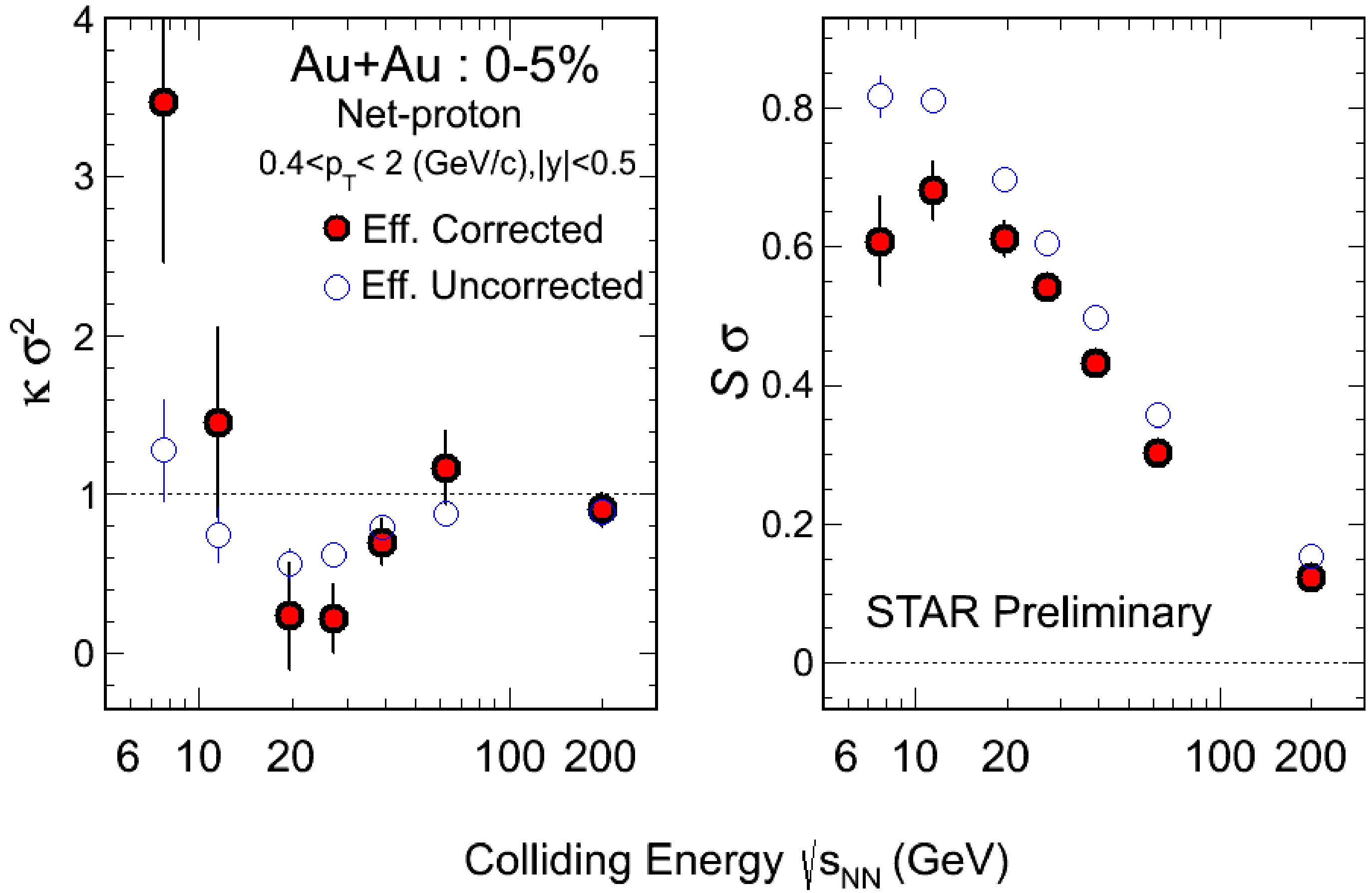
Non-binomial efficiency corrections, plus comments on multi-particle correlations

Adam Bzdak

AGH University of Science and Technology, Krakow

Efficiency correction is important

STAR (thanks to X. Luo)



If efficiency is driven by binomial with p (or ϵ)

true measured

$$\left\langle \frac{N!}{(N-i)!} \right\rangle = \frac{1}{p^i} \left\langle \frac{n!}{(n-i)!} \right\rangle$$

$$F_i = \frac{1}{p^i} f_i$$

So we express true cumulants through factorial moments F_i, which are known from the above equality (f_i is measured, p is known)

