Freeze-out Dynamics In Relativistic Heavy-Ion Collisions

Outline:
- Chemical Freeze-out
- Kinetic Freeze-out
- Summary

See also talks at -
Chemical FO – 28th June 2016 – Parallel sessions
Freeze-out – 29th June 2016 – Plenary sessions
Particle production – 30th June 2016 – Parallel session

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Dynamics of Relativistic Heavy Ion Collisions

Time Evolution of Heavy-ion Collisions:

- Initial state
- QGP and Hydro. expansion
- Hadronization
- Colliding ions
- De-confined state
- Chemical freeze-out
- Kinetic freeze-out

Information of QCD phase diagram and particle production
Chemical Freeze-out

Inelastic Collisions Ceases

Statistical Model

\[ n = \frac{1}{V} \frac{\partial(T \ln Z)}{\partial \mu} = \frac{VTm_i^2 g_i}{2\pi^2} \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k} \left( e^{\beta k \mu_i} \right) K_2 \left( \frac{km_i}{T} \right) \]


Parameters:
- Temperature \(T_{ch}\), Chemical Potentials \(\mu_B, \mu_S, \mu_Q\), \(\gamma_S\) and Volume
- Obtained by fitting the particle yields

Model Features:
- Assumes non-interacting hadrons and resonances
- Assumes thermodynamically equilibrium system
- Ensembles: Grand Canonical - average conservation of B, S, and Q
  - Strangeness Canonical - exact conservation of S
  - Canonical - exact conservation of B, S, and Q

J. Cleymans and K. Redlich, PRC 60 (1999)054908

STAR: NPA 757 (2005)102
**Freeze-out Conditions – Particle Yields**


AGS results: Does not include Multi-strange hadrons

New aspect from BES RHIC Centrality dependence of $T_{ch}$ vs. $\mu_B$
Multiple Freeze-out Scenarios


- Advent of LHC data
  - Difficulty in explaining Strange-to-non-Strange Particle/nuclei ratios
    - Led to new developments

- Departure from equilibrium physics:
  - Hadronization followed by hadronic afterburner within the hybrid UrQMD model.
  - FO with nonequilibrium quark phase space factors for light and strange quarks were used.

- Within ambit of equilibrium physics:
  - Flavor dependent freeze-out surfaces

F. Becattini, et al., PRC 90 (2014) 054907
M. Petran, et al., PRC 88 (2013) 034907
S. Chatterjee, et al., PLB 727 (2013) 554
K. Bugaev, EPL 104 (2013) 22002
S. Chatterjee & B. Mohanty, PRC 90 (2014) 034908
S. Gupta, R. Sharma, PRC 89 (2014) 057904

Y – T ~ 250 MeV
Strange hadrons Freeze-out at different time compared to non-strange hadrons?

- Preferred for AA collisions (medium) and NOT preferred for pA or pp

\[ \langle dN_{ch}/d\eta \rangle_{|\eta|<0.5} \]

\( \chi^2/\text{ndf} \)

\( \sqrt{s_{NN}} \):
- Pb-Pb, \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \)
- p-Pb, \( \sqrt{s_{NN}} = 5.02 \text{ TeV} \)
- pp, \( \sqrt{s} = 7 \text{ TeV} \)

\[ |<0.5 \eta| > \eta / dN / d\eta \]
Freeze-out Conditions from Higher Moments of Multiplicity Distributions

Ratios of Susceptibilities of conserved quantities (B, Q, S) calculated in QCD/Models are related to products of moments of corresponding distributions measured in experiments – Use this to extract Freeze-out conditions

<table>
<thead>
<tr>
<th>Theory (Susceptibilities)</th>
<th>Experiment (moments/Cumulants)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_3/\chi_2$</td>
<td>$S\sigma$</td>
</tr>
<tr>
<td>$\chi_4/\chi_2$</td>
<td>$K\sigma^2$</td>
</tr>
<tr>
<td>$\chi_1/\chi_2$</td>
<td>$M/\sigma^2$</td>
</tr>
<tr>
<td>$\chi_3/\chi_1$</td>
<td>$S\sigma^3/M$</td>
</tr>
</tbody>
</table>

One strategy: $\chi_3/\chi_1$ is independent of $\mu_B$ – estimate temperature
LO in $\chi_2/\chi_1$ is linear in $\mu$ - estimate chemical potential

