Electroweak Precision Measurements and BSM Physics: (A) Triplet Models (B) The 3- and 4-Site Models

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S. Dawson and C. Jackson, arXiv:0810.5068; hep-ph/0703299

M. Chen, S. Dawson, and C. Jackson, arXiV:0809.4185

WARNING: THIS IS A THEORY TALK

Standard Model Renormalization

- EW sector of SM is SU(2) x U(1) gauge theory
  - 3 inputs needed: g, g', v, plus fermion/Higgs masses
  - Trade g, g', v for precisely measured G_μ, M_Z, α
  - SM has $\rho=M_W^2/(M_Z^2 c_\theta^2)=1$ at tree level
    - s_θ is derived quantity
  - Models with ρ=1 at tree level include
    - MSSM
    - Models with singlet or doublet Higgs bosons
    - Models with extra fermion families

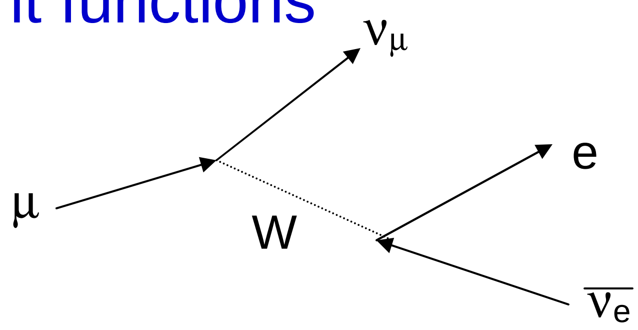
Muon Decay in the SM

- At tree level, muon decay related to input parameters:
- One loop radiative corrections included in parameter Δr_{SM}

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}\left(1-\frac{M_W^2}{M_Z^2}\right)M_W^2}(1+\Delta r_{SM})$$

- Dominant contributions from 2-point functions

Δr is a physical parameter



Part A: Triplet Model
Models with $\rho \neq 1$ at tree level are different from the SM

$$\rho = M_W^2/(M_Z^2 c_\theta^2) \neq 1$$

- SM with Higgs Triplet
- Left-Right Symmetric Models
- Little Higgs Models
- …..many more
- These models need additional input parameter
- Decoupling is not always obvious beyond tree level

Higgs Triplet Model

Simplest extension of SM with $\rho \neq 1$

- Add a real triplet

$$H = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + h^0 + i\chi^0) \end{pmatrix} \qquad \Phi = \begin{pmatrix} \eta^+ \\ v' + \eta^0 \\ \eta^- \end{pmatrix}$$

  – $v_{SM}^2 = (246\ \text{GeV})^2 = v^2 + 4v'^2$
  – Real triplet doesn't contribute to M_Z

$$M_W^2 = \frac{g^2 v^2}{4}\left(1 + \frac{4v'^2}{v^2}\right)$$

- At tree level, $\rho = 1 + 4v'^2/v^2 \neq 1$
- PDG: $v' < 12$ GeV

Neglects effects of scalar loops

Motivated by Little Higgs models

Scalar Potential

$$V = \mu_1^2|H|^2 + \mu_2^2|\Phi|^2 + \lambda_1|H|^4 + \frac{\lambda_2}{4}|\Phi|^4 + \frac{\lambda_3}{2}|H|^2|\Phi|^2 + \lambda_4 H^+\sigma^a H\Phi_a$$

Forbidden by T-parity

- λ_4 has dimensions of mass → doesn't decouple
- Mass Eigenstates:

$$\begin{pmatrix} H^0 \\ K^0 \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix}\begin{pmatrix} h^0 \\ \eta^0 \end{pmatrix} \qquad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} c_\delta & s_\delta \\ -s_\delta & c_\delta \end{pmatrix}\begin{pmatrix} \phi^\pm \\ \eta^\pm \end{pmatrix}$$

- 6 parameters in scalar sector: Take them to be:

M_{H0}, M_{K0}, M_{H+}, v, δ, γ

$\tan\delta = 2\,v'/v$

δ small since it is related to ρ parameter

Decoupling at Tree Level

- Require no mixing between doublet-triplet sectors for decoupling

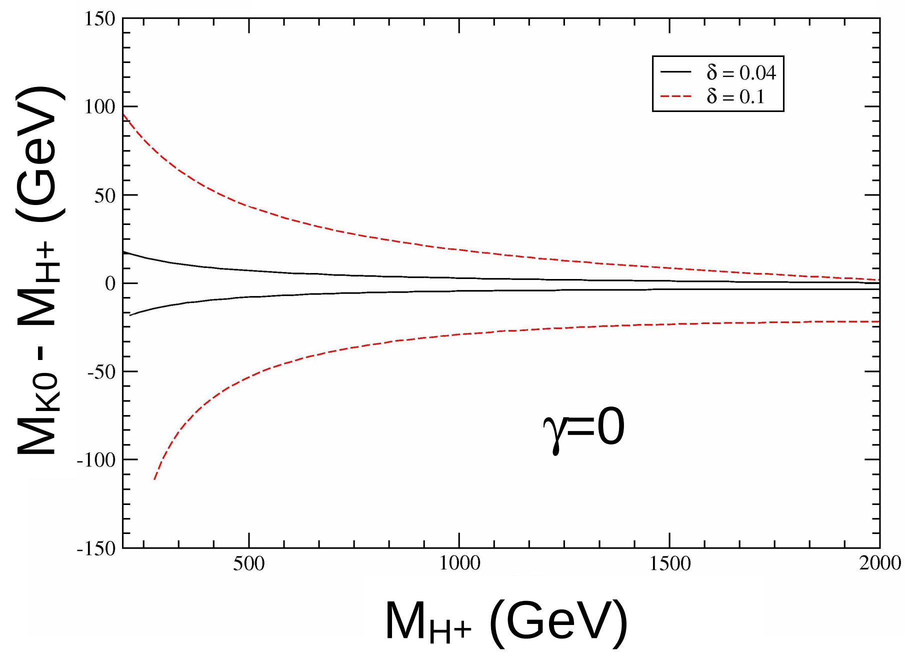
$$\Rightarrow \quad \boxed{v' = \frac{\lambda_4}{\lambda_3}}$$

