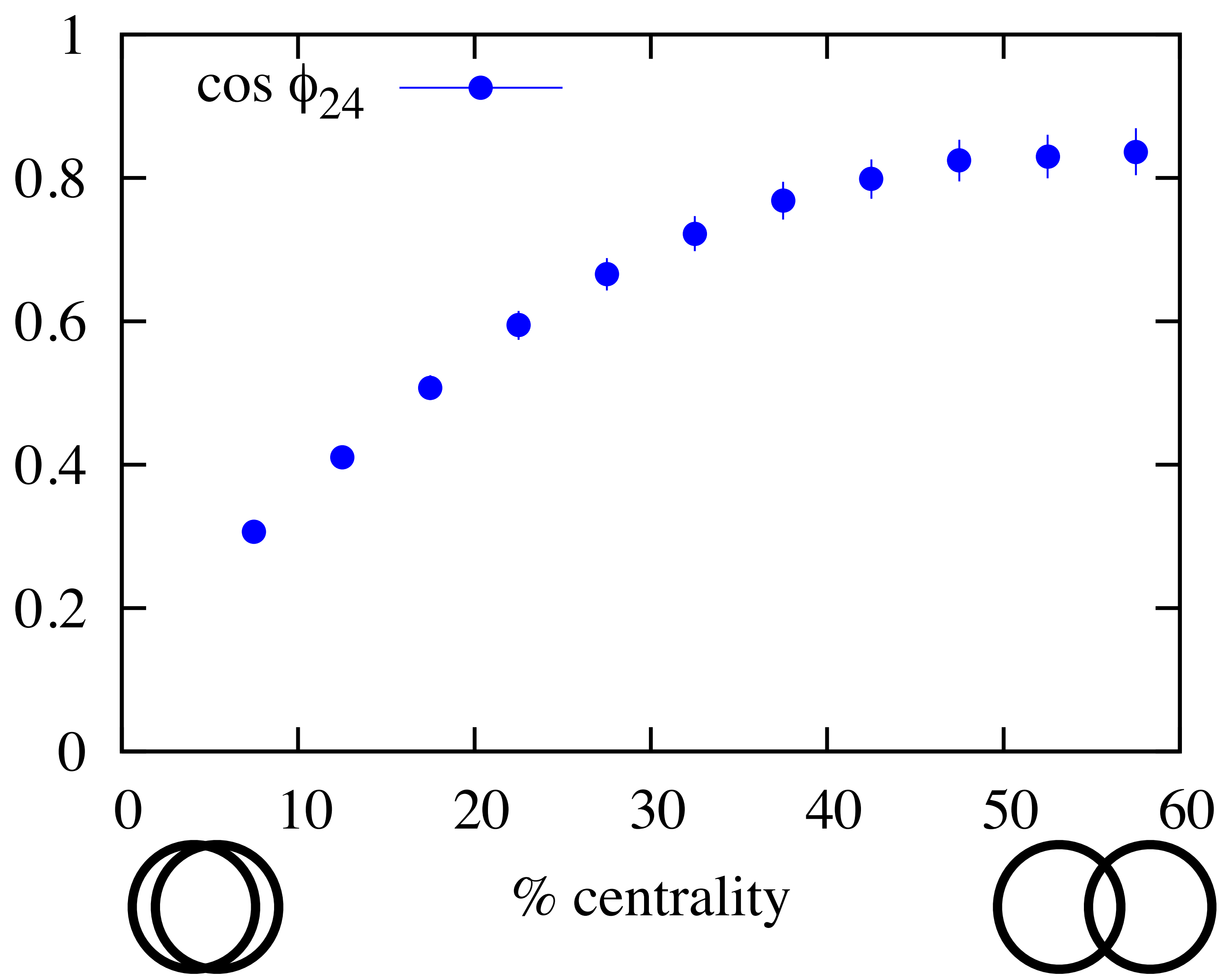
- The *event-plane correlation* measured by ATLAS is in fact a linear (Pearson) correlation between the complex flow coefficients V_4 and $(V_2)^2$

$$\cos\Phi_{24} \equiv \frac{\mathrm{Re}\langle V_4 (V_2^*)^2\rangle}{\sqrt{\langle v_4^2\rangle\langle v_2^4\rangle}}$$

- It is also a measure of the correlation between V_4 and V_2, which involves the relative angle *and* the magnitudes.

