CP phases in SUSY

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„From the LHC to a future collider“

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CP violation in the Standard Model

Described by the CKM matrix:
3 angles, 1 phase, unitarity $\Delta s$

$$
\begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 - \frac{\lambda^2}{2}
\end{pmatrix}
$$

Observed in $K$ and $B$ systems, both direct and indirect CPV.

On the other hand, for the strong CP phase $|\bar{\theta}| < 10^{-9}$.

$$
\bar{\theta} = \theta + \text{Arg Det}M_q
$$
Baryon asymmetry of the Universe

\[ \eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.14 \pm 0.25) \times 10^{-10} \]

Sakharov conditions, 1967:
- baryon number violation
- C and CP violation
- departure from equilibrium

\( \Rightarrow \) need BSM contributions

see e.g. Dine & Kusenko, 2003
Cline, 2006
Electric dipole moments

\[ |d_{T1}| < 9 \times 10^{-25} e \text{ cm} \]
\[ |d_{\text{Hg}}| < 2 \times 10^{-28} e \text{ cm} \]
\[ |d_n| < 6 \times 10^{-26} e \text{ cm} \]

90% CL

\[ d_{T1} = -585 d_e - e 43 \text{ GeV} \times (C_{S}^{(0)} - 0.2 C_{S}^{(1)}) \]
\[ d_{\text{Hg}} = -(1.8 \times 10^{-4} \text{ GeV}^{-1}) e g_{\pi NN}^{(1)} + 10^{-2} d_e \]
\[ + (3.5 \times 10^{-3} \text{ GeV}) e C_{S}^{(0)} \]

SM prediction orders of magnitudes below the experimental limits

Strong constraints but still ample room for new physics contributions

e.g. Pospelov, Ritz, 2005
CP violation in SUSY

- **Explicit CPV (Lagrangian)**
  - Scalar-pseudoscalar Higgs mixing
  - Changes in cross sections and branching ratios
  - CP-odd observables at colliders
  - Dipole moments, flavour observables
  - Neutralino relic density
  - Electroweak baryogenesis

- **Spontaneous CPV (VEVs)**
  - SUSY GUTs, SO(10) ?
  - Strong CP problem
  - Neutrino masses, leptogenesis, ..

*Early 1980's: Ellis, Ferrara, Nanopoulos; Buchmüller, Wyler; del Aguila; Polchinski, Wise; Dugan, Grinstein, Hall; Frere, Gavela; Gerard,…*
MSSM with explicit CP violation

In the general MSSM, many parameters can be complex, thus inducing explicit CP violation in the model:

\[ M_i = |M_i| e^{i\phi_i}, \quad \mu = |\mu| e^{i\phi_\mu}, \quad A_f = |A_f| e^{i\phi_f} \]

(assuming \(B_\mu\) to be real by convention).

The physical phases are \(\text{Arg}(M_i \mu)\) and \(\text{Arg}(A_f \mu)\). They

- affect sparticle masses and couplings through mixings
- induce CP mixing of \((h, H, A)\) through radiative corrections
- influence CP-even observables like cross sections and BRs
- lead to interesting CP-odd observables at colliders
- etc...
Neutralino system

neutralino mass matrix in the ($\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0$) basis

$$\mathcal{M}_N = \begin{pmatrix}
|M_1|e^{i\phi_1} & 0 & -M_Zc_\beta s_W & M_Zs_\beta s_W \\
0 & M_2 & M_Zc_\beta c_W & -M_Zs_\beta c_W \\
-M_Zc_\beta s_W & M_Zc_\beta c_W & 0 & |\mu|e^{i\phi_\mu} \\
M_Zs_\beta s_W & -M_Zs_\beta c_W & |\mu|e^{i\phi_\mu} & 0
\end{pmatrix}$$

Neutralino mass eigenstates

$$N^* \mathcal{M}_N N^\dagger = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}), \quad m_{\tilde{\chi}_1^0} < \ldots < m_{\tilde{\chi}_4^0}.$$  

$$\tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W} + N_{13} \tilde{H}_1 + N_{14} \tilde{H}_2$$

