Lattice QCD results on freeze-out

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- Limiting the validity range of HRG model calculations
  characterization of bulk thermodynamics and
  fluctuations of conserved charges in the crossover region
- Taylor expansion of the equation of state and cumulant ratios
- relations between skewness and kurtosis ratios in the non-critical region

BEST COLLABORATION

June 2016

Probing the properties of matter through the analysis of conserved charge fluctuations

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

the pressure in hadron resonance gas (**HRG**) models:

$$\frac{p}{T^4} = \sum_{m\in meson} \ln Z_m^b(T, V, \mu) + \sum_{m\in baryon} \ln Z_m^f(T, V, \mu)$$

$$\sim \mathrm{e}^{-m_H/T} \mathrm{e}^{(B\mu_B + S\mu_S + Q\mu_Q)/T}$$

Chiral transition, hadronization and freeze-out

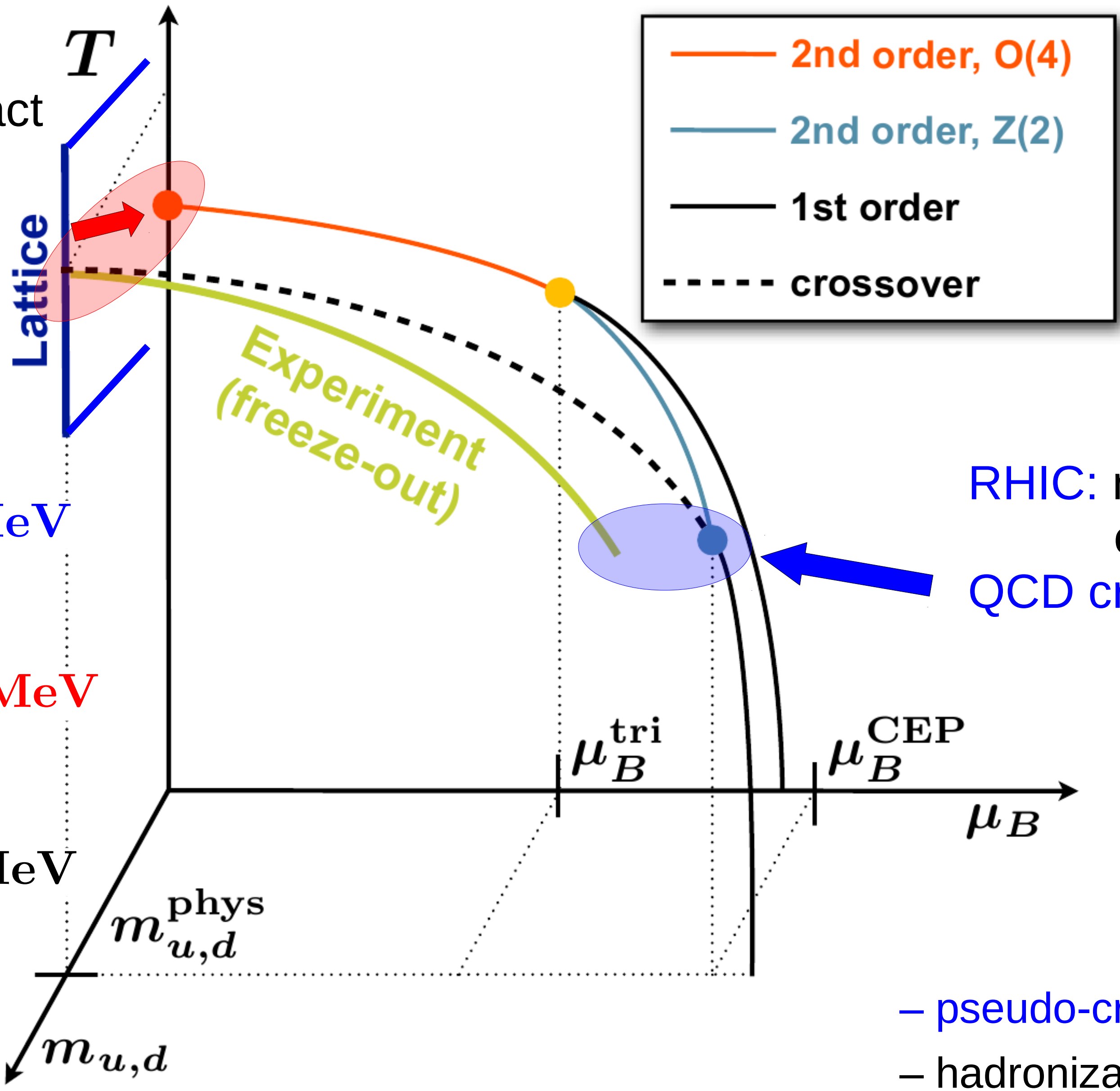
LHC: may establish contact with the QCD chiral PHASE transition

lattice QCD:
$T_c = 154(9)$ MeV

ALICE/LHC:
$T_{fo} = 156(3)$ MeV

Becattini et al:

$T_h = 164(3)$ MeV



RHIC: may establish evidence for the QCD critical end point

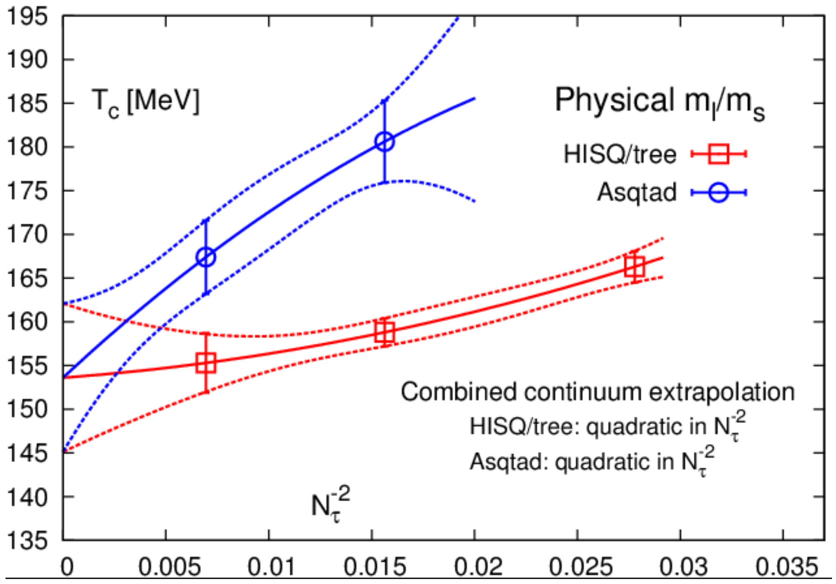
– pseudo-critical temperature

– hadronization temperature

– freeze-out temperature

The QCD crossover transition
– extracting the pseudo-critical temperature –

Crossover transition temperature



$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

– well defined pseudo-critical temperature
– quark mass dependence of susceptibilities consistent with O(4) scaling

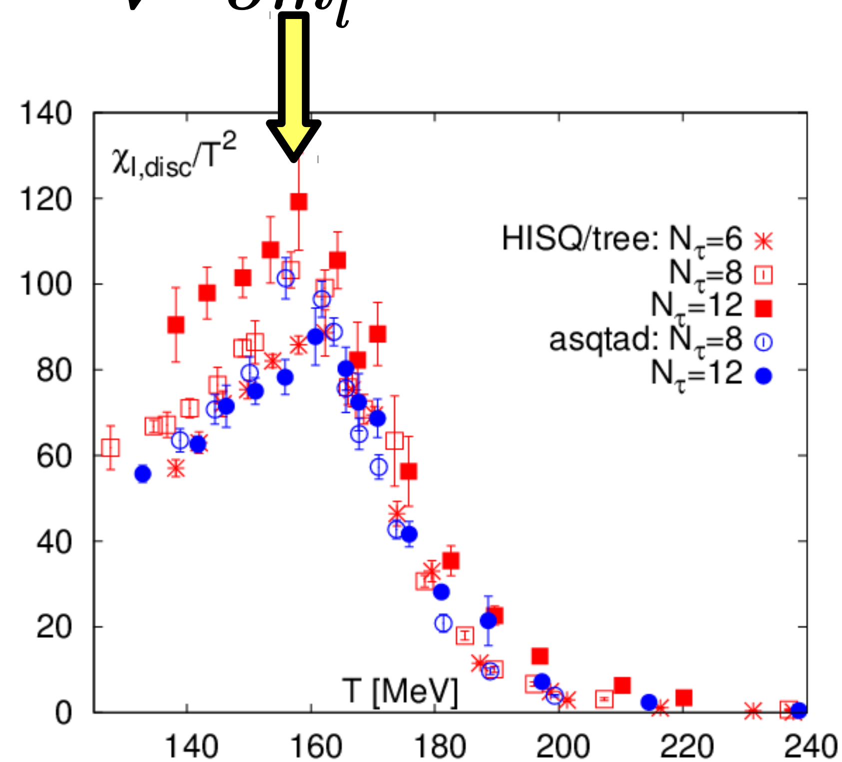
A. Bazavov et al. (hotQCD),
Phys. Rev. D85, 054503 (2012), arXiv:1111.1710

lattice: $N_\sigma^3 \cdot N_\tau$
temperature: $T = 1/N_\tau a$

Critical temperature from location of peak in the fluctuation of the chiral condensate (order parameter):

Chiral susceptibility

$$\chi_l = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} = \chi_{l,disc} + \chi_{l,con}$$



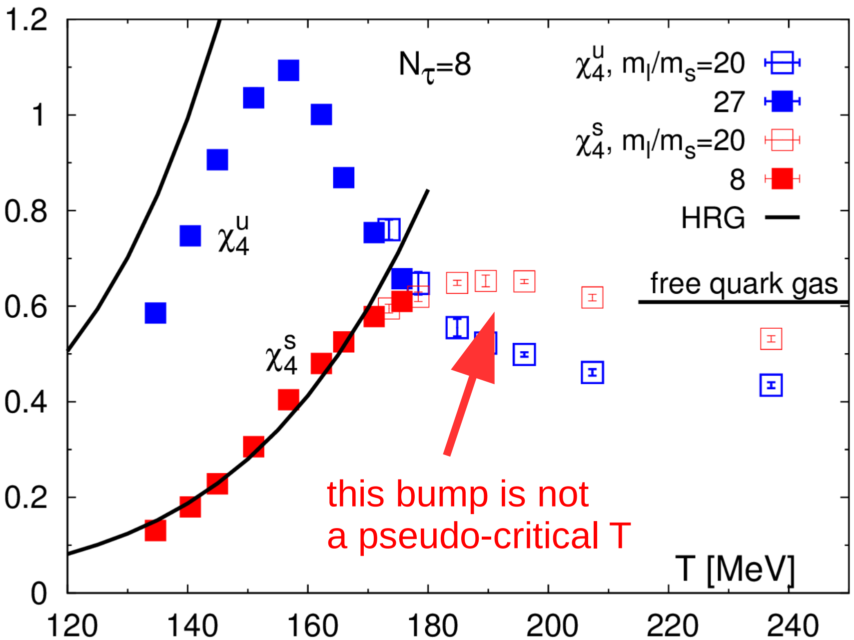
consistent with Y. Aoki et al, JHEP 0906 (2009) 088

Pseudo-critical temperature(s) in QCD

Definition: A pseudo-critical temperature is a temperature defined in the presence of a symmetry breaking field that becomes identical to the critical temperature in the limit of vanishing symmetry breaking field

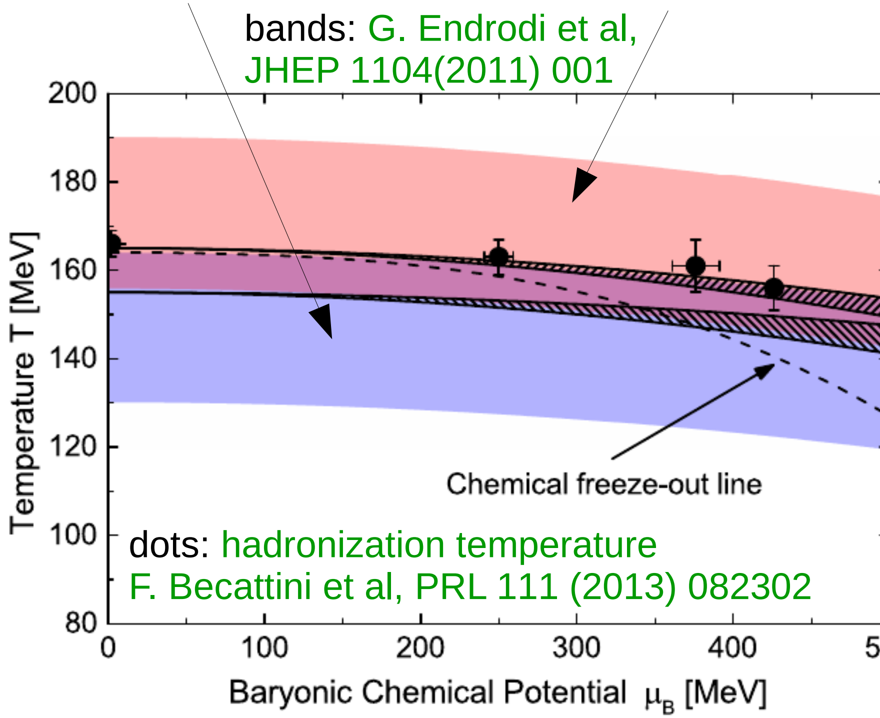
Corollary: Not every bump or inflection point in an observable is suitable to define a pseudo-critical temperature