Luzum JYO arXiv:1209.2323

ATLAS « event-plane correlation »



ATLAS Collaboration, arXiv:1403.0489

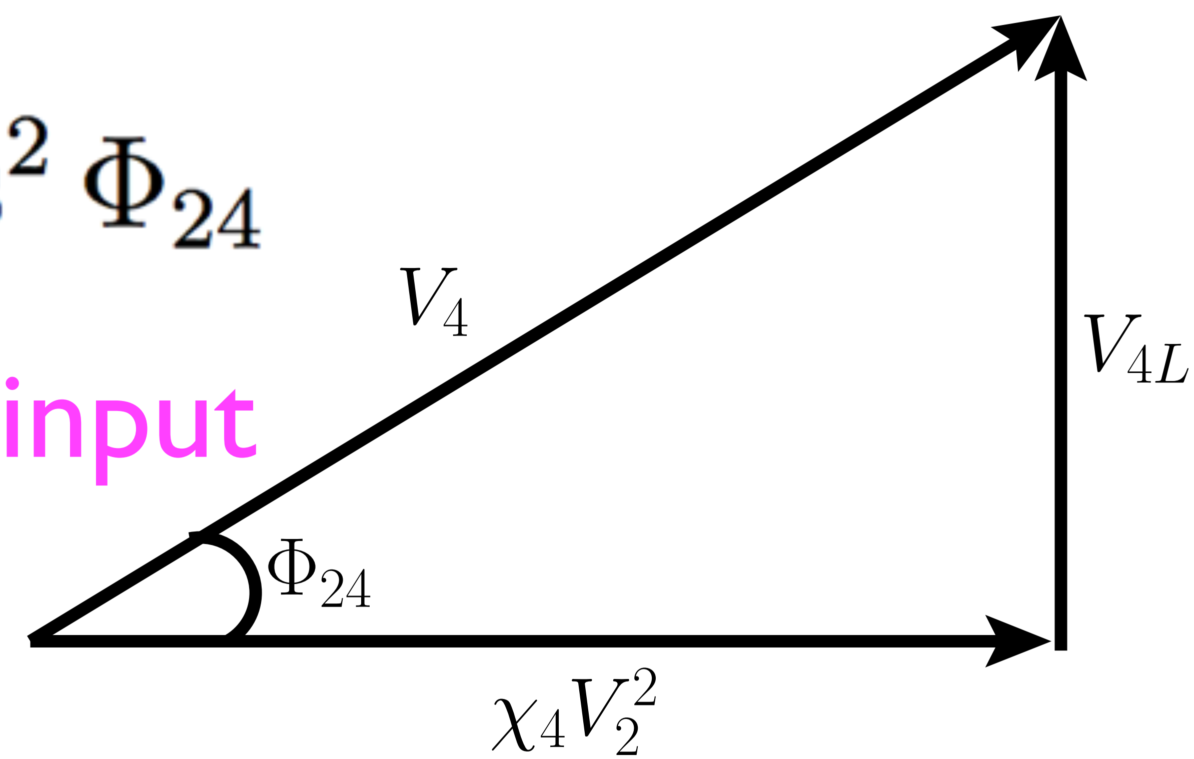
- We have two measures of the correlation between V_4 and V_2, the *symmetric cumulant* and the *event-plane correlation*
- I derive a *quantitative relation* between these two measures, test it on hydro calculations and then on data.

Linear and nonlinear hydro response

- Decompose $V_4 = V_{4L} + \chi_4 (V_2)^2$, with χ_4 = constant fixed so that linear correlation between the two terms = 0.
- Then Φ_{24} measures the relative magnitude of the 2 terms:

$$\chi_4^2 \langle v_2^4 \rangle = \langle v_4^2 \rangle \cos^2 \Phi_{24}$$

- Just math, no physics input



Linear and nonlinear hydro response

- Hydrodynamics also predicts the decomposition $V_4 = V_{4L} + \chi_4 (V_2)^2$, where
- V_{4L} = response to initial fluctuations in 4th harmonic
- $\chi_4 (V_2)^2$ = nonlinear response induced by hydrodynamic evolution

Teaney & Yan arXiv:1206.1905

Yan & JYO arXiv:1502.02502

Linear and nonlinear hydro response

- $V_4 = V_{4L} + \chi_4 (V_2)^2$
- We assume that linear and nonlinear are *independent* (stronger than *uncorrelated*)
- Then: only correlation between $(v_4)^2$ and $(v_2)^2$ is from the nonlinear part:

$$\langle v_4^2 v_2^2 \rangle - \langle v_4^2 \rangle \langle v_2^2 \rangle = \chi_4^2 \left(\langle v_2^6 \rangle - \langle v_2^4 \rangle \langle v_2^2 \rangle \right)$$