CP phases induce shifts in masses and couplings
Chargino system

Mass matrix

\[ \mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & \mu \end{pmatrix} \]

diagonalized by the two unitary matrices \( U \) and \( V \):

\[ U^* \mathcal{M}_C V^\dagger = \text{diag}(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}) , \]

Interaction with neutralino and W

\[ \mathcal{L}_{\tilde{\chi}\tilde{\chi}W} = g W_\mu^{-\tilde{\chi}_j^0} \tilde{\chi}_j^0 \gamma^\mu (O^L_{ji} P_L + O^R_{ji} P_R) \tilde{\chi}_i^+ \]

\[ O^L_{ji} = N_{j2} V_{i1}^* - \frac{1}{\sqrt{2}} N_{j4} V_{i2}^* \quad \text{and} \quad O^R_{ji} = N_{j2}^* U_{i1} + \frac{1}{\sqrt{2}} N_{j3}^* U_{i2} \]

\text{wino} \quad \text{higgsino fractions} \quad \text{(coupling vanishes in the pure bino case)}
Higgs CP mixing

Loop-induced mixing \( (h,H,A) \rightarrow (H_1,H_2,H_3) \) with indefinite CP,
Size of mixing is proportional to

\[
\frac{3}{16\pi^2} \frac{\text{Im}(A_f \mu)}{m_H^2 - m_{H'}^2}
\]

⇒ Drastic changes in Higgs phenomenology

Review:
- CPNSH report, hep-ph/0608079

Public codes:
- CPsuperH by J.S. Lee et al.
- FeynHiggs by S. Heinemeyer et al.
LEP limits in CPX scenario

Arg(A)=60°

Arg(A)=90°

$M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3} = M_{\text{SUSY}}$, 
$|\mu| = 4 M_{\text{SUSY}}$, $|A_{t,b,\tau}| = 2 M_{\text{SUSY}}$, $|M_3| = 1$ TeV.

light green: 95% CL
dark green: 99.7% CL

$mt = 174.3$ GeV

LEP Higgs WG, P. Bechtle et al.
hep-ex/0602042
LHC potential in CPX scenario

ATLAS discovery potential with 300 fb$^{-1}$
Light relic neutralino in the CPX window?

\[ 7 \text{ GeV} \lesssim M_{H_1} \lesssim 10 \text{ GeV} \quad \text{and} \quad 3 \lesssim \tan \beta \lesssim 5, \]

\[ 2.93 \text{ GeV} \lesssim m_\chi \lesssim 5 \text{ GeV}. \]
Higgs CP at ILC

\[ \frac{d\sigma}{d\cos\theta_Z} \propto \beta_{\phi_Z} \left[ 1 + \frac{s^2_{\beta_{\phi_Z}}}{8M_Z^2} \sin^2\theta_Z + \eta \frac{2s_{\beta_{\phi_Z}}}{M_Z} \cos\theta_Z + \eta^2 \frac{s^2_{\beta_{\phi_Z}}}{8M_Z^2} (1 + \cos^2\theta_Z) \right] \]

Figure 2.2.8: a): The \( \cos\theta \) dependence of \( e^+e^- \rightarrow ZH, e^+e^- \rightarrow ZA, e^+e^- \rightarrow ZZ \) for \( \sqrt{s} = 500 \text{ GeV} \), assuming \( M_H = M_A = 120 \text{ GeV} \) [58] and b): the dependence of the expectation value of the optimal observable and the total cross-section on \( \eta \) for \( M_H = 120 \text{ GeV}, \sqrt{s} = 350 \text{ GeV} \) and \( \mathcal{L} = 500 \text{ fb}^{-1} \) after applying the selection cuts. The shaded bands show the 1\( \sigma \) uncertainty in the determination of \( \langle O \rangle \) and the total cross section.
$H \rightarrow ZZ \rightarrow 4\text{leptons}$

$g_{HZZ} = \frac{ig}{m_Z \cos \theta_W} \left[ a g_{\mu\nu} + b (k_{2\mu}k_{1\nu} - k_1 \cdot k_2 g_{\mu\nu}) + c \epsilon_{\mu\nu\alpha\beta} k_1^{\alpha} k_2^{\beta} \right]$