quartic up-ness and strangeness fluctuations



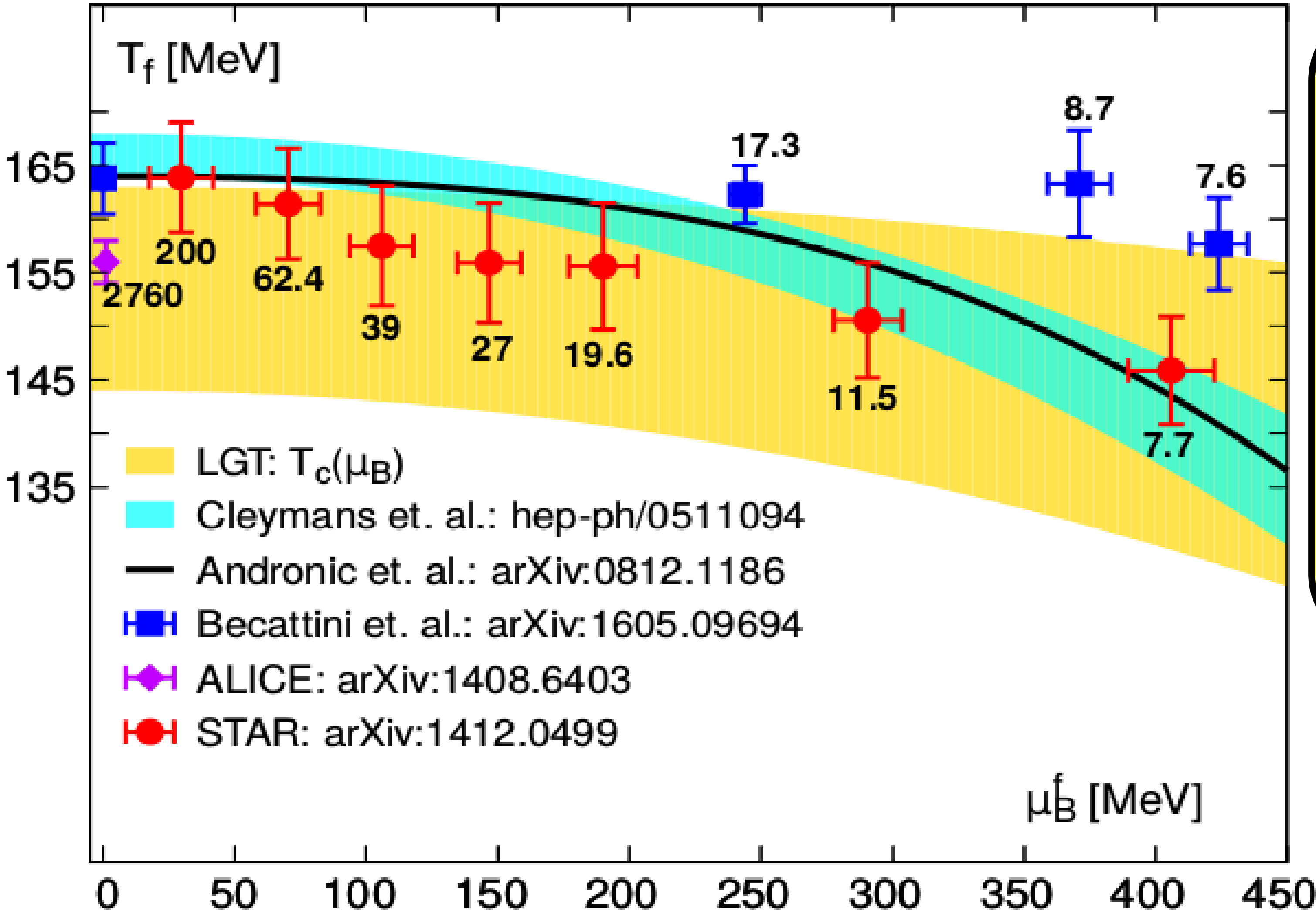
peak of chiral susceptibility

inflection point of χ_2^S



Chiral transition, hadronization and freeze-out

LGT: $T_c(\mu_B) = 154(9)(1 - [0.006; 0.014](\mu_B/T)^2)\mathrm{MeV}$



phenomenological freeze-out / hadronization curve, QCD transition line and experimental data (obtained by <u>assuming the validity of the HRG model</u>) are consistent for

$$\mu_B/T \lesssim 3$$

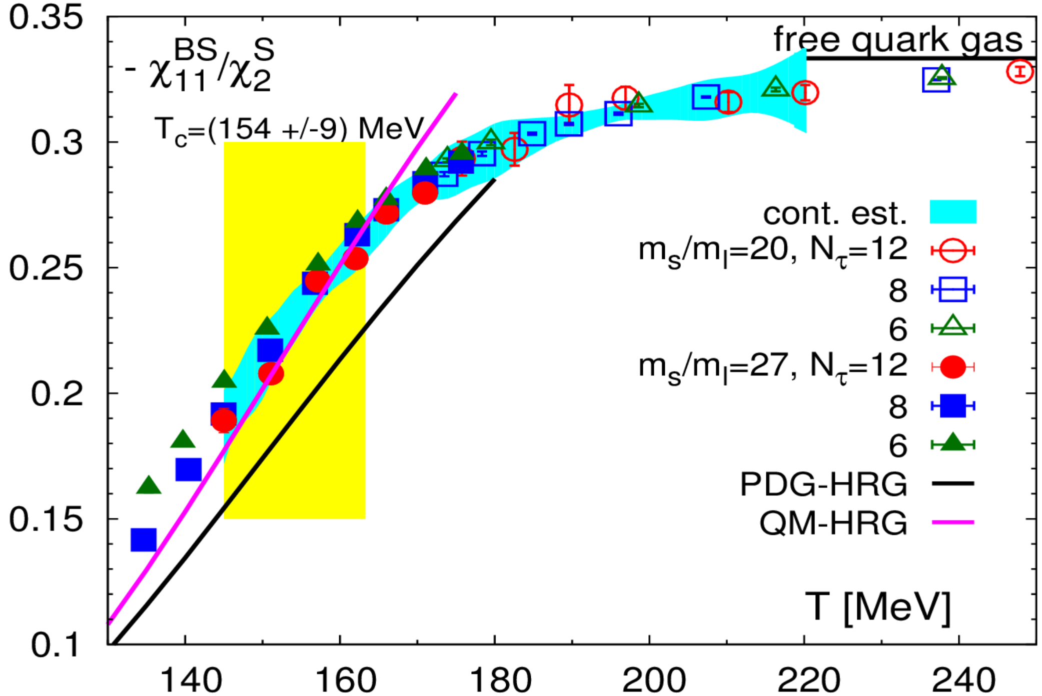
HOWEVER

physics is quite different at lower and upper end of the current error bar on Tc

→ **probed with net-charge correlations&fluctuations**

Probing the properties of matter through the analysis of conserved charge fluctuations

Koch ratio



$$\chi_{11}^{BS} = \frac{\partial^2 P/T^4}{\partial\hat{\mu}_B \partial\hat{\mu}_S} = \langle B\cdot S\rangle - \langle B\rangle\langle S\rangle$$

$$\chi_2^S = \frac{\partial^2 P/T^4}{\partial\hat{\mu}_S^2} = \langle S^2\rangle - \langle S\rangle^2$$

S. Borsanyi et al., JHEP 1201 (2012) 138
A. Bazavov et al.,
PRL 113 (2014) 072001, arXiv:1404.6511

V. Koch, A. Majumder, J. Randrup, PRL 95 (2005) 182301

<u>Abstract:</u>

The correlation between baryon number and strangeness elucidates the nature of strongly interacting matter, such as that formed transiently in high-energy nuclear collisions.

...The analysis of present lattice results above the critical temperature severely limits the presence of q-qbar bound states, thus supporting a picture of independent (quasi)quarks

Probing the properties of matter through the analysis of conserved charge fluctuations

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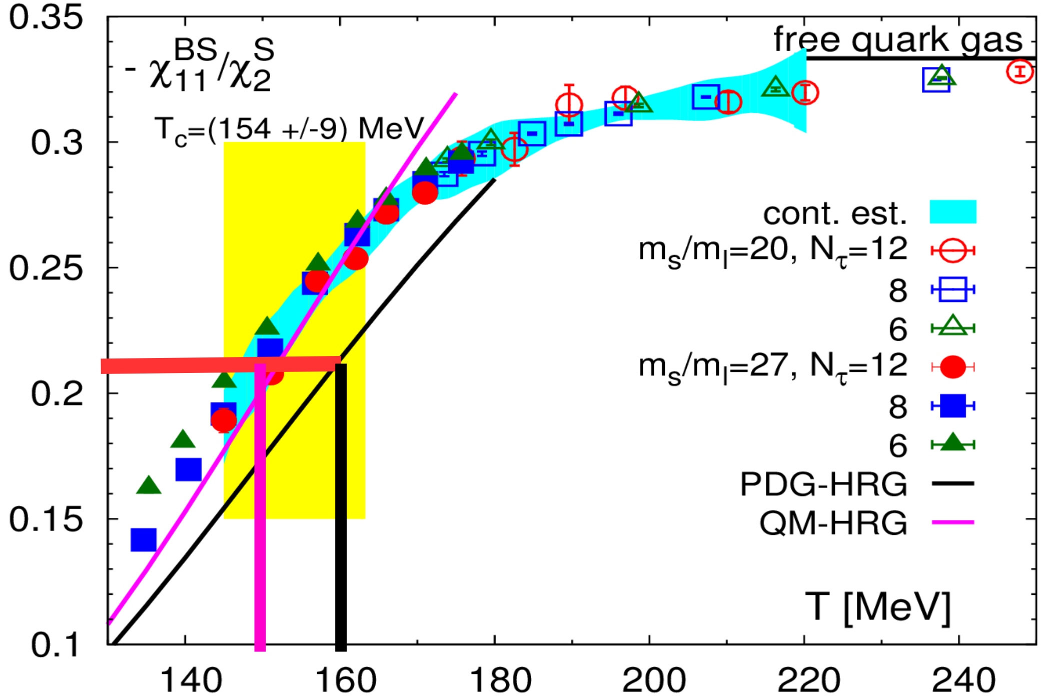

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Gedankenexperiment: **ALICE measures** $\chi_{11}^{BS}/\chi_2^S = 0.21$

comparing the measurement with HRG gives: $T_f = 160\ \mathrm{MeV}$

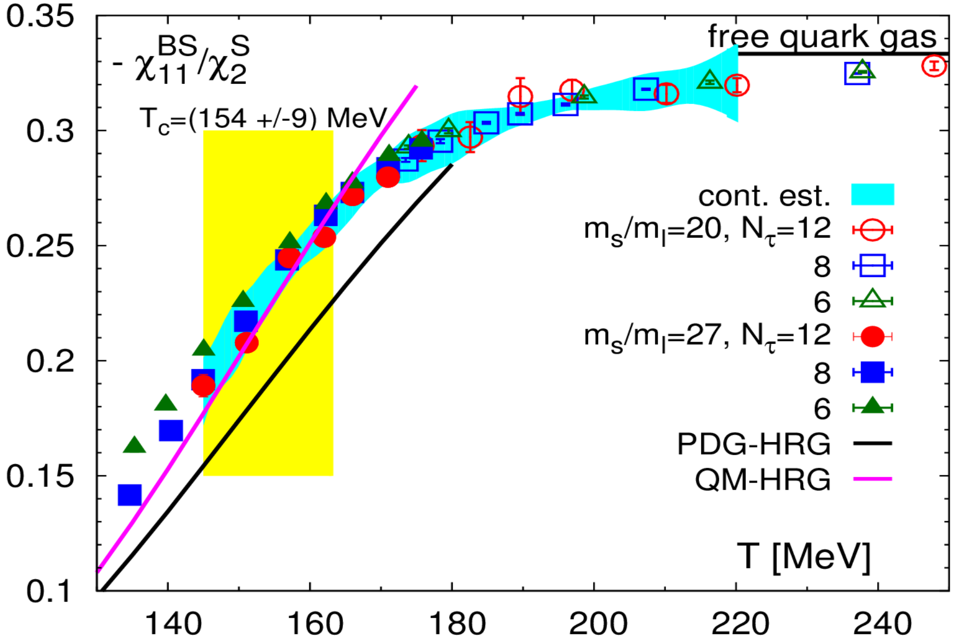
freeze-out param. from QCD

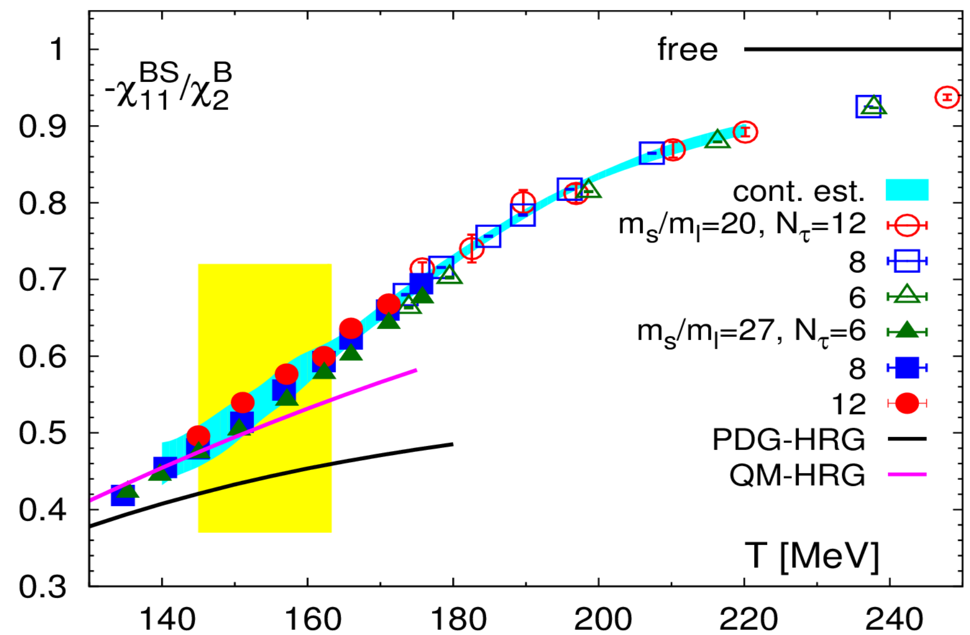
comparing the measurement with QCD gives: $T_f = 150\ \mathrm{MeV}$

Can we cure the HRG model by adding a few more resonances?