Result

Expressing χ_4 as a function of the event-plane correlation, we obtain:

$$SC(4,2) \quad = \quad \left(\frac{\langle v_2^6 \rangle}{\langle v_2^4 \rangle \langle v_2^2 \rangle} - 1 \right) \cos^2 \Phi_{24}$$

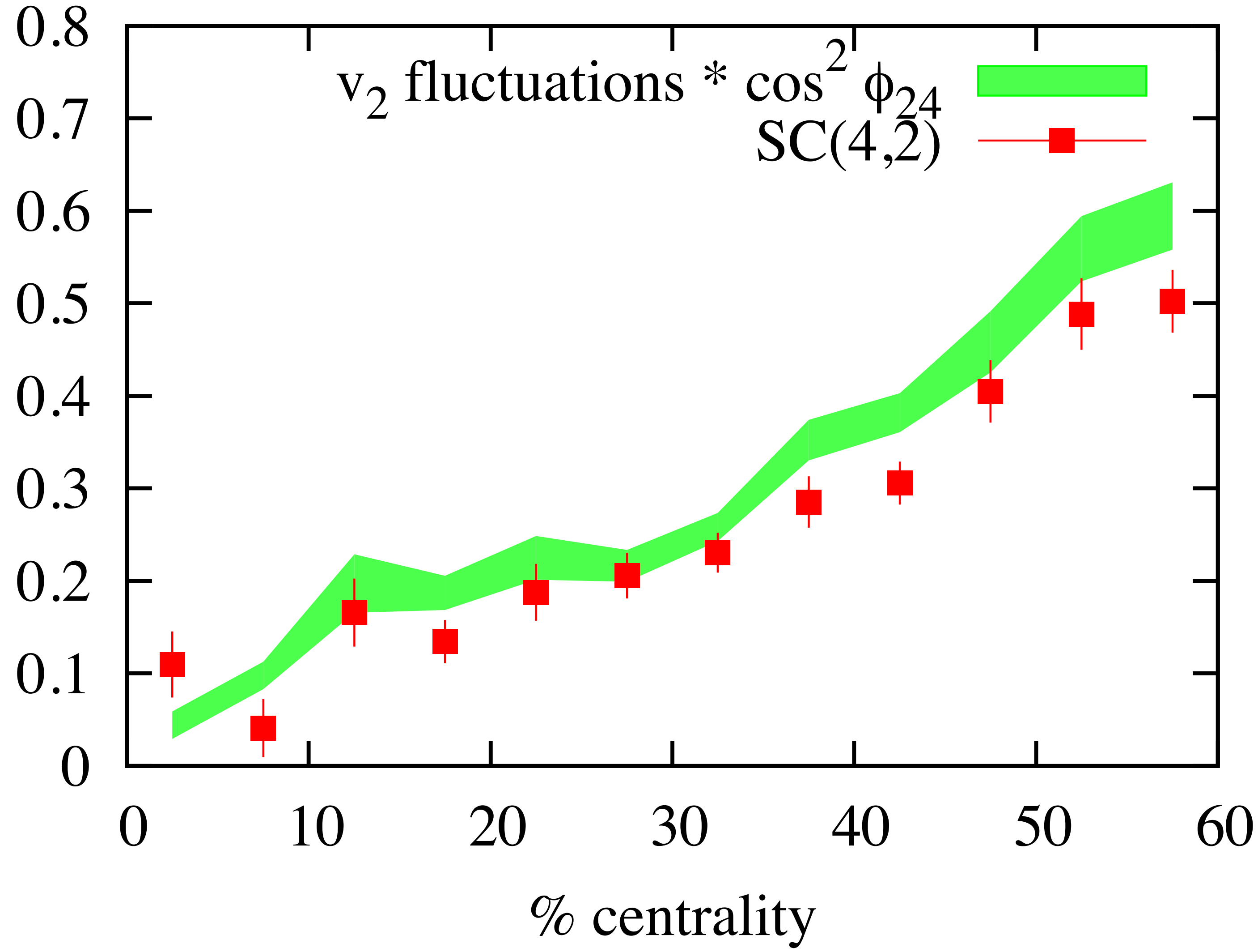
symmetric cumulant

elliptic flow fluctuations

event-plane correlation

Event-by-event hydrodynamics

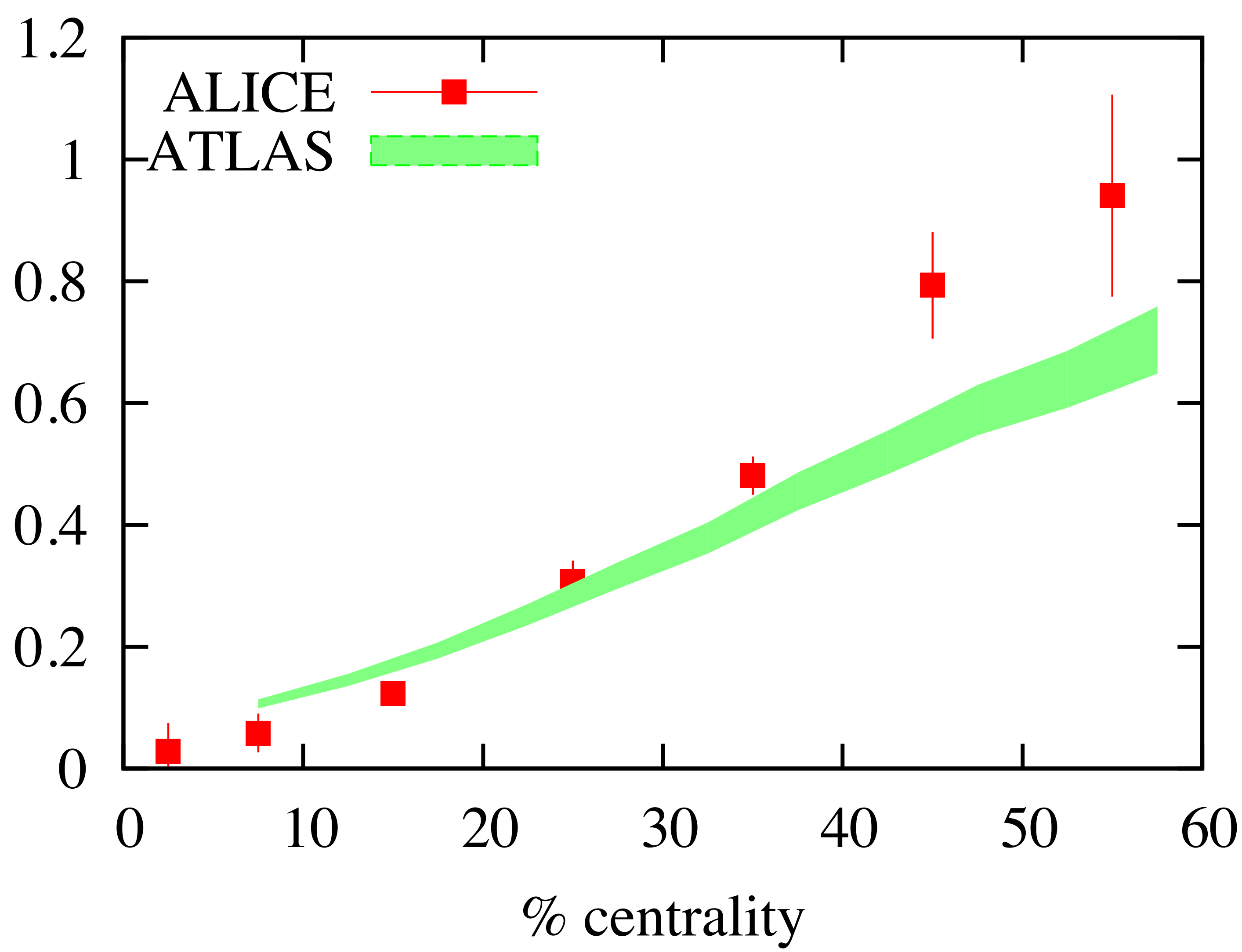
We compute both sides of the equation independently in event-by-event viscous hydro with Glauber initial conditions



The relation is satisfied to a good approximation for all centralities

ALICE versus ATLAS

Using elliptic flow fluctuations (cumulants) and event plane correlations from ATLAS:



ALICE versus ATLAS

- Agreement not as good as in hydro. why?
- ATLAS event-plane correlations are measured with a large pseudorapidity gap and over a wide interval -4.8 to 4.8
- ALICE SC(4,2) is measured without any gap and over the interval -0.8 to 0.8
- Longitudinal flow fluctuations induce a decoherence which may explain why the ATLAS result is smaller.

