Recent development on heavy quark theory in heavy-ion collisions

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Abstract. A brief overview is presented for the theory of open heavy flavor dynamics in ultra-relativistic nuclear collisions, including a summary of different transport models, recent development on heavy quark phenomenology, in particular possible solutions to the "heavy vs. light flavor" puzzle and the "$R_{\rm AA}$ vs. v_2" puzzle in the field.

1. Introduction

Heavy quarks serve as ideal probes of the color deconfined quark-gluon plasma (QGP) matter produced in relativistic nuclear collisions. Since their thermal production is suppressed by their large mass, they are primarily produced at the primordial stage of heavy-ion collisions via hard scatterings and then travel through and interact with the medium with their flavors conserved, and therefore probe the entire evolution history of the QGP fireballs. Over the past decade, experimental observations at both RHIC and the LHC have revealed a great many interesting data of heavy flavors, among which the most surprising ones are the small values of the nuclear modification factor $R_{\rm AA}$ and large values of the elliptic flow coefficient v_2 of heavy mesons which are almost comparable to those of light hadrons [1, 2, 3]. This seems contradictory to one's earlier expectation of the mass hierarchy of parton energy loss and is known as a "heavy flavor puzzle". Apart from this "heavy vs. light flavor puzzle", another puzzle related to not only heavy quarks but also to all hard partons is the difficulty in simultaneously describing their $R_{\rm AA}$ and v_2. Therefore, it still remains a great challenge to fully understand the heavy flavor dynamics in the same framework with light flavor partons.

Various transport models have been developed to study the heavy quark evolution inside the dense nuclear matter, such as the parton cascade model based on the Boltzmann equation [4, 5, 6], the linearized Boltzmann approach coupled to a hydrodynamic background [7, 8, 9] and the Langevin-based transport models [10, 11, 12, 13, 14, 15]. In this talk, I will first briefly summarize different theoretical treatments of heavy flavor dynamics in heavy-ion collisions and then discuss recent theoretical progress towards solving the heavy flavor puzzles in the field.

2. Transport Theory of Heavy Flavor in Heavy-Ion Collisions

In the most general form, the evolution of a heavy quark can be described using the Boltzmann equation

$$p\cdot\partial f_Q(x,p)=EC[f_Q], \tag{1}$$

where the right hand side represents the collision term which can be expressed as a gain term minus a loss term

$$C\left[f_Q\right] = \int d^3k \left[w(\vec{p}+\vec{k},\vec{k}) f_Q(\vec{p}+\vec{k}) - w(\vec{p},\vec{k}) f_Q(\vec{p})\right], \tag{2}$$

in which $w(p,k)$ represents the transition rate of a heavy quark from momentum p to $p-k$ and can be directly evaluated from the microscopic cross sections.

For the quasi-elastic scattering process, one may assume the momentum change of heavy quark during each of its scattering with a light parton is small ($|\vec{k}| \ll |\vec{p}|$) and thus the collision term can be simplified as

$$C\left[f_Q\right] \approx \int d^3k \left(k_i \frac{\partial}{\partial p_i} + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}\right) w(\vec{p},\vec{k}) f_Q(\vec{p}), \tag{3}$$

and the Boltzmann equation is reduced to the following Fokker-Planck equation:

$$\frac{\partial}{\partial t} f_Q = \frac{\partial}{\partial p_i} \left\{ A_i(\vec{p}) f_Q + \frac{\partial}{\partial p_j} \left[B_{ij}(\vec{p}) f_Q\right] \right\}. \tag{4}$$

Additionally, one may assume every heavy quark scatters multiple times with the medium background along its propagation, and thus the Fokker-Planck equation can be stochastically realized by the Langevin equation:

$$dx_i = \frac{p_i}{E_{\vec{p}}} dt, \tag{5}$$

$$dp_i = -\eta_D(\vec{p}) p_i dt + \xi_i dt. \tag{6}$$

The first term in Eq. (6) is known as drag and the second is related to the thermal random force. One may refer to Ref. [16] for calculations of the above mentioned transport coefficients – A_i, B_{ij}, η_D and ξ_i. Note that these simplifications from the Boltzmann equation to the Fokker-Planck equation and then to the Langevin equation are only valid for the collisional energy loss, but not for their radiative energy loss because the energy of each radiated gluon is not necessarily small and the number of emitted gluons from each heavy quark may not be large.