- $v' \to 0$ requires $\lambda_4 \to 0$ (custodial symmetry), or $\lambda_3 \to \infty$ (invalidating perturbation theory)

$$M_{K^0}^2 = \mu_2^2 + 12 v'^2 \lambda_2 + \frac{1}{2} v^2 \lambda_3$$
$$M_{H^+}^2 = \mu_2^2 + 4 v'^2 \lambda_2 + \frac{1}{2} v^2 \lambda_3 + 2 v' \lambda_4$$

- $v' \to 0$ implies $M_{K0} \sim M_{H+}$

Heavy Scalars → Small Mass Splittings



Allowed region is between curves

- Plots are restriction $\lambda_2 < (4\pi)^2$

Forshaw, Vera, & White, hep-ph.0302256

Renormalization of Triplet Model

- At tree level, W mass related to input parameters:

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2} s_\theta^2 G_\mu}(1+\Delta r) \qquad \rho = \frac{M_W^2}{c_\theta^2 M_Z^2} \neq 1$$

- One loop radiative corrections included in parameter Δr

For $\rho \neq 1$, 4 input parameters

Input 4 Measured Quantities

(M_Z, α, G_μ, $\sin\theta^{eff}$)

- Use effective leptonic mixing angle at Z resonance as 4th parameter

$$L = -i\bar{e}\gamma_\mu(v_e + a_e\gamma_5)eZ^\mu \qquad v_e = \frac{1}{2} - 2s_\theta^{eff\,2}, \qquad a_e = \frac{1}{2}$$

- Could equally well have used ρ or M_W as 4th parameter

- At tree level, SM and triplet model are identical in s_θ^{eff} scheme (SM inputs α, $G\mu$, $\sin\theta eff$ here)

$$M_W^2 = \frac{\alpha\pi}{\sqrt{2}s_\theta^{eff\,2}G_\mu}(1 + \Delta r)$$

This scheme discussed by: Chen, Dawson, Krupovnickas, hep-ph/0604102; Blank and Hollik hep-ph/9703392

Triplet Results

- Compare with SM in effective mixing angle scheme
- Input parameters: M_Z, $\sin\theta^{eff}$, α, G_μ, M_{H0}, M_{K0}, M_{H+}, γ
  - $\rho=1/\cos^2\delta = (M_W / M_Z \cos\theta^{eff})^2$ predicts $\sin\delta = .07$ (v'=9 GeV)
  - System is overconstrained (can't let v' run)
- Triplet model has extra contributions to Δr from K^0, H^+
- SM couplings are modified by factors of $\cos\delta$, $\cos\gamma$

Quadratic dependence on Higgs mass

- Triplet model with $M_{H0} << M_{K0} \approx M_{H\pm}$ and small mixing

$$\Delta r^{triplet} \approx \Delta\tilde{r}^{SM} + \frac{\alpha}{24\pi s_\theta^2}\frac{M_{K^0}^2 - M_{H^+}^2}{M_{H^+}^2} + \sin\delta(\ldots) + \sin\gamma(\ldots)$$

Inputs different in triplet model and SM

Triplet model: M_Z=91.1876 GeV is input

SM (in this scheme): M_Z is calculated = 91.453 GeV

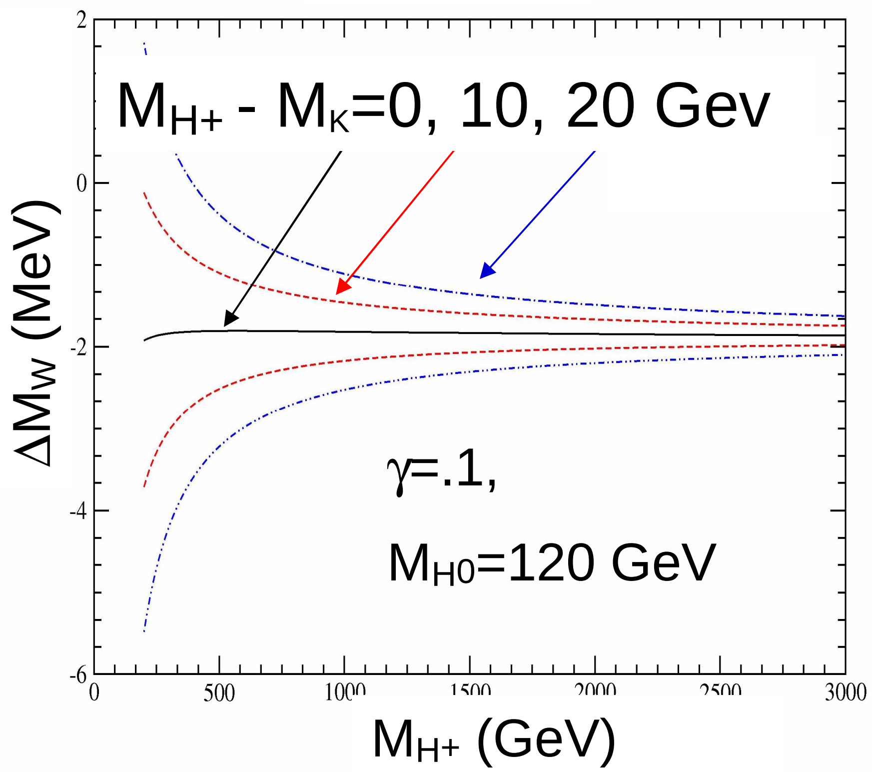
Perturbativity requires $M_{K0} \sim M_{H+}$ for large M_{H+}