Predictions

Same methodology applied to different orders:

$$SC(4,3) = \left(\frac{\langle v_2^4 v_3^2\rangle}{\langle v_2^4\rangle\langle v_3^2\rangle} - 1\right)\cos^2\Phi_{24}$$

$$SC(5,2) = \left(\frac{\langle v_2^4 v_3^2\rangle}{\langle v_2^2 v_3^2\rangle\langle v_2^2\rangle} - 1\right)\cos^2\Phi_{235}$$

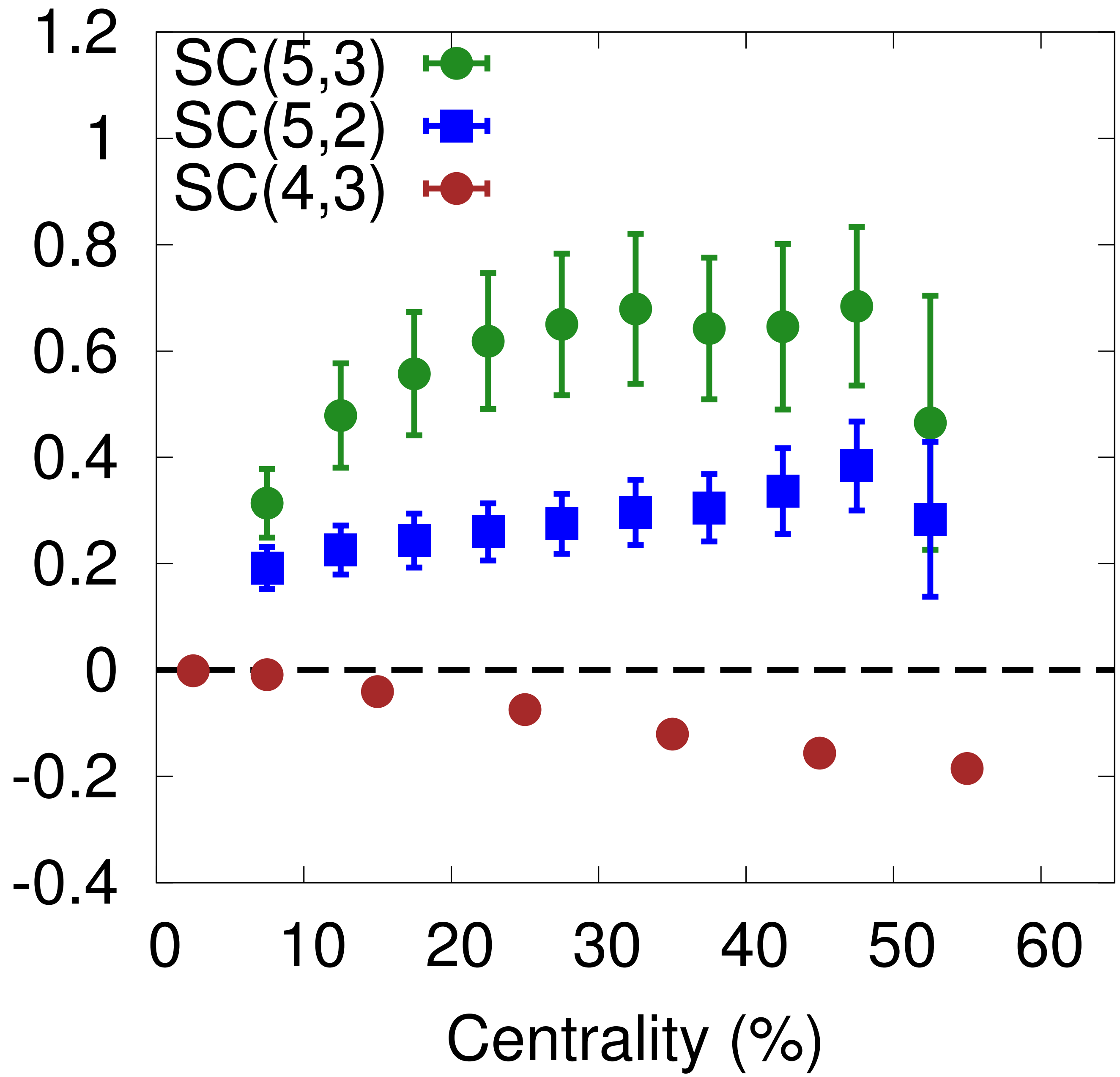
$$SC(5,3) = \left(\frac{\langle v_2^2 v_3^4\rangle}{\langle v_2^2 v_3^2\rangle\langle v_3^2\rangle} - 1\right)\cos^2\Phi_{235}$$

symmetric cumulants ↑ — flow fluctuations ↑ — event-plane correlations ↑

Predictions

Data-driven predictions (no hydro calculation!) using ATLAS results on v_n fluctuations and event-plane correlations.

ALICE measurement is likely to be higher than our prediction due to the narrower η window.



Conclusions

- We have derived relations between symmetric cumulants (4- particle correlations) and event-plane correlations (3-particle correlations).
- Relations are well satisfied in hydro.
- ALICE sees stronger correlations than ATLAS, which we interpret as an effect of the narrower η window and longitudinal flow fluctuations.