HRG vs. QCD
Strangeness-Baryon number Correlations

Koch ratios





A. Bazavov et al.,
Phys. Rev. Lett. 113, 072001 (2014), arXiv:1404.6511

continuum extrapolated results on strangeness-baryon correlations **do NOT agree** with a conventional hadron resonance gas, based on experimentally known resonances listed in the particle data tables

in the crossover region (and above):

PDG-HRG $\neq$ QCD

QM-HRG does better

HRG vs. QCD
electric Charge-Baryon number Correlations

– chiral transition temperature: Tc= 154 (8) (1) MeV

scale uncertainty
statistical uncertainty

– error band on Tc is mainly statistical;
physics is quite different at lower and upper end of the current error bar

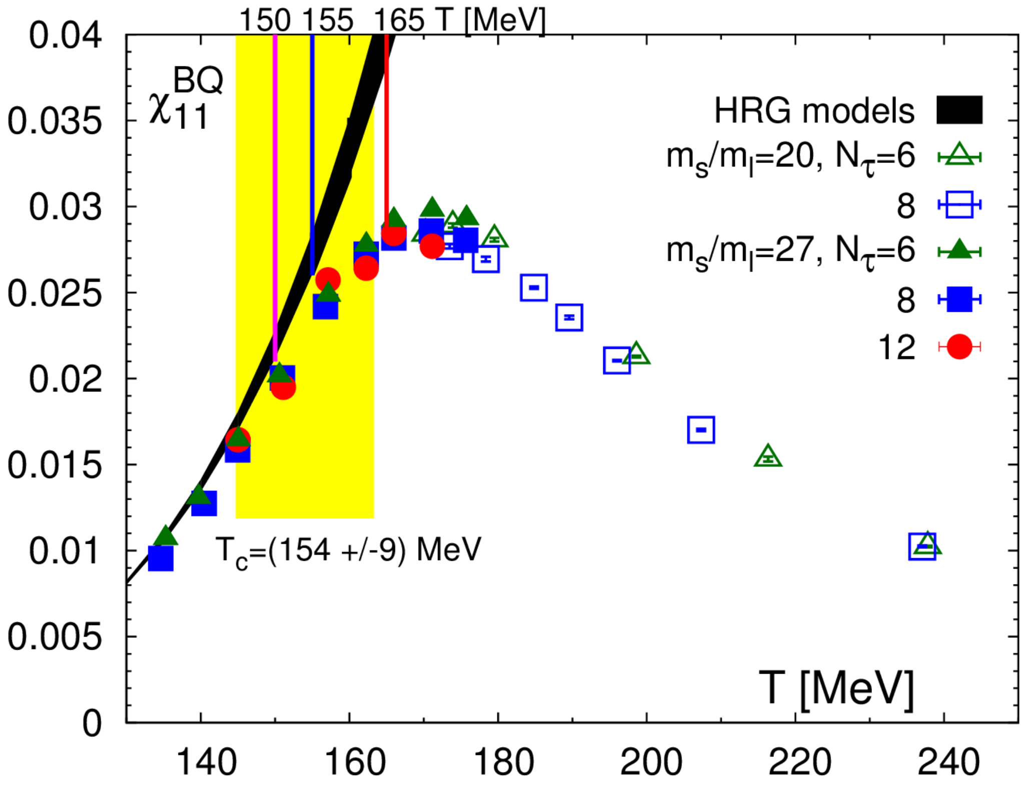
baryon number – electric charge correlations

$T \simeq 150\ \mathrm{MeV}:$

correlations **similar to HRG**

$T \simeq 165\ \mathrm{MeV}:$

correlations **very different from HRG**



HRG vs. QCD
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– chiral transition temperature: Tc= 154 (8) (1) MeV

(8): statistical uncertainty
(1): scale uncertainty

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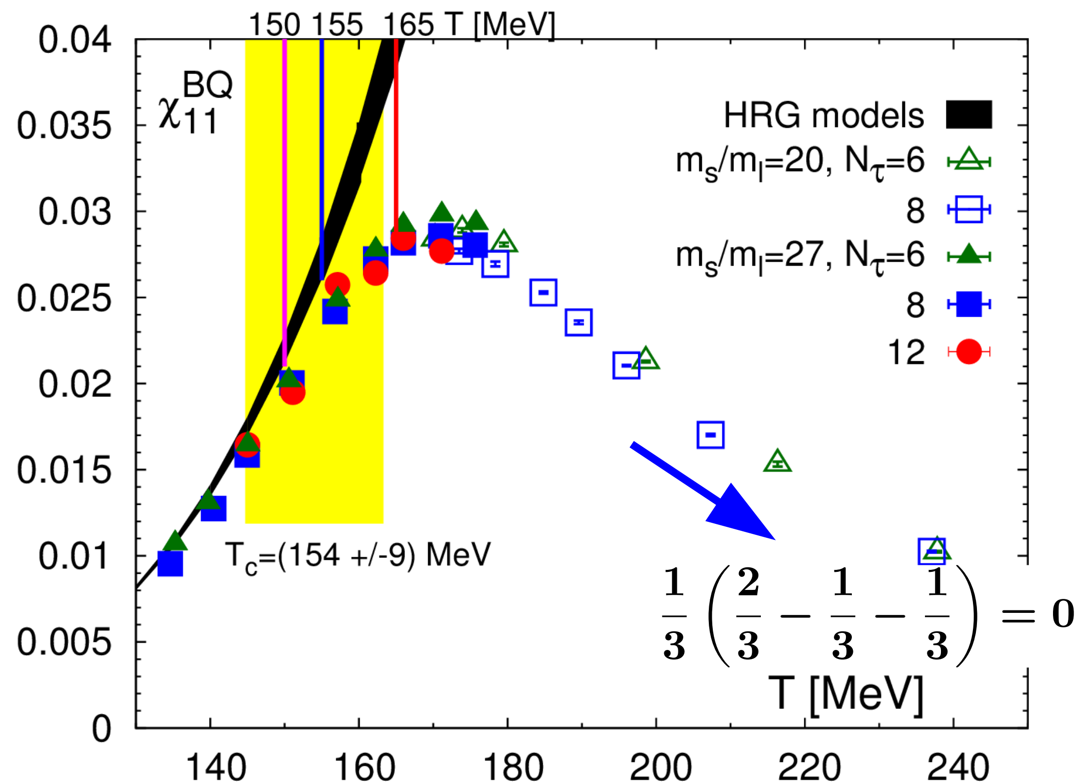
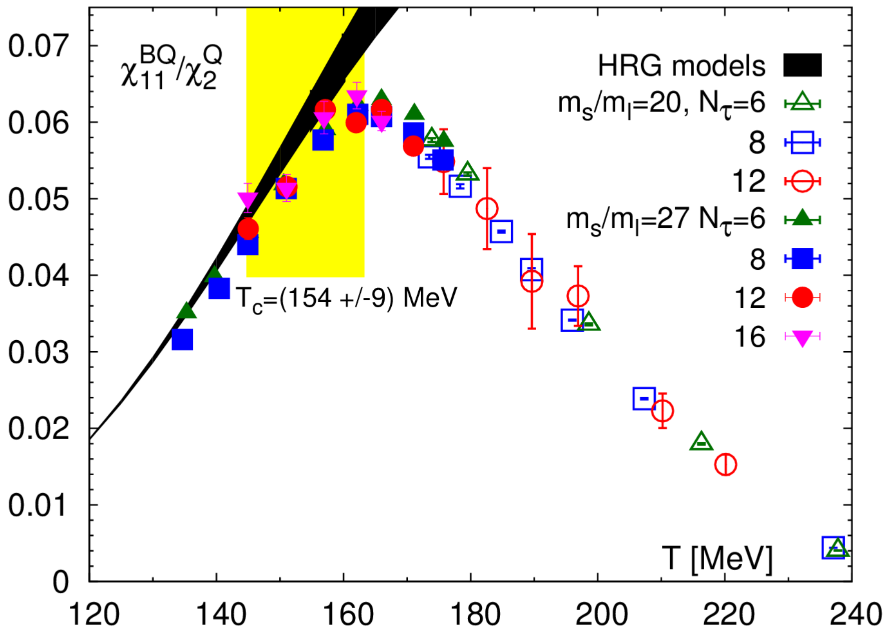
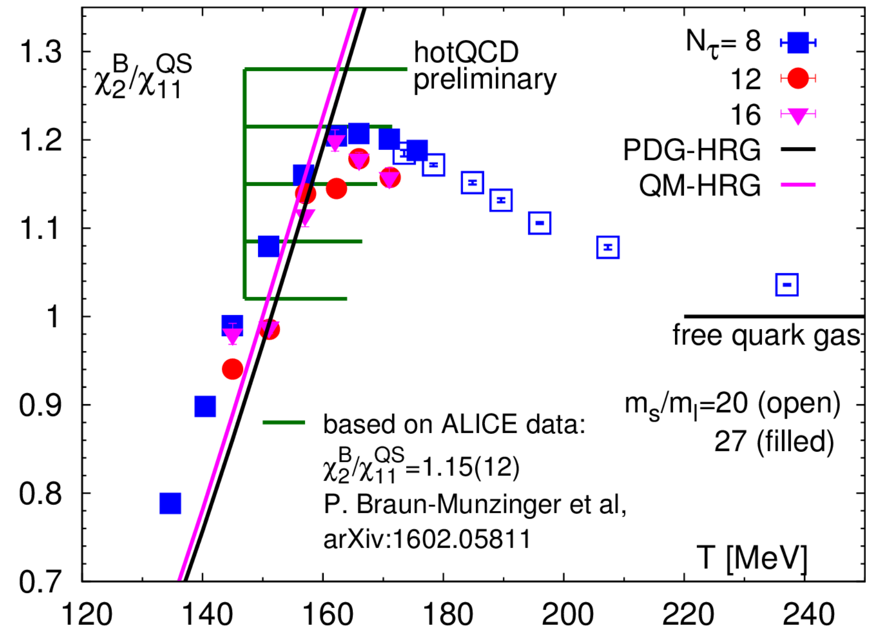


$$\frac{1}{3}\left(\frac{2}{3}-\frac{1}{3}-\frac{1}{3}\right)=0$$

Correlations of net-electric charge fluctuations with net-baryon number and net-strangeness

$$\mu_B/T = 0$$



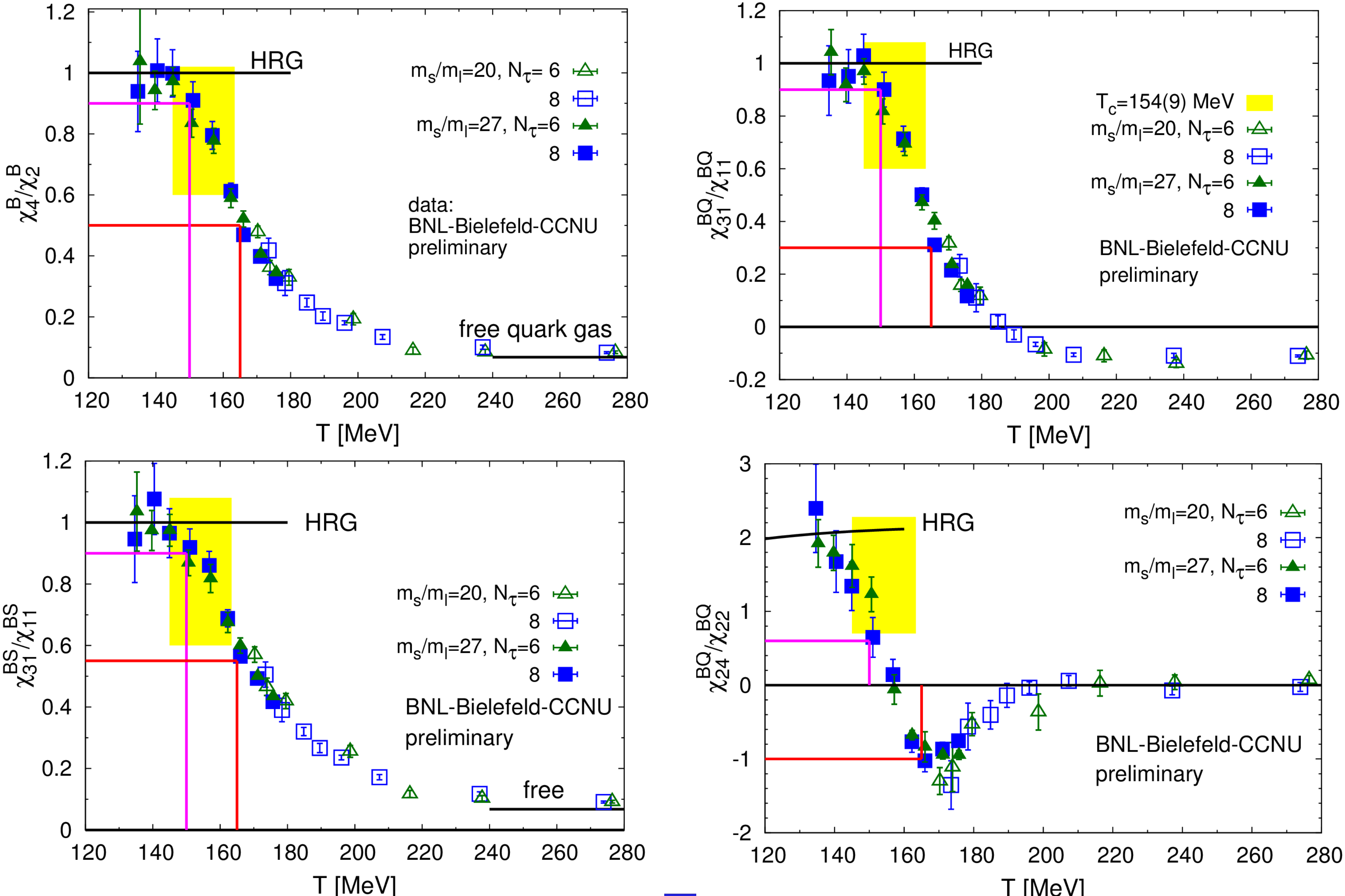


– **HRG models fail for T>165 MeV**

– **QCD bounds on conserved charge correlations** that can (will) be tested by ALICE:

$$\frac{\chi_{11}^{BQ}}{\chi_2^Q} < 0.06 \qquad \frac{\chi_{11}^{QS}}{\chi_2^B} > 0.84$$