Theory: Lattice QCD
HRG

Phys.Rev.Lett. 113 (2014) 052301
Phys.Rev.Lett. 113 (2014) 072001
## Experimental data on Higher Moments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Net-Charge</th>
<th>Net-Proton</th>
<th>Net-Kaon</th>
</tr>
</thead>
</table>
| STAR       | Obs: $M, \sigma, S, \kappa$<br>Beam Energy (evts): 7.7 (1.4M), 11.5 (2.4M), 19.6 (15.5), 27 (24M), 39 (56M), 62.4 (32M), 200 (75M)  
$\eta$: +/- 0.5; $\Phi$: 2$\pi$  
$p_T$: 0.2 – 2 GeV/c | Obs: $M, \sigma, S, \kappa$<br>Beam Energy (evts): 7.7 (3M), 11.5 (6.6M), 19.6 (15M), 27 (30M), 39 (86M), 62.4 (47M), 200 (238M)  
$\eta$: +/- 0.5; $\Phi$: 2$\pi$  
$p_T$: 0.4 – 0.8 (2) GeV/c | Obs: $M, \sigma, S, \kappa$<br>Beam Energy (evts): 7.7 (3M), 11.5 (6.6M), 14.5 19.6 (15), 27 (30), 39 (86M), 62.4 (47M), 200 (238M)  
$\eta$: +/- 0.5; $\Phi$: 2$\pi$  
$p_T$: 0.2 – 1.6 GeV/c |
| PHENIX     | Obs: $M, \sigma, S, \kappa$<br>Beam Energy (evts): 7.7 (2M), 19.6 (6M), 27 (21M), 39 (154M), 62.4 (474M), 200 (1681M)  
$\eta$: +/- 0.35  
$\Phi$: $\pi/2$  
$p_T$: 0.3 – 2 GeV/c |  |  |

Data: Finite Acceptance ($y, \phi, p_T$)<br>Efficiency Corrected<br>net-proton – proxy for net-baryon<br>net-kaon – proxy for net-strangeness

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STAR:QM2015

1603.09057
LQCD: Freeze-out Conditions from Higher Moments of Multiplicity Distributions

Assumption: If the freeze-out can be described by the same temperature and chemical potentials for charge and protons.

1. Consistency in freeze-out parameters for various observables.
2. Fluctuations Freeze-out at a slightly lower temperature than yields?
1. Fluctuations Freeze-out at a lower temperature than yields?
2. Consistency in freeze-out parameters for various observables at higher energies (indication of chiral critical fluctuations at lower energies?)
3. Freeze-out temperature from fluctuations – explains anti-proton yields better, misses the multi-strange baryons (need for strangeness fluctuations?)
Compilation of Freeze-out Conditions

Does Particle yields and Fluctuations freeze-out differently?

Or

It is a simple question of Sensitive observable?

Or

Strange and non-Strange Freeze-out at different Times?
Freeze-out line and Transition Line

Assumption:
- Charge-conjugation invariance at $\mu_B=0$
- Analyticity at the point $\mu_B=0$
- Low Baryon densities and lowest order expansion

\[
\frac{T(\mu_B)}{T_c(0)} = 1 - \kappa \left( \frac{\mu_B}{T(\mu_B)} \right)^2 + \Sigma_{r}^{\mu_B} = \Sigma_{r}^{\mu_B} + \left( \Sigma_{r}^{\mu_B} - \kappa \mathcal{T}_{r,0} \left( \Sigma_{r}^{\mu_B} \right) \right) \mu_B^2
\]

\[\kappa > 0 \text{ Transition line}\]
\[\kappa \sim 0 \text{ or } < 0 \text{ Freeze-out line}\]

<table>
<thead>
<tr>
<th>$\kappa$-values (error)</th>
<th>Calculation</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.020(4)</td>
<td>PRD 93, 014507 (2016) : LQCD, im. $\mu$</td>
<td>Transition Line</td>
</tr>
<tr>
<td>0.0135(20)</td>
<td>PRD 92, 054503 (2015): LQCD im. $\mu$</td>
<td>Transition line</td>
</tr>
<tr>
<td>0.059(2)(4)</td>
<td>PRD 83, 014504 (2011): LQCD T. exp.</td>
<td>Transition line</td>
</tr>
<tr>
<td>0.0089(14) ($\chi_s$); 0.0066(20) ($\psi_c$)</td>
<td>JHEP 1104, 001 (2011): LQCD</td>
<td>Transition line</td>
</tr>
<tr>
<td>0.015(6) (l); 0.017(5) ($\epsilon$) 0.018(7) ($c_s$); 0.016(4) (s)</td>
<td>JHEP 1208, 053 (2012) LQCD, T. exp.</td>
<td>Transition line</td>
</tr>
<tr>
<td>0.023(3)</td>
<td>PRC 73, 034905 (2006): Fit to Yields</td>
<td>Freeze-out line</td>
</tr>
<tr>
<td>Weaker than above</td>
<td>PRL 111, 082302 (2013): Fit to yields</td>
<td>Freeze-out line</td>
</tr>
<tr>
<td>$\sim 0$</td>
<td>NPA 772, 167 (2006): Fit to Yields</td>
<td>Freeze-out line</td>
</tr>
<tr>
<td>-0.073(16)- STAR pub.</td>
<td>Phys.Rev. D93 (2016), 014512: Fit to higher moments</td>
<td>Freeze-out line</td>
</tr>
<tr>
<td>-0.012(15) STAR-prel.</td>
<td></td>
<td>&lt; 0.011</td>
</tr>
<tr>
<td>-0.056(67) STAR+PHENIX</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Freeze-out line and Transition Line