AB, V. Koch,
PRC 86 (2012) 044904; PRC 91 (2015) 027901

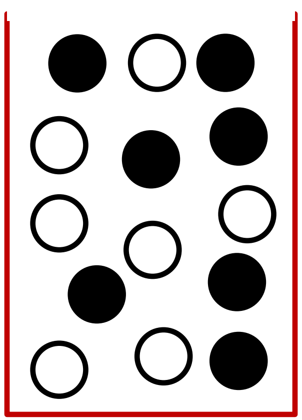
Binomial efficiency

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + c_2(y_1, y_2)]$$

two-particle density

single-particle densities

reduced correlation function



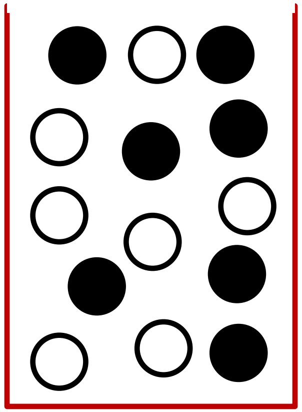
$$B(n, N) = \frac{N!}{n!\,(N-n)!}[p]^n[1-p]^{N-n}$$

Binomial efficiency modifies only single particle distribution, it does not influence physics

Non-binomial efficiency

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + c_2(y_1, y_2)]$$

non-binomial influences the correlation function!

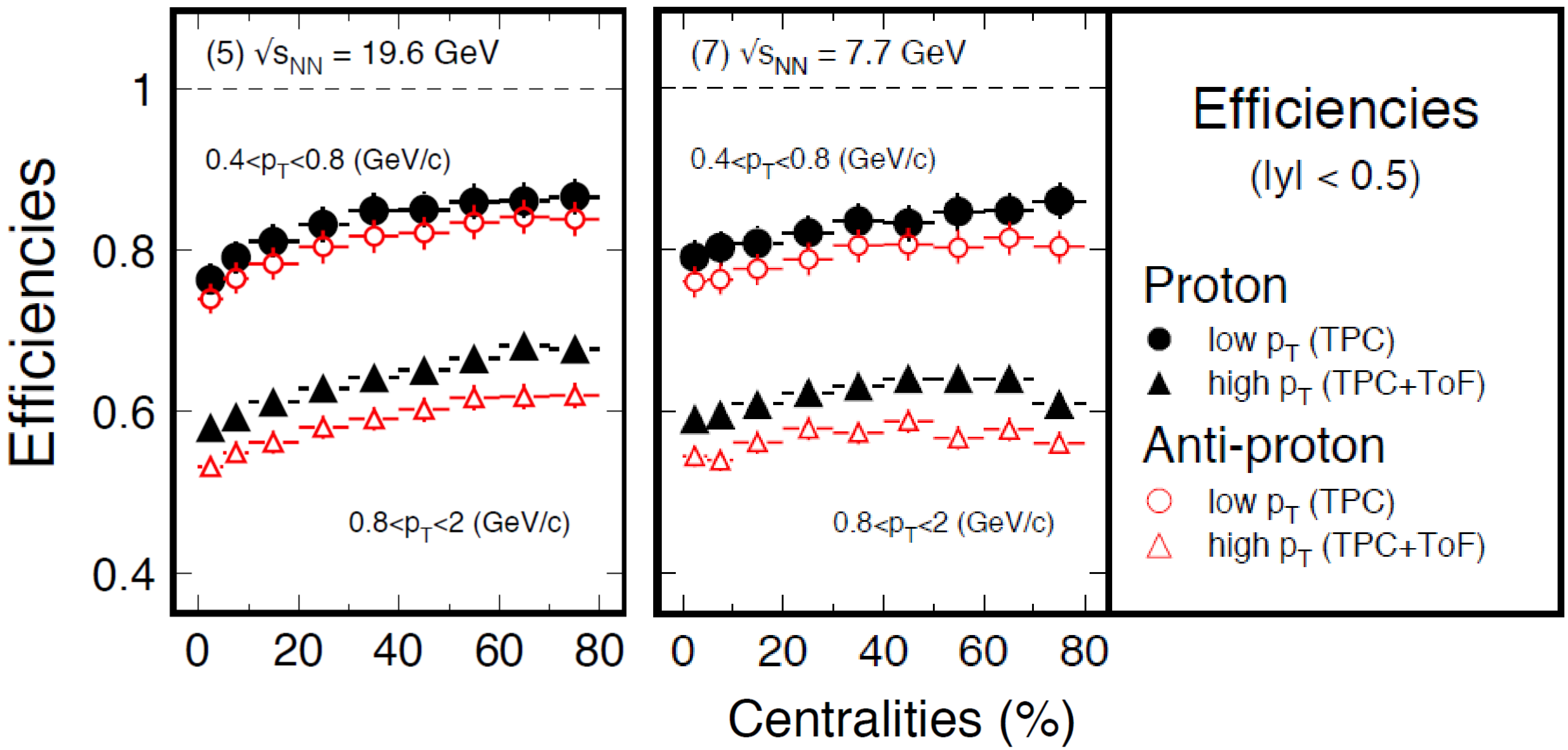


$$\frac{\rho_2(y_1, y_2)}{\rho(y_1)\rho(y_2)}$$ is not efficiency independent (unless we have binomial)

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3)[1 + c_2(y_1, y_2) + \cdots + c_3(y_1, y_2, y_3)]$$

STAR efficiencies at 19.6 GeV and 7.7 GeV

X. Luo [STAR Collaboration] arXiv:1503.02558 [nucl-ex]].



We see non-binomial efficiency.

