An Application of Functional Renormalization Group Method for Superdense Nuclear Matter

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Abstract. We proposed a method, using the expansion of the effective potential in a base of harmonic functions, to study the Functional Renormalization Group (FRG) method at finite chemical potential. Within this theoretical framework we determined the equation of state and the phase diagram of a simple model of massless fermions coupled to scalars through Yukawa-couling at the zero-temperature limit. Here, we use our FRG-based equation of state to describe the superdense nuclear matter inside compact astrophysical objects. We calculated the mass-radius relation for a compact star using the TOV equation, which was compared to other results.

1. Introduction

Compact astrophysical objects, such as neutron-, quark-, or hybrid stars are the most extreme, high-density creatures of the present-day Universe. It is a challenging task to explore the inner structure of these *compact stars* due to the lack of direct probes or measurements of their interior. Recent spectroscopic radius measurements using X-ray data analysis [1] and even the gravitational-wave discoveries [2, 3] may provide such constraints, which led us to develop more reliable equation of state (EoS) of the superdense matter.

Modeling the high-density nuclear matter and providing its equation of state is still an research field. On the other hand the success of the above task is shaded by the *masquerade problem*, since different complex and sophisticated EoS result similar behavior and observables of the compact celestial bodies [4]. This motivates us not only to provide perfect EoS, but describe the phase structure of the cold and high-density nuclear matter [5].

The calculation of the equation of state in the high-density and zero temperature limit is usually considered in the mean-field or one-loop approximation. Functional Renormalization Group (FRG) method can extend this description in an exact way, taking into account the effect of quantum fluctuations in the effective action of the system.

In this work we use the Wetterich-equation to compute the EoS and Litim's regulator is applied regulating the scale dependence [6]. The Local Potential Approximation (LPA) is used to obtain the EoS for the *ansatz* containing a Yukawa-type interaction as described in Ref. [7]. We present there the calculated EoS which has a Maxwell-construction as an inner nature.

The calculated EoS is tested by solving the corresponding Tolman – Oppenheimer – Volkov (TOV) equations and investigating the properties by the mass-radius relation, $M(R)$ of compact stars. Comparison of the FRG-based equation of state to other high-density zero-temperature nuclear matter EoS and to the calculated $M(R)$ by various models are given.

2. The FRG Method for a Yukawa-type Model

The functional renormalization group method is a general way to find the effective action of a system. This formalism led us to calculate low-energy effective (observable) quantities by gradual momentum integration of a theory defined at some high-energy scale, k. Since low-scale effective quantities incorporate quantum fluctuations, using FRG at finite temperature one may calculate the equation of state of the system including the quantum fluctuations as well.

Within the FRG framework the quantum n-point correlation function is calculated by the gradual path integration. This can be achieved by introducing a regulator term, R_k in the generator functional, $Z_k[J]$, which acts as a mass term and suppresses modes below scale, k as explained in Refs. [8, 9]. Thanks to this regulator term, the effective action becomes scale-dependent, which scale dependence is given by the Wetterich-equation [10]

$$\partial_k \Gamma_k = \frac{1}{2} \int \mathrm{d}p^D \,\mathrm{STr} \left[(\partial_k R_k) \left(\Gamma_k^{(2)} + R_k \right)^{-1} \right], \tag{1}$$

where $\Gamma_k^{(2)}$ is the second derivative matrix of the effective action. The term 'STr' stands for the normal *trace* operation but includes a negative sign for fermionic fields and sums over all indices. The low-scale (observable) effective action is computed by integrating the Wetterich-equation (1), from the classical limit at some UV-scale $k = \Lambda$ to the IR-scale $k = 0$, where quantum effects are taken into account. The initial condition in this integration is the UV-scale (classical) action $\Gamma_{k=\Lambda}$, which has to be chosen in a way, that the low-scale effective action reproduces physical quantities, correctly.

Here, we use a simple Yukawa-type model with one bosonic and one fermionic degree of freedom described by the bare action. This is described by the bare action at scale Λ,

$$\Gamma_\Lambda[\varphi, \psi] = \int \mathrm{d}^4x \left[\bar{\psi}(i\partial\!\!\!/ - g_0\varphi)\psi + \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m_0^2}{2}\varphi^2 - \frac{\lambda_0}{24}\varphi^4 \right]. \tag{2}$$

As we described in Ref. [7] this model has two phases: (i) in the symmetric phase the fermion is massless, (ii) in the Spontaneous Symmetry Breaking (SSB) phase the fermion mass is $g\langle\varphi\rangle$.

