BULK PROPERTIES OF QCD MATTER FROM LATTICE SIMULATIONS

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SQM Berkeley 2016

Lattice QCD

- Best first principle-tool to extract predictions for the theory of strong interactions in the non-perturbative regime
- Uncertainties:
  - Statistical: finite sample, error $\sim 1/\sqrt{\text{sample size}}$
  - Systematic: finite box size, unphysical quark masses
- Given enough computer power, uncertainties can be kept under control
- Results from different groups, adopting different discretizations, converge to consistent results
- Unprecedented level of accuracy in lattice data

Low temperature phase: HRG model

Dashen, Ma, Bernstein; Prakash, Venugopalan, Karsch, Tawfik, Redlich

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas
- The pressure can be written as:

$$p^{HRG}/T^4 = \frac{1}{VT^3}\sum_{i\in\ mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3}\sum_{i\in\ baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a})$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{Vd_i}{2\pi^2}\int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

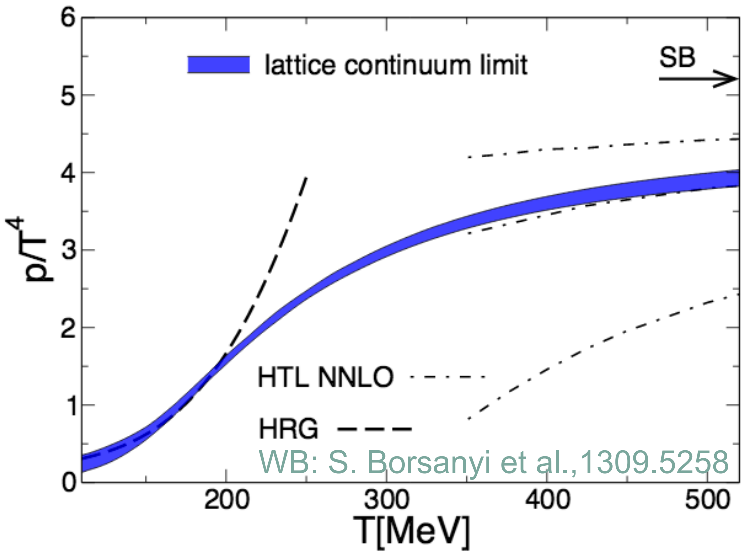
with energies $\varepsilon_i = \sqrt{k^2+m_i^2}$, degeneracy factors d_i and fugacities

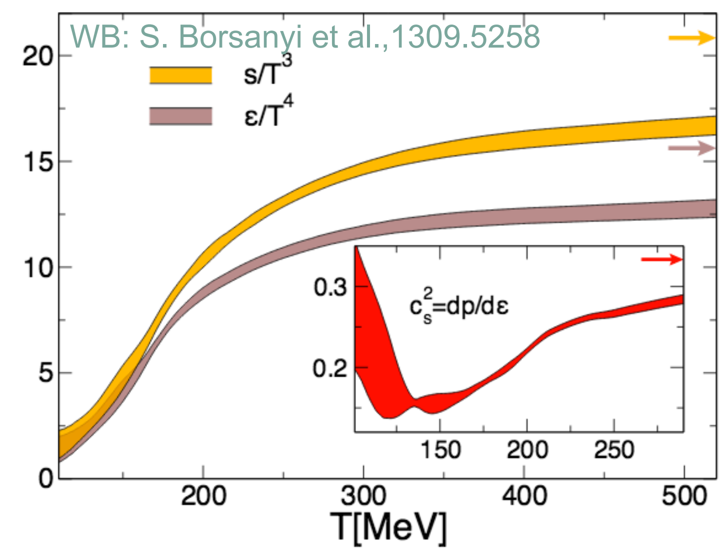
$$z_i = \exp\left((\sum_a X_i^a \mu_{X^a})/T \right) \quad .$$

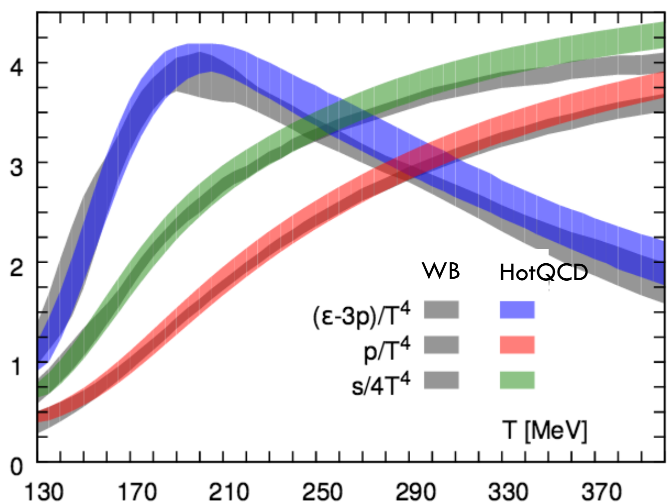
X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

- Needs knowledge of the hadronic spectrum

QCD Equation of state at μ_B=0







- EoS available in the continuum limit, with realistic quark masses
- Agreement between stout and HISQ action for all quantities

WB: S. Borsanyi et al., 1309.5258, PLB (2014)
HotQCD: A. Bazavov et al., 1407.6387, PRD (2014)

Sign problem

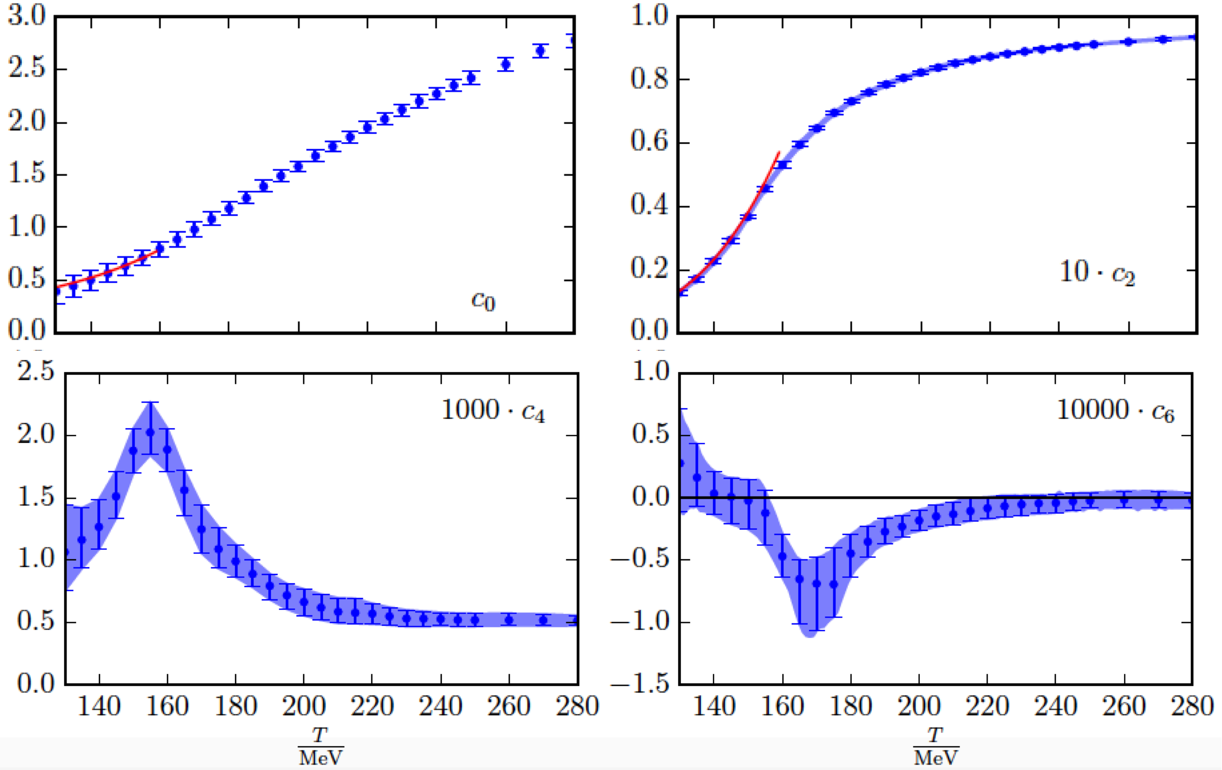
- The QCD path integral is computed by Monte Carlo algorithms which samples field configurations with a weight proportional to the exponential of the action

$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}} - \mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

- $\det M[\mu_B]$ complex → Monte Carlo simulations are not feasible

- We can rely on a few approximate methods, viable for small μ_B/T:
  - Taylor expansion of physical quantities around $\mu_B=0$ (Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003)
  - Reweighting (complex phase moved from the measure to observables) (Barbour et al. 1998; Z. Fodor and S, Katz, 2002)
  - Simulations at imaginary chemical potentials (plus analytic continuation) (Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003)

Equation of state at $\mu_B>0$



- Expand the pressure in powers of μ_B

$$\frac{p(\mu_B)}{T^4} = c_0 + c_2\left(\frac{\mu_B}{T}\right)^2 + c_4\left(\frac{\mu_B}{T}\right)^4 + c_6\left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$

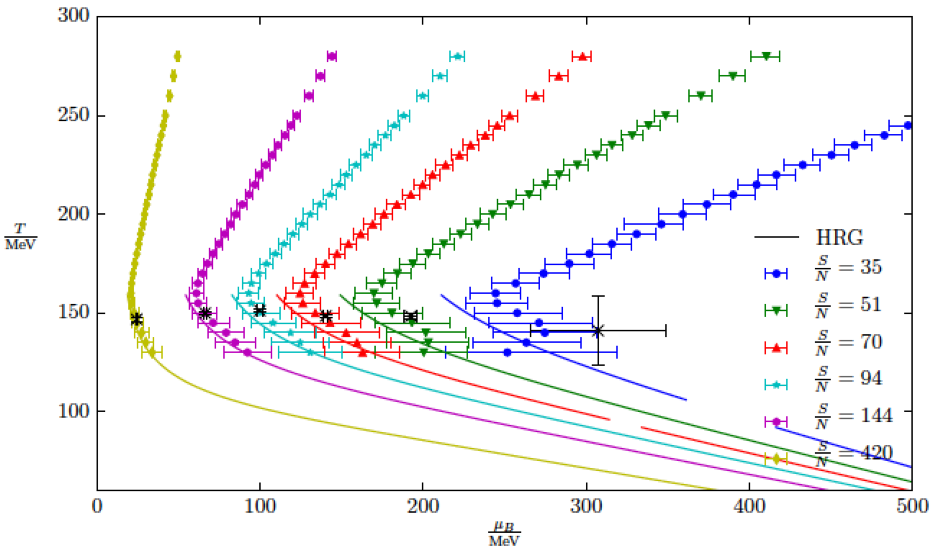
- Continuum extrapolated results for c_2, c_4, c_6 at the physical mass

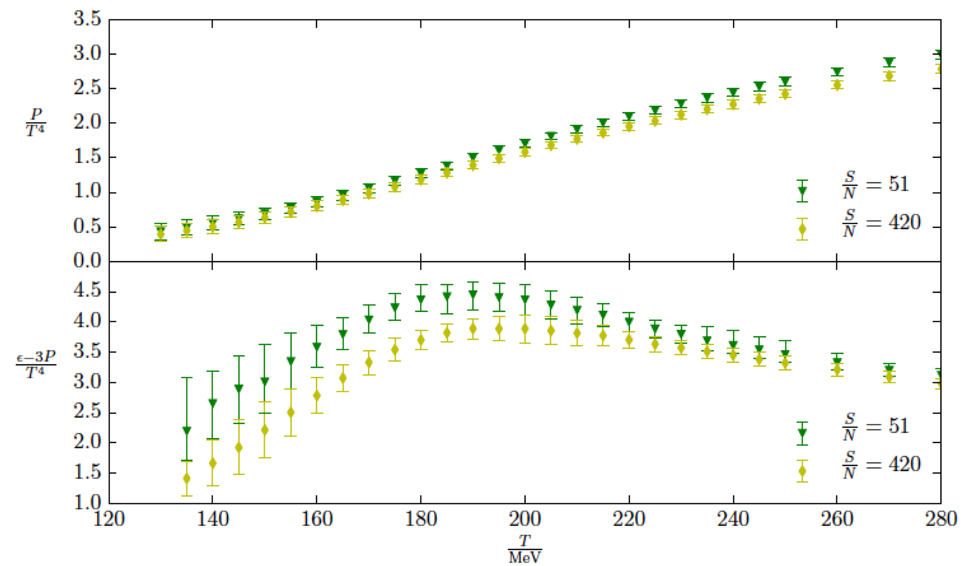
- Enables us to reach $\mu_B/T\sim2.5$

WB: S. Borsanyi et al., preliminary

Equation of state at μ_B>0

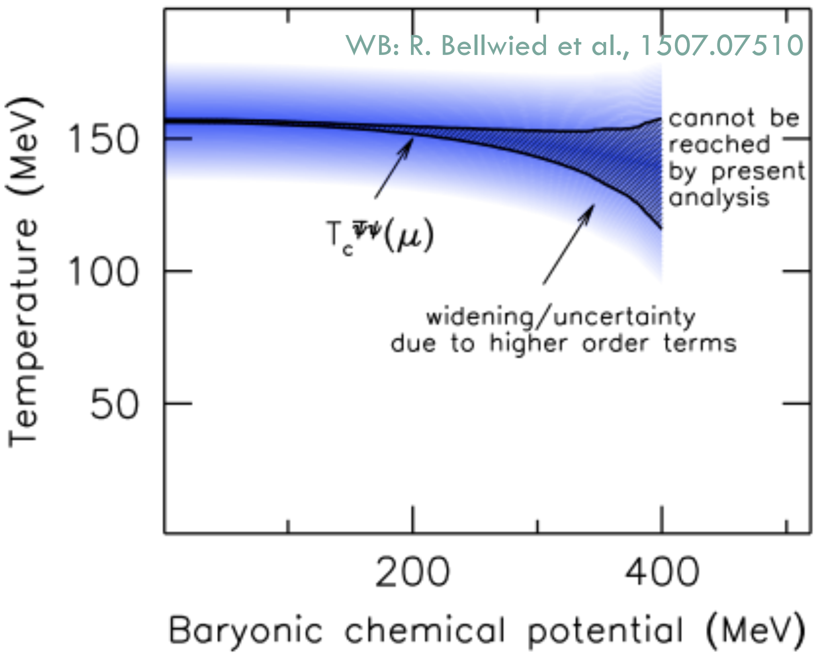
- Extract the isentropic trajectory that the system follows in the absence of dissipation
- Calculate the EoS along these constant S/N trajectories