Toussaint, PRD18 (1978) 1626

M_W(SM)-M_W(Triplet)

- For heavy H^+, perturbativity requires $M_{H+} \sim M_{K^0}$, and predictions of triplet model approach SM

- No large effects in perturbative regime

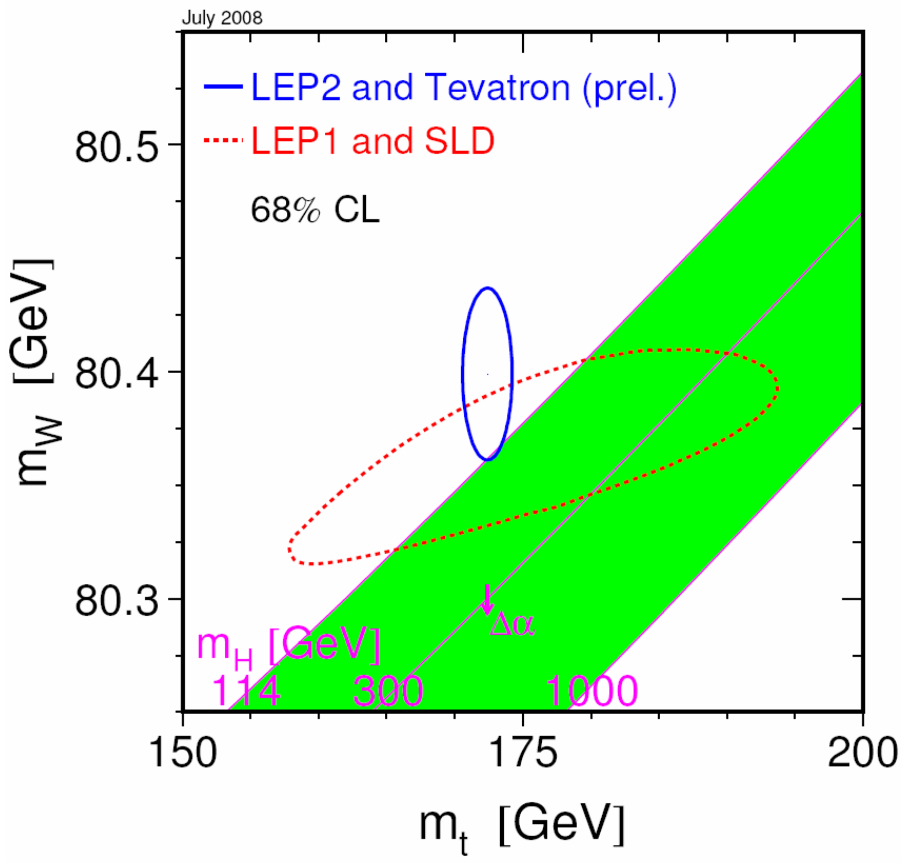


- SM not exactly recovered at large M_H^+ due to different M_Z inputs for 1-loop corrections

Similar conclusions from Chivukula, Christensen, Simmons: arXiv:0712.0546

Conclusions on Triplets

M_W=80.399 ± 0.025 GeV



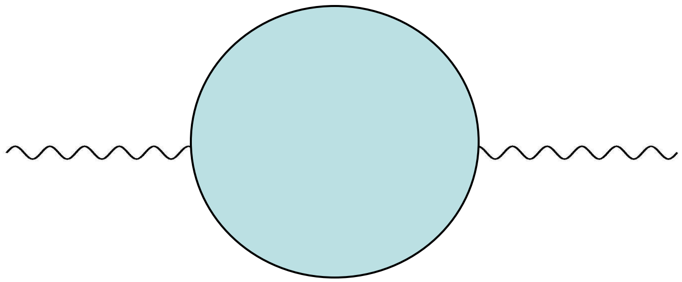
Triplet model consistent with experimental data if $M_K \sim M_{H+}$

Small mixing angles required

- Part B: Higgsless Models and Effective Lagrangians
- What can we learn from precision electroweak measurements?

The Usual Approach

- Build the model of the week
- Assume new physics contributes primarily to gauge boson 2-point functions
- Calculate contributions of new particles to S, T, U
- Extract limits on parameters of model



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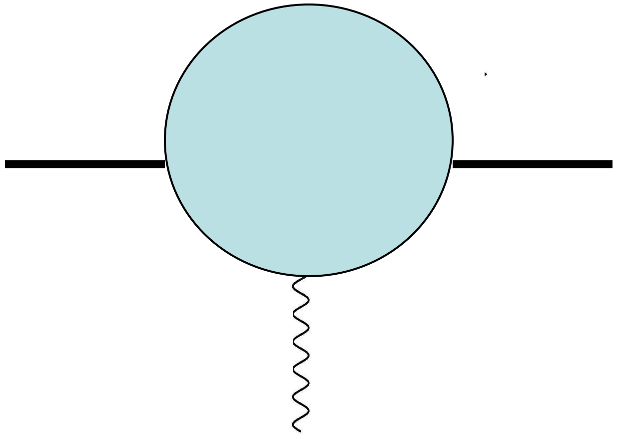


NOT THIS

STU Assumptions

- Assume dominant contribution of new physics is to 2-point functions
- Assume scale of new physics, $\Lambda >> M_Z$
  - This means no new low energy particles
  - Taylor expand in M_Z/Λ
  - Symmetry is symmetry of SM
- Assume reference values for M_H, M_t
- Assume $\rho=1$ at tree level
  - Otherwise you need 4-input parameters to renormalize