$O_1 \equiv \cos \theta_1 = \frac{(\vec{p}_{f_1} - \vec{p}_{f_2}) \cdot (\vec{p}_{f_1} + \vec{p}_{f_2})}{|\vec{p}_{f_1} - \vec{p}_{f_2}| |\vec{p}_{f_1} + \vec{p}_{f_2}|}$

$A_1 = \frac{\Gamma(\cos \theta_1 > 0) - \Gamma(\cos \theta_1 < 0)}{\Gamma(\cos \theta_1 > 0) + \Gamma(\cos \theta_1 < 0)}.$
Hjj at LHC

Use distribution of azimuthal angle between the jets to determine the CP properties of the Higgs

Figure 2: Normalized distributions of the jet-jet azimuthal angle difference as defined in the text. The curves are for the SM CP-even case ($\alpha_3 = 0$), a pure CP-odd ($\alpha_2 = 0$) and a CP-mixed case ($\alpha_2 = \alpha_3 \neq 0$).
Gauginos and sfermions at ILC

- CP-even observables: masses, cross sections, branching ratios
  - Parameter determination ($M_i$, $\mu$, $A_f$,...) in principle possible
  - Beam polarization is essential
  - Ambiguities for phases remain

- CP-odd / T-odd observables
  - Triple product asymmetries (example p.18)
  - Charge asymmetries

$$A_{CP} = \frac{\Gamma(+)(\tilde{\chi}_i^+ \to \tilde{\chi}_j^0 W^+) - \Gamma(-)(\tilde{\chi}_i^- \to \tilde{\chi}_j^0 W^-)}{\Gamma(+)(\tilde{\chi}_i^+ \to \tilde{\chi}_j^0 W^+) + \Gamma(-)(\tilde{\chi}_i^- \to \tilde{\chi}_j^0 W^-)}$$

through loops  [Eberl et al]
Threshold behaviour: neutralino production

- Threshold curve of $e^+ e^- \rightarrow \chi_i \chi_j$ ($i \neq j$) depends on the relative CP-parity of the 2 neutralinos.

- $\text{CP}(\chi_i \chi_j) = +1$: P-wave
  $\text{CP}(\chi_i \chi_j) = -1$: S-wave

- In case of CPV, we are somewhere in between

$\sigma(e^+ e^- \rightarrow \chi_i \chi_j) = \sigma_{Z}^{ij} + \sigma_{\ell Z}^{ij} + \sigma_{\ell}^{ij}$

Exploit threshold scans

Petcov, PLB178 (1986)
Triple products

Spin correlations between production and decay $\rightarrow$ CP asymmetries.
Have been analyzed for neutralinos/charginos with 2 and 3body decays

Example $e^+e^- \rightarrow$ charginos

$$\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 \nu \ell^+ , \quad T_\ell = \vec{p}_{\ell^+} \cdot (\vec{p}_e^- \times \vec{p}_{\tilde{\chi}_1^+})$$

$$\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 \bar{s} c , \quad T_q = \vec{p}_s \cdot (\vec{p}_c \times \vec{p}_{e^-})$$

$$A_T(T_{\ell,q}) = \frac{N[T_{\ell,q} > 0] - N[T_{\ell,q} < 0]}{N[T_{\ell,q} > 0] + N[T_{\ell,q} < 0]}$$

$$A_{CP}(T_{\ell,q}) = \frac{A_T(T_{\ell,q}) - \bar{A}_T(T_{\ell,q})}{2}$$

[Bartl et al]
EDM constraints

At 1 loop:

\[ d_e = \frac{e m_e}{16 \pi^2 M_{\text{SUSY}}^2} \left[ \left( \frac{5g_2^2}{24} + \frac{g_1^2}{24} \right) \tan \beta \sin[\text{Arg}(\mu M_2 m_{\text{h}}^2)] + \frac{g_1^2}{12} \sin[\text{Arg}(M_1^* A_e)] \right] \]

\( \text{Arg}(A_{\text{tbr}}) \) enters at 2 loops, ca factor 10 suppressed

For O(100) GeV masses and O(1) phases, the EDMs are typically 3 orders of magnitude too large.