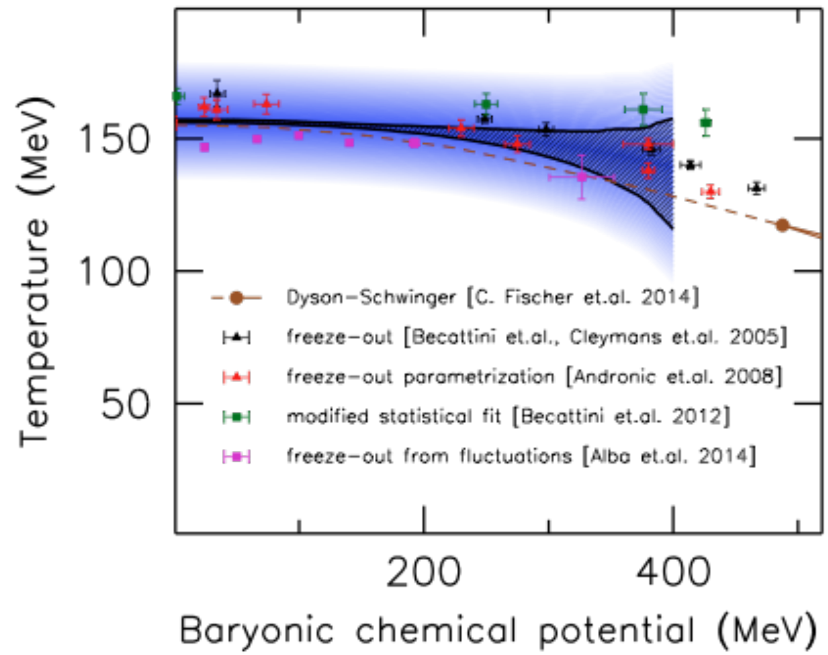




WB: S. Borsanyi et al., premilinary

QCD phase diagram





Curvature κ defined as:

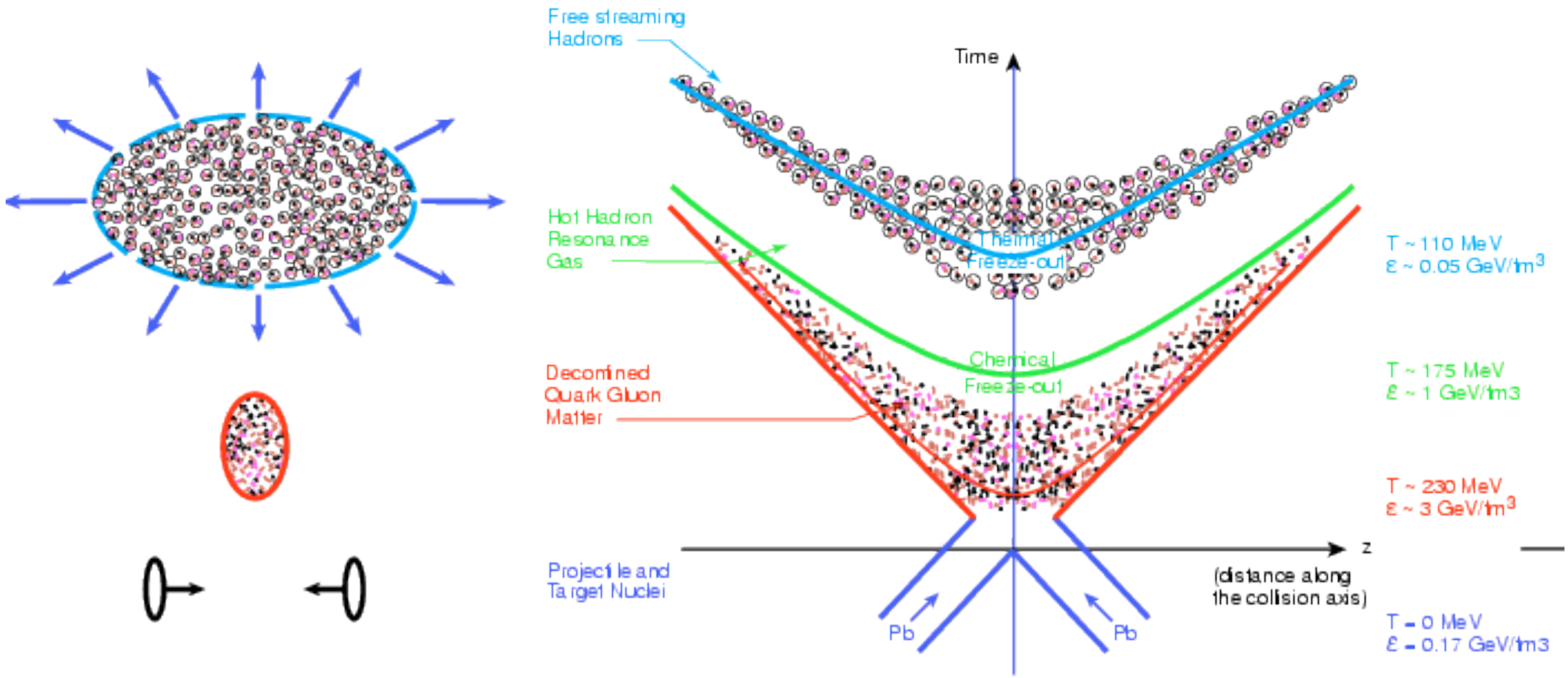
$$\frac{T_c(\mu_B)}{T_c(\mu=0)} = 1 - \kappa\left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + \lambda\left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 \ldots$$

Recent results:

$$\kappa = 0.0149 \pm 0.0021$$

WB: R. Bellwied et al., 1507.07510

Evolution of a Heavy Ion Collision

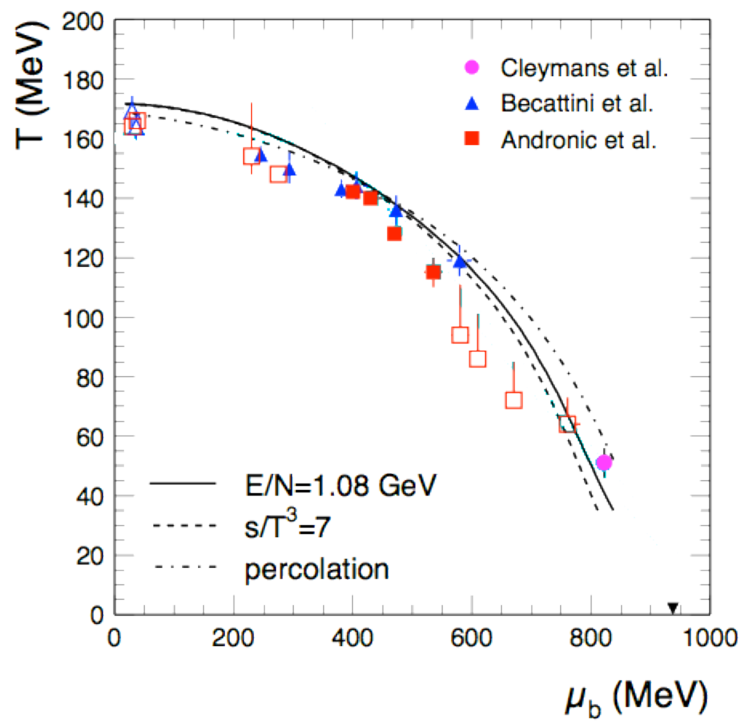


- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

The thermal fits

See talk by A. Andronic on Wednesday Morning

- $E=mc^2$: lots of particles are created
- Particle counting (average over many events)
- Take into account:
  - detector inefficiency
  - missing particles at low p_T
  - decays



- HRG model: test hypothesis of hadron abundancies in equilibrium

$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}$$

A. Andronic, P. Braun-Munzinger, J. Stachel; J. Cleymans; F. Becattini

Fluctuations of conserved charges

- Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\begin{aligned}
\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\
\mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\
\mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.
\end{aligned}$$

- They can be calculated on the lattice and compared to experiment

Connection to experiment

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

- Lattice QCD results are functions of temperature and chemical potential
  - By comparing lattice results and experimental measurement we can extract the freeze-out parameters from first principles

F. Karsch: Centr. Eur. J. Phys. (2012)

“Baryometer and Thermometer”

Let us look at the Taylor expansion of R^B_{31}

$$R_{31}^B(T,\mu_B) = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = \frac{\chi_4^B(T,0) + \chi_{31}^{BQ}(T,0)q_1(T) + \chi_{31}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)} + \mathcal{O}(\mu_B^2)$$

- To order μ^2_B it is independent of μ_B: it can be used as a thermometer
- Let us look at the Taylor expansion of R^B_{12}

$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

- Once we extract T from R^B_{31}, we can use R^B_{12} to extract μ_B

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

V. Skokov et al., PRC (2013)

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

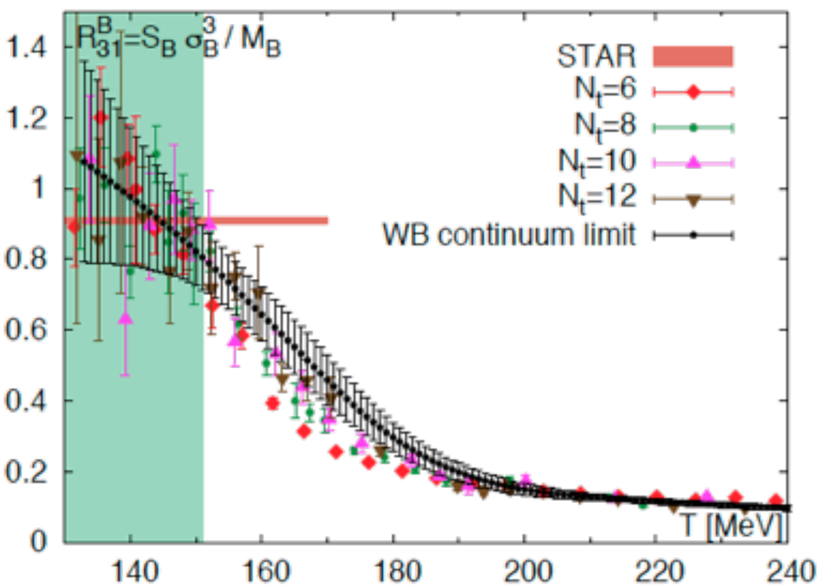
M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

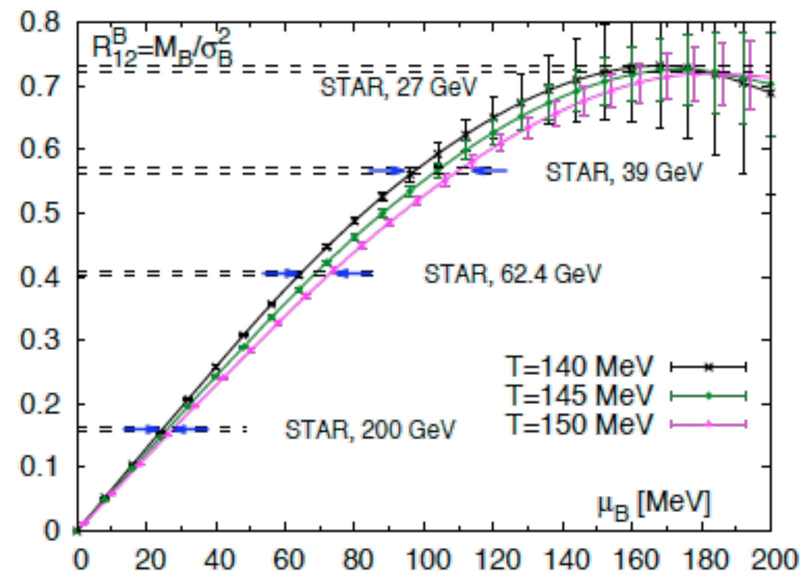
- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

J.Steinheimer et al., PRL (2013)

Freeze-out parameters from B fluctuations

- Thermometer: $\frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}$ $=S_B\sigma_B^3/M_B$ Baryometer: $\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}$ $=\sigma_B^2/M_B$



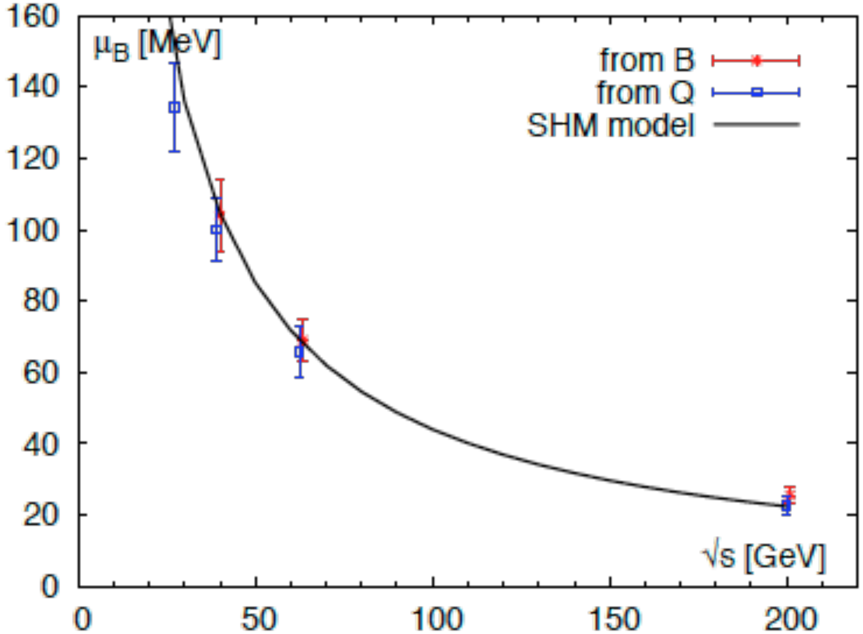


WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

- Upper limit: $T_f \leq 151 \pm 4$ MeV
- Consistency between freeze-out chemical potential from electric charge and baryon number is found.