R. Holzmann,
talk at HIC for FAIR

If ϵ depends on N the whole method brakes down.

$$f_i = \sum_N \underline{\epsilon^i(N)} P(N) \frac{N!}{(N-i)!}$$

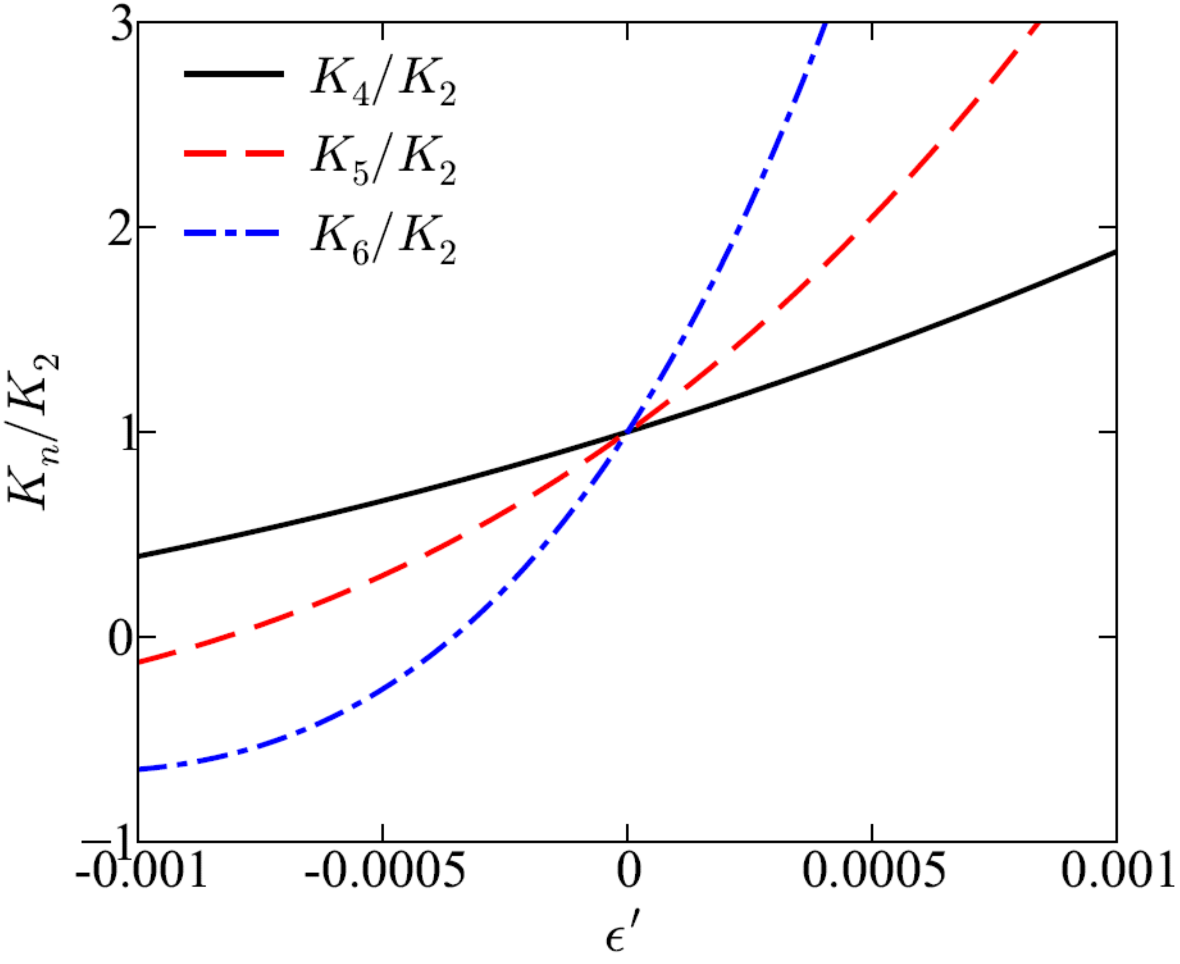
Let's test it. Suppose that

$$P(N) = \frac{\langle N \rangle^N}{N!} e^{-\langle N \rangle},$$
$$\epsilon(N) = \epsilon_0 + \epsilon'(N - \langle N \rangle)$$

with $\langle N \rangle = 40, \epsilon_0 = 0.65$ and plot K_n/K_2 as a function of ϵ'.
We calculate exact f_i and correct using constant efficiency
$F_i \ = \ f_i/\epsilon_0^i$.