While collisional energy loss alone is successful in describing heavy flavor observables in the low p_{T} region where the phase space for the medium-induced gluon radiation is restricted by the large mass of heavy quarks [17, 18], it has been shown insufficient [14, 15] at high p_{T}. To incorporate gluon radiation into the Boltzmann transport model, one needs to evaluate the pQCD diagrams for the $2 \to 3$ processes for the collision term. Although a full evaluation is available [19], the result is tedious and hard to efficiently implement in numerical calculations. For this reason, the Gunion-Bertsch approximation is adopted by Refs. [7, 8, 20] that is derived at high energy limit and reproduce the exact calculation of the matrix elements over a wide rapidity range. The LO pQCD calculation does not include the LPM effect due to the coherent scatterings. To mimic this effect in the numerical simulation, Ref. [21] requires that the heavy quark mean free path is larger than the formation time of radiated gluons times an X factor.

An alternative approach to implement radiative energy loss is calculating the inelastic scattering probability based on the average number of medium-induced gluon, which has been successfully applied in the improved Langevin framework [14, 15] and the linear Boltzmann transport (LBT) model [9]. The average gluon number during each time interval Δt is obtained by integrating the radiated gluon spectrum:

$$\langle N_g \rangle (E,T,t,\Delta t) = \Delta t \int dx dk_\perp^2 \frac{dN_g}{dx dk_\perp^2 dt}, \tag{7}$$

and the gluon spectrum can be adopted from a higher-twist energy loss calculation [22, 23, 24]

$$\frac{dN_g}{dx dk_\perp^2 dt} = \frac{2\alpha_s C_A \hat{q} P(x)}{\pi k_\perp^4} \left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^4 \sin^2\left(\frac{t - t_i}{2\tau_f}\right), \tag{8}$$

in which x is the fractional energy of the radiated gluon from its parent hard parton, $k_\perp$ is the gluon transverse momentum. $\hat{q}$ is known as the quark or gluon transport coefficient. P is the splitting function and τ_f is the splitting time for gluon emission. The quartic term in Eq. (8) is the dead cone factor for heavy quarks, denoting the mass effect on parton energy loss. Multiple gluon emissions during each time step is allowed and the number of radiated gluons during Δt obeys a Poisson distribution with the mean value N_g. Thus the probability of inelastic scattering during Δt is $P_{\rm inel} = 1 - e^{-\langle N_g \rangle}$. If gluon radiation happens based on this probability, the energy and momentum of each radiated gluon is determined based on the differential spectrum Eq. (8). It has been shown in Ref. [15] that in 2.76 TeV central Pb-Pb collisions, quasi-elastic scattering dominates the energy loss of heavy quarks with low initial energy while gluon radiation dominates the high energy region. The crossing points are around 7 GeV for charm quark and 18 GeV for bottom quark. This indicates that including both energy loss mechanisms is necessary to study the heavy quark phenomenology at high $p_{\rm T}$ as observed at the LHC experiment.

One key ingredient of transport models is the transport coefficient. Apart from perturbative QCD calculations, evaluations based on quasi-particle models [25], non-perturbative T-matrix method [26, 27, 11, 28] and lattice QCD [29, 30, 31] have been investigated and shown successful. In addition to these first principle driven calculations of heavy quark transport coefficient, a data driven framework based on the Bayesian analysis [32] has also been developed for heavy-ion physics in which one is able to precisely extract the transport coefficient of hard probes from transport model to experimental data comparison and place systematic constraints on our understanding of the heavy quark – medium interaction.

After heavy quarks travel outside the QGP medium, they hadronize into color neutral bound states. Hybrid models of fragmentation plus heavy-light quark coalescence have been established to convert heavy quarks into heavy flavor hadrons. High $p_{\rm T}$ heavy quarks tend to fragment directly into hadrons. One may use either a proper fragmentation function to calculate the corresponding hadron spectra or use PYTHIA to simulate this process. On the other hand, it is more probable for lower $p_{\rm T}$ heavy quarks to combine with thermal partons from the medium to form new hadrons. This mechanism can be described using either an instantaneous coalescence model [33, 34, 15] or a resonance recombination model [26, 27, 11]. It has been shown in Refs. [11, 15] that while fragmentation dominates the high $p_{\rm T}$ region of heavy quark hadronization, coalescence significantly enhances the heavy flavor hadron production rate at medium $p_{\rm T}$. For this reason, the coalescence mechanism could generate the bump structure of the D meson $R_{\rm AA}$. In addition, coalescence also enhances the D meson v_2 since it adds the momentum space anisotropy of light partons onto heavy quarks when D mesons are formed.