Some 4th and 6th order cumulants



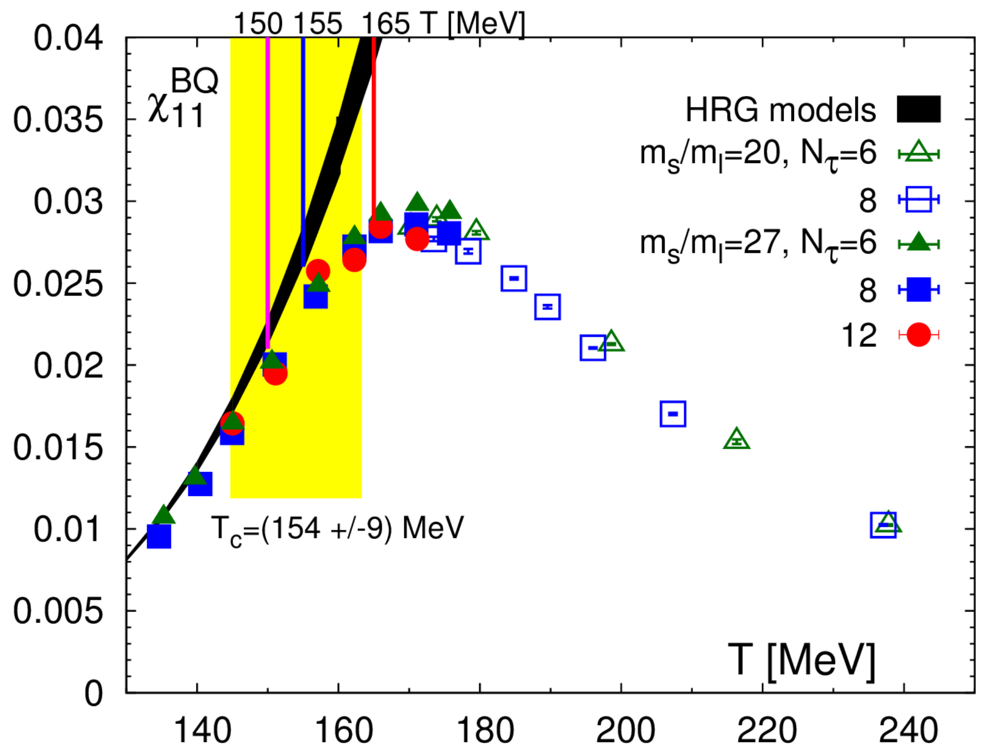
HRG vs. QCD
electric Charge-Baryon number Correlations

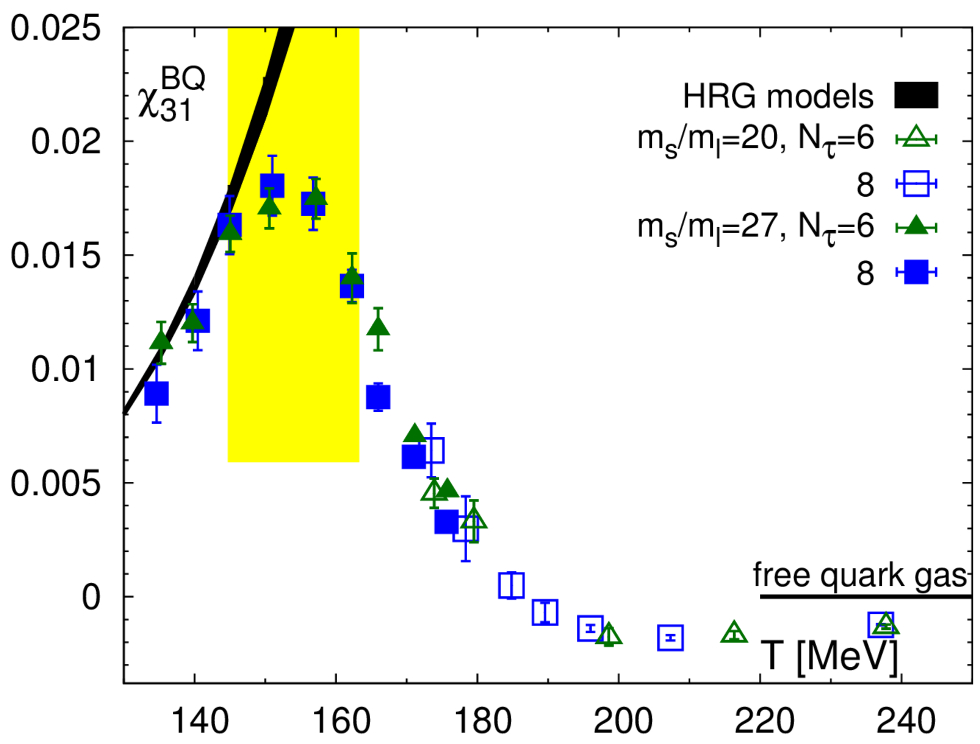
$$\mu_B/T > 0$$

for simplicity: $\mu_Q = \mu_S = 0$

$$\chi_{11}^{BQ}(T,\mu_B) = \chi_{11}^{BQ} + \frac{1}{2}\chi_{31}^{BQ}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

– agreement between HRG and QCD will start to deteriorate for T>150 MeV





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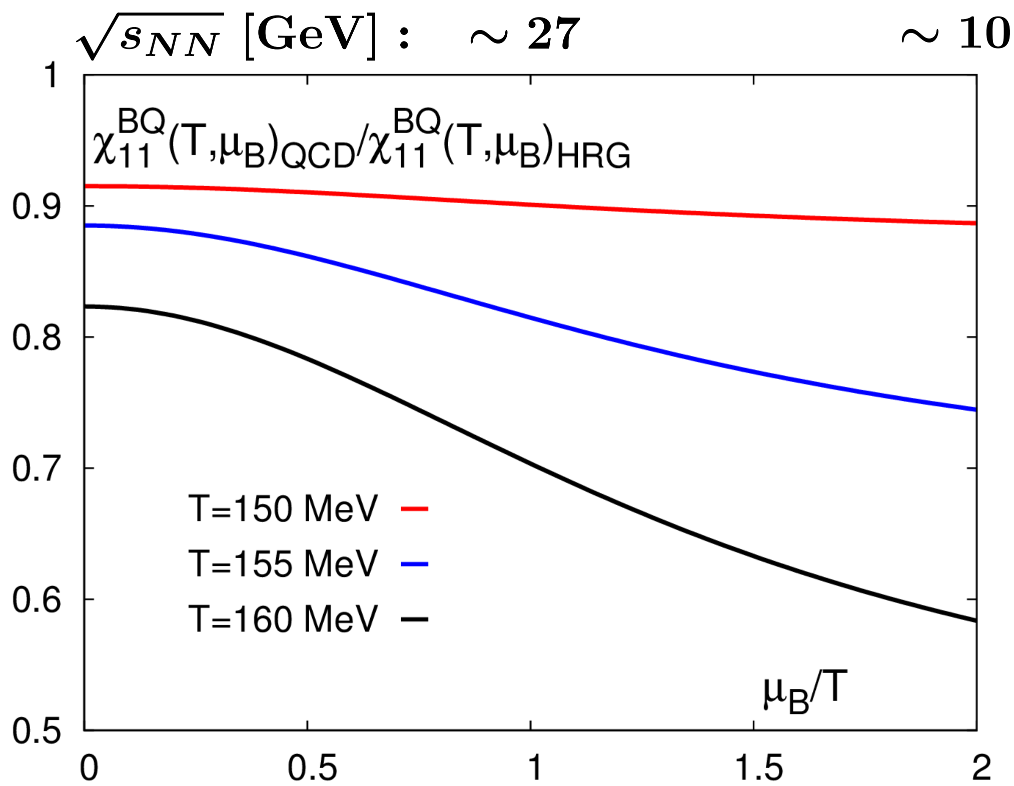

Chiral transition, hadronization and freeze-out

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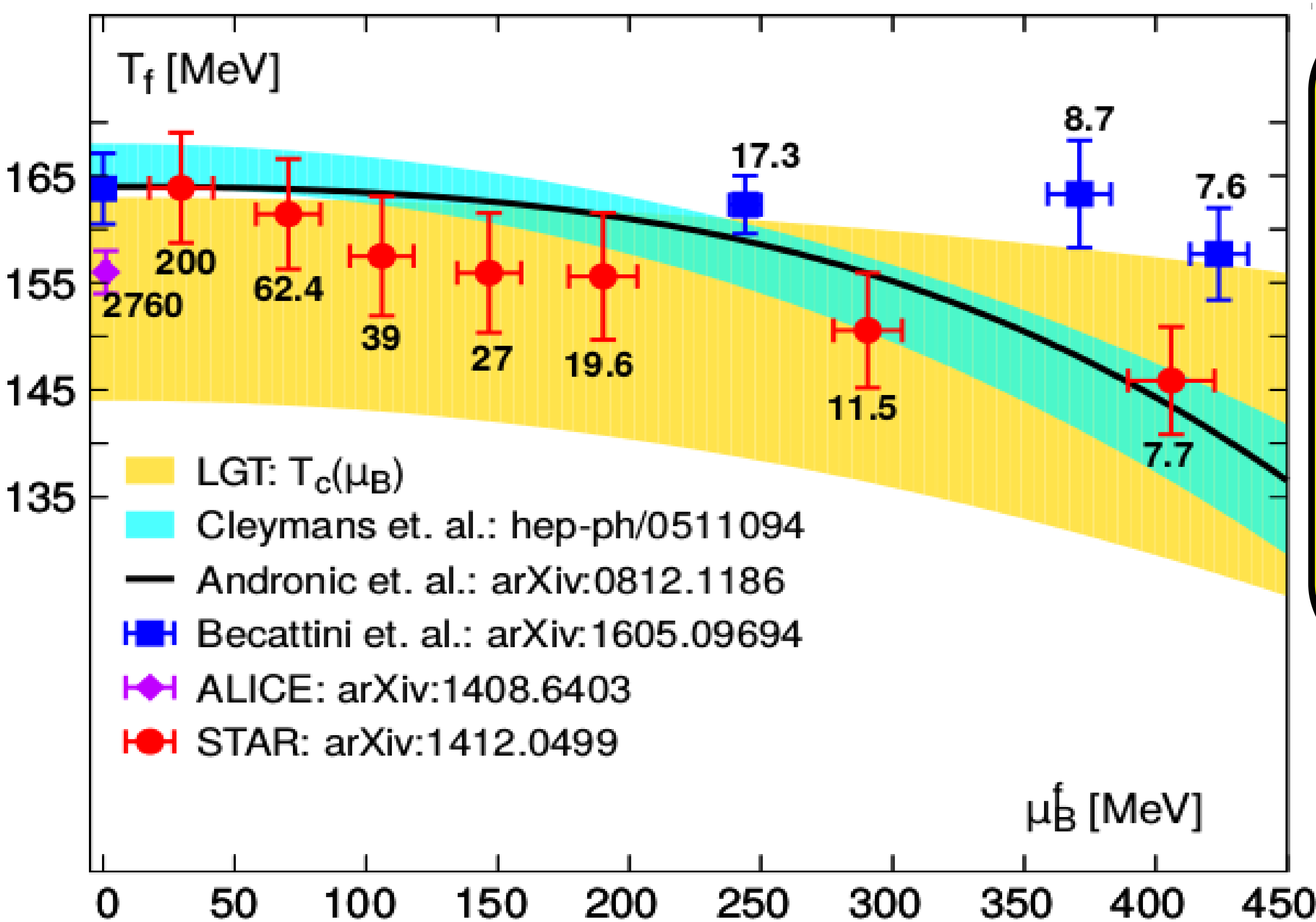


phenomenological freeze-out / hadronization curve, QCD transition line and experimental data (obtained by <u>assuming the validity of the HRG model</u>) are consistent for

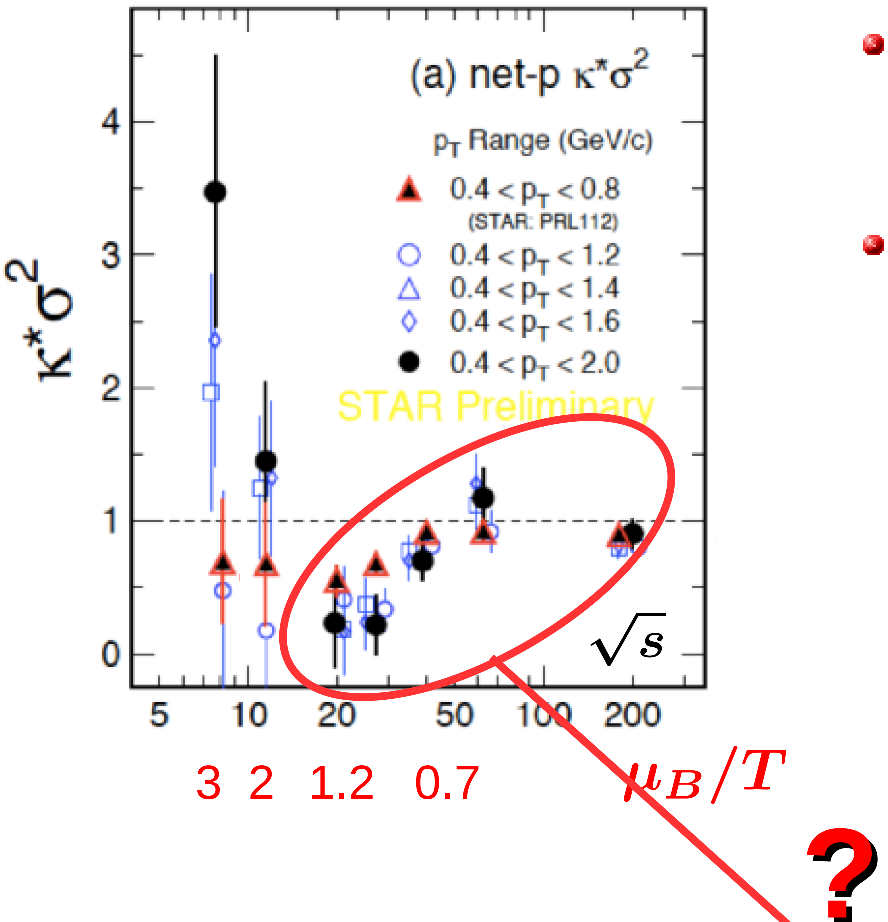
$$\mu_B/T \lesssim 3$$

<u>HOWEVER</u>

physics is quite different at lower and upper end of the current error bar on Tc

probed with net-charge correlations&fluctuations

Exploring the QCD phase diagram



- Can we understand the systematics seen in cumulants of charge fluctuations in terms of QCD thermodynamics ?
- How far do we get with low order Taylor expansions of **QCD** in explaining the obvious deviations from HRG model behavior ?