We obtain

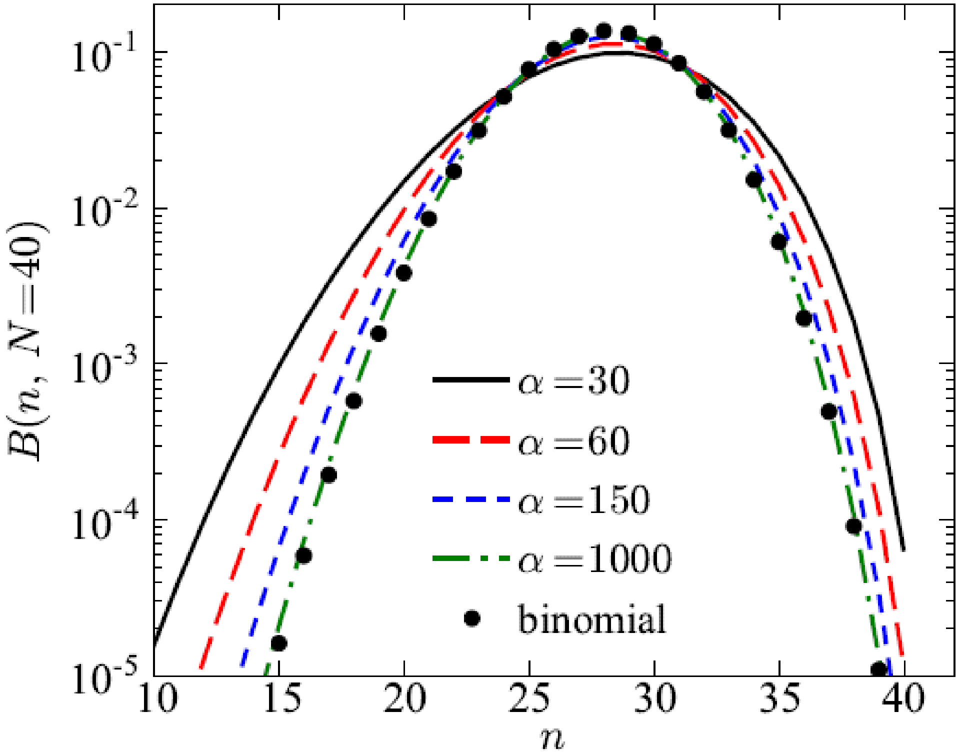
AB, R.Holzmann, V.Koch
arXiv:1603.09057



Large corrections for small ϵ'

Another example of non-binomial distribution
Beta-binomial distribution (we return 2 balls)

AB, R.Holzmann, V.Koch
arXiv:1603.09057



Beta-binomial	$\alpha = 30$	$\alpha = 60$	$\alpha = 150$	$\alpha = 1000$
K_3/K_2	1.28	1.24	1.13	1.02
K_4/K_2	0.82	1.45	1.35	1.07
K_5/K_2	-1.11	1.15	1.63	1.16
K_6/K_2	5.71	-0.44	1.80	1.32

We need to do proper unfolding

For example:

$$p(n) = \sum_{N=n}^{\infty} P(N) \frac{N!}{n!(N-n)!} \epsilon^n (1-\epsilon)^{N-n}$$

