The fluctuations of quadrangular flow

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Abstract. The ATLAS Collaboration has measured for the first time the fourth cumulant of quadrangular flow, $v_4\{4\}^4$. Unlike the fourth cumulants of elliptic and triangular flows, it presents a change of sign above 30% centrality. We show that this change of sign is predicted by event-by-event hydrodynamics. We argue that it results from the combined effects of a nonlinear hydrodynamic response, which couples quadrangular flow to elliptic flow, and elliptic flow fluctuations.

1. Introduction

The bulk of particle production in ultrarelativistic nucleus-nucleus collisions is described by the flow paradigm [1], which states that particles are emitted independently from an underlying probability distribution. In particular, the flow paradigm naturally explains the long-range azimuthal correlations, which are a salient feature of heavy-ion collisions, as resulting from the fluctuations of the underlying azimuthal probability distribution $P(\varphi)$ [2]. This azimuthal distribution is traditionally written as a Fourier series:

$$P(\varphi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} V_n \mathrm{e}^{-in\varphi}, \tag{1}$$

where $V_n = v_n \exp(in\Psi_n)$ is the (complex) anisotropic flow coefficient in the nth harmonic and $V_{-n} = V_n^*$. Both the magnitude and phase of V_n fluctuate event to event [3]. Experimental observables involving anisotropic flow can be recast as statistical properties of the distribution of V_n. For instance, the cumulants $v_n\{2\}^2$ and $v_n\{4\}^4$ are defined by [4]:

$$\begin{aligned} v_n\{2\}^2 &\equiv \langle v_n^2 \rangle, \\ v_n\{4\}^4 &\equiv 2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle, \end{aligned} \tag{2}$$

where angular brackets denote an average over events in a centrality class. The lowest order cumulant, $v_n\{2\}^2$, is simply the mean square value of v_n, while the fourth cumulant, $v_n\{4\}^4$, is a nontrivial combination of moments. In particular, despite the unfortunate notation, $v_n\{4\}^4$ can be either positive or negative, depending on the probability distribution of v_n. Both $v_2\{4\}^4$ [5] and $v_3\{4\}^4$ [6] are observed to be positive across all centralities in Pb+Pb collisions at the LHC. However, a striking observation, which seems to have received little attention from the theory community, is that $v_4\{4\}^4$ changes sign [7]: it is positive only up to $\sim 30\%$ centrality.

In this article, we show that the change of sign of $v_4\{4\}^4$ is predicted by hydrodynamics. In hydrodynamics, the fluctuations of the anisotropic flow coefficients, V_n, result from the fluctuations of the energy density profile released after the collisions [8]. v_2 and v_3 are determined to a good approximation [9, 10] by linear response to the initial anisotropies in the corresponding harmonics. This, in turn, explains why both $v_2\{4\}^4$ and $v_3\{4\}^4$ are positive, even though they originate from different mechanisms: $v_2\{4\}$ is driven by the reaction-plane eccentricity [11] while $v_3\{4\}$ is driven by non-Gaussian fluctuations of the initial triangularity [12–14]. By contrast, simple linear response to the initial anisotropy in the fourth harmonic is unable to explain the observed fluctuations of v_4 [15, 16]. In Sec. 2, we recall why linear response does not apply to v_4 and how a significant nonlinear response can be taken into account [17]. We infer the nonlinear response from experimental data and we show that its magnitude is correctly predicted by hydrodynamics. In Sec. 3, we calculate $v_4\{4\}^4$ in hydrodynamics.

2. Linear and nonlinear hydrodynamic response

V_4 and $(V_2)^2$ transform identically under azimuthal rotations. Therefore, rotational symmetry allows for a coupling between these two quantities, which is indeed predicted by hydrodynamics [18]. We take this coupling into account by writing V_4 as the sum of a term proportional to $(V_2)^2$ (the nonlinear response) and a term uncorrelated with $(V_2)^2$, which we dub the linear part, V_{4L} [19, 20]:

$$V_4 = V_{4L} + \chi_4 (V_2)^2. \tag{3}$$

The condition that linear and nonlinear parts are uncorrelated, $\langle V_{4L}(V_2)^{*2}\rangle = 0$, uniquely defines the proportionality coefficient, χ_4, i.e.,

$$\chi_4 = \frac{\langle V_4 (V_2^*)^2\rangle}{\langle |V_2|^4\rangle}. \tag{4}$$