- For $\sqrt{s} \geq 19\ \mathrm{GeV}$ **:** Structure of (net-electric charge and) net-proton cumulants is inconsistent with HRG thermodynamics, but can it eventually be understood in terms of

QCD thermodynamics in a next-to-leading order Taylor expansion ?

Probing the properties of matter through the analysis of conserved charge fluctuations

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0}$$

$M_X \sim \chi_1^X$: mean

$\sigma_X^2 \sim \chi_2^X$: variance

$S_X \sim \chi_3^X / (\chi_2^X)^{3/2}$: skewness

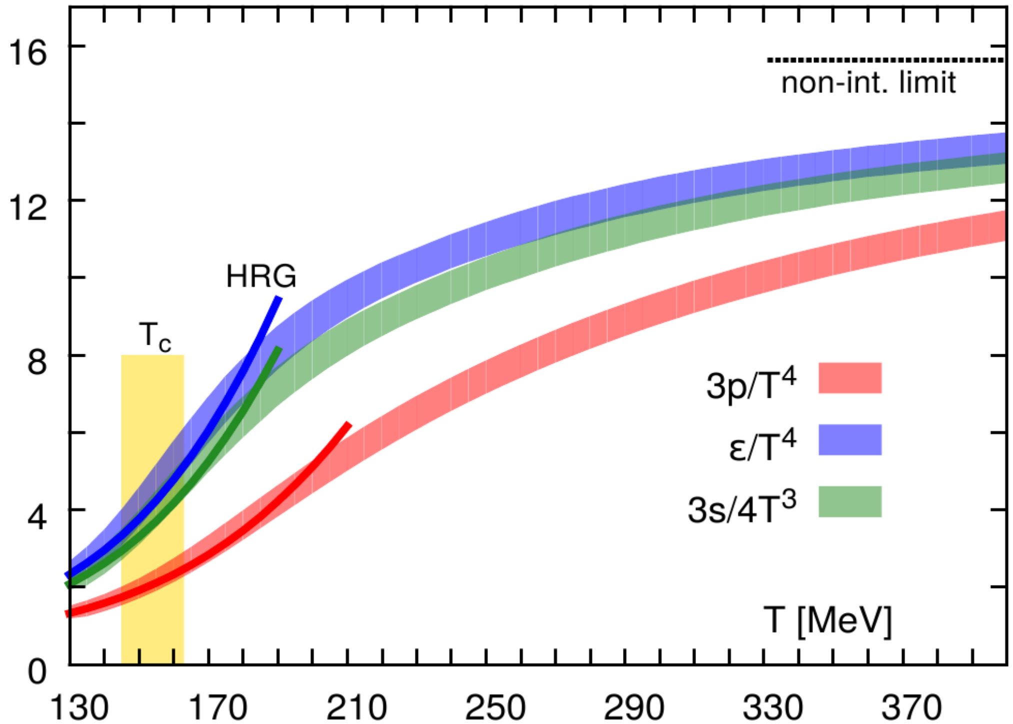
$\kappa_X \sim \chi_4^X / (\chi_2^X)^2$: kurtosis

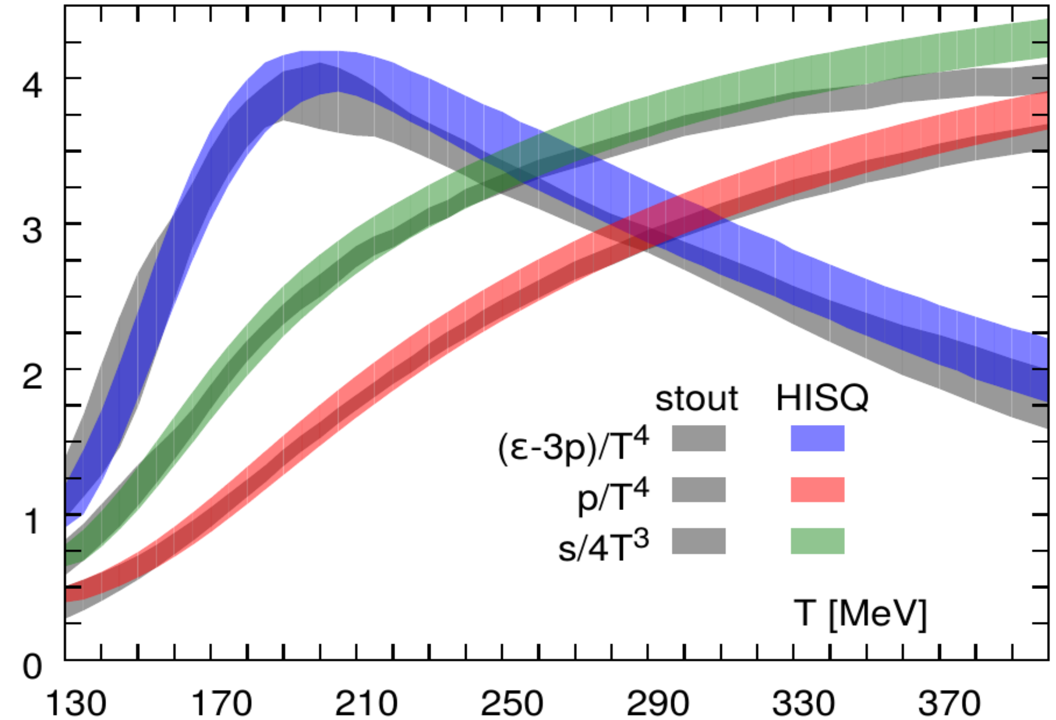
cumulant ratios: $R_{nm}^X(T, \mu_B) = \frac{\chi_n^X(T, \mu_B)}{\chi_m^X(T, \mu_B)}$

→ Taylor expansion of cumulant ratios

Equation of state of (2+1)-flavor QCD: $\mu_B/T = 0$

pressure, **entropy** & **energy** density





A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

– improves over earlier hotQCD calculations:
A. Bazavov et al., Phys. Rev. D80, 014504 (2009)

– consistent with results from Budapest-Wuppertal (stout): S. Borsanyi et al., PL B730, 99 (2014)

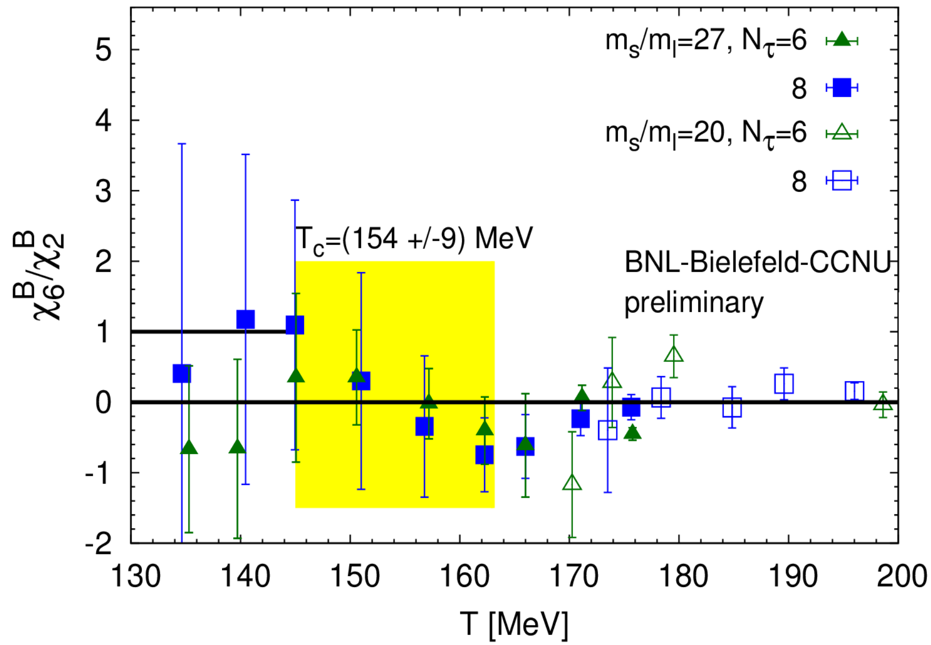
– up to the crossover region the QCD EoS agrees quite well with hadron resonance gas (HRG) model calculations; **However**, QCD results are systematically above HRG

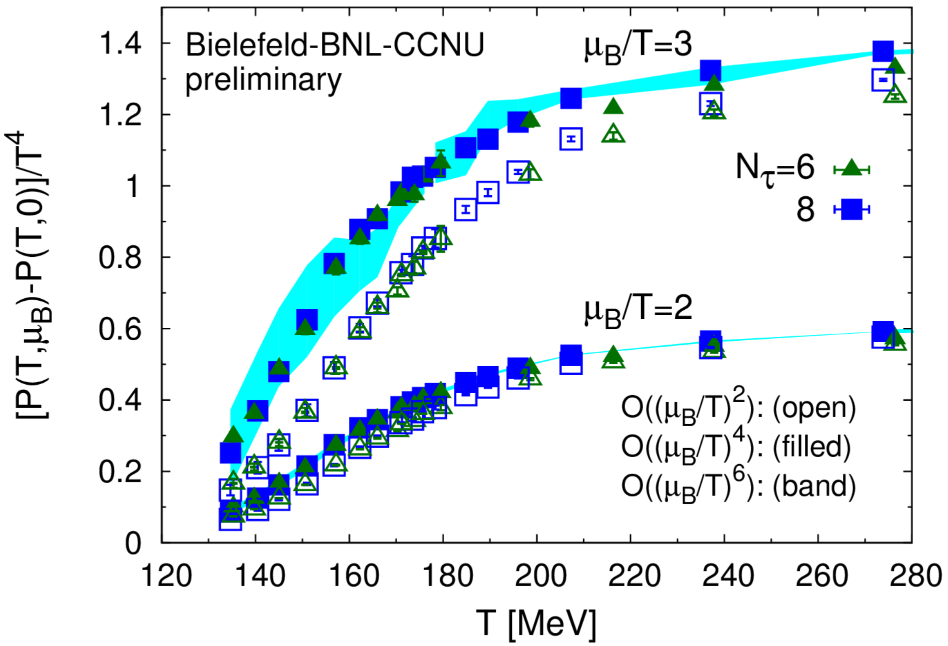
Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

for simplicity: $\mu_Q = \mu_S = 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

estimating the $\mathcal{O}((\mu_B/T)^6)$ correction: $\sim \frac{1}{720}\frac{\chi_6^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^6$





$\mathcal{O}(\mu_B^6)$: FK, CPOD 2016, ICPAQGP 2015

$\mathcal{O}(\mu_B^4)$: P. Hegde (BNL-Bielefeld-CCNU), PoS Lattice2014 (2014) 226, arXiV:1412.6727

The EoS is well controlled for $\mu_B/T \leq 2$ or equivalently $\sqrt{s_{NN}} \geq 20$ GeV