Freeze-out parameters from B fluctuations

- Thermometer: $\frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}$ $=S_B\sigma_B^3/M_B$ Baryometer: $\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}$ $=\sigma_B^2/M_B$



$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

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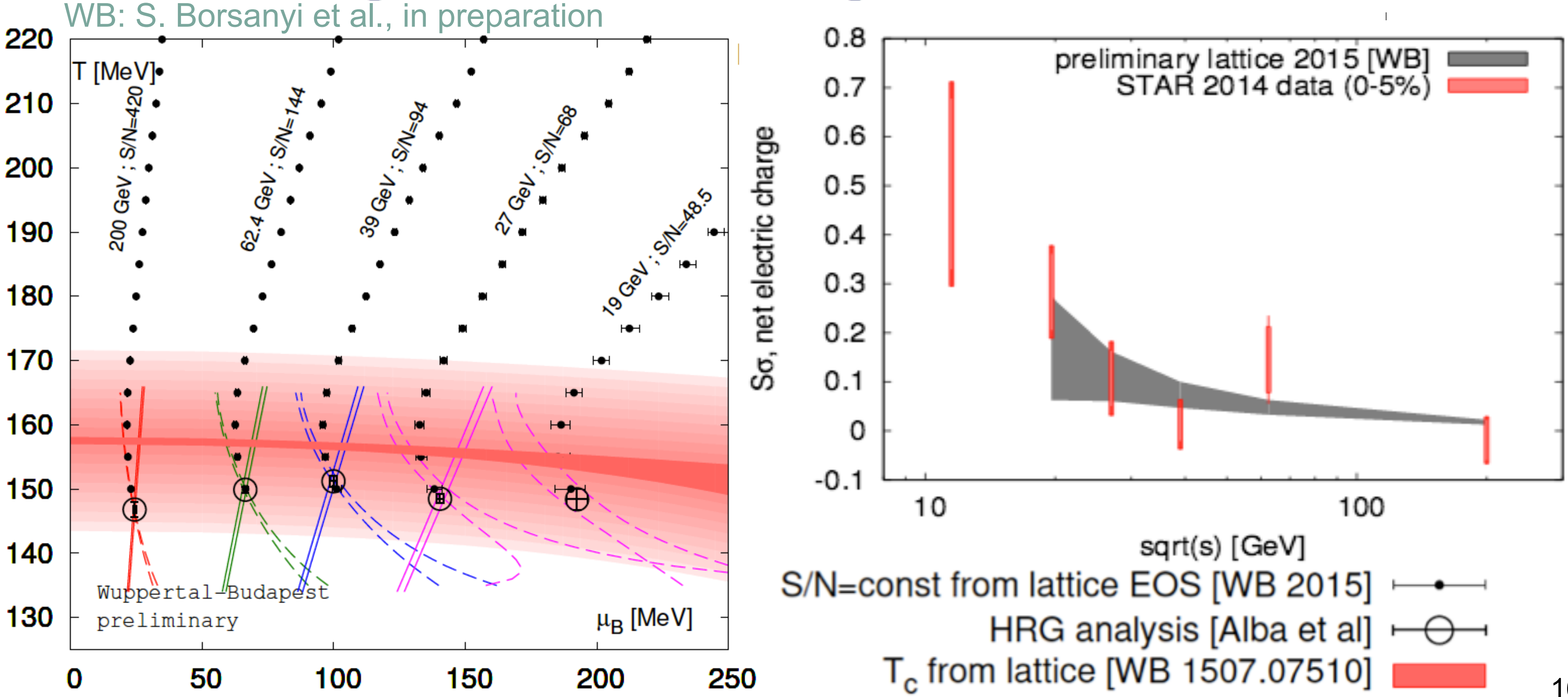
Freeze-out line from first principles

- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

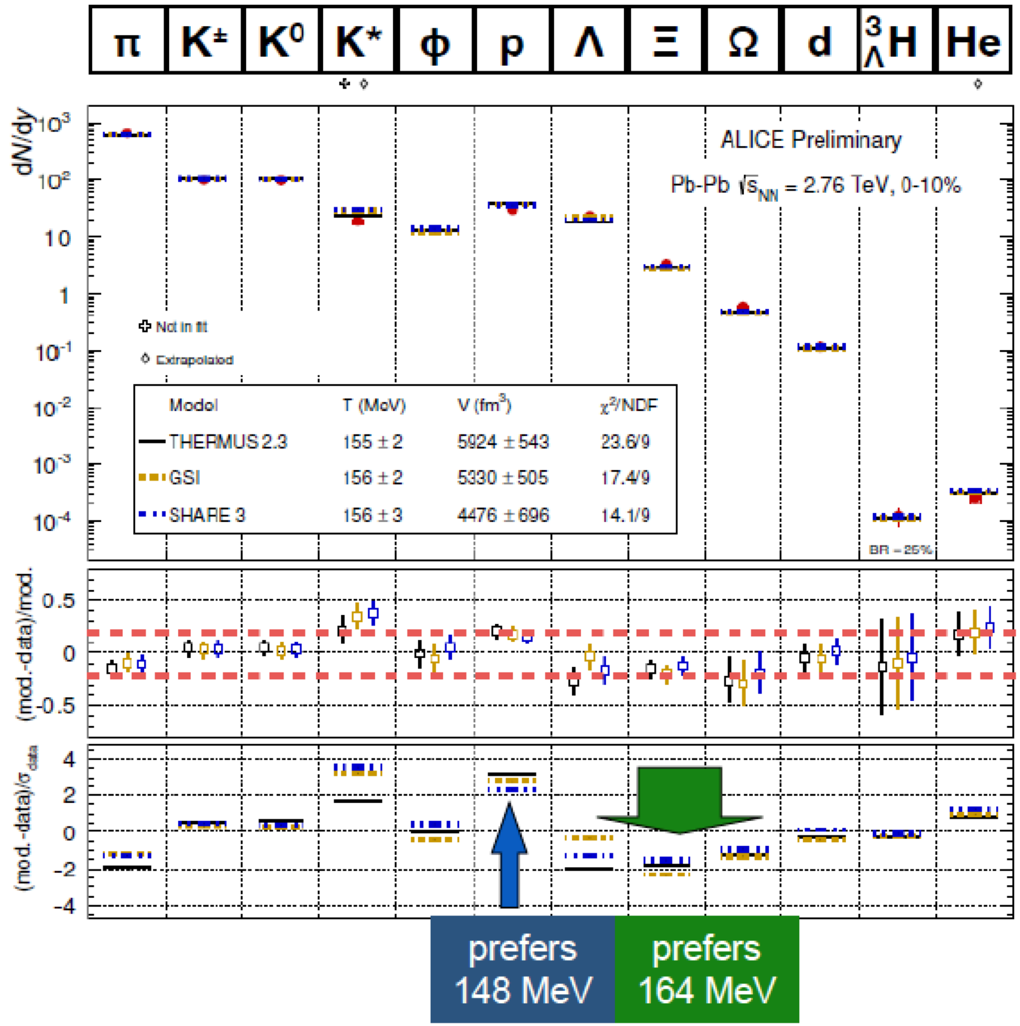
$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

WB: S. Borsanyi et al., in preparation



What about strangeness freeze-out?

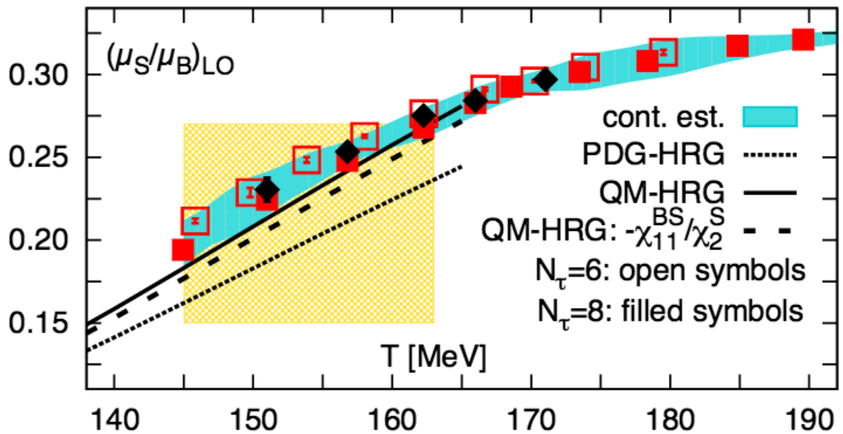
- Yield fits seem to hint at a higher temperature for strange particles

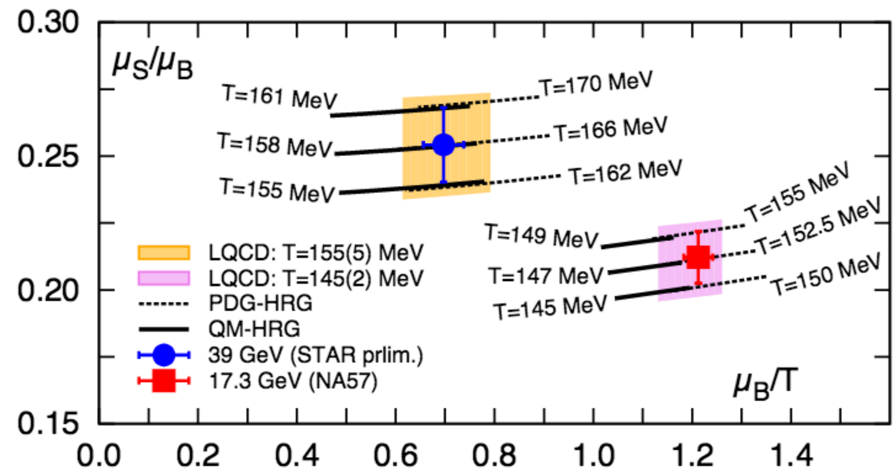


M. Floris: QM 2014

Missing strange states?