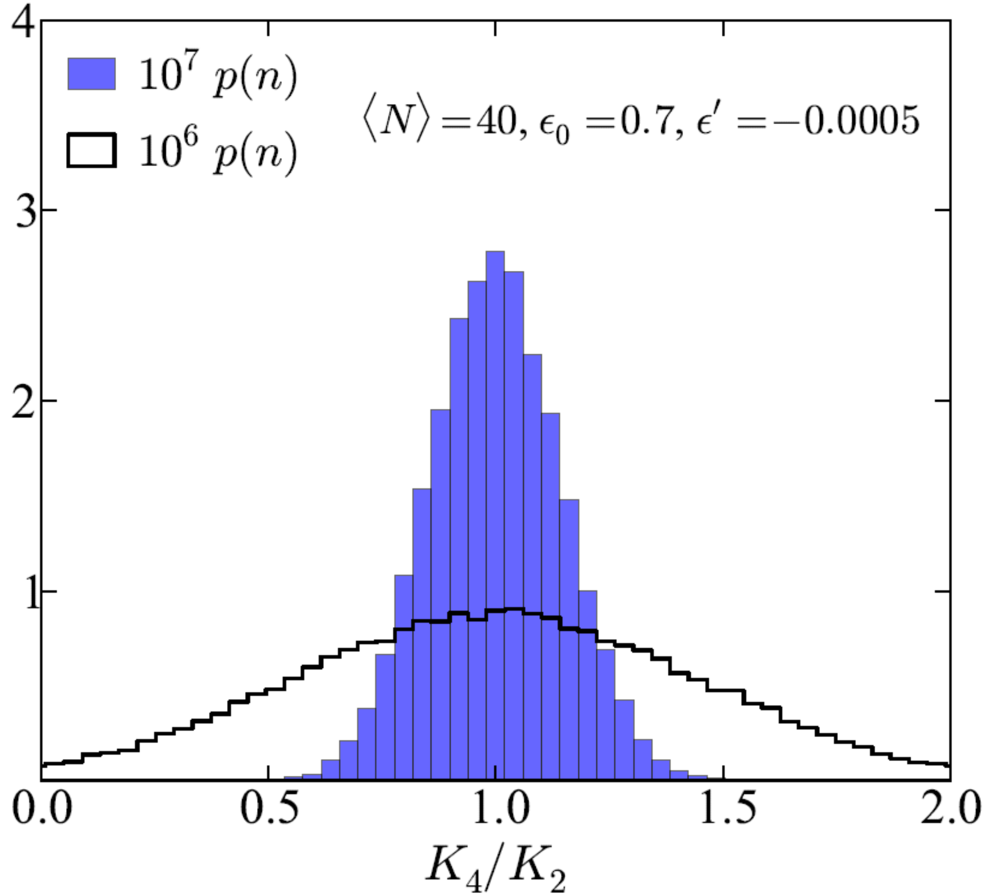
$$\begin{pmatrix} p(0) \\ p(1) \\ p(2) \\ p(3) \\ p(4) \end{pmatrix} = \begin{pmatrix} 1 & 1-\epsilon & (1-\epsilon)^2 & (1-\epsilon)^3 & (1-\epsilon)^4 \\ 0 & \epsilon & 2\epsilon(1-\epsilon) & 3\epsilon(1-\epsilon)^2 & 4\epsilon(1-\epsilon)^3 \\ 0 & 0 & \epsilon^2 & 3\epsilon^2(1-\epsilon) & 6\epsilon^2(1-\epsilon)^2 \\ 0 & 0 & 0 & \epsilon^3 & 4\epsilon^3(1-\epsilon) \\ 0 & 0 & 0 & 0 & \epsilon^4 \end{pmatrix} \begin{pmatrix} P(0) \\ P(1) \\ P(2) \\ P(3) \\ P(4) \end{pmatrix}$$

matrix is pseudo-singular

We can easily use $\epsilon(N)$, matrix is much more complicated but it is not a big deal.

In general $$p(n) = \sum_{N=n} P(N) B(n; N)$$

The method works for $\epsilon(N)$



It works very well, statistical errors are under control

Take-home message

- Non-binomial efficiency is (most likely) important

- Technique based on correcting factorial moments is not good enough

- Proper unfolding is warranted – see R. Holzmann’s talk

Multi-particle correlation functions

AB, V.Koch, in preparation

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + c_2(y_1, y_2)]$$

$$\langle N(N-1)\rangle = \langle N\rangle^2 + \langle N\rangle^2 c_2$$

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1 dy_2}{\int \rho(y_1)\rho(y_2)dy_1 dy_2}$$

coupling

and the second order cumulant

$$K_2 = \langle N\rangle + \langle N\rangle^2 c_2$$

In the same way

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3)[1 + c_2(y_1, y_2\ ) + \cdots + c_3(y_1, y_2, y_3)]$$

$$F_3 = \langle N(N-1)(N-2)\rangle = \langle N\rangle^3 + 3\langle N\rangle^3 c_2 + \langle N\rangle^3 c_3$$

$$c_3 = \frac{\int \rho(y_1)\rho(y_2)\rho(y_3)c_3(y_1, y_2, y_3)dy_1dy_2dy_3}{\int \rho(y_1)\rho(y_2)\rho(y_3)dy_1dy_2dy_3}$$

coupling

and the third order cumulant

$$K_3 = \langle N\rangle + 3\langle N\rangle^2 c_2 + \langle N\rangle^3 c_3$$

Finally we obtain

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1 dy_2}{\int \rho(y_1)\rho(y_2)dy_1 dy_2}$$

$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

cumulants mix
correlation functions

$$K_3 = \langle N \rangle + 3\langle N \rangle^2 c_2 + \langle N \rangle^3 c_3$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

Let's fix the rapidity bin:

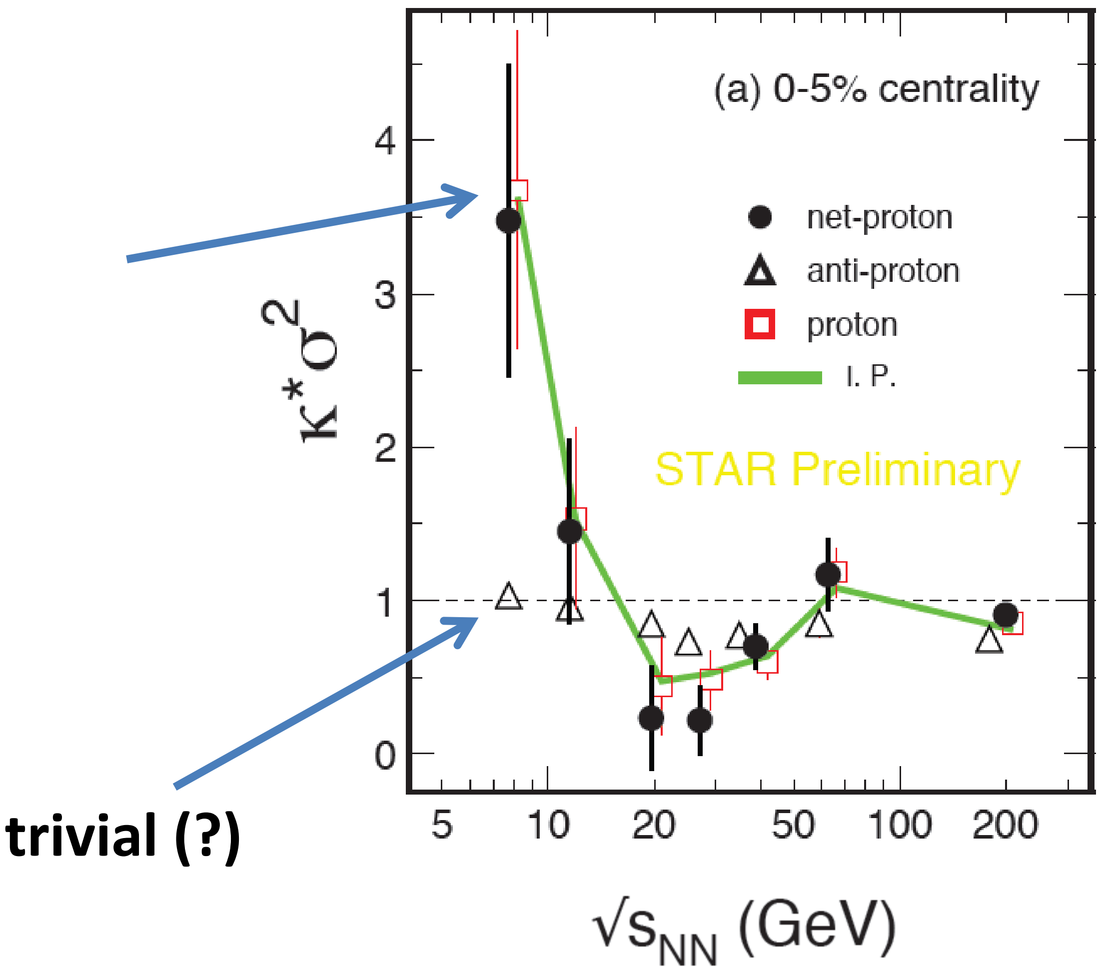
- K_i scales like $\langle N \rangle^i$ if correlation function do not depend on $\langle N \rangle$.

- Comparing different centralities is tricky. Physics ($c_{2,3,4}$) could be the same but K_4/K_2 can be different because of changing $\langle N \rangle$.

- Same with energy!

- If $\langle N \rangle < 1$ (anti-protons at low energy) higher order correlation functions are getting less visible, $K_i \sim \langle N \rangle$, and the ratios $\rightarrow 1$.

Anti-protons and protons are different at low energies



$$\langle N_{\mathrm{proton}} \rangle \sim 40, \quad \langle N_{\mathrm{anti-proton}} \rangle \sim 0.25$$

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1 dy_2}{\int \rho(y_1)\rho(y_2)dy_1 dy_2}$$

$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_3 = \langle N \rangle + 3\langle N \rangle^2 c_2 + \langle N \rangle^3 c_3$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