Conserved charge fluctuations and freeze-out

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

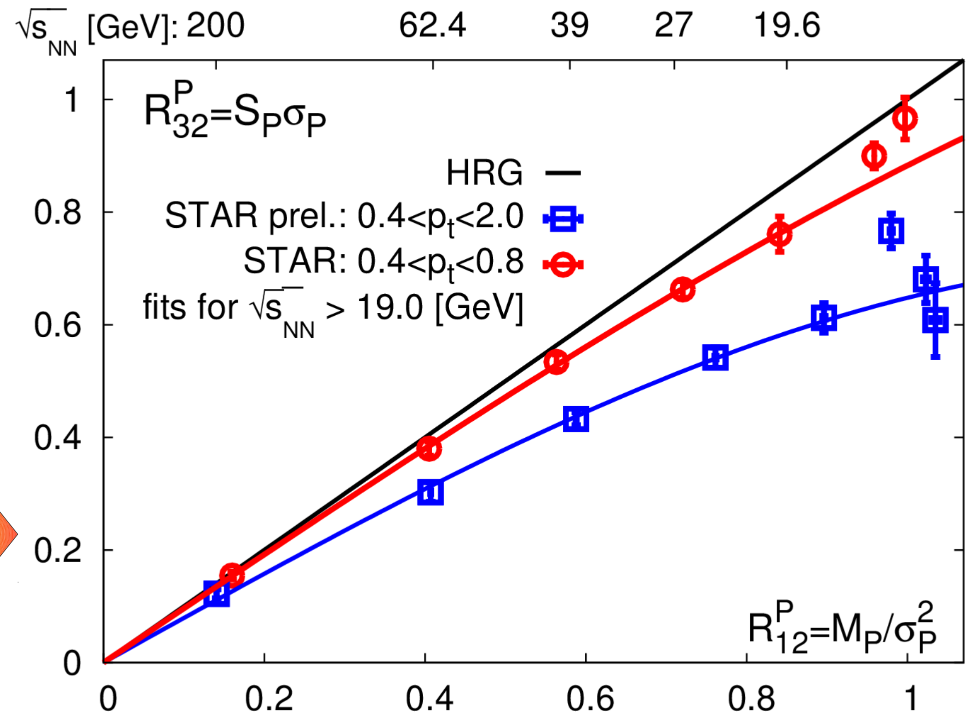
$$R_{12}^B \equiv \frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

see backup for $\mu_Q \neq \mu_S \neq 0$

$$R_{32}^B \equiv S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$

no need for talking about a chemical potential

$$S_B\sigma_B = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$



$$\frac{\chi_4^B}{\chi_2^B} < 1$$

slope < 1

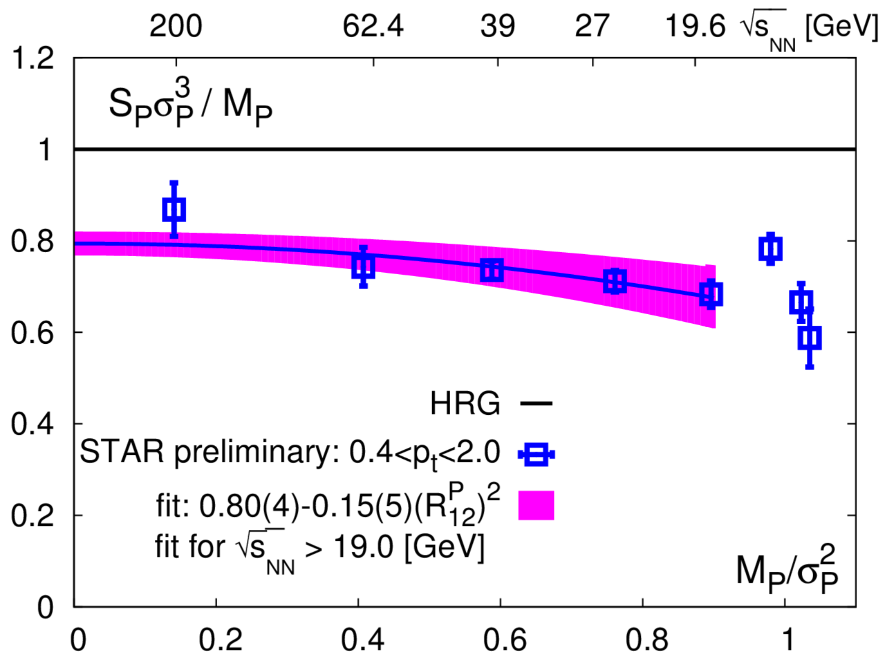
Conserved charge fluctuations and freeze-out
mean, variance and skewness

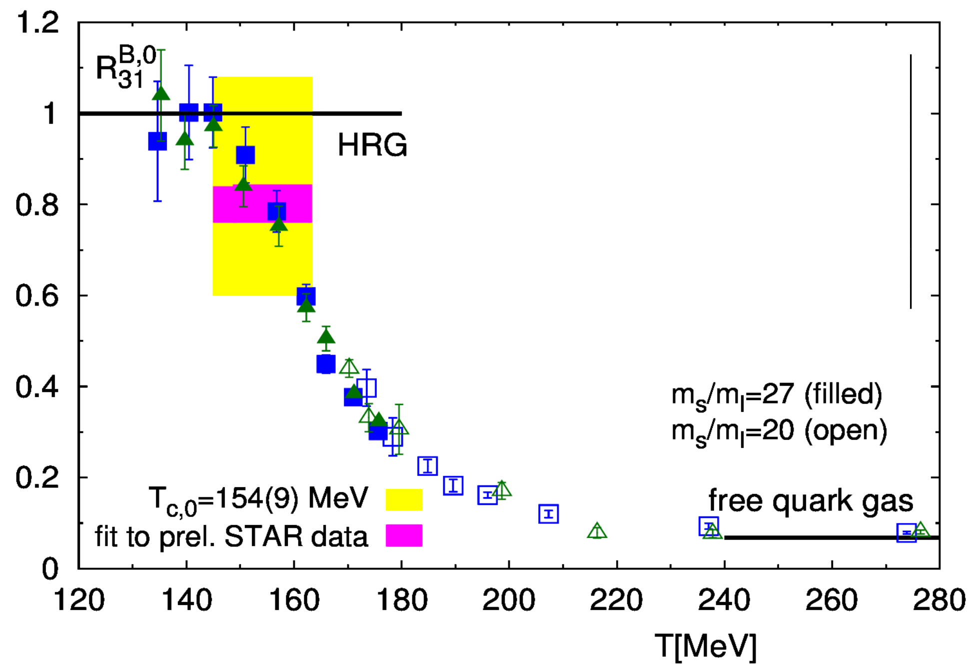
NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots \qquad \mu_Q = \mu_S = 0$$

$$\Leftrightarrow \quad R_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = R_{31}^{B,0} + \frac{R_{31}^{B,2}}{(R_{12}^{B,1})^2}\left(\frac{M_B}{\sigma_B^2}\right)^2$$

F. Karsch et al., arXiv:1512.06987





$$\text{e.g.,}\quad \mu_Q \neq \mu_S \neq 0: \quad R_{31}^{B,0} = \frac{\chi_4^B + \frac{\mu_S}{\mu_B}\chi_{31}^{BS} + \frac{\mu_Q}{\mu_B}\chi_{31}^{BQ}}{\chi_2^B + \frac{\mu_S}{\mu_B}\chi_{11}^{BS} + \frac{\mu_Q}{\mu_B}\chi_{11}^{BQ}}\ ,\quad R_{31}^{B,2} = \ldots$$

Conserved charge fluctuations and freeze-out
mean, variance and skewness

NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \dots \qquad \mu_Q = \mu_S = 0$$

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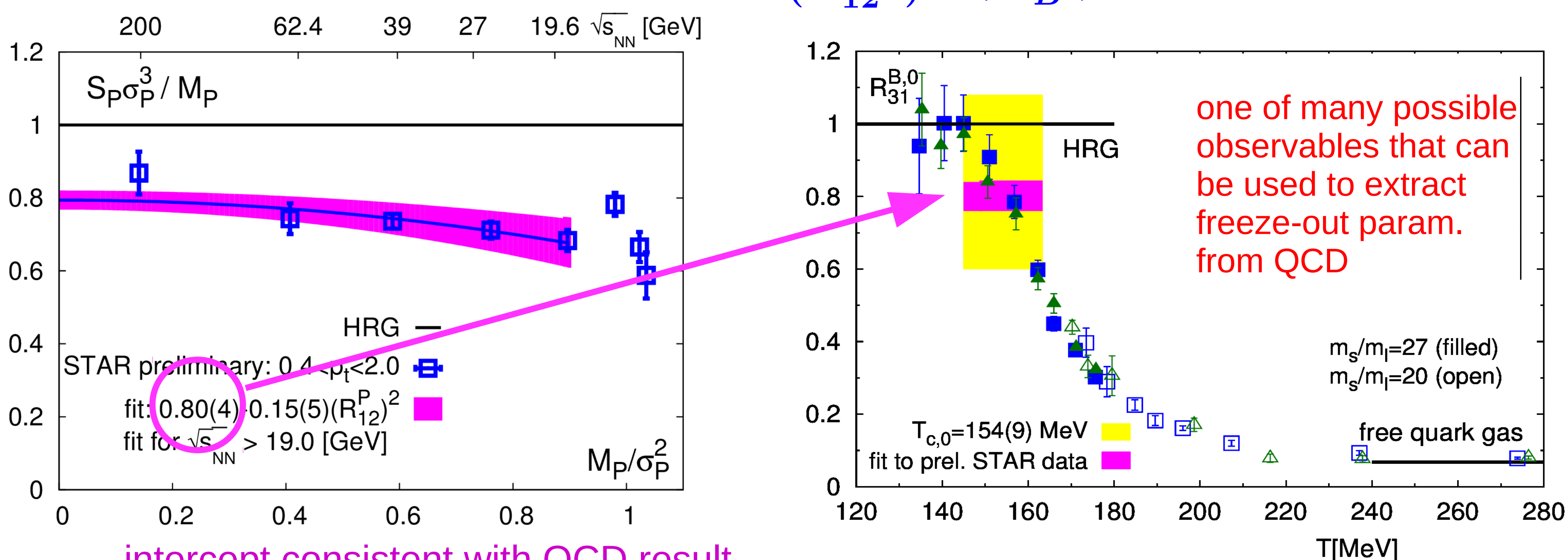


– intercept consistent with QCD result,

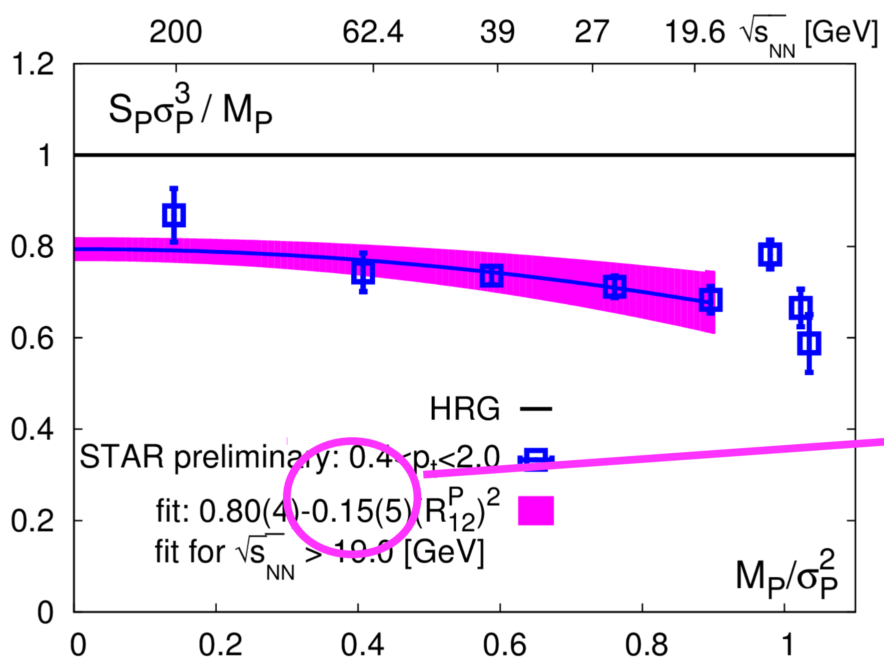
Conserved charge fluctuations and freeze-out
mean, variance and skewness

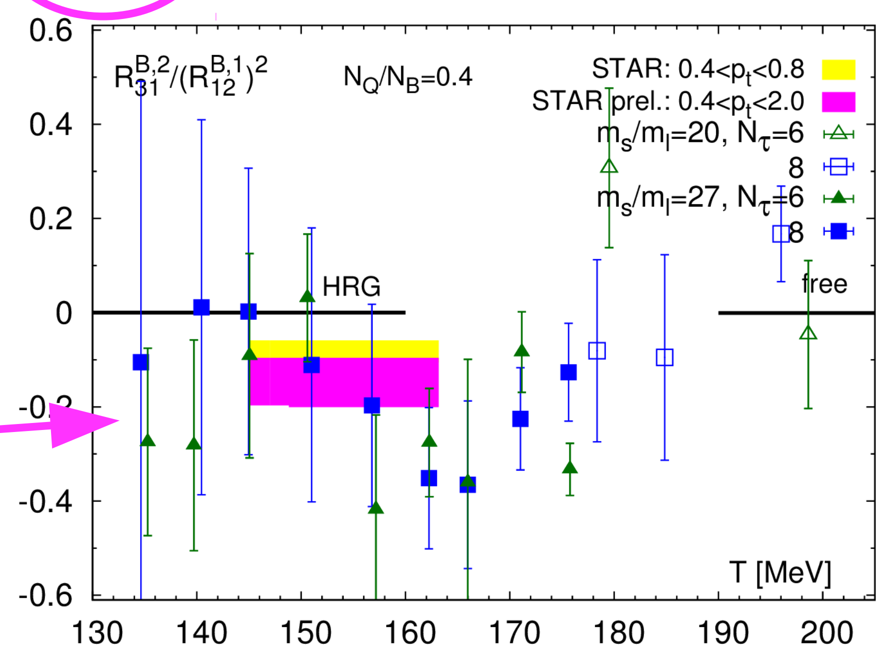
NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots \qquad \mu_Q = \mu_S = 0$$

$$\Leftrightarrow \quad R_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = R_{31}^{B,0} + \frac{R_{31}^{B,2}}{(R_{12}^{B,1})^2}\left(\frac{M_B}{\sigma_B^2}\right)^2$$

lattice QCD calculation involves 6th order cumulants





– intercept consistent with QCD result,

– curvature consistent with QCD result (still noisy)

Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B \sigma_B^3 / M_B$, $R_{42}^B \equiv \kappa_B \sigma_B^2$ } are closely related