- Quark Model predicts not-yet-detected (multi-)strange hadrons





- QM-HRG improves the agreement with lattice results for the baryon-strangeness correlator:

$$(\mu_S/\mu_B)_{LO}=-\chi_{11}^{BS}/\chi_2^S+\chi_{11}^{QS}\mu_Q/\mu_B$$

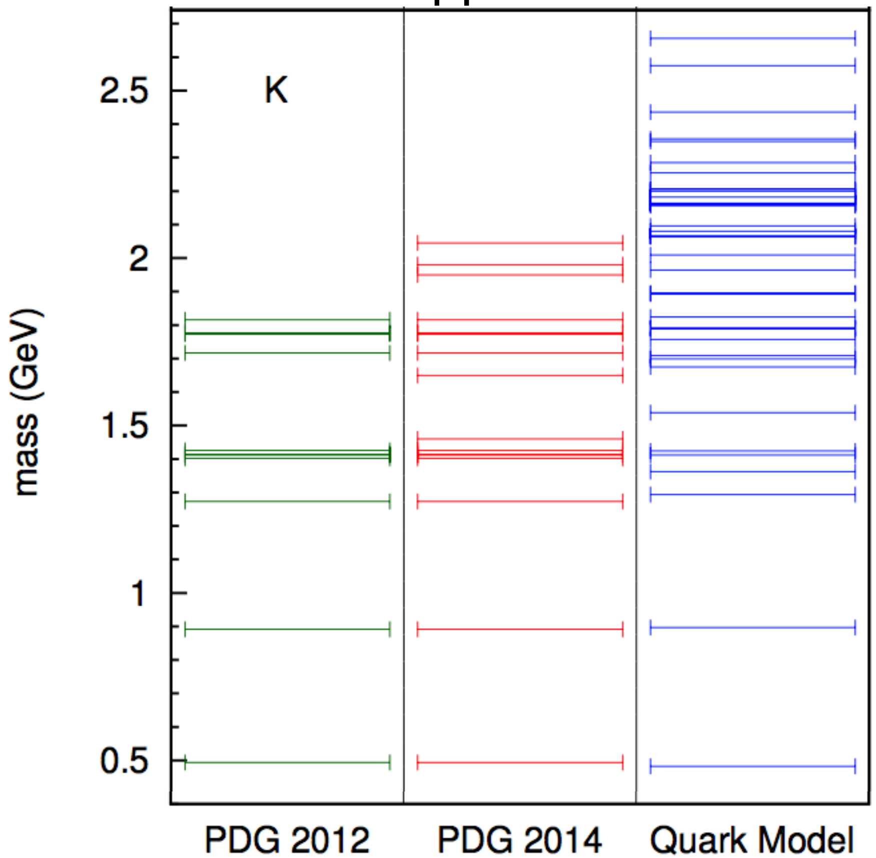
- The effect is only relevant at finite μ_B

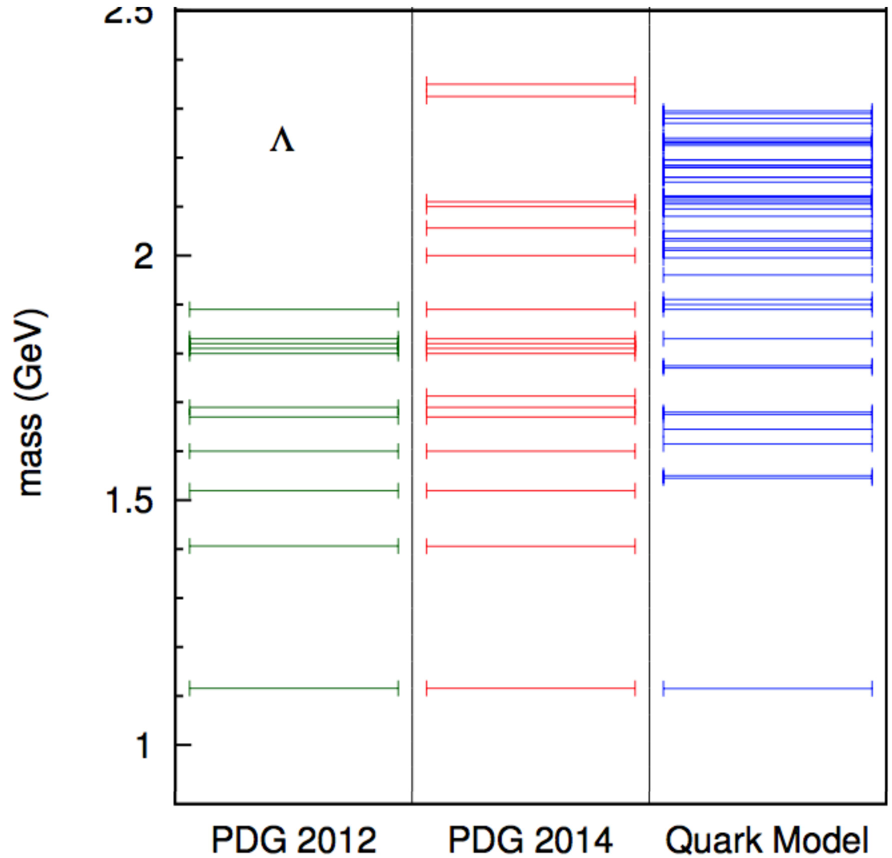
- Feed-down from resonance decays not included

A. Bazavov et al., PRL (2014)

Missing strange states?

- New states appear in the 2014 version of the PDG





WB collaboration, in preparation

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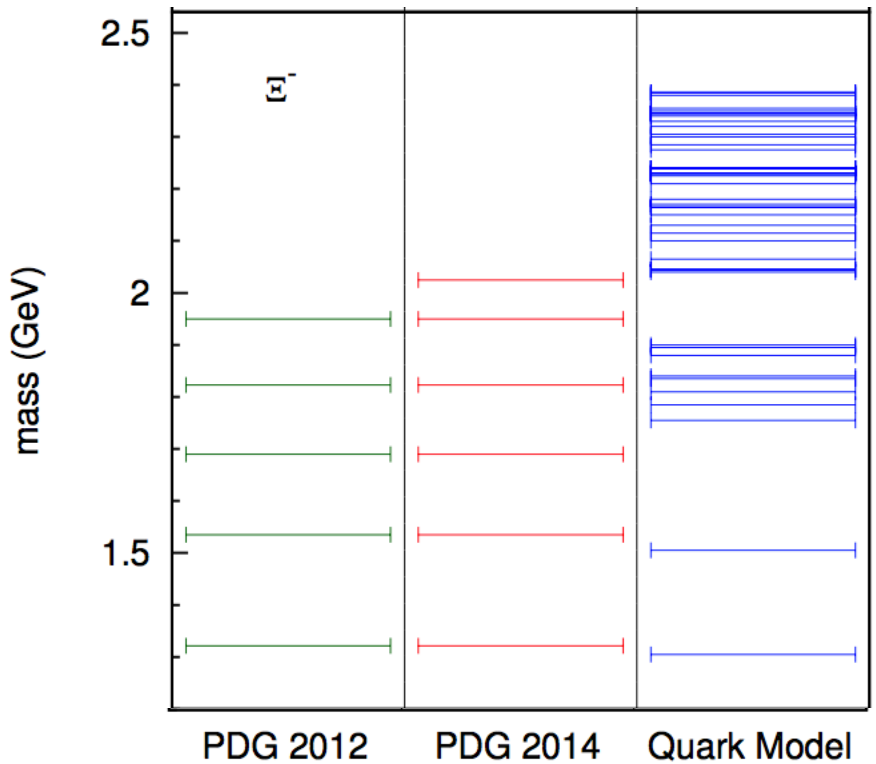
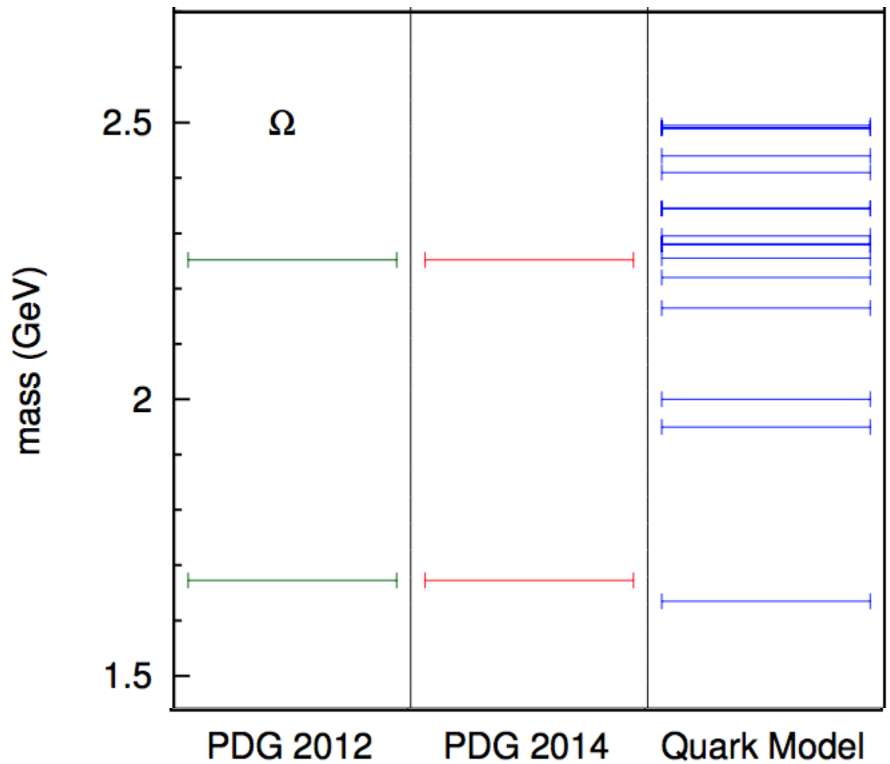
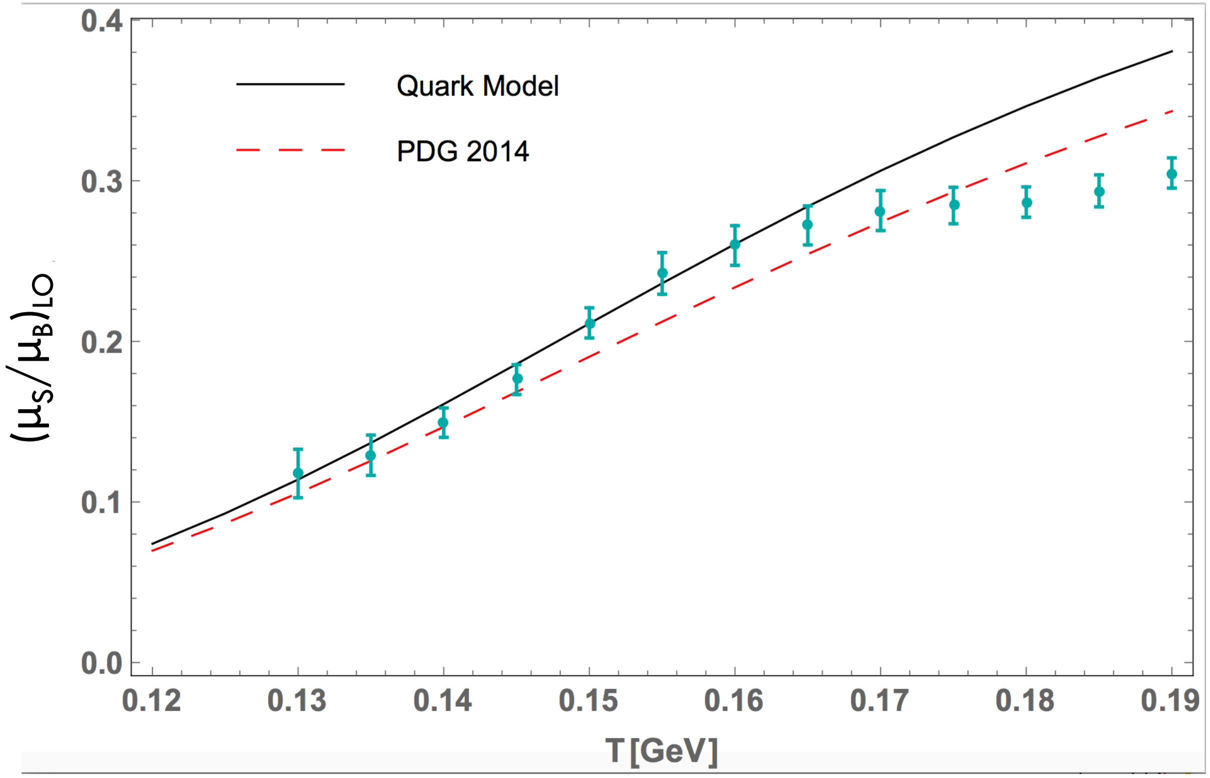




WB collaboration, in preparation

Missing strange states?

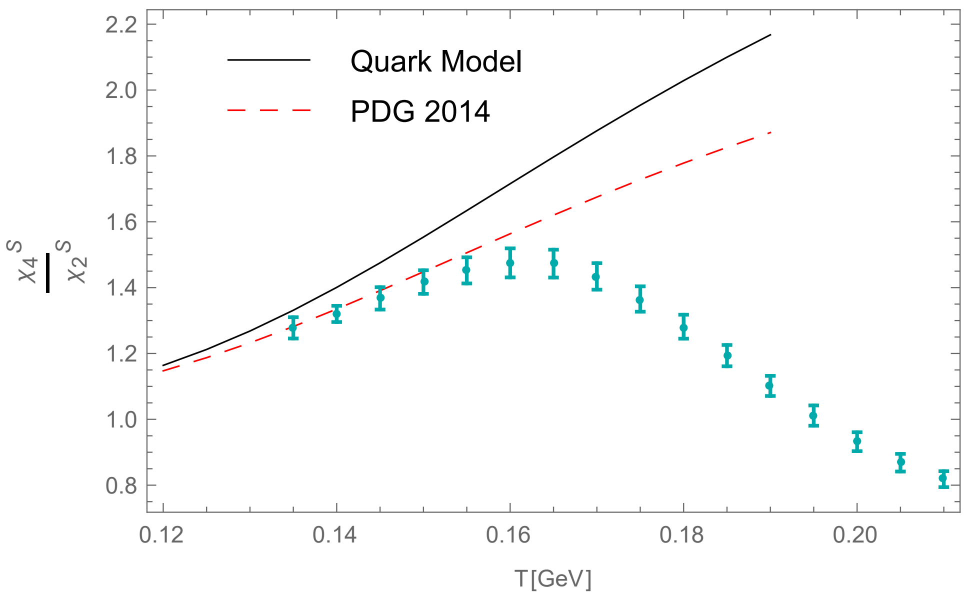
- The comparison with the lattice is improved for the baryon-strangeness correlator:

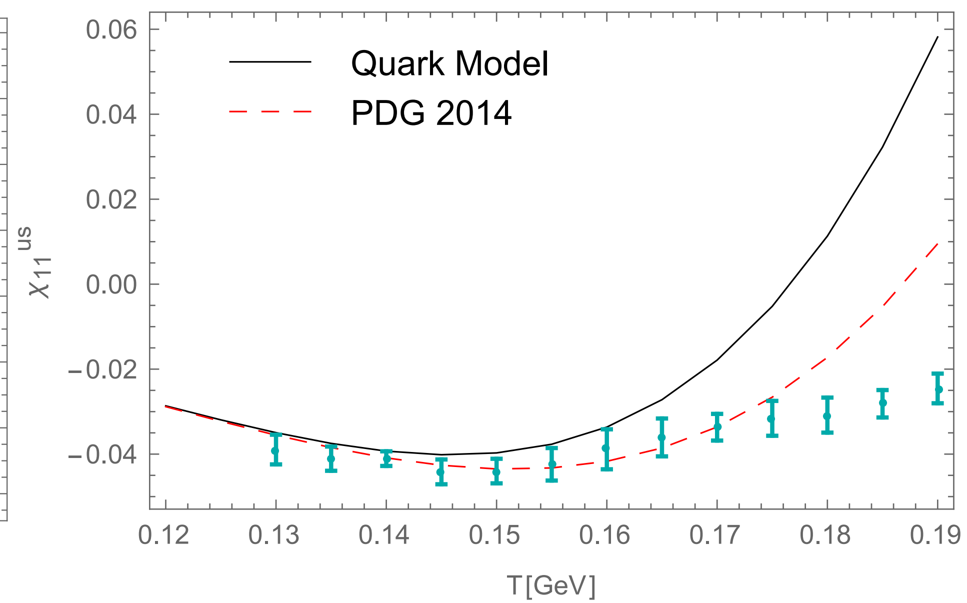


WB collaboration, in preparation

Missing strange states?