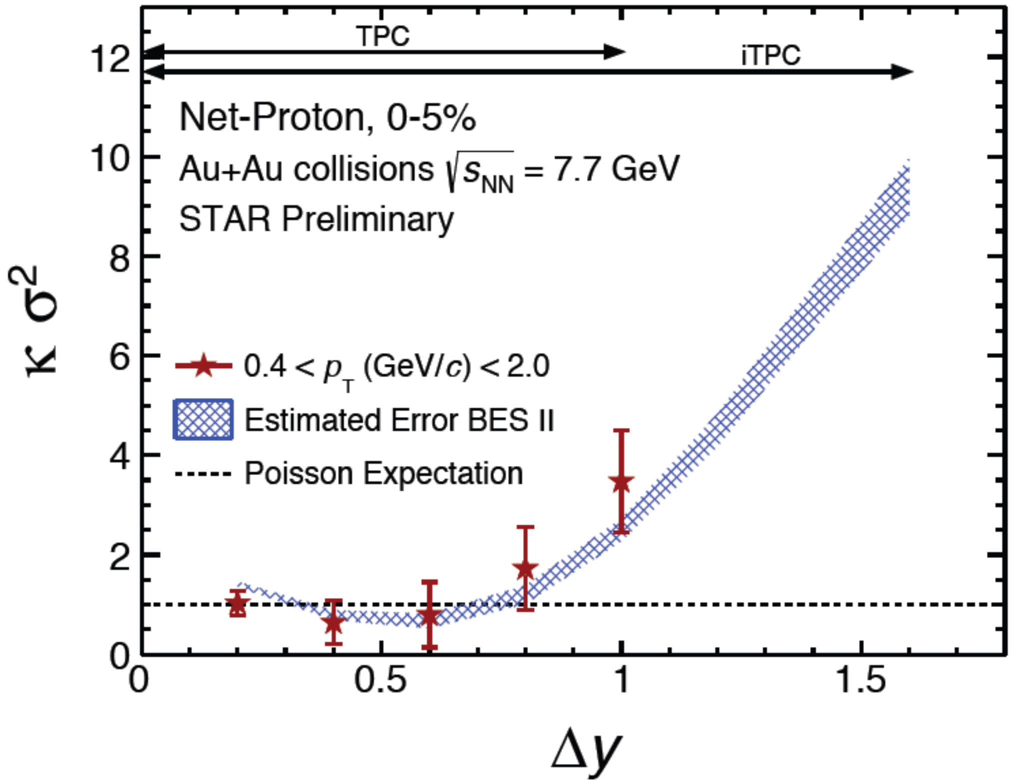
Now let's change the rapidity bin, Δy.
Assume that $c_n(y_1, \dots, y_n)$ changes slowly within measured rapidity bin.
In this case c_n does not depend on Δy.

$\langle N \rangle \sim \Delta y$ and $K_2 \sim \Delta y^2$, $K_4 \sim \Delta y^4$.