Note that the decomposition defined in Eqs. (3) and (4) is purely mathematical and holds irrespective of the hydrodynamical setup.[1] The nonlinear part thus defined can be isolated by analyzing V_4 with respect to the direction of elliptic flow [22, 23]. The resulting observable is dubbed $v_4\{\Psi_2\}$ [24] and is defined by [25]:

$$v_4\{\Psi_2\} \equiv \frac{\langle V_4 (V_2^*)^2\rangle}{\sqrt{\langle |V_2|^4\rangle}}. \tag{5}$$

From Eqs. (4) and (5), one obtains $v_4\{\Psi_2\} = \chi_4\sqrt{\langle |V_2|^4\rangle}$, that is, $v_4\{\Psi_2\}$ is the rms value of the nonlinear contribution to V_4 in Eq. (3). The triangular inequality guarantees that $v_4\{\Psi_2\} \leq v_4\{2\}$ [20], where $v_4\{2\}$ is defined in Eq. (2). The mean square value of V_{4L} can be obtained by subtracting $v_4\{\Psi_2\}^2$ from $v_4\{2\}^2$ [26, 27]. Figure 1 displays CMS data [28] for $v_4\{\Psi_2\}$ and $v_4\{2\}$. They satisfy $v_4\{\Psi_2\} < v_4\{2\}$ for all centralities, which is a nontrivial test of the flow paradigm. Further, because of the increase of elliptic flow in the reaction plane, the relative weight of the nonlinear contribution, $v_4\{\Psi_2\}$, increases with centrality. In order to illustrate that these trends are captured by hydrodynamics [29–31], we carry out event-by-event viscous relativistic hydrodynamic calculations within the code v-USPhydro [32, 33]. Initial conditions are given by the Monte Carlo Glauber model [34]. The setup is the same as in Ref. [35]: In particular, the shear viscosity over entropy ratio is $\eta/s = 0.08$ [36]. Anisotropic flow, V_n, is calculated at freeze-out [37] for pions in every event. We calculate both $v_4\{2\}$ and $v_4\{\Psi_2\}$ by averaging over events. Results are shown in Fig. 1. Our calculation matches

[1] As an example, hydrodynamics predicts a similar decomposition [21], where the linear part results from initial fluctuations in the fourth harmonic. In this case, there is no requirement concerning how the linear and the nonlinear part are correlated.

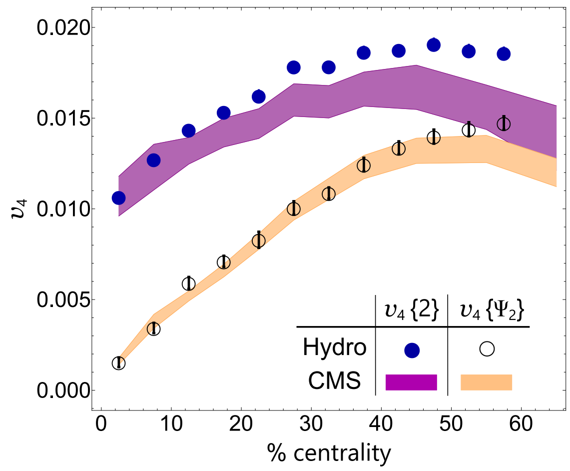

Figure 1. $v_4\{2\}$ (full symbols and dark shaded band) and $v_4\{\Psi_2\}$ (open symbols and light shaded band) as a function of centrality percentile in Pb+Pb collisions at 2.76 TeV. Bands: CMS data for charged particles in the p_t range $0.3 < p_t < 3$ GeV [28]. Symbols: hydrodynamic calculations (pions, same p_t range).