- Some observables are in agreement with the PDG 2014 but not with the Quark Model:





- χ_4^S/χ_2^S is proportional to <S²> in the system
- It seems to indicate that the quark model predicts too many multi-strange states

WB collaboration, in preparation

Missing strange states?

See Talk by J. Noronha-Hostler on Thursday afternoon

- Idea: define linear combinations of correlators which receive contributions only from particles with a given quantum number

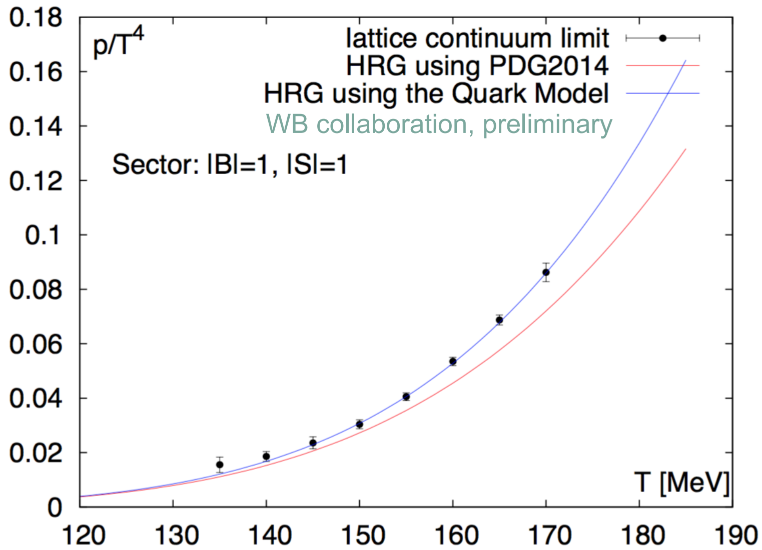
- They allow to compare PDG and QM prediction for each sector separately

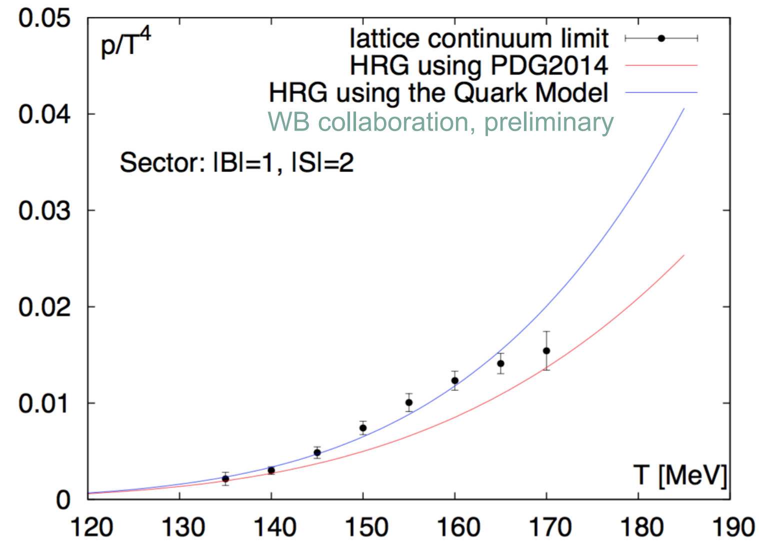
$$\begin{aligned} P_S(\hat{\mu}_B,\hat{\mu}_S) &= P_{0|1|}\cosh(\hat{\mu}_S) \\ &+ P_{1|1|}\cosh(\hat{\mu}_B-\hat{\mu}_S) \\ &+ P_{1|2|}\cosh(\hat{\mu}_B-2\hat{\mu}_S) \\ &+ P_{1|3|}\cosh(\hat{\mu}_B-3\hat{\mu}_S) \end{aligned}$$

$$\begin{aligned} P_{0|1|} &= \chi_2^S-\chi_{22}^{BS} \\ P_{1|1|} &= \frac{1}{2}\left(\chi_4^S-\chi_2^S+5\chi_{13}^{BS}+7\chi_{22}^{BS}\right) \\ P_{1|2|} &= -\frac{1}{4}\left(\chi_4^S-\chi_2^S+4\chi_{13}^{BS}+4\chi_{22}^{BS}\right) \\ P_{1|3|} &= \frac{1}{18}\left(\chi_4^S-\chi_2^S+3\chi_{13}^{BS}+3\chi_{22}^{BS}\right) \end{aligned}$$

A. Bazavov et al., PRL (2013)

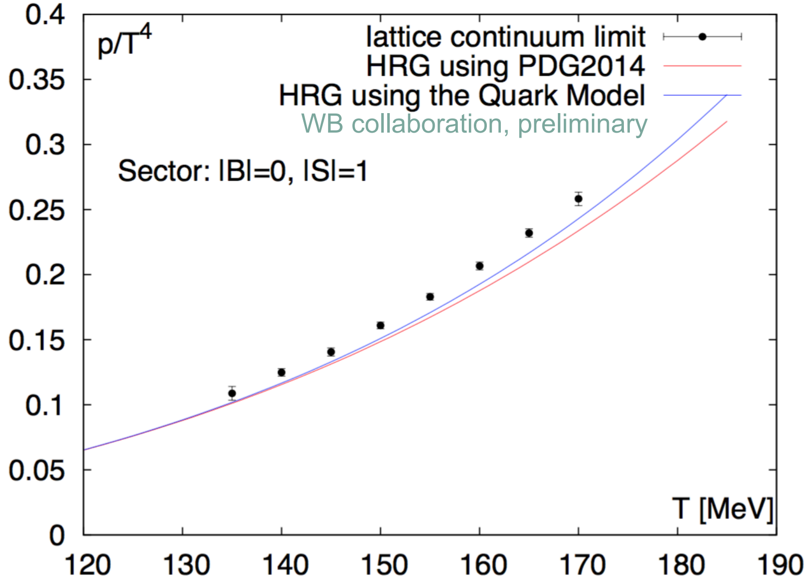
Missing strange states?

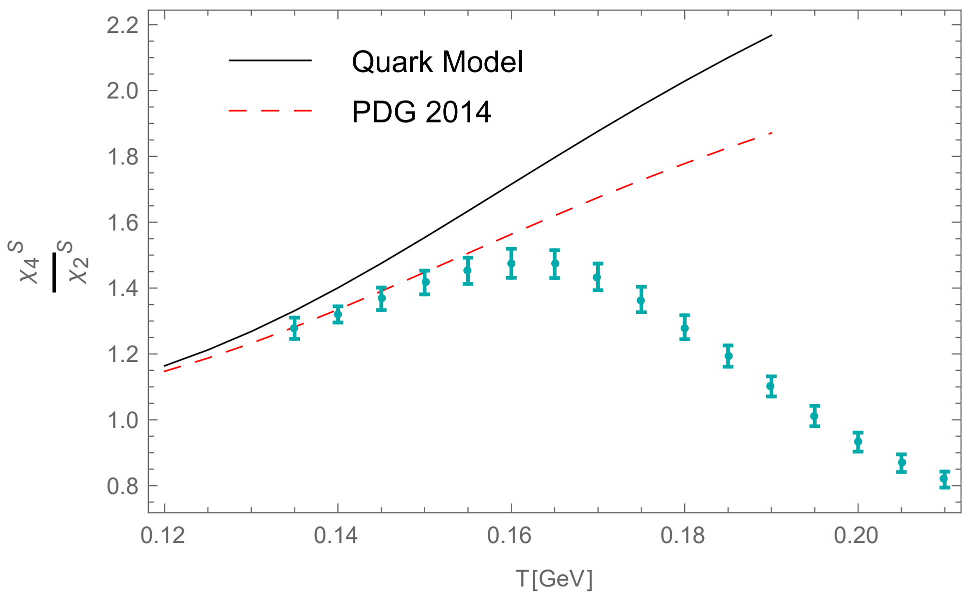




- The precision in the lattice results can allow to distinguish between the two scenarios
- Quark model yields better agreement with the data for the strange baryons

Not enough strange mesons





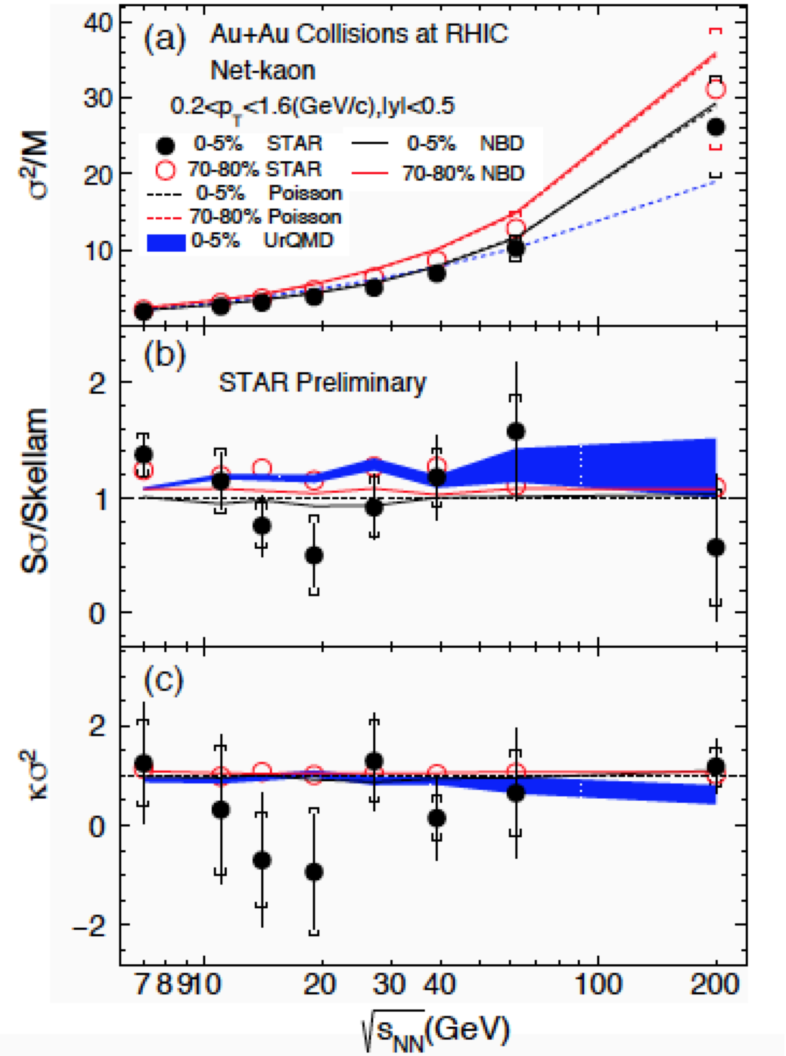
- Both Quark Model and PDG 2014 underestimate the partial pressure due to strange mesons
- This explains why the QM overestimates χ_4^S/χ_2^S: more strange mesons would bring the curve down

P.M. Lo, K. Redlich, C. Sasaki, PRC (2015)