To treat this model with the FRG method we need an *ansatz* for the effective action at scale k. As a consequence of LPA, we choose the simplest possible one, where only the bosonic effective potential depends on the scale:

$$\Gamma_k[\varphi, \psi] = \int \mathrm{d}^4x \left[\bar{\psi}(i\partial\!\!\!/ - g\varphi)\psi + \frac{1}{2}(\partial_\mu \varphi)^2 - U_k(\varphi) \right]. \tag{3}$$

Note, neither wave function renormalization, nor the running of the Higgs coupling are taken into account, however, both effects can be easily adapted into the present method.

The integrated Wetterich-equation (1) for this model can be rewritten after applying the three-dimensional Litim regulator at finite temperature T and at finite chemical potential μ,

$$\partial_k U_k = \frac{1}{2}\,\mathrm{STr}\ln\left[R_k + \Gamma_k^{(2)}\right] = \frac{k^4}{12\pi^2}\left[\frac{1+2n_B(\omega_B)}{\omega_B} + 4\frac{-1+n_F(\omega_F-\mu)+n_F(\omega_F+\mu)}{\omega_F}\right], \tag{4}$$

where n_B and n_F are the Bose – Einstein and the Fermi – Dirac distributions, respectively

$$n_{B/F}(\omega) = \frac{1}{1 \mp e^{-\beta\omega_{B/F}}}, \text{ while } \omega_B^2 = k^2 + \partial_\varphi^2 U, \;\; \omega_F^2 = k^2 + g^2\varphi^2, \text{ and } \beta = \frac{1}{T}. \tag{5}$$

In the case of $T = 0$ and $\mu > 0$ the Bose – Einstein distribution does not give contribution, but the Fermi – Dirac distribution reduces $n_F(\omega) \to \Theta(-\omega)$ and this simplifies equation (4) to:

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - 4\frac{\Theta(\omega_F - \mu)}{\omega_F}\right] \quad \text{with initial condition} \quad U_\Lambda(\varphi) = \frac{m_0^2}{2}\varphi^2 + \frac{\lambda_0}{24}\varphi^4. \tag{6}$$

The presence of the step function, $\Theta(\omega)$, generates two different domains, where two different differential equations evolve the potential in k. The boundary of these domains is called Fermi-surface S_F, which can be determined from requiring $\omega_F(k,\varphi)|_{S_F} = \mu$. The surface can be characterized either by $k = k_F(\varphi)$ or by $\varphi = \varphi_F(k)$ conditions. In our case these read as

$$k_F = \sqrt{\mu^2 - g^2\varphi^2} \qquad \text{and} \qquad \varphi_F = g^{-1}\sqrt{\mu^2 - k^2}. \tag{7}$$

The surface S_F, in terms of k and $g\varphi$, is a circle with radius μ, and for $\mu = 0$ it disappears. The Fermi-surface divides the coordinate space into two parts; we will denote the high energy regime by $\mathcal{D}_>$, the low energy regime by $\mathcal{D}_<$, where the following differential equations hold:

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - \frac{4}{\omega_F}\right], \qquad \text{if } (k,\varphi) \in \mathcal{D}_> := \left\{(k,\varphi)\,|\,k^2 + g^2\varphi^2 > \mu^2\right\}, \tag{8a}$$

$$\partial_k U_k = \frac{k^4}{12\pi^2}\frac{1}{\omega_B}, \qquad \text{if } (k,\varphi) \in \mathcal{D}_< := \left\{(k,\varphi)\,|\,k^2 + g^2\varphi^2 < \mu^2\right\}. \tag{8b}$$

and the solution is continuous at $k = k_F$.

For the solution we need to use standard FRG techniques, e.g. with discretization or with polynom expansion. The value of the potential at the boundary can be determined by cutting out the Fermi-surface from the zero chemical potential solution. In Ref. [7] we introduced a coordinate transformation which maps the circle-like Fermi-surface to a rectangular one, while it keeps the symmetries of the differential equations. Applying this circle-to-rectangle transformation and a harmonic expansion the Wetterich-equation can be solved numerically.