STU Definitions

Taylor expand 2-point functions:

$$\Pi_{\gamma\gamma}(q^2) = \Pi_{\gamma\gamma}(0) + q^2\Pi'_{\gamma\gamma}(q^2)$$

Vanishes by EM gauge invariance

$$\Pi_{\gamma Z}(q^2) = \Pi_{\gamma Z}(0) + q^2\Pi'_{\gamma Z}(q^2)$$

$$\Pi_{WW}(q^2) = \Pi_{WW}(0) + q^2\Pi'_{WW}(q^2)$$

$$\Pi_{ZZ}(q^2) = \Pi_{ZZ}(0) + q^2\Pi'_{ZZ}(q^2)$$

Fermion & scalar contributions vanish; gauge boson contributions non-zero

6 unknown functions to this order in M_Z/Λ

Peskin & Takeuchi, PRD46 (1992) 381

STU Definitions

6 unknowns:

3 fixed by SM renormalization, 3 free parameters

$$\alpha S = \frac{4s_W^2 c_W^2}{M_Z^2}\left\{\Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0) - \Pi_{\gamma Z}(M_Z^2) + \frac{c_W^2 - s_W^2}{c_W s_W}\left(\Pi_{\gamma Z}(M_Z^2) 0 \Pi_{\gamma Z}(0)\right)\right\}$$

$$\alpha T = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} - 2\frac{s_W}{c_W}\frac{\Pi_{\gamma Z}(0)}{M_Z^2}$$

$$\alpha U = 4s_W^2\left\{\frac{\Pi_{WW}(M_W^2) - \Pi_{WW}(0)}{M_W^2} - c_W^2\left(\frac{\Pi_{ZZ}(M_W^2) - \Pi_{ZZ}(0)}{M_Z^2}\right) + 2c_W s_W\left(\frac{\Pi_{\gamma Z}(M_W^2) - \Pi_{\gamma Z}(0)}{M_Z^2}\right) - s_W^2\frac{\Pi_{\gamma\gamma}(0)}{M_Z^2}\right\}$$

- S is scaling of Z 2-point function from $q^2=0$ to M_Z^2
- T is isospin violation
- U contributes mostly to M_W

Peskin & Takeuchi, PRD46 (1992) 381

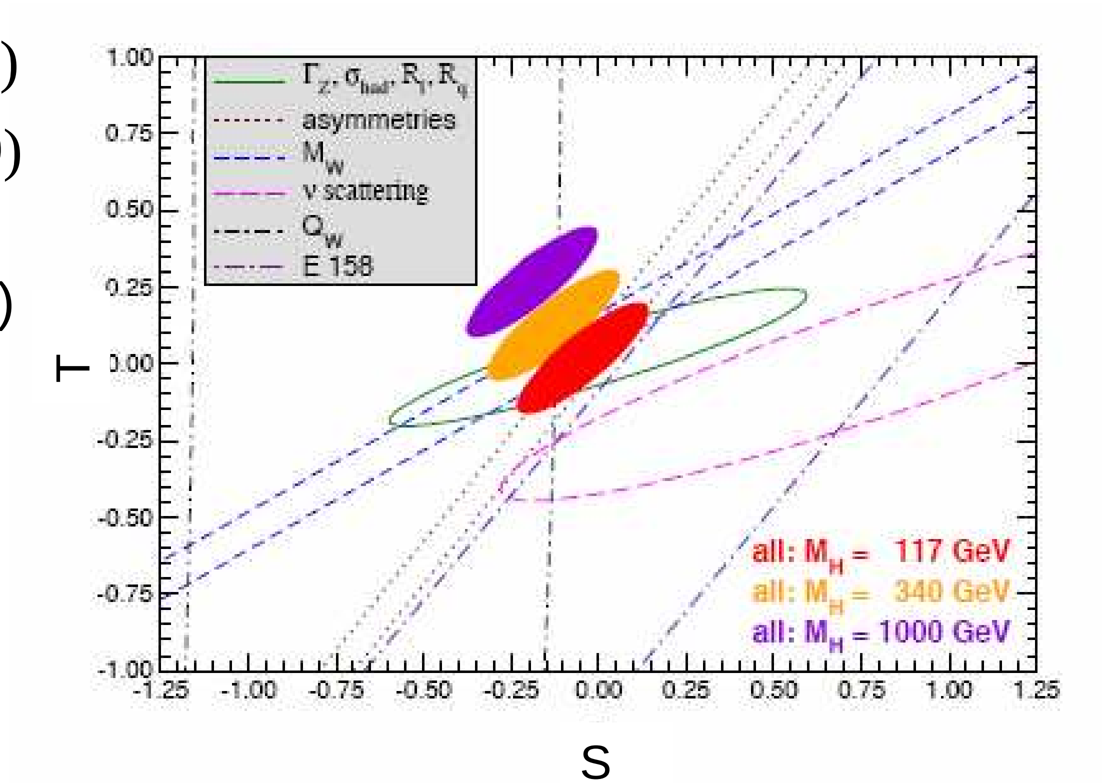
3 and 4 Site Higgsless Models
PDG Fits

- Data are 1σ constraints with M_H=117 GeV
- Ovals are 90% CL contours

$$\Delta S = -0.04 \pm 0.09 \ (-0.07)$$

$$\Delta T = +0.02 \pm 0.09 \ (-0.09)$$

$M_{H,ref}$=117 GeV (300 GeV)



What if There is No Higgs?