3. Progress on Heavy Flavor Phenomenology

To obtain a thorough understanding of heavy flavor dynamics in heavy-ion collisions and investigate the mass effect on parton energy loss, one is expected to establish a unified theoretical framework for both heavy and light parton evolution inside the QGP. This has been realized in a Monte-Carlo based transport model for the first time within the LBT framework [9] in which both elastic and inelastic scatterings of heavy and light partons are treated on the same footing.

Figure 1 displays the flavor hierarchy of the nuclear modification factor at both parton and hadron levels. In Fig. 1(a), one observes that due to the mass effect of radiative energy loss, light quarks are slightly more suppressed than charm quarks in all the 3 colliding systems –

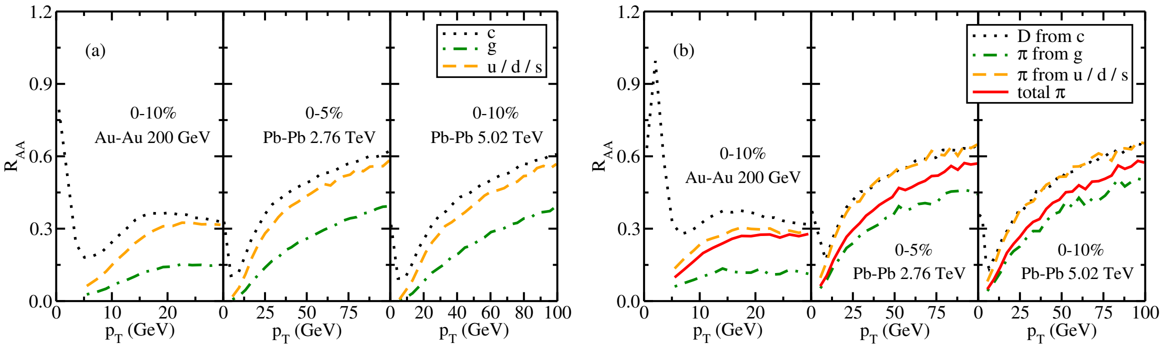

Figure 1. (Color online) Flavor dependence of R_{AA} from RHIC to the LHC energies at (a) parton and (b) hadron levels.

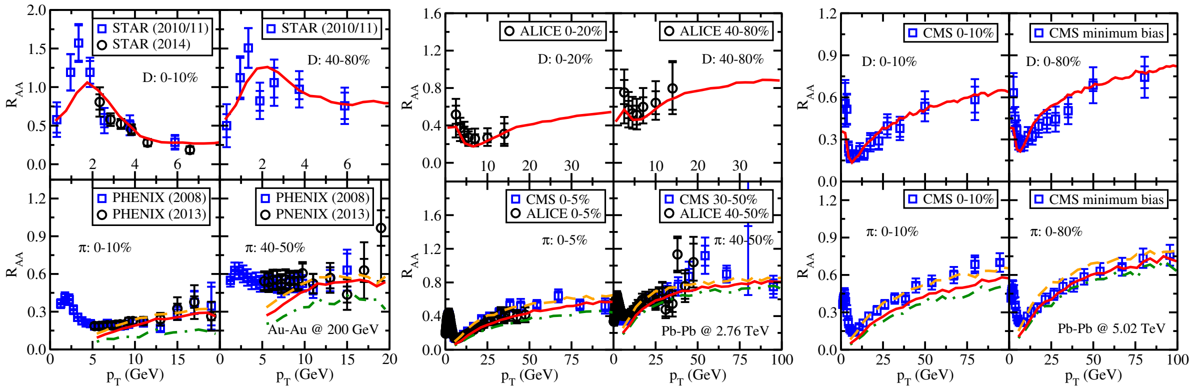

Figure 2. (Color online) R_{AA} of D and π from RHIC to the LHC energies. Data are taken from Refs. [2, 35, 36, 37, 38, 39, 40, 41].

200 GeV Au-Au, 2.76 TeV Pb-Pb and 5.02 TeV Pb-Pb collisions. Gluons are significantly more suppressed due to its larger color factor. However, this hierarchy can be slightly changed at the hadron level. As shown in Fig. 1(b), due to the harder fragmentation function of charm quark than that of light quark, together with the harder initial parton spectra that leads to the fast increasing parton R_{AA} with p_{T} at the LHC energy, D meson and π from quark jet have almost the same R_{AA}. On the other hand, the effect of different fragmentation functions on the flavor hierarchy of hadron R_{AA} is not so significant at the RHIC energy due to the soft initial spectra that leads to the relatively flat parton R_{AA} at high p_{T}. π from gluon jet is always significantly more suppressed in all the 3 systems.