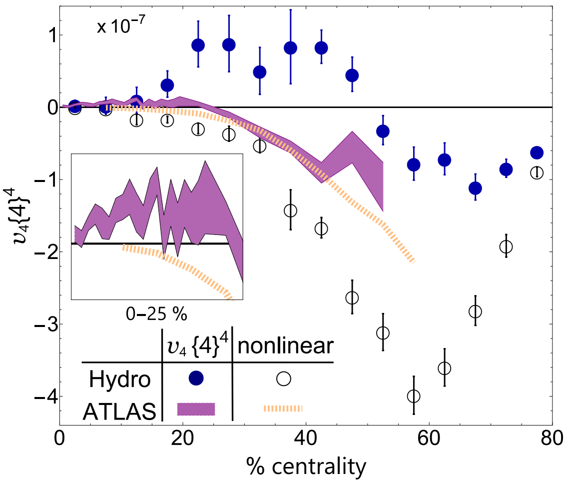

Figure 2. Dark shaded band: ATLAS data [7] for $v_4\{4\}^4$ as a function of centrality percentile for charged particles in the p_t range $0.5 < p_t < 5$ GeV. Full symbols: hydrodynamic calculation (pions, $0.5 < p_t < 3$ GeV). Dashed line and open symbols: last term of Eq. (6), from ATLAS data and hydrodynamic calculations.

experimental data on $v_4\{\Psi_2\}$ but slightly overestimates $v_4\{2\}$, meaning that our hydrodynamical setup overestimates the linear part, V_{4L}. We stress that, in our calculation, we implement a very low value of η/s and that the linear part, V_{4L}, is more strongly damped by viscosity than the nonlinear part [21]. Agreement with data is likely to be improved with a larger value of η/s.

3. Explaining $v_4\{4\}^4$

Figure 2 presents $v_4\{4\}^4$, as defined in Eq. (2), from ATLAS data [7] (the plotted quantity is $c_4\{4\} \equiv -v_4\{4\}^4$). It is positive up to 25% centrality (see inset in Fig. 2) and then negative. This change of sign is also observed in hydrodynamic calculations (full symbols), although it occurs around 50% centrality. We now argue that the change of sign is driven by the nonlinear response. Neglecting the linear part, V_{4L}, in Eq. (3), one obtains

$$v_4\{4\}^4 = \chi_4^4\left(2\langle v_2^4\rangle^2 - \langle v_2^8\rangle\right) = v_4\{\Psi_2\}^4\left(2 - \frac{\langle v_2^8\rangle}{\langle v_2^4\rangle^2}\right), \tag{6}$$

where, in the last equality, we have used Eqs. (4) and (5). We compute the last term of Eq. (6) both in hydrodynamics and using experimental data. The estimate of hydrodynamics is shown as open symbols in Fig. 2. As for experimental data, we employ the relation $v_4\{\Psi_2\} \equiv v_4\{2\}\langle\cos(4(\Phi_4 - \Phi_2))\rangle_w$, where $\langle\cos(4(\Phi_4 - \Phi_2))\rangle_w$ is the event-plane correlation [19, 38] measured by the ATLAS Collaboration. Higher-order moments of v_2 are instead obtained from the measured higher-order cumulants of elliptic flow [7, 19]. The resulting estimate is plotted as a dashed line in Fig. 2. Both estimates show that large fluctuations of v_2 lift the value of $\langle v_2^8\rangle/\langle v_2^4\rangle^2$, causing the contribution of the nonlinear term to be negative for all centralities. The nonlinear term increases in magnitude as a function of centrality percentile and drives the change of sign of $v_4\{4\}^4$. We find that the hydrodynamic calculation overestimates both $v_4\{4\}^4$

and the difference between $v_4\{4\}^4$ and the nonlinear contribution. Moreover, the change of sign of $v_4\{4\}^4$ occurs at a centrality percentile which is too large. These issues are consistent with the conclusion drawn in Fig. 1: Our hydrodynamical setup overestimates the linear part, V_{4L}.

We have shown that the peculiar centrality dependence of $v_4\{4\}^4$ observed in Pb+Pb collisions at the LHC is understood in hydrodynamics as resulting from the combined effects of a nonlinear hydrodynamic response coupling v_4 to v_2, and large v_2 fluctuations. This provides further evidence for a fluidlike behavior of the matter created in ultrarelativistic Pb+Pb collisions.

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