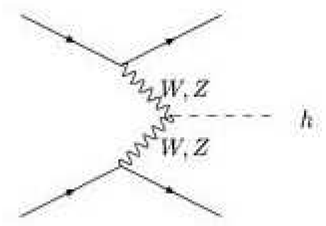
- Simplest possibility: No Higgs / No new light particles / No expanded gauge symmetry at EW scale
  - Electroweak chiral Lagrangian

$$L_{eff} = L_{SM}^{nl} + \sum L_i$$

- L_{SM}^{nl} doesn't include Higgs, so it is non-renormalizable
- Assume global symmetry SU(2) x SU(2) $\rightarrow SU(2)_V$ or U(1)
  - SU(2) x SU(2) $\rightarrow SU(2)_V$ is symmetry of Goldstone Boson sector of SM
- L_i is an expansion in $(\text{Energy})^2/\Lambda^2$

Remember Han talk

No Higgs → Unitarity Violation



- Consider $W^+W^- \to W^+W^-$
- Unitarity conservation requires $|\mathrm{Re}(a_l)| \leq \frac{1}{2}$
- $M_H \to \infty$ $a_0^0 \to -\frac{s}{32\pi v^2}$

Λ ~1.7 TeV

→ New physics at the TeV scale

- If all resonances (Higgs, vector mesons…etc) much heavier than ~ few TeV

$$A(W^+W^- \to W^+W^-) = \frac{s+t}{v^2} + O\left(\frac{s^2}{v^4}\right)$$

Electroweak Chiral Lagrangian

- Terms with 2 derivatives:

$$L_2 = \frac{v^2}{4} Tr\left(D_\mu \Sigma^+ D^\mu \Sigma\right) \qquad \Sigma = \exp(i\vec{\omega}\cdot\vec{\tau}/v)$$

$$D_\mu \Sigma = \partial_\mu \Sigma + \frac{ig}{2}\tau_i \cdot W_{i\mu} - \frac{ig'}{2} B_\mu \Sigma \tau_3$$

- Unitary gauge: $\Sigma=1$
  - SM masses for W/Z gauge bosons
- This is SM without Higgs
  - SM W/Z/γ interactions

General Framework for studying BSM physics without a Higgs

E^4 Terms in Chiral Lagrangian

- 3 operators contribute at tree level to gauge boson 2-point functions

$$L_1' = \frac{1}{4}\beta_1\left[Tr\left(TV_\mu\right)\right]^2$$

Gives tree level isospin violation

$$L_1 = \frac{1}{2}\alpha_1 g g' Tr\left(B_{\mu\nu} T W^{\mu\nu}\right)$$

$$L_8 = \frac{1}{4}\alpha_8 g^2\left[Tr\left(TW_{\mu\nu}\right)\right]^2$$

Also contribute to gauge boson 3-point functions

⇒ Limits from LEP2/Tevatron

$$T = 2\Sigma T^3 \Sigma$$

$$V_\mu = (D_\mu \Sigma)\Sigma^+$$

Apologies: my normalization is different from Han… $\alpha \sim l(v/\Lambda)^2$

E^4 Terms continued

- Contribute to WWγ, WWZ vertices (but not to 2-point functions)

$$L_2 = \frac{ig'}{2}\alpha_2 B_{\mu\nu} Tr\left(T\left[V^\mu, V^\nu\right]\right)$$
$$L_3 = ig\alpha_3 Tr\left(W_{\mu\nu}\left[V^\mu, V^\nu\right]\right)$$
$$L_9 = \frac{i}{2} g\alpha_9 Tr\left(TW_{\mu\nu}\right) Tr\left(T\left[V^\mu, V^\nu\right]\right)$$

- Only contribute to quartic interactions

$$L_4 = \alpha_4\left[Tr\left(V_\mu V_\nu\right)\right]^2$$
$$L_5 = \alpha_5\left[Tr\left(V_\mu V^\mu\right)\right]^2$$
$$L_6 = \alpha_6\left[Tr\left(V_\mu V_\nu\right) Tr\left(TV^\mu\right) Tr\left(TV^\nu\right)\right]$$
$$L_7 = \alpha_7\left[Tr\left(V_\mu V^\mu\right) Tr\left(TV_\nu\right) Tr\left(TV^\nu\right)\right]$$
$$L_{10} = \frac{1}{2}\alpha_{10}\left[TrTr\left(TV_\mu\right) Tr\left(TV^\nu\right)\right]^2$$

- Conserves CP, violates P

$$L_{11} = \alpha_{11} g \varepsilon^{\mu\nu\rho\sigma} Tr\left(TV_\mu\right) Tr\left(V_\nu W_{\rho\sigma}\right)$$

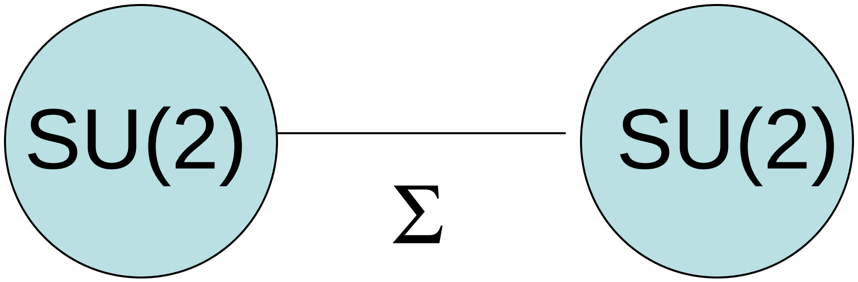
Appelquist & Longhitano

12 E^4 Operators

- Assume custodial $SU(2)_V$ (ρ=1at tree level)
  - L_1', L_6, L_7, L_8, L_9, L_{10} vanish
  - 6 operators remain
  - Assume P conservation, L_{11} vanishes