F. Karsch et al., arXiv:1512.06987

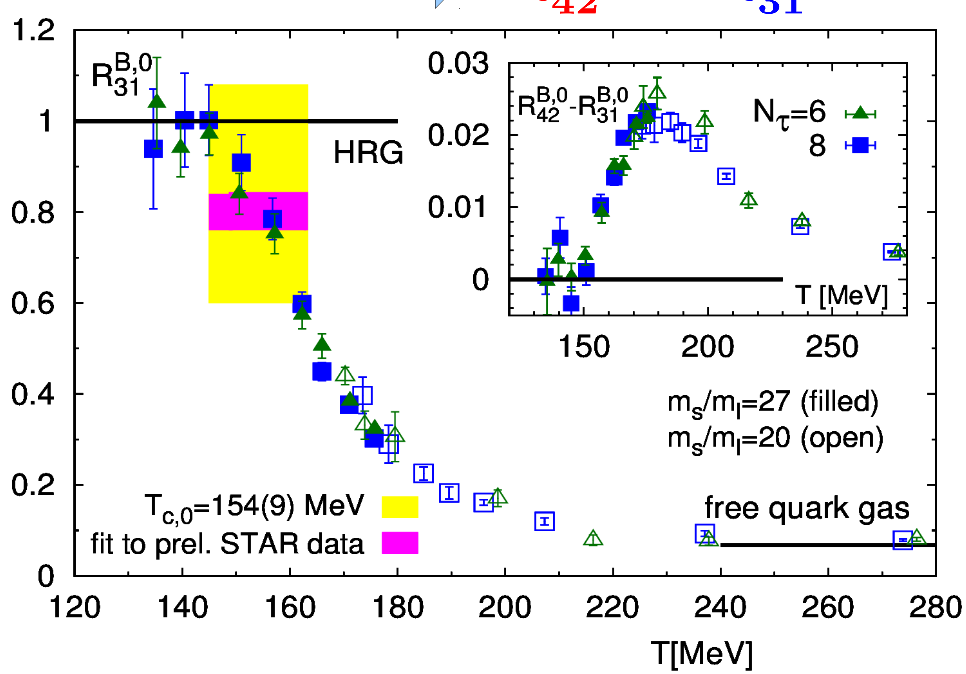
$$R_{31}^B = R_{31}^{B,0} + R_{31}^{B,2}\left(\frac{\mu_B}{T}\right)^2$$
$$R_{42}^B = R_{42}^{B,0} + R_{42}^{B,2}\left(\frac{\mu_B}{T}\right)^2$$

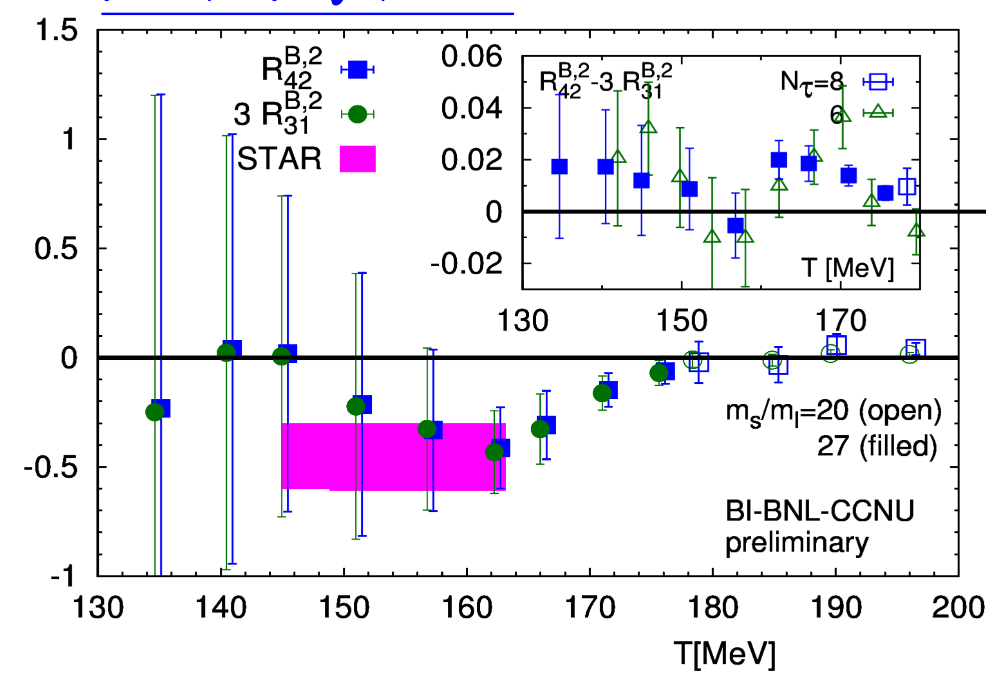
$\Rightarrow$ $R_{42}^{B,0} \simeq R_{31}^{B,0}$

$\mu_S = \mu_Q = 0$:

$$R_{42}^{B,2} = 3R_{31}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$$

$\mu_S \neq \mu_Q \neq 0$:





Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B\sigma_B^3/M_B$, $R_{42}^B \equiv \kappa_B\sigma_B^2$ } **are closely related**

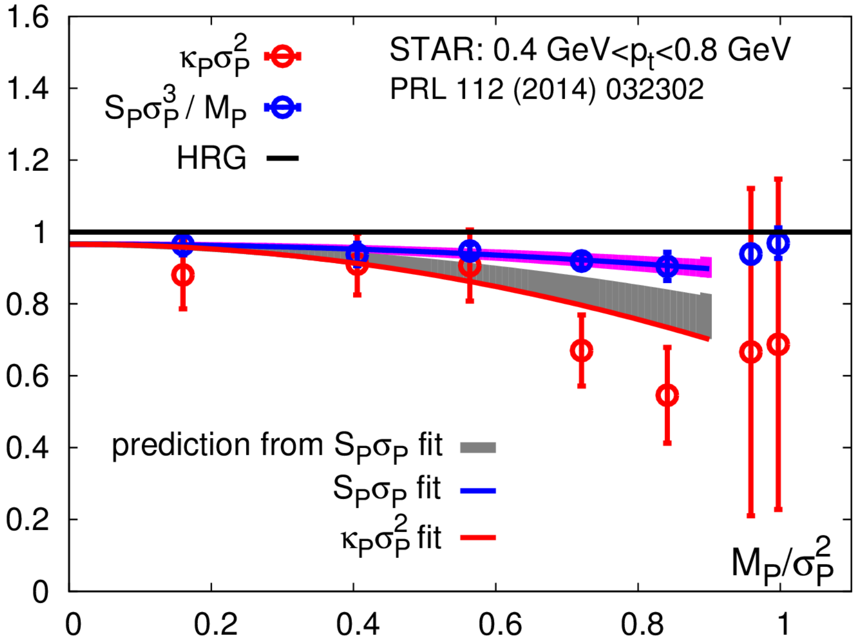
F. Karsch et al., arXiv:1512.06987

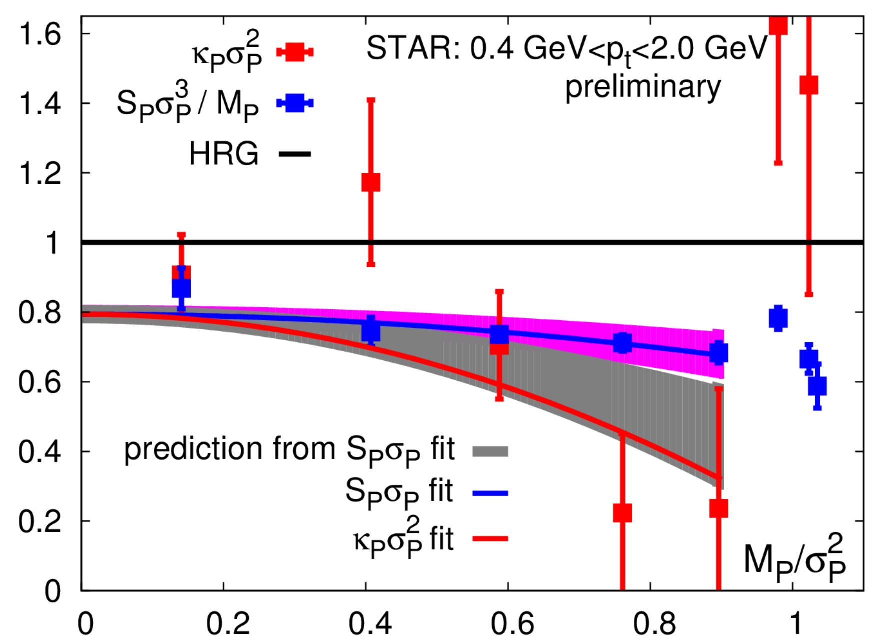
$$R_{31}^B = R_{31}^{B,0} + R_{31}^{B,2}\left(\frac{\mu_B}{T}\right)^2$$

$$R_{42}^B = R_{42}^{B,0} + R_{42}^{B,2}\left(\frac{\mu_B}{T}\right)^2$$

$\mu_S = \mu_Q = 0:$

$$R_{42}^{B,2} = 3R_{31}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$$





Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B\sigma_B/M_B$ and $R_{42}^B \equiv \kappa_B\sigma_B^2$ } **are closely related**

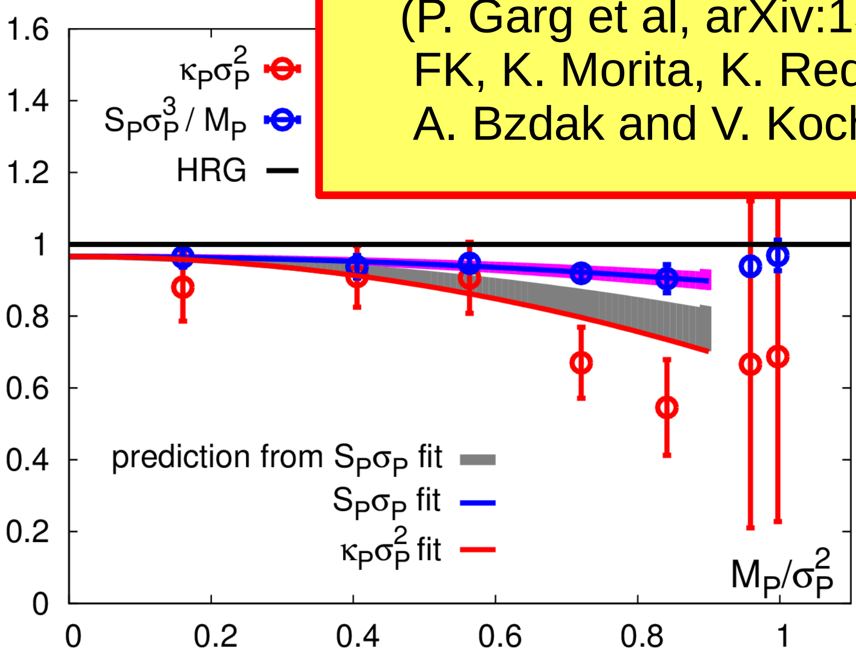
F. Karsch et al., arXiv:1512.06987

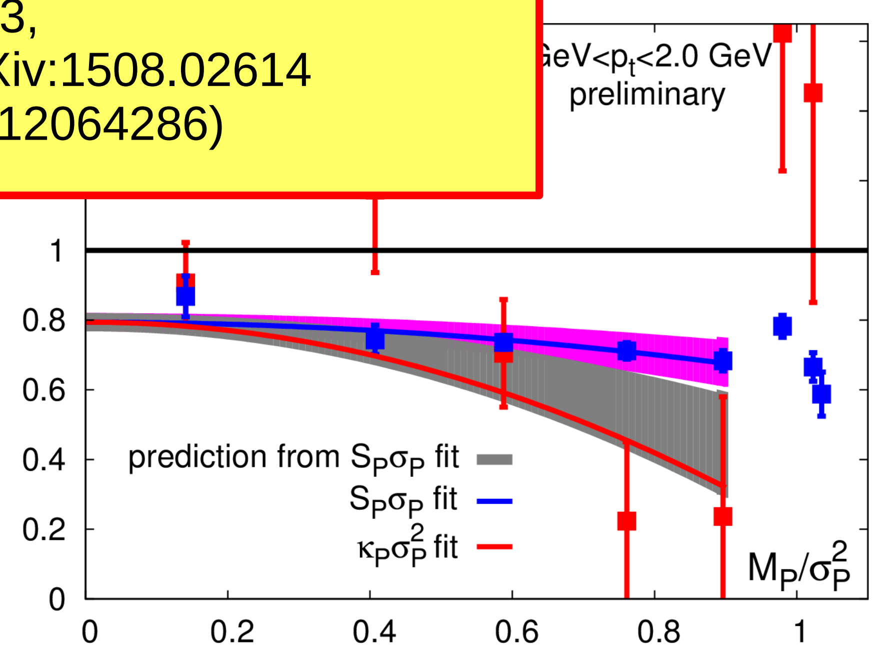
$R_{31}^B = R_{31}^{B,}$ …

$R_{42}^B = R_{42}^{B,0}$ … $\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2 \Bigg)$

<u>need to understand systematics :</u>

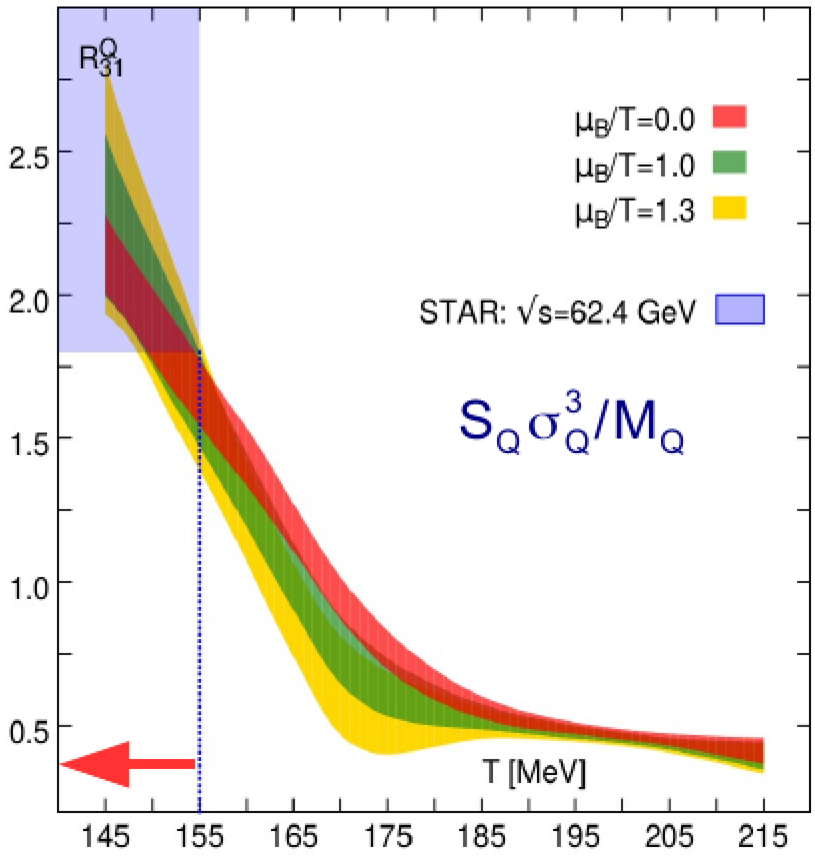
– non-equilibrium effects
(S. Mukherjee et al., arXiv:1506.00645)
– proton vs. baryon number distributions
(M. Kitazawa et al, arXiv:1205.3292, arXiv:1303.3338)
– acceptance and pt-cuts
(P. Garg et al, arXiv:1304.7133,
FK, K. Morita, K. Redlich, arXiv:1508.02614
A. Bzdak and V. Koch, arXiv:12064286)

