On the Use of the Tsallis Distribution at LHC Energies.

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Abstract. Numerous papers have appeared recently showing fits to transverse momentum (p_T) spectra measured at the Large Hadron Collider (LHC) in proton - proton collisions.This talk focuses on the fits extending to very large values of the transverse momentum with p_T values up to 200 GeV/c as measured by the ATLAS and CMS collaborations at $\sqrt{s} = 0.9$ and 7 TeV. A thermodynamically consistent form of the Tsallis distribution is used for fitting the transverse momentum spectra at mid-rapidity. The fits based on the proposed distribution provide an excellent description over 14 orders of magnitude. Despite this success, an ambiguity is noted concerning the determination of the parameters in the Tsallis distribution. This prevents drawing firm conclusions as to the universality of the parameters appearing in the Tsallis distribution.

It was shown in numerous recent publications that fits based on the Tsallis distribution [1] give a good description of transverse momentum distributions measured at the LHC [2, 3, 4, 5, 6, 7, 8, 9, 10]. Some of these fits extend to values of p_T up to 200 GeV/c [11, 12, 13, 14, 15] and provide an excellent description over 14 orders of magnitude in the transverse momentum spectrum.

Of the many forms proposed, one in particular [16, 17] leads to a consistent version of thermodynamics for the particle number, energy density and pressure. In particular, the momentum distribution of particles is given by the following expression

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3}\left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}. \tag{1}$$

where g is the degeneracy factor, E the energy, p the momentum, V the volume of the system and μ is the chemical potential. The advantages are that all thermodynamic consistency conditions are satisfied:

$$N = V\left.\frac{\partial P}{\partial \mu}\right|_{T,V} \qquad \text{etc...,}$$

and the parameter T truly deserves its name since

$$T = \left.\frac{\partial E}{\partial S}\right|_{V,N}.$$

In terms of the rapidity and transverse mass variables, $E = m_T \cosh y$, Eq. (1) becomes (at mid-rapidity $y = 0$ and for $\mu = 0$)

$$\left.\frac{d^2N}{dp_T \, dy}\right|_{y=0} = gV \frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}}. \tag{2}$$

Integrating the above expression over the rapidity variable leads to the equivalent form first derived in [18]

$$
\begin{aligned}
\left.\frac{d^2N}{dp_T \, dy}\right|_{y=0} = {} & \left.\frac{dN}{dy}\right|_{y=0} \frac{p_T m_T}{T} \left[1 + (q-1)\frac{m_T}{T}\right]^{-q/(q-1)} \\
& \times \frac{(2-q)(3-2q)}{(2-q)m^2 + 2mT + 2T^2} \\
& \times \left[1 + (q-1)\frac{m}{T}\right]^{1/(q-1)}
\end{aligned}
\tag{3}
$$

or, showing only the dependence on the transverse variables:

$$
\begin{aligned}
\left.\frac{d^2N}{dp_T \, dy}\right|_{y=0} = {} & \left.\frac{dN}{dy}\right|_{y=0} p_T \frac{m_T}{T} \left[1 + (q-1)\frac{m_T}{T}\right]^{-q/(q-1)} \\
& \times (\text{factors independent of } p_T)
\end{aligned}
\tag{4}
$$

At large transverse momenta the asymptotic behavior is

$$\lim_{p_T \to \infty} \left.\frac{d^2N}{dp_T \, dy}\right|_{y=0} \propto p_T \left[\frac{p_T}{T}\right]^{-1/(q-1)}$$

which shows that the scale is being set by the temperature T and the asymptotic behavior is set by q. It is highly sensitive to small deviations from 1 in this last variable. The upper limit for q is given by

$$q < \frac{4}{3}. \tag{5}$$

For larger values of q the integrals become divergent [19].

The above power law has been used to fit the p_T spectra of charged particles measured by the ATLAS [20] and CMS [21] collaborations in [14]. The ATLAS collaboration has reported the transverse momentum in an inclusive phase space region taking into account at least two charged particles in the kinematic range $|\eta| < 2.5$ and $p_T > 100$ MeV [20]. The CMS collaboration has presented the differential transverse momentum distribution covering a p_T range up to 200 GeV/c, the largest range ever measured in a colliding beam experiment [21]. The fits are presented in Figs. (1) and (2).

The results can be compared to those obtained in [7, 11, 12] where very good fits to transverse momentum distributions were presented. The quality of the fits is confirmed albeit with different values of the parameters since a different version is being used. The resulting parameters are listed in table 1. To show the quality of the fits we also present the ratio of the measured values over the fitted values in Fig. (3). It is quite remarkable that the transverse momentum distributions measured up to 200 GeV/c in p_T can be described consistently over 14 orders of magnitude by a straightforward Tsallis distribution.