Simple format for BSM physics

Estimate coefficients in your favorite model



Tree Level

- 2-point functions

$$\alpha\Delta S_{tree} = -4e^2\alpha_1$$
$$\alpha\Delta T_{tree} = 2\beta_1$$
$$\alpha\Delta U_{tree} = -4e^2\alpha_8$$

- SM fit assumes a value for M_H
  - Contribution from heavy Higgs: $\Delta S_H = \frac{1}{6\pi}\ln\left(\frac{M_H}{M_Z}\right)$, $\Delta T_H = \frac{3}{8\pi c^2}\ln\left(\frac{M_H}{M_Z}\right)$
- Scale theory from Λ to M_Z, add back in contribution from M_H(ref)
  - Approach assumes logarithms dominate

$$\alpha\Delta S_{tree} = \frac{\alpha}{6\pi}\ln\left(\frac{\Lambda}{M_{H,ref}}\right) - 4e^2\alpha_1$$
$$\alpha\Delta T_{tree} = \frac{3\alpha}{8\pi c^2}\ln\left(\frac{\Lambda}{M_{H,ref}}\right) + 2\beta_1$$
$$\alpha\Delta U_{tree} = -4e^2\alpha_8$$

$$\boxed{\Lambda = 3\,TeV \quad .0034 < \alpha_1 < .0074}$$

Extended Gauge Symmetries

- General model with gauged SU(2) x SU(2)^N x U(1)

$$L_G = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \sum_{i=1}^{N+1} W^a_{i,\mu\nu} W_i^{a,\mu\nu}$$

- After electroweak symmetry breaking, massless photon, plus tower of massive $W_n^{\pm}$, Z_n vector bosons

$$W_i^{\pm,\mu} = \sum_{n=1}^{N+1} a_{in} W_n^{\pm,\mu} \qquad W_{3,i}^{\mu} = b_{i0}\gamma^{\mu} + \sum_{n=1}^{N+1} b_{in} Z_n^{\mu} \qquad B^{\mu} = b_{00} + \sum_{n=1}^{N+1} b_{0n} Z_n^{\mu}$$

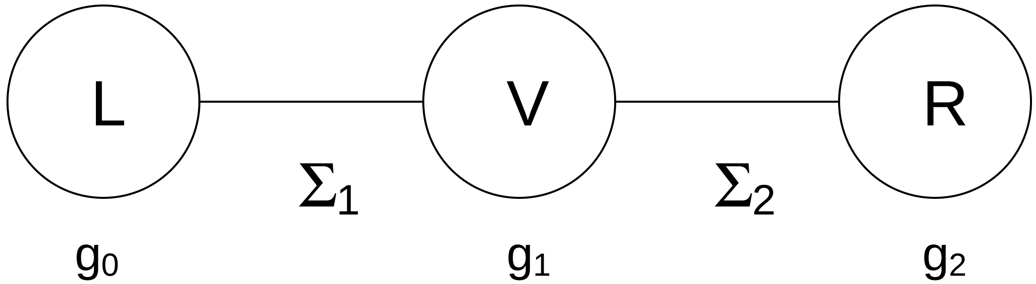
- Fermions:

$$L_f = -\sum_{n=1}^{N+1} \sum_{ij} \frac{g_{ijW_n^{\pm}}}{2\sqrt{2}} \overline{\psi}_i \gamma^{\mu} (1-\gamma_5) \psi_j W^{\pm}_{n,\mu} + h.c$$

$$-\sum_{n=1}^{N+1} \sum_{i} g_{ijV_j^0} \overline{\psi}_i \gamma^{\mu} (g_{V_i}^{V_n^0} + g_{A_i}^{V_n^0} \gamma_5) \psi_i V^0_{n,\mu}$$

- Calculate for general couplings, then apply to specific model

3-Site Higgsless Model (aka BESS)



- Global SU(2) x SU(2) x SU(2) $\rightarrow SU(2)_V$ symmetry
- Gauged $SU(2)_1$ x $SU(2)_2$ x U(1)

$$L_2 = \frac{f^2}{4}\sum_{i=1}^{2} Tr\left[D_\mu \Sigma_i^{+} D^\mu \Sigma_i\right] - \frac{1}{2g_0^2} Tr\left[L_{\mu\nu}\right]^2 - \frac{1}{2g_1^2} Tr\left[V_{\mu\nu}\right]^2 - \frac{1}{2g_2^2} Tr\left[R_{\mu\nu}\right]^2$$

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 - i L_\mu \Sigma_1 + i \Sigma_1 V_\mu$$

$$D_\mu \Sigma_2 = \partial_\mu \Sigma_2 - i V_\mu \Sigma_2 + i \Sigma_2 R_\mu$$

- Model looks like SM in limit

$$x = \frac{g_0}{g_1} << 1 \qquad y = \frac{g_2}{g_1} << 1 \quad \Rightarrow \quad g_0^2 \approx \frac{4\pi\alpha}{s_W^2} \qquad g_2^2 \approx \frac{4\pi\alpha}{c_W^2}$$

Scales

$\Lambda \sim 10$ TeV — Strong Coupling

$M_{W'} \sim 1$ TeV — Weakly coupled non-linear σ model

M_W

Calculate log-enhanced contributions to S

3-Site Higgsless Model

- At tree level, $\alpha S_{tree} = \frac{4\alpha}{g_1^2} = \frac{4 s_W^2 M_W^2}{M_{W'}^2}$

Requires $M_{W'}$> 3 TeV

Problem with unitarity

- Get around this by delocalizing fermions

$$L_f = -x_1 \overline{\psi}_L i(D\Sigma_1)\Sigma_1^+ \psi_L \Rightarrow L_f = \vec{J}_L^\mu \cdot \left\{ (1 - x_1) L_\mu + x_1 V_\mu \right\} + J_Y^\mu B_\mu$$

- Ideal delocalization: pick x_1 to make S vanish at tree level

$$\alpha S_{tree} = \frac{4 s_W^2 M_W^2}{M_{W'}^2} \left(1 - \frac{x_1 M_{W'}^2}{2 M_W^2} \right)$$

What happens at 1-loop?