Kaon fluctuations

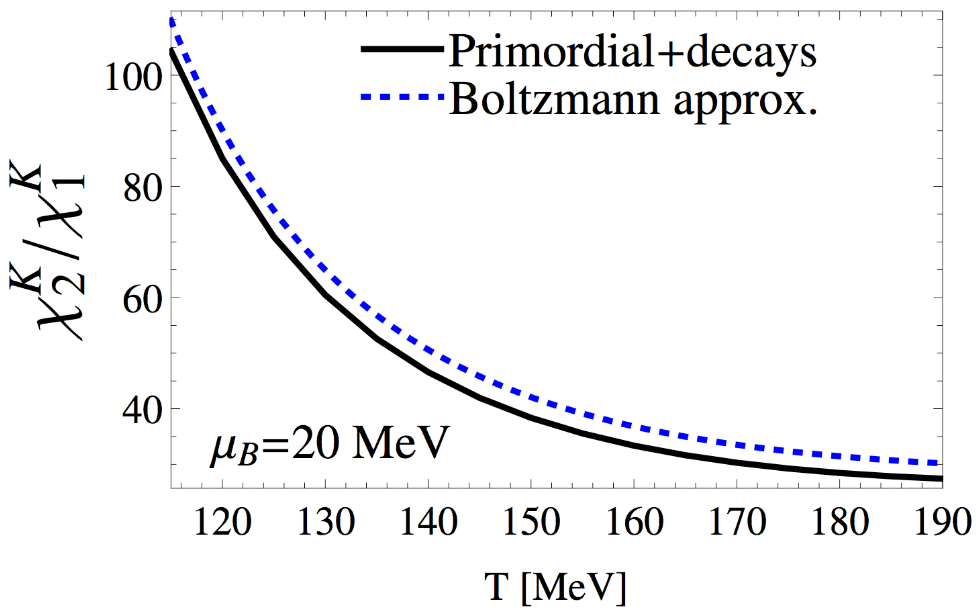
Talk by Ji XU on Tuesday

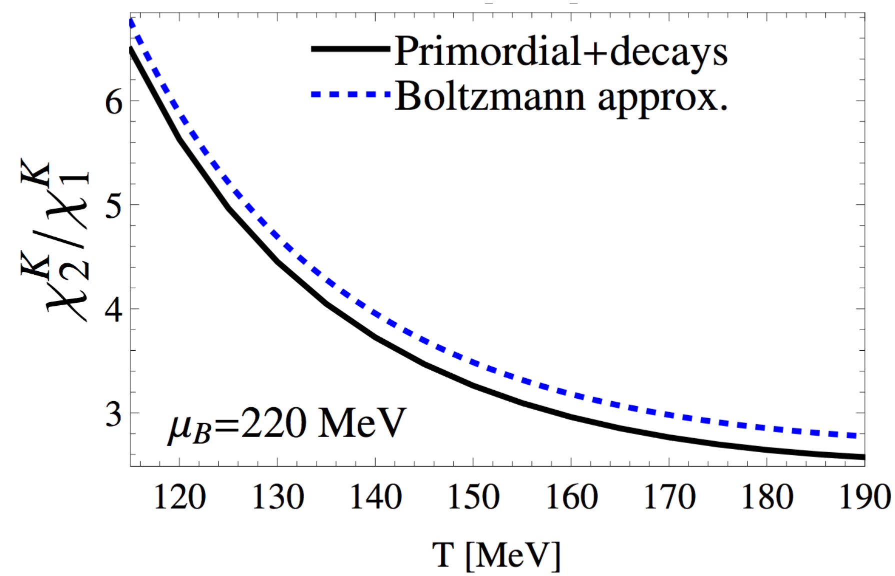
- Experimental data are becoming available.
- Exciting result but presently hampered by systematic errors
- BES-II will help
- Kaon fluctuations from HRG model will be affected by the hadronic spectrum and decays



Kaon fluctuations on the lattice?

J. Noronha-Hostler, C.R. et al. (2016)





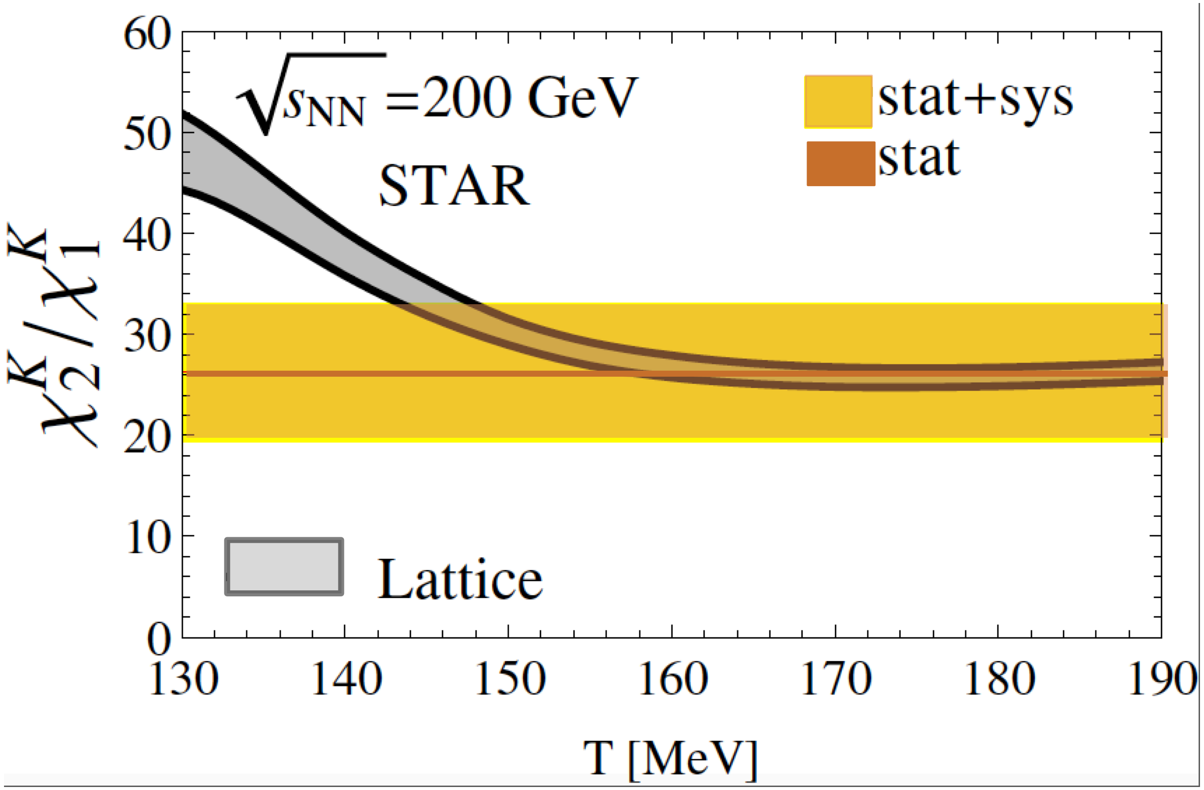
- Boltzmann approximation works well for lower order kaon fluctuations

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

- χ_2^K/χ_1^K from primordial kaons + decays is very close to the one in the Boltzmann approximation

Kaon fluctuations on the lattice?

J. Noronha-Hostler, C.R. et al. (2016)

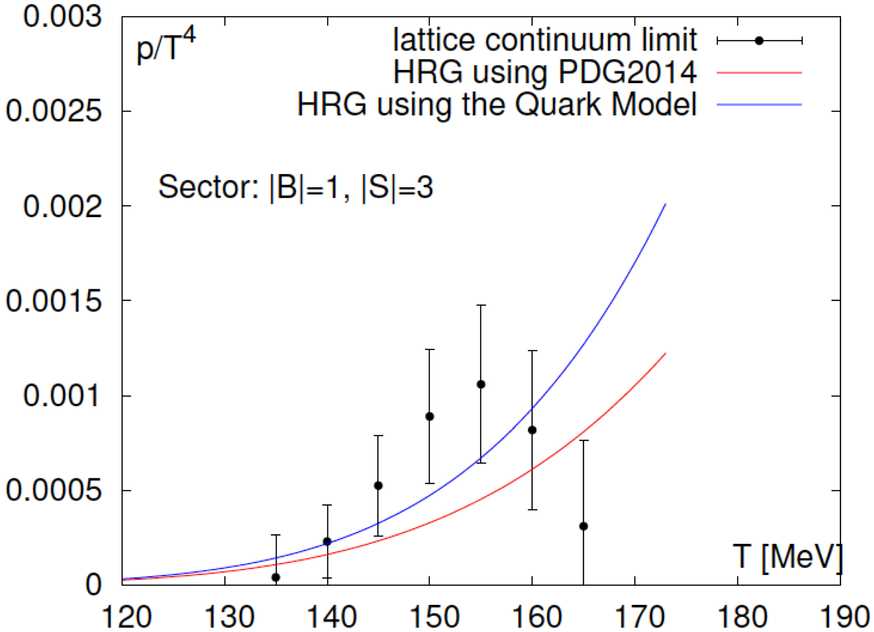


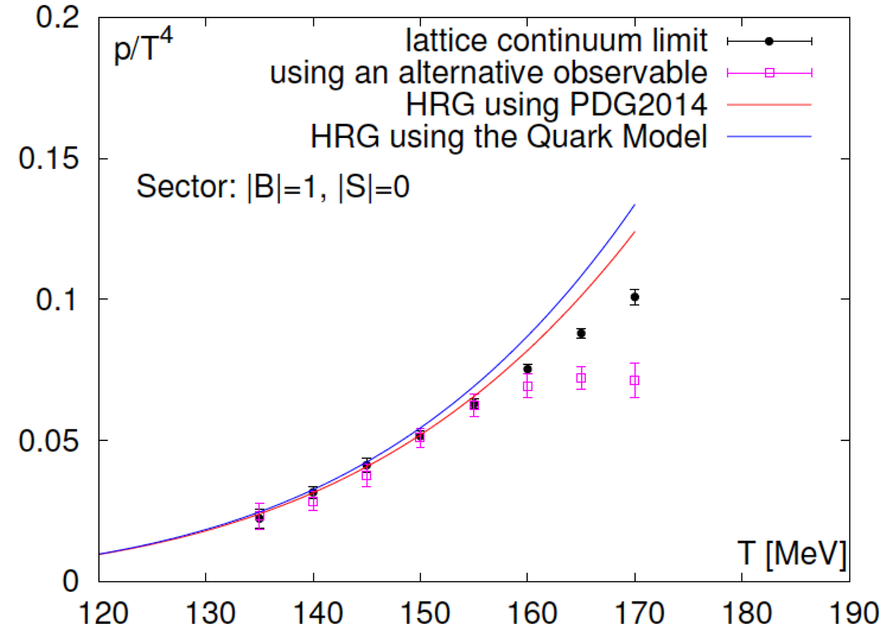
- Experimental uncertainty does not allow a precise determination of T_f^K

Conclusions

- Unprecedented precision in lattice QCD data allows a direct comparison to experiment for the first time
- QCD thermodynamics at μ_B=0 can be simulated with high accuracy
- Extensions to finite density are under control up to $O(\mu_B^6)$
- Quark Model states are needed for all particle families
- Effect of decays on freeze-out parameters from yields under investigation
- Kaon T_f can be determined from lattice QCD

Missing strange states?