After taking all effects into account, the LBT model provides a simultaneous description of heavy and light hadron suppression from RHIC to the LHC energies. In Fig. 2, upper panels show the R_{AA} of D mesons, and lower panels show the R_{AA} of π in which three curves are presented separately – upper for π from quark jet, lower for π from gluon jet and middle for the mixture. Note that since all heavy quarks are produced from initial hard scatterings, their full p_{T} spectra of R_{AA} can be obtained. On the other hand, reliable calculation for light hadron is only available at high p_{T} at this moment; contribution from the soft bulk matter at low p_{T} will be included in a future effort.

Another challenge for not only heavy flavor study but other hard probes as well is the simultaneous description of their R_{AA} and v_2. As presented in Ref. [42], although most model calculations provide reasonable descriptions of the D meson R_{AA} with proper tunings of heavy

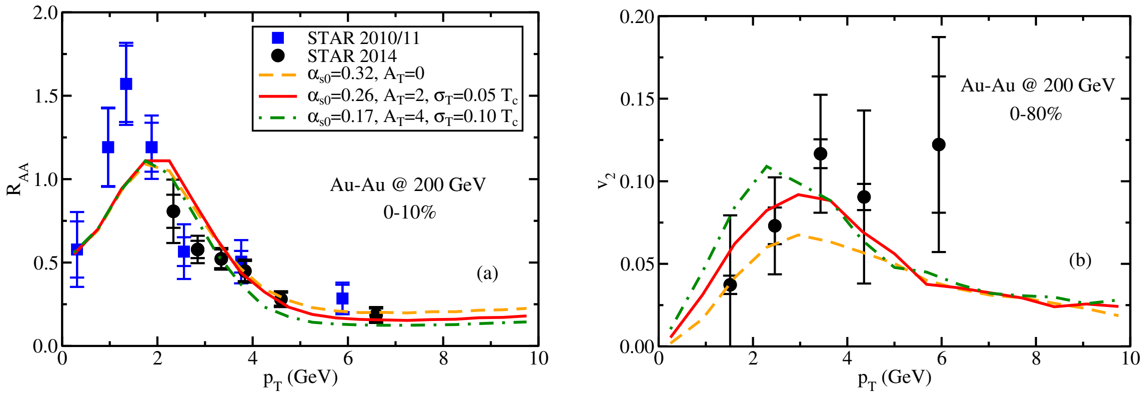

Figure 3. (Color online) Effects of the temperature dependence of the transport coefficient on (a) R_{AA} vs. (b) v_2 of D mesons.

quark transport coefficients, v_2 is often underestimated. Several recent studies target on solving this "R_{AA} vs. v_2" puzzle. For instant, the effect of different temperature dependence of quark transport coefficient on the D meson v_2 is investigated in Ref. [9]. The strong coupling constant is rescaled with a factor that peaks around the critical temperature T_{c} and returns to a constant α_{s0} at large T:

$$\alpha_s(T) = \alpha_{s0}\left[1 + A_T e^{-(T-T_{\mathrm{c}})^2/2\sigma_T^2}\right]. \tag{9}$$

As shown in Fig. 3(a), by adjusting α_s, different momentum dependences of the transport coefficient can provide similar R_{AA} of D mesons. However, as shown in Fig. 3(b), while their R_{AA} is fixed, the stronger α_s is around T_{c}, the larger v_2 one obtains. The physical picture is that if the interaction around T_{c} is stronger, larger part of heavy quark energy loss will be shifted towards the freeze-out hypersurface of the QGP where the anisotropic flow of the bulk matter is stronger, and therefore heavy quarks pick up a larger v_2 from the medium. This is consistent with the findings presented in Refs. [43, 44].

Other interesting topics on open heavy flavor presented at this conference include: effects of different path length dependence of heavy quark energy loss and event-by-event fluctuation on heavy flavor v_2 and v_3 [45]; heavy flavor production from soft collinear effective theory [46]; D-meson observables in p-Pb collisions; angular correlation between heavy and light mesons [47]; and effect of strong magnetic field on heavy quark diffusion [48].

4. Summary

To conclude, I have summarized different transport models and their implementations to heavy quark energy loss in heavy-ion collisions. Recent progress on open heavy flavor phenomenology has been discussed. In particular, it has been shown that with a delicate transport model that consistently incorporate elastic and inelastic scatterings of heavy and light partons inside the QGP, one can naturally obtain a simultaneous description of heavy and light hadron suppression from RHIC to the LHC energies and the "heavy vs. light flavor puzzle" no longer exists. A possible solution to the "R_{AA} vs. v_2 puzzle" has also been discussed: a more careful investigation of the temperature dependence of the interaction strength between hard probes and the QGP medium.

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