Despite this success, an ambiguity has been noted concerning the determination of the parameters [23] in the Tsallis distribution. This is shown explicitly in Fig. (3) where fits are presented to the distributions of proton measured by the CMS collaboration [24]. In the figure,
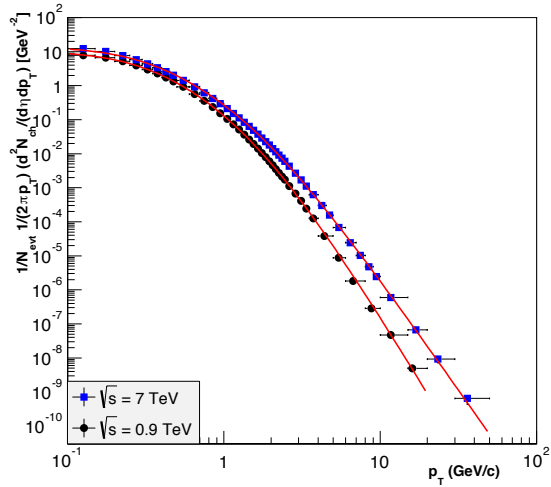
Figure 1. Charged particle multiplicities as a function of the transverse momentum measured by the ATLAS collaboration for events with $n_{ch} \geq 2$, $p_T >$100 MeV and $|\eta| < 2.5$ at $\sqrt{s} = 0.9$ and 7 TeV in proton - proton collisions [20] fitted with Tsallis distribution [14].
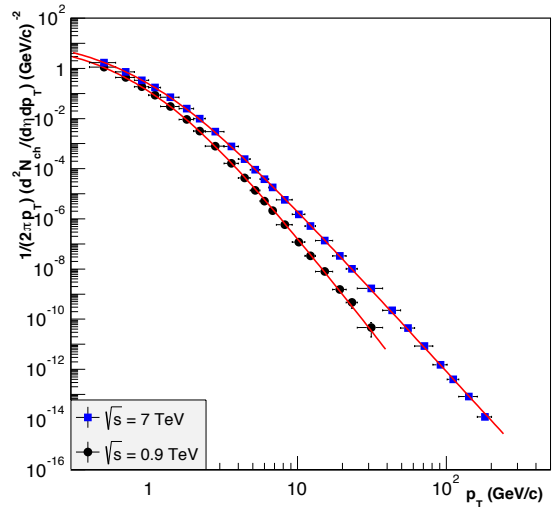

Figure 2. Charged particle differential transverse momentum yields measured within $|\eta| < 2.4$ by the CMS collaboration in proton - proton collisions at $\sqrt{s} = 0.9$ and 7 TeV [21] fitted with the Tsallis distribution [14].

Experiment	$\sqrt{s}$ (TeV)	q	T (MeV)	R (fm)	χ^2/NDF
ATLAS	0.9	1.129 ± 0.005	74.21 ± 3.55	4.62 ± 0.29	0.66/36
ATLAS	7	1.150 ± 0.002	75.00 ± 3.21	5.05 ± 0.07	4.35/41
CMS	0.9	1.129 ± 0.003	76.00 ± 0.17	4.32 ± 0.29	0.65/17
CMS	7	1.153 ± 0.002	73.00 ± 1.42	5.04 ± 0.27	0.52/24

Table 1. Values of the q, T and R parameters and χ^2/NDF obtained from fits to the p_T spectra measured by the ATLAS [20] and CMS [21] collaborations.

as an example, three different sets of variables are shown reproducing the measurements. This shows clearly that a larger interval in the transverse momentum is needed before the parameters can be determined accurately. The solid black line corresponds to a temperature $T = 29.6$ MeV, the dashed line has $T = 52.3$ MeV and the dash-dotted line has $T = 73.00$ MeV as determined from the fit to charged particles at the same energy. The first two choices for T and the corresponding changes in q and dN/dy give equally acceptable fits despite the large change especially in the temperature T. Thus no firm conclusion can be drawn at present as to the universality of the parameters appearing in the Tsallis distribution as given above.

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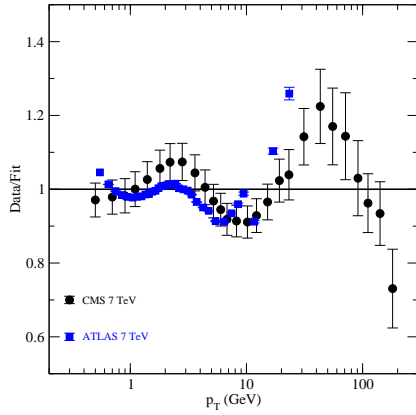**Figure 3.** Ratio of the transverse momenta of charged particles as measured by the CMS [22] and ATLAS [20] collaborations in $p - p$ collisions at $\sqrt{s} = 7$ TeV and the fit values listed in Table 1.
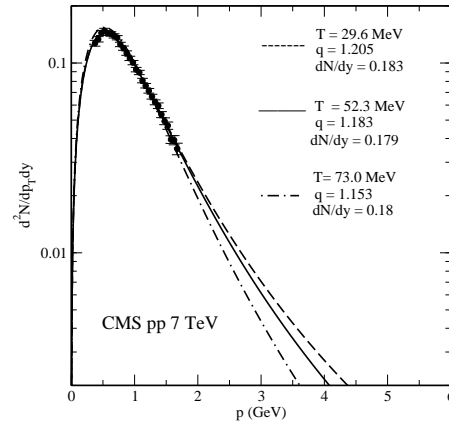

Figure 4. Distribution of protons measured by the CMS collaboration [24] in $p - p$ collisions at 7 TeV. Three different Tsallis fits are shown with the parameters indicated in the figure.

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