\( \Rightarrow \) Need suppression mechanism:

- small phases
- heavy sparticles
  (in particular 1st and 2nd generation)
- cancellations
Neutralino relic density

$\chi^0$ LSP as thermal relic: relic density computed as thermally avaraged cross section of all annihilation channels $\rightarrow \Omega h^2 \sim \langle \sigma v \rangle^{-1}$

Neutralino couplings depend on phases $\rightarrow$ expect large influence on $\langle \sigma v \rangle$

Caution: need to single out kinematic effects

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LHC2FC: CP phases in SUSY
Scan over phases in $M_1$-$\mu$ plane:
- $M_1 \sim \mu$ main channel is annihilation into WW
- in WMAP region, LSP has $\sim 25\%$ higgsino component

Blue: WMAP-allowed range for phases=0
Green: same arbitrary phases of $M_1$, $\mu$, ...

\[ \Delta = \frac{(m_{\chi_1^+} - mLSP)}{mLSP} \]

In CPV case, much smaller mass differences can give the right $\Omega h^2$
Direct DM detection

\[ \mathcal{L}_{\chi q} = d_q (\bar{\chi} \gamma^\mu \gamma_5 \chi)(\bar{q} \gamma_\mu \gamma_5 q) + f_q (\bar{\chi} \chi)(\bar{q} q). \]

Spin independent (SI): t-channel neutral Higgs and s-channel squark exchange
Spin dependent (SD): t-channel Z and s-channel squark exchange

Nihei, Sasagawa

Orders of magn. effects in SI cross sections
Well-tempered neutralino with CPV, relic density

Figure 1: (a) Bands of $0.094 < \Omega h^2 < 0.136$ in the $|M_1|$ vs $|\mu|$ plane for $\tan \beta = 10$ and $m_{H^+} = m_{\tilde{g}} = |A_f| = 1$ TeV, in blue for all phases zero, in light blue for $\phi_{\mu} = 180^\circ$ (or $\mu < 0$) and all other phases zero, in green for $\phi_1 = 180^\circ$ and all other phases zero, and in yellow for all other cases, i.e. arbitrary phases. (b) The corresponding relative mass difference $\Delta m$ of $\tilde{\chi}_1^0$ and $\tilde{\chi}_1^+$ as function of $m_{\tilde{\chi}_1^0}$ in the WMAP-preferred region with the same colour code as in (a).
Well-tempered neutralino w/ CPV, direct detection

Figure 2: Cross sections for direct detection in the WMAP-prefered region of Fig. 1 as a function of the LSP mass. On the left the spin-independent case, $\sigma_{\chi p}^{SI}$, and on the right the spin-dependent case, $\sigma_{\chi p}^{SD}$. Same colour code as in Fig. 1.
Electroweak baryogenesis

Sakharov conditions:

✓ baryon number violation
  ⇒ shaleron processes

✓ C and CP violation
  ⇒ Resonant CPV in chargino sector, $M_2 \sim \mu$

✓ departure from equilibrium
  ⇒ Light stop, $m < m_t$, for strong 1st-order phase transition

Balazs et al; Konstandin et al; Cirigliano et al.
Carena et al.
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Balazs et al; Konstandin et al; Cirigliano et al; Carena et al.
Spontaneous CP violation

- Arises through complex VEV of extra Higgs field
- Leads to vanishing $\theta_{\text{QCD}}$ at tree level
- Can lead to a complex CKM matrix

- SCPV is a very elegant idea but difficult to realize in SUSY; not possible in the MSSM
- Extra Higgses invoke FCNC $\rightarrow$ suppress by heavy mass scale (and/or extra SM singlet fermions)
- Consider L-R symmetric models, SUSY GUTs,...

- Interesting recent work on SUSY SO(10);
  link with neutrino seesaw and leptogenesis
  $\Rightarrow$ exciting case for model building
Conclusions

- **Explicit CPV** (CP phases in Lagrangian)
  - Affects sparticle masses and couplings through mixing
  - Scalar-pseudoscalar Higgs mixing
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  - Changes in cross sections and branching ratios
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- Go $e^+e^-$ (ILC, CLIC)
- Simulations still missing

Need precision measurements!