Freeze-out parameters using thermal model without multi-strange and strange baryons yields closer to FO parameters from net-p and net-Q fluctuation data.

Freeze-out points closely follow the parton-hadron phase boundary.
Kinetic Freeze-out

Elastic Collisions Ceases

Blast-Wave Model

\[
\frac{dN}{p_T dp_T} \propto \int_0^R r dr m_T I_0 \left( \frac{p_T \sinh \rho(r)}{T_{kin}} \right) \times K_1 \left( \frac{m_T \cosh \rho(r)}{T_{kin}} \right)
\]

Parameters: Temperature \((T_{kin})\) and transverse radial velocity \((\beta)\)

Obtained by fitting the momentum distribution of particles

Features:
- Approximates Hydrodynamic based model
- Assumes particles are locally thermal and moving with a common velocity

Recent work: Viscous Blast-Wave Model:

arXiv:1508.05878 , A. Jaiswal and V. Koch

SQM2016 – 28th July 2016
Kinetic Freeze-out Conditions

- Central collisions: lower $T_{kin}$ and larger collectivity $\beta$
- Stronger collectivity at higher energy

Kinetic Freeze-out temperature lower than Chemical Freeze-out temperature
Reflects Interactions in hadronic phase


STAR: V. Bairathi QM L. Kumar QM2014

Experimental evidence of re-scattering

R. Holzmann talk – 28th June $\sqrt{s_{NN}}$ (GeV)

$T$ (MeV)

$T_{kin}$
Kinetic Freeze-out New Observation: RHIC-BES

Particles: $\pi^+$, $K^+$, p, $\Lambda$, $\Xi^-$;
Antiparticles: $\pi^-$, $K^-$, $\bar{p}$, $\bar{\Lambda}$, $\bar{\Xi}$


Differences between particle and anti-particles?
Summary

- New data (LHC, RHIC BES)
- New Observables (Higher Moments)
- New Models/Approaches
**New:** Freeze-out parameters using Lattice QCD calculations and HRG vs. experimental data on high moments

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**Difference in Freeze-out parameters from yields and fluctuations:**
- Yields and fluctuations freeze-out at different times?
- Fluctuations more sensitive to Freeze-out dynamics than yields?
- Role of strangeness (Strangeness data on fluctuations important)?

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**Discussion**
**Discussion**

**Fluctuations: Lattice and HRG**

- **Strange (s) vs. Non-Strange (u,d) FO**

  - Are there sufficient evidence that non-strange hadrons freeze-out at different time compared to strange hadrons? – May be?

  
  
- **Yields: Statistical Model**

  
  
- **T_s > T_u,d**

  
  
- **T_{net-K} > T_{net-p} \sim T_{net-Q}**

  
  
- **Role of additional Strange Hadrons?**

  
  
- **Phys.Rev.Lett. 113 (2014) 072001**

  
  
- **R. Bellwied, S. Borsanyi, Z. Fodor, S. D. Katz**

  
  

  
  
- **S. Chatterjee, A. Dash & BM**

  
  
- **R. Bellwied – WWND2016**

  
  
- **\( \chi^2_{\text{ndf}} \)**

  
  
- **\( \gamma_s \)**
Landmark Point (CP) on Phase Diagram

S. Gupta
Lattice 2013

Expectation

Measurement

Au + Au Collisions at RHIC
$\sqrt{s_{NN}} < 0.5, 0.4 < p_t < 2$ (GeV)

(a) 0-5% centrality
(b) 70-80% centrality

STAR: CPOD 2015

Exciting times ahead....

Thanks