Consequently $K_4/K_2 \sim \Delta y^2$

STAR data on Δy dependence at 7.7 GeV

K_4/K_2

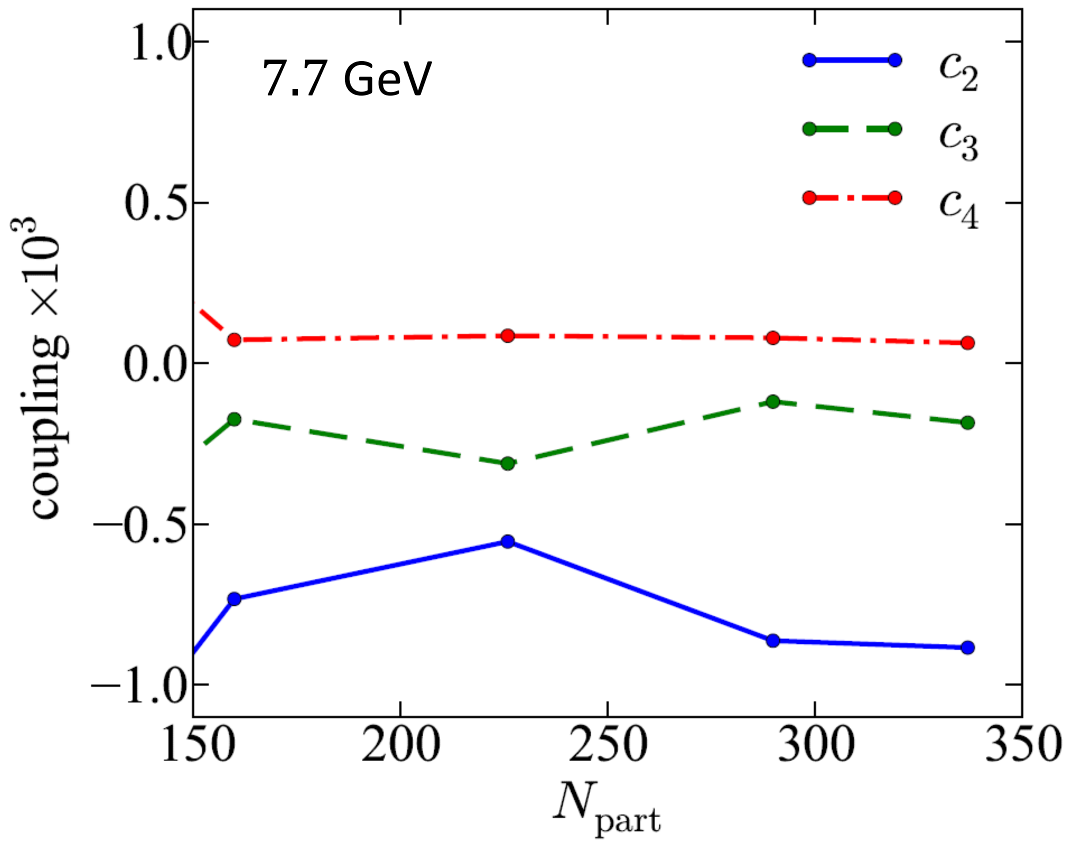


Any initial state effect (i.e., correlation long-range in rapidity, e.g., volume fluctuation) can give this.

This plot only proves that correlation between particle is not very short-range in rapidity.

AB, V.Koch
in preparation

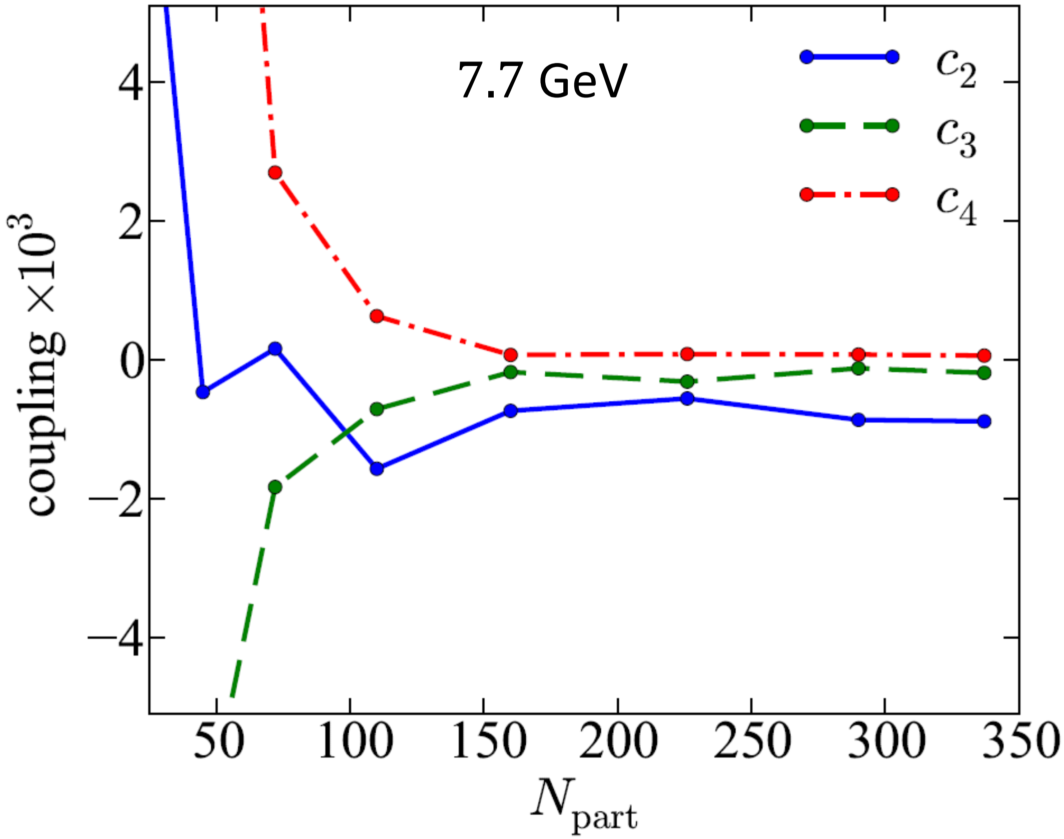
We extracted multi-particle correlation couplings



Physics rather flat as a function of $N_{\rm part}$

$c_2 > c_3 > c_4$ (what do we expect for the critical point?)

… and for smaller N_{part}



Non-trivial dependence for smaller N_{part}.
Something changes around $N_{\text{part}} = 100$.

less stopping
for smaller N_{part}

Take-home message

- Cumulants ratios are tricky to interpret, they mix correlations function of different order and ratios can depend on average number of particles.

- Multi-particle correlation couplings are much cleaner.

- In data $c_2 > c_3 > c_4$ at 7.7 GeV.
  What do we expect for the critical point?

Conclusions

Non-binomial efficiency is a serious problem.
We need to do proper unfolding.

Studying multi-particle correlation functions is much cleaner.