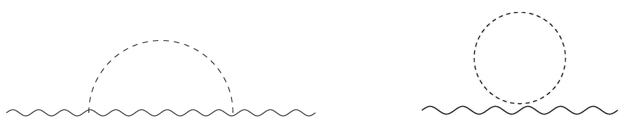
The Problem with Gauge Boson Loops

$$D_S^{\mu\nu}(q^2) = \frac{ig^{\mu\nu}}{q^2 - M_S^2}$$

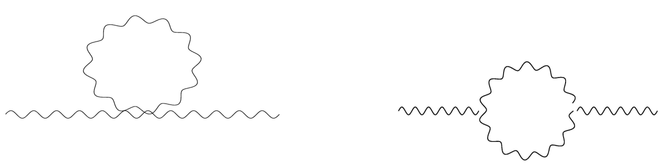
$$D_{G_i}^{\mu\nu}(q^2) = \frac{ig^{\mu\nu}}{q^2 - \xi M_{G_i}^2}$$

$$D_V^{\mu\nu}(q^2) = \frac{i}{q^2 - M_V^2}\left(-g^{\mu\nu} + (1-\xi)\frac{q^\mu q^\nu}{q^2 - \xi M_V^2}\right)$$

ξ=1 (Feynman), ξ=0 (Landau), $\xi \to \infty$ (Unitary)



Independent of ξ



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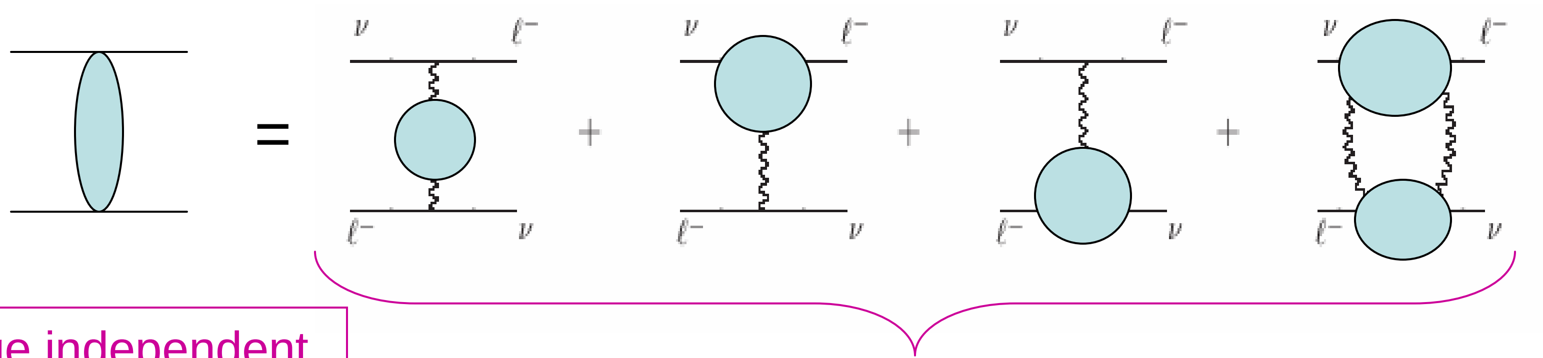
Depends on ξ

Pinch Technique



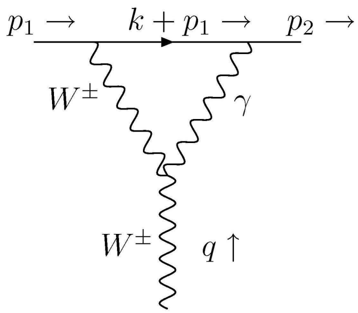
Gauge independent

Individually gauge dependent

- STU extracted from 4-fermion interactions
- Idea:
  - Isolate gauge-dependent terms in vertex/box diagrams
  - Combine with 2-point diagrams
  - Result is gauge-independent
  - q^4 and q^6 terms cancel in unitary gauge

Degrassi, Kniehl, & Sirlin, PRD48 (1993) 3963;
Papavassiliou & Sirlin, PRD50 (1994) 5951

Pinch Technique

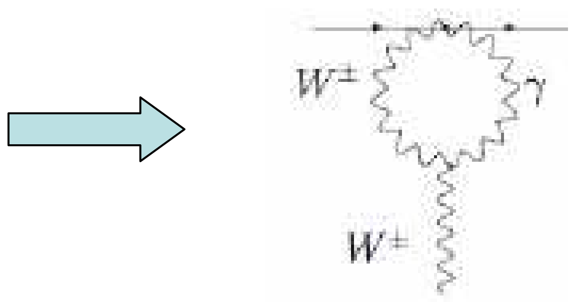


$$\approx \int \frac{d^n k}{(2\pi)^n} \bar{u}(p_2)\{\ldots(k+p_1)k\ldots\}u(p_1)\frac{1}{(k^2-M_W^2)(k+p_1)^2(k-q)^2}$$

$$= \int \frac{d^n k}{(2\pi)^n} \bar{u}(p_2)\{\ldots(k+p_1)(k+p_1-p_1)\ldots\}u(p_1)\frac{1}{(k^2-M_W^2)(k+p_1)^2(k-q)^2}$$

$$= \int \frac{d^n k}{(2\pi)^n} \bar{u}(p_2)\{\ldots(k+p_1)^2\ldots\}u(p_1)\frac{1}{(k^2-M_W^2)(k+p_1)^2(k-q)^2} + \ldots$$