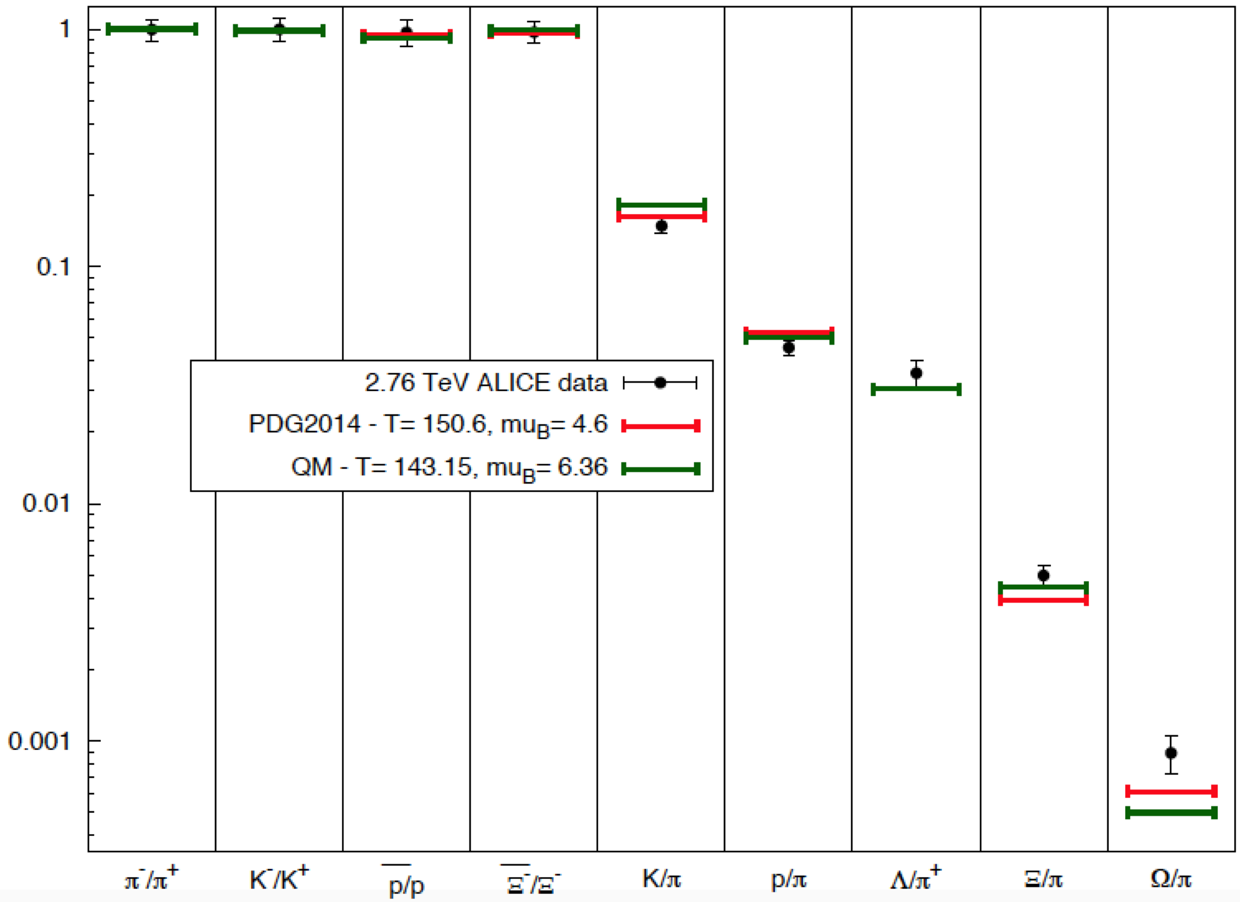




WB collaboration, in preparation

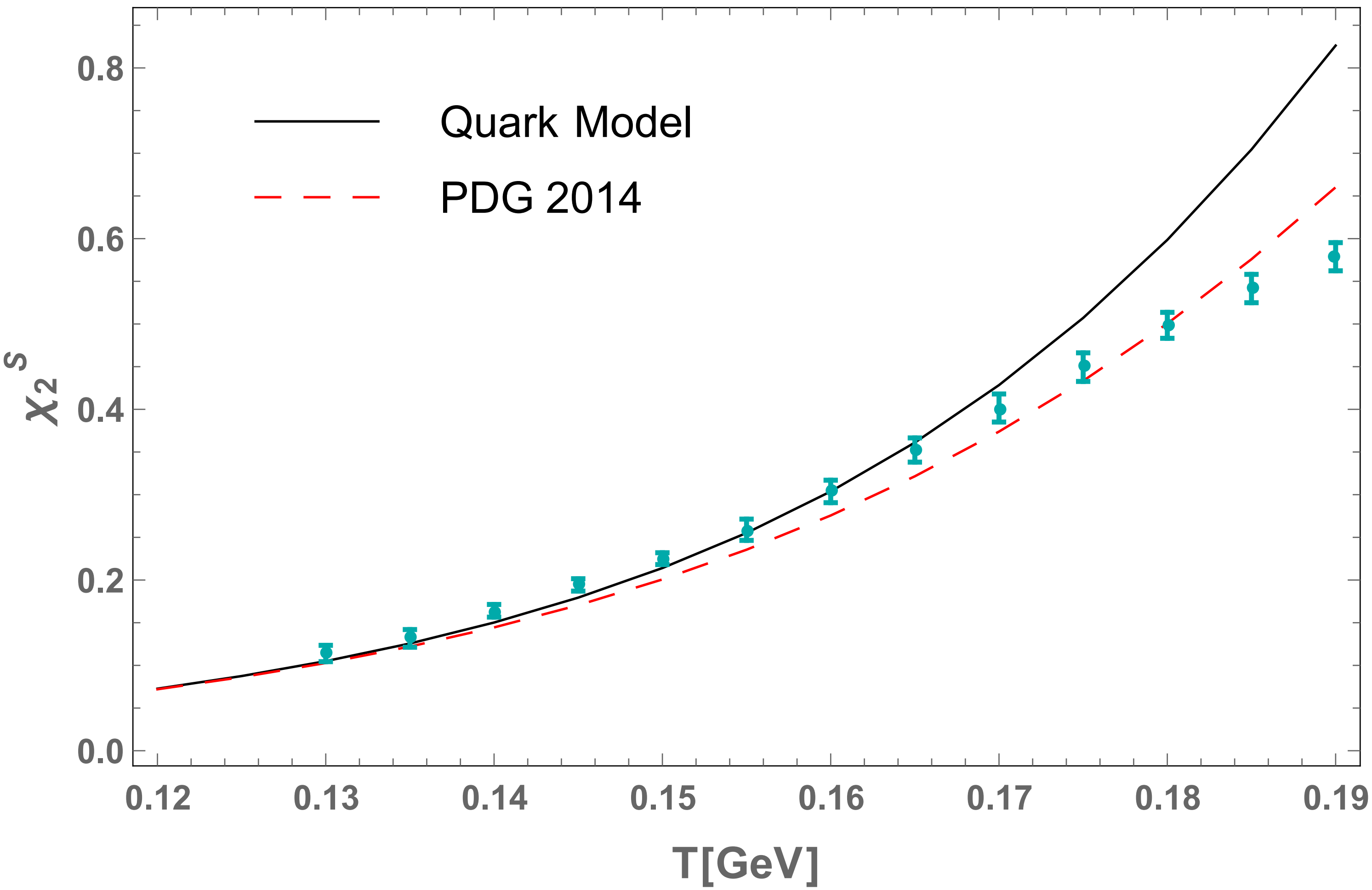
Effect of resonance decays

- The decays have a big effect on the freeze-out parameters



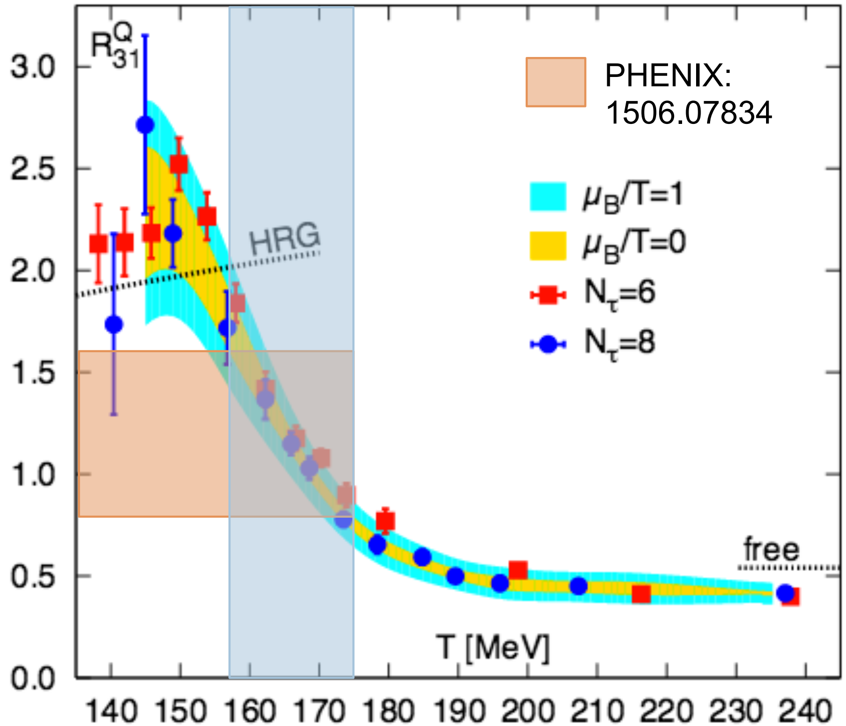
P. Alba et al., in preparation

Missing strange states?

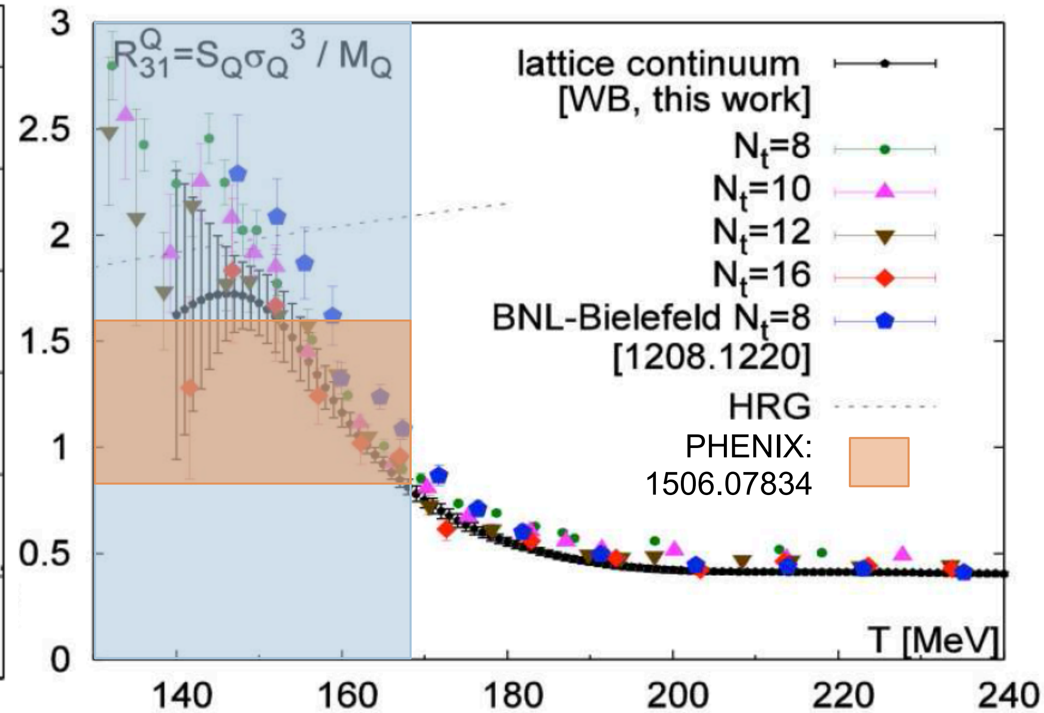


WB collaboration, in preparation

Freeze-out parameters from Q fluctuations



A. Bazavov et al. (2014)



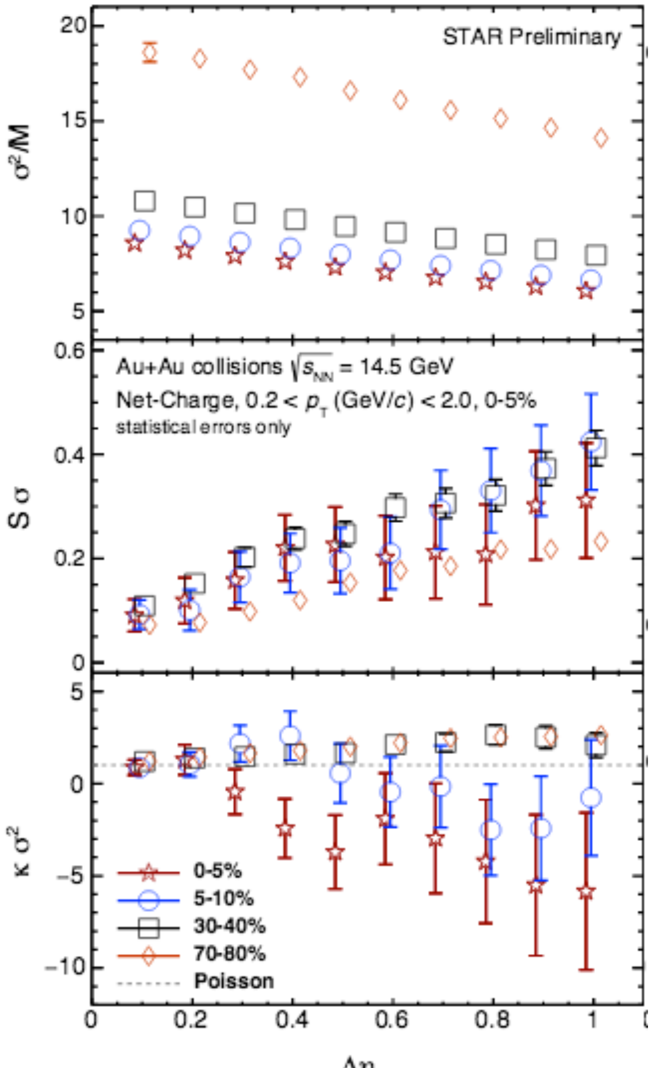
WB: Borsanyi et al. PRL (2013)

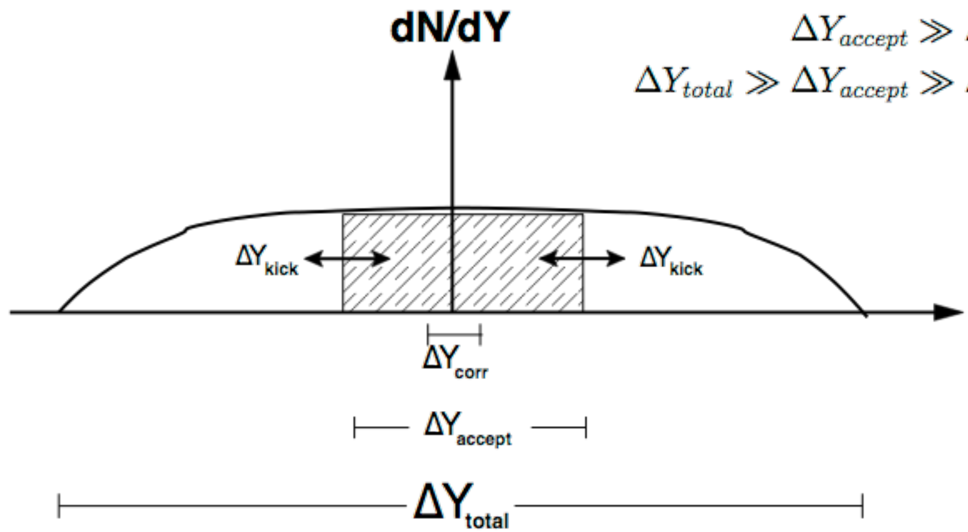
- Studies in HRG model: the different momentum cuts between STAR and PHENIX are responsible for more than 30% of their difference

F. Karsch et al., 1508.02614

- Using continuum extrapolated lattice data, lower values for T_f are found

Effects of kinematic cuts





$$\Delta Y_{accept} \gg \Delta Y_{corr}$$
$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