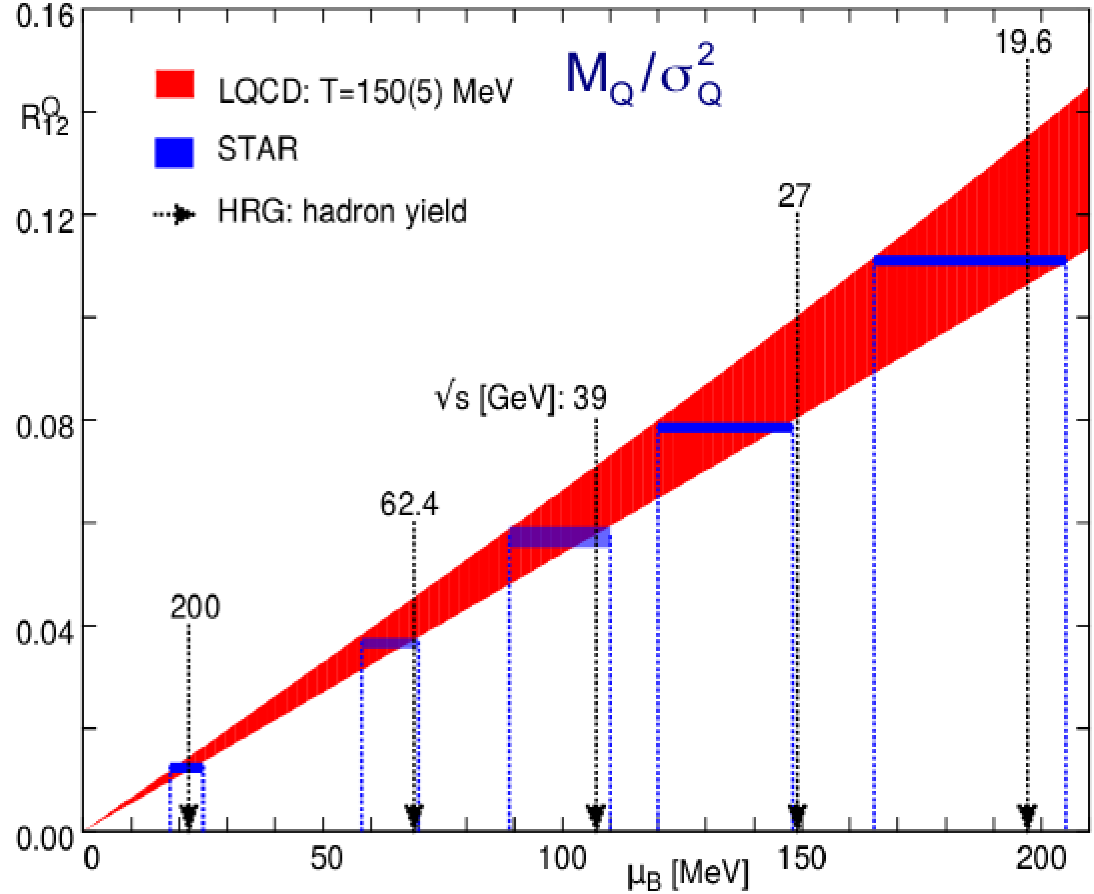




Freeze-out parameter from conserved charge fluctuations

cumulant ratios of electric charge fluctuations





constrains freeze-out temperature

$$T_f \simeq (150 \pm 5)\ \mathrm{MeV}$$

determines freeze-out chemical potential

FK, Central Eur.J.Phys. 10 (2012) 1234
BI-BNL, PRL 109, 192302 (2012)
S. Mukherjee, M. Wagner, PoS CPOD2013 (2013) 039
S. Borsanyi et al., PRL 111 (2013) 062005

Conclusions

– attempts to understand freeze-out/hadronization in terms of HRG model based calculations at temperatures T > 160 MeV are difficult to conciliate with QCD;

QCD thermodynamics is quite different from HRG thermodynamics at T > 160 MeV

– results on bulk thermodynamics coming from Taylor expansion of the QCD partition function are already now reliable in the range $0 \le \mu_B/T \le 2$

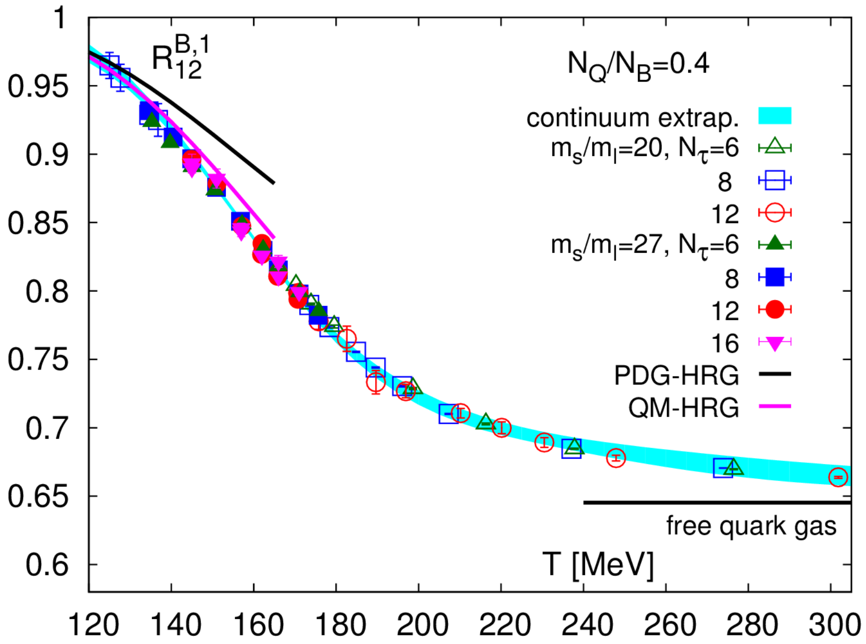
bulk QCD thermodynamics in the entire parameter range accessible to BES I and II may soon be accessible also through Taylor expansions

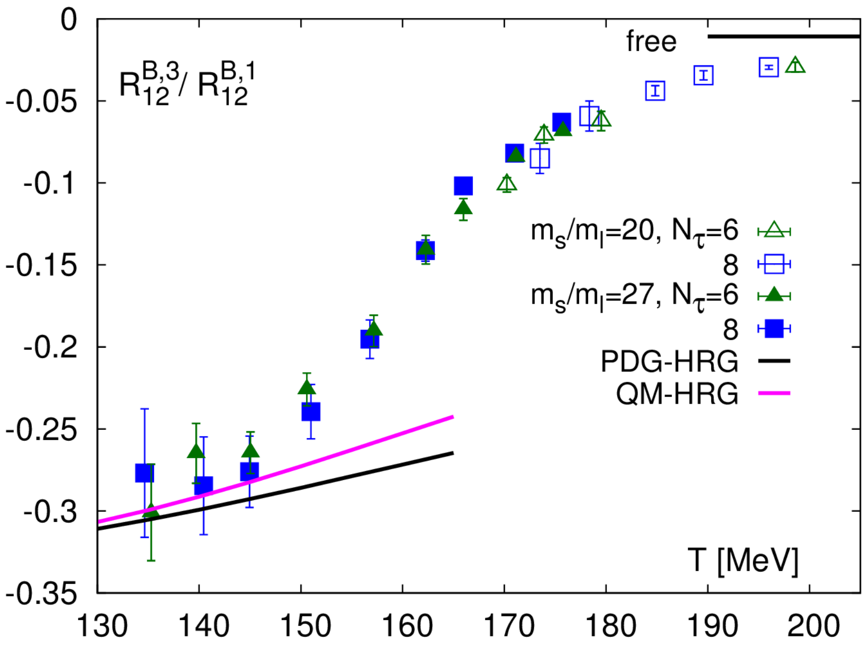
– properties of cumulants measured in BES-I for $\sqrt{s_{NN}} \le 19\ \mathrm{GeV}$ clearly differ from HRG thermodynamics but are consistent with QCD thermodynamics close to the crossover transition temperature

$$S_B\sigma_B < M_B/\sigma_B^2 \ \ , \ \ \kappa_B\sigma_B^2 - S_B\sigma \sim \left(M_B/\sigma_B^2\right)^2$$

Conserved charge fluctuations and freeze-out
mean over variance squared

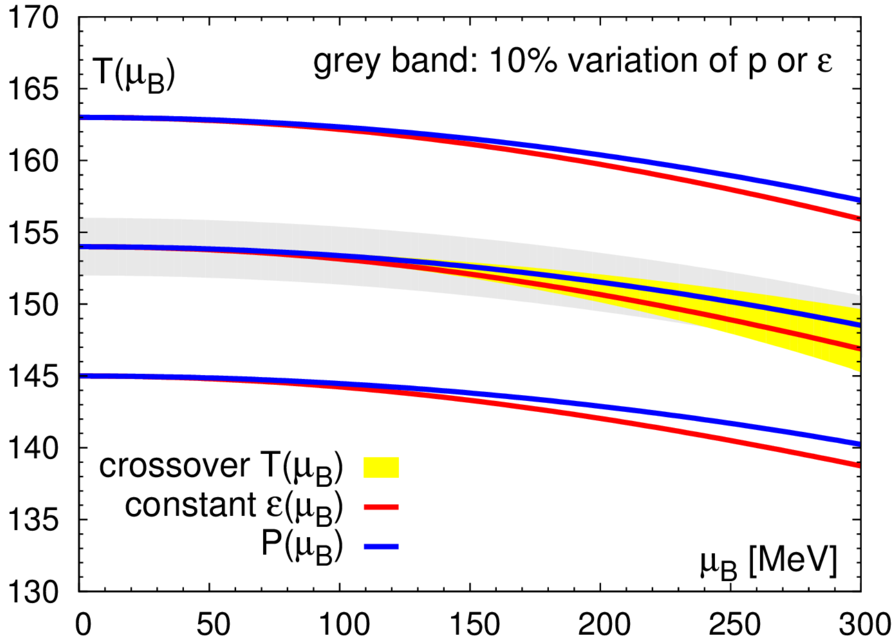
$$R_{12}^{B} = R_{12}^{B,1}\frac{\mu_B}{T} + R_{12}^{B,3}\left(\frac{\mu_B}{T}\right)^3$$

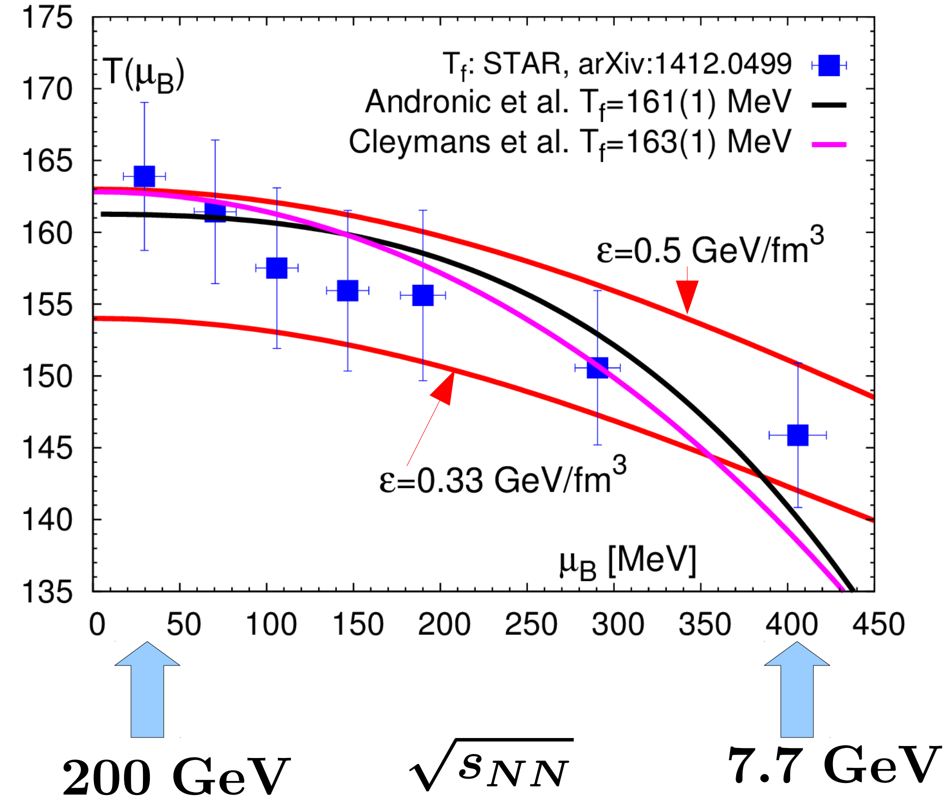




Lines of constant physics and freeze-out

$$T_f(\mu_B) = T_f(0)\left(1 - \kappa_{f,2}\hat{\mu}_B^2 - \kappa_{f,4}\hat{\mu}_B^4\right)$$





$\mu_Q = \mu_S = 0:$

constant pressure: $\kappa_{2,p} \simeq 0.011$

constant energy density: $\kappa_{2,\epsilon} \simeq 0.013$

crossover line: $\kappa_{2,c} \simeq 0.006 - 0.013$