Extract terms from vertex/box corrections that look like 2-point functions

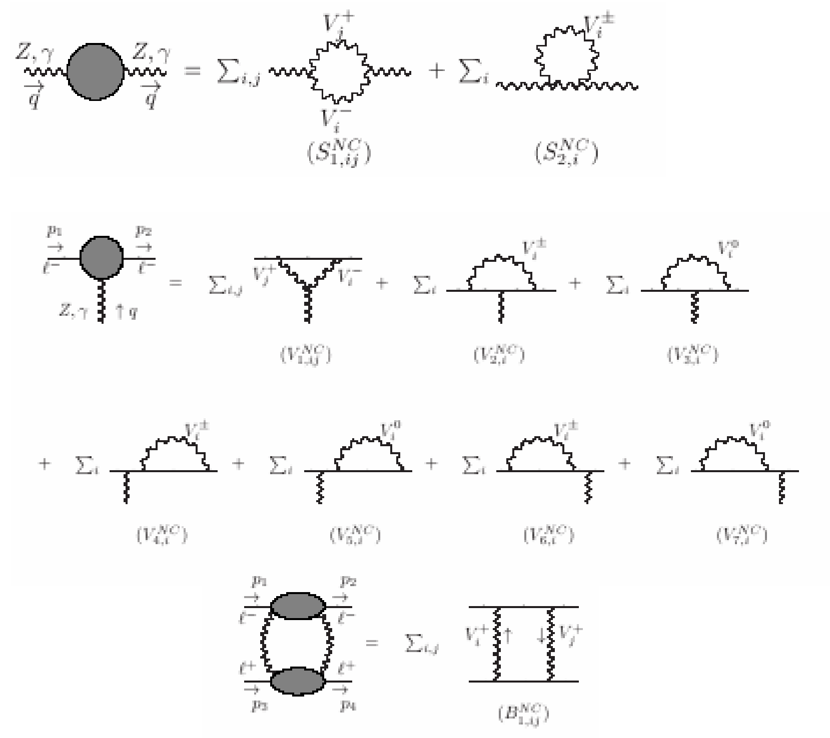


Associate this piece with propagator

Pinch Technique

- Calculate pinch 2-point functions with SM gauge sector plus W’,Z’
- Unitary gauge minimizes number of diagrams
- Neutral gauge boson 2-point functions:

$$A_{\gamma}^{1-loop} = \frac{A_{\gamma}^{0}}{q^2}\left[\Pi_{\gamma\gamma} + 2q^2 V_{\gamma} + q^4 B_{\gamma\gamma}\right] \equiv \frac{A_{\gamma}^{0}}{q^2}\Pi_{\gamma\gamma}^{Pinch}$$



Unknown Terms

- Unknown higher-dimensional operators
  - Generalization of α_1 term

$$L_4 = c_1 g_1 g_2 Tr\left[V_{\mu\nu}\Sigma_2 B^{\mu\nu}\Sigma_2^+\right] + c_2 g_1 g_0 Tr\left[L_{\mu\nu}\Sigma_1 V^{\mu\nu}\Sigma_1^+\right]$$

- L_2 gives poles at one loop: $1/\varepsilon \rightarrow \mathrm{Log}\,(\Lambda^2/M_W^2)$
  - Poles absorbed in redefinition of arbitrary couplings, c_1 and c_2
- ***Approach only makes sense if logarithms dominate***
- Results have scheme dependence

$$S_0 = -8\pi\left(c_1(\Lambda^2) + c_2(\Lambda^2)\right)$$

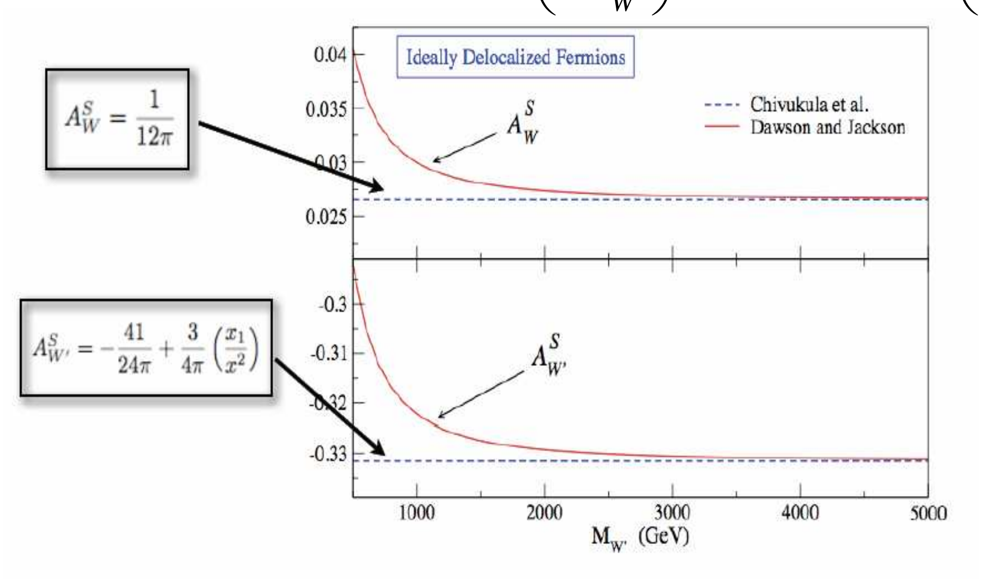
Leading Chiral Logarithm

- To leading order in $x=2M_W/M_{W'}$
  - Landau and also Feynman gauge (Chivukula, Simmons….)