V. Koch, 0810.2520

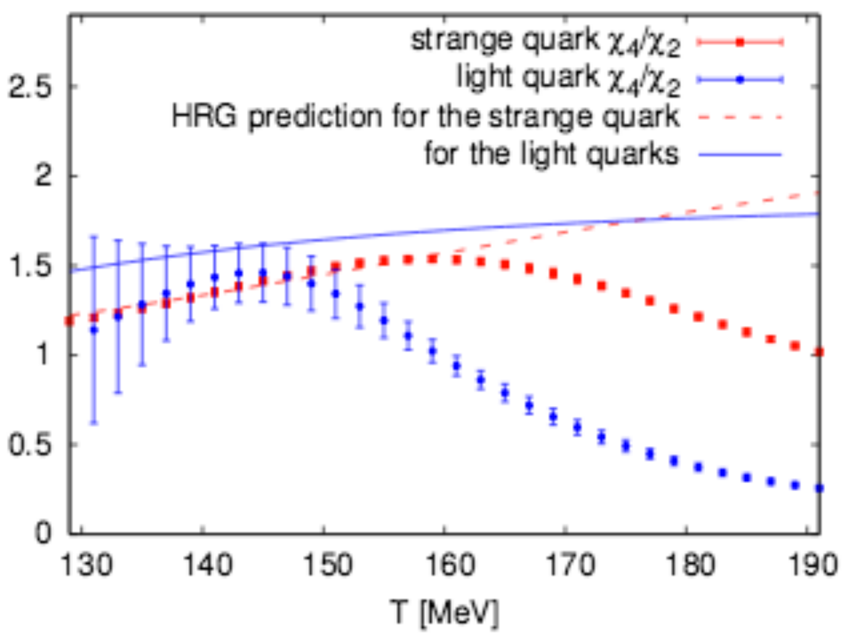
- Rapidity dependence of moments needs to be studied for 1<Δη<2
- Difference in kinematic cuts between STAR and PHENIX leads to a 5% difference in T_f

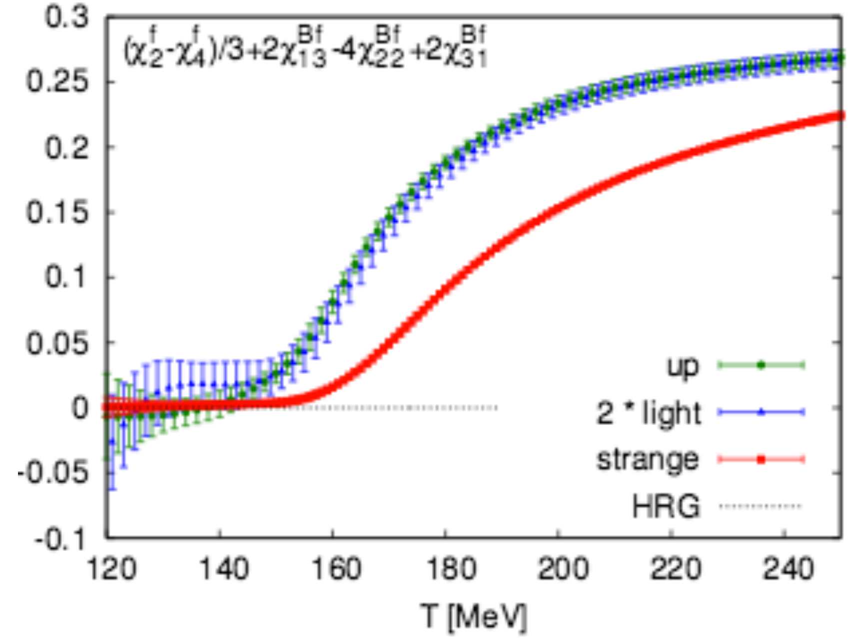
Talk by J. Thaeder on Monday

Talk by F. Karsch on Monday

Strangeness fluctuations

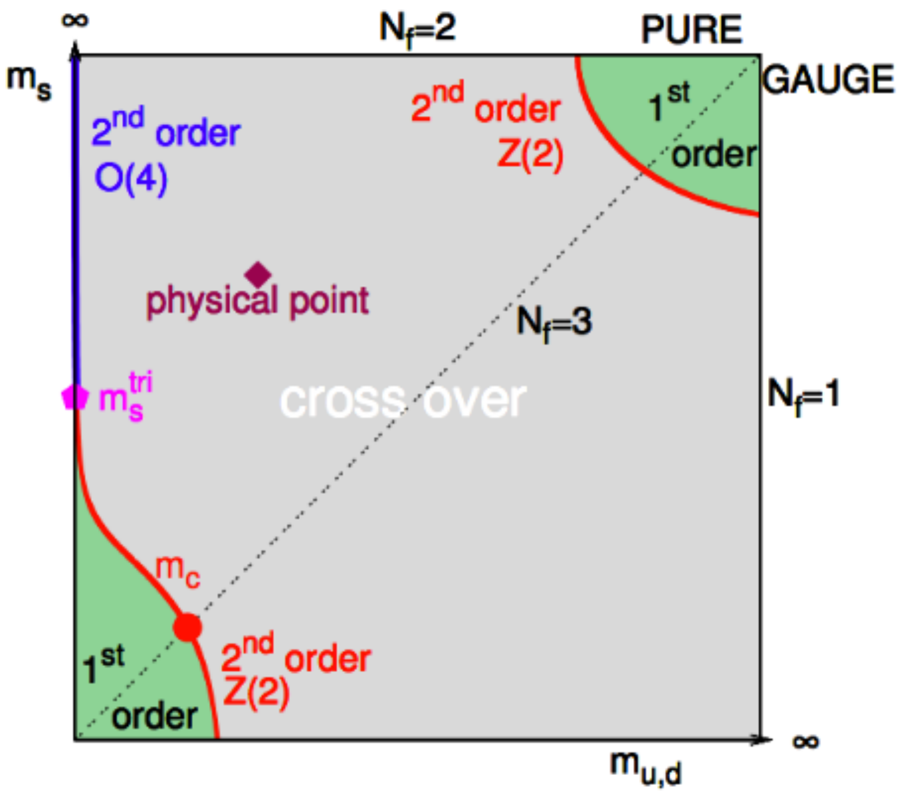
WB: R. Bellwied et al, PRL (2013)





- Lattice data hint at possible flavor-dependence in transition temperature
- Possibility of strange bound-states above T_c?

Columbia plot



- Pure gauge theory: Tc=294(2) MeV
  Francis et al., 1503.05652
- N_f=2 QCD at $m_\pi > m_\pi^{phys}$:
  - O(a) improved Wilson, N_t=16
    - m_π=295 MeV T_c=211(5) MeV
    - m_π=220 MeV T_c=193(7) MeV

    Brandt et al., 1310.8326
  - Twisted-mass QCD

    - m_π=333 MeV T_c=180(12) MeV

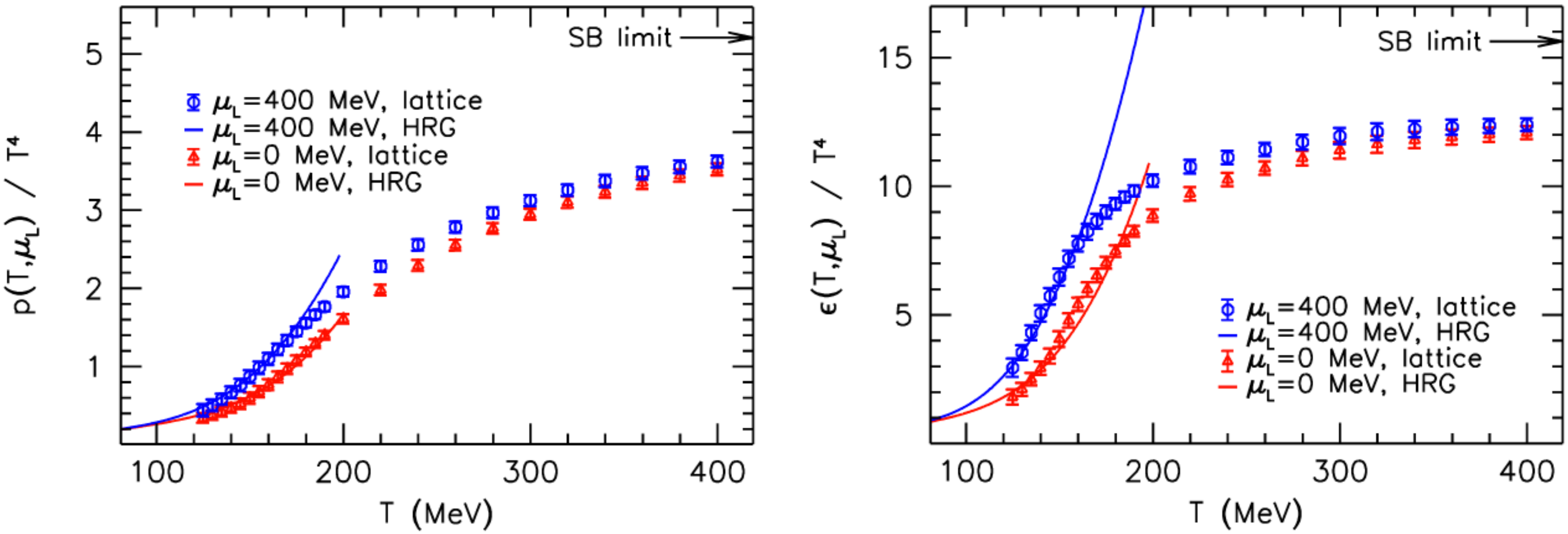
    Burger et al., 1412.6748
- N_f=2+1 O(a) improved Wilson
  - Continuum results

    Borsanyi et al., 1504.03676
- HISQ action, N_t=6, no sign of 1^{st} order phase transition at m_π=80 MeV
  HotQCD, 1312.0119, 1302.5740

Equation of state at $\mu_B>0$

- Expand the pressure in powers of μ_B (or $\mu_L=3/2(\mu_u+\mu_d)$)

$$\frac{p(T,\{\mu_i\})}{T^4} = \frac{p(T,\{0\})}{T^4} + \frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij} \quad \text{with} \quad \chi_2^{ij} \equiv \frac{T}{V}\frac{1}{T^2}\left.\frac{\partial^2 \log \mathcal{Z}}{\partial\mu_i\partial\mu_j}\right|_{\mu_i=\mu_j=0}$$

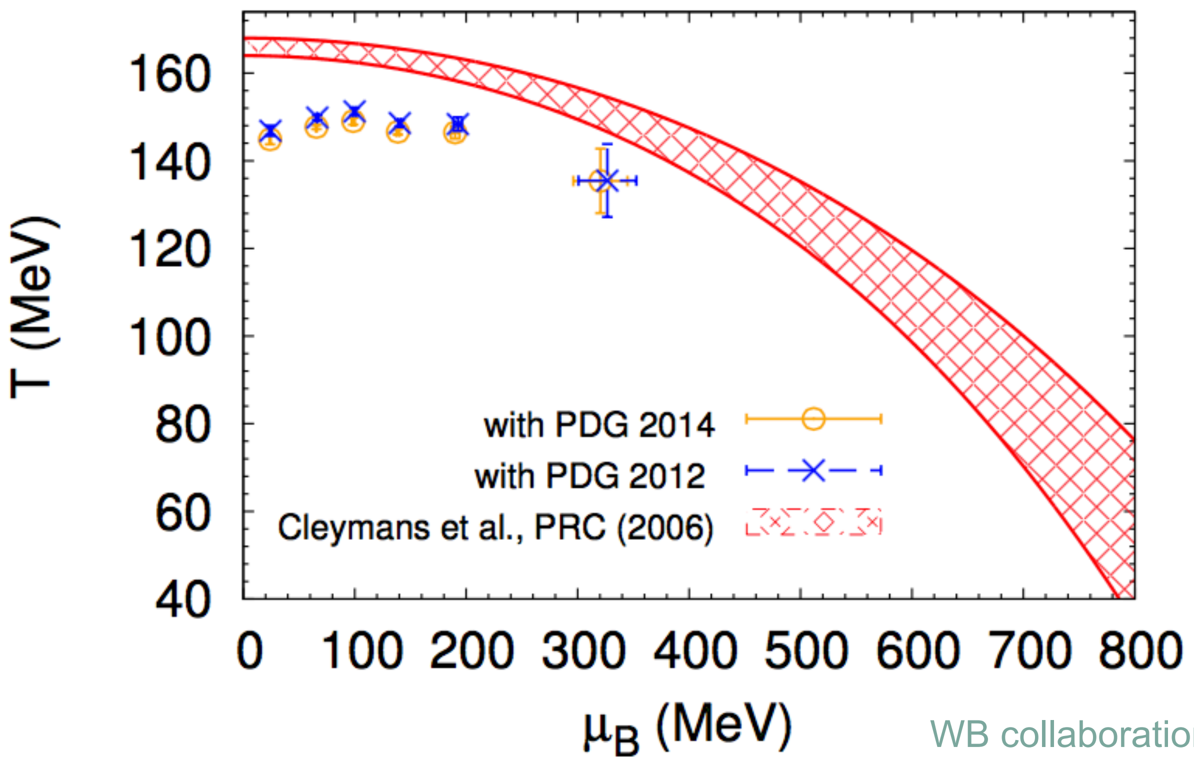


S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass

Effect of resonance decays

- We used the PDG2014 to estimate the effect of resonance decays on the fit to proton and charge fluctuations
- The results agree with the ones obtained with the PDG2012 within errorbars



WB collaboration, in preparation