Since exact solution can be given in the mean field approximation, we took $v = f_\pi$, $g = m_N/v$, and $\lambda = 3m_\sigma^2/v^2$ with the values $m_N = m_\sigma = 0.938$ GeV and $f_\pi = 0.093$ GeV. We choose the chemical potential, μ_{MF}, close to the value of the first-order phase transition $\mu_{MF} \approx 0.6177 m_N$.

3. Results: Equation of State and Mass-radius Relation of Compact Stars

The potential solution provided by the FRG-method was found to converge quickly, where the potential is convex. On contrary it converges slowly at the concave part and contains a native Maxwell-construction, too. Comparison of the FRG-based EoS results to the mean-field and one-loop approximation is plotted on Fig. 1 as $p(\mu)$ in the zero temperature limit and at finite chemical potential. One can see, that FRG-based EoS is the softest, followed by the stiffer one-loop and finally the mean field approximation result.

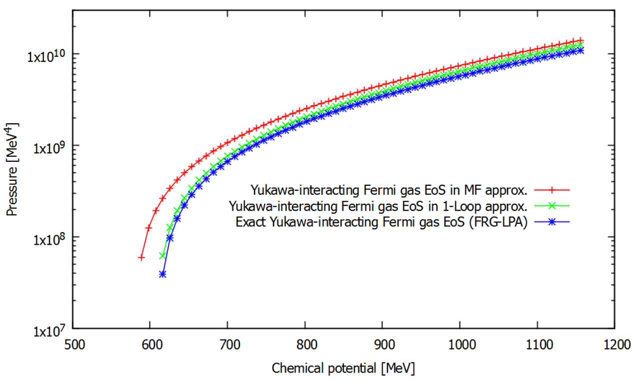

Figure 1. Equation of state $p(\mu)$, calculated from the functional renormalization group method.

We compared our FRG-based EoS results to some other EoS taken from Ref. [1], which are typically used in compact star models. On the *left panel* of Fig. 2, we found, our EoS calculations is the closest to the 'SQM3' model predictions at the highest, but differs at lower energy-density values. *Right panel* of Fig. 2 presents the calculated mass-radius diagram, $M(R)$ based on the Tolman – Oppenheimer – Volkov (TOV) equation combined together with the calculated FRG-based EoS. All these were drawn together with other model predictions from Ref. [1]. However these simple FRG-based EoS predict smaller but consistent compact stars with $M < 1.5M_{\odot}$ and $R < 8$ km, by this way we could successfully validate our concept described in Ref. [7].

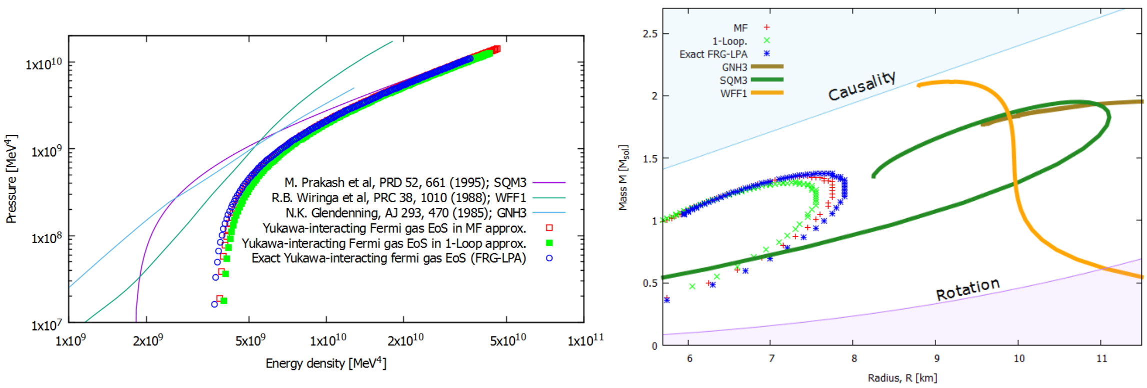

Figure 2. Comparison of different Equation of States and the corresponding $M(R)$ diagram for compact stars. Points were calculated by our method [7] and lines are from Ref. [1].

4. Summary

Using the Wetterich-equation the zero-temperature and high-density cold nuclear matter EoS was calculated in the FRG-framework based on the method and concept described in Ref. [7]. The calculated equation of state and the $M(R)$ diagram of compact stars were found to be consistent with the ones listed in Ref. [1], however with smaller stellar mass $M < 1.5M_{\odot}$ and radius $R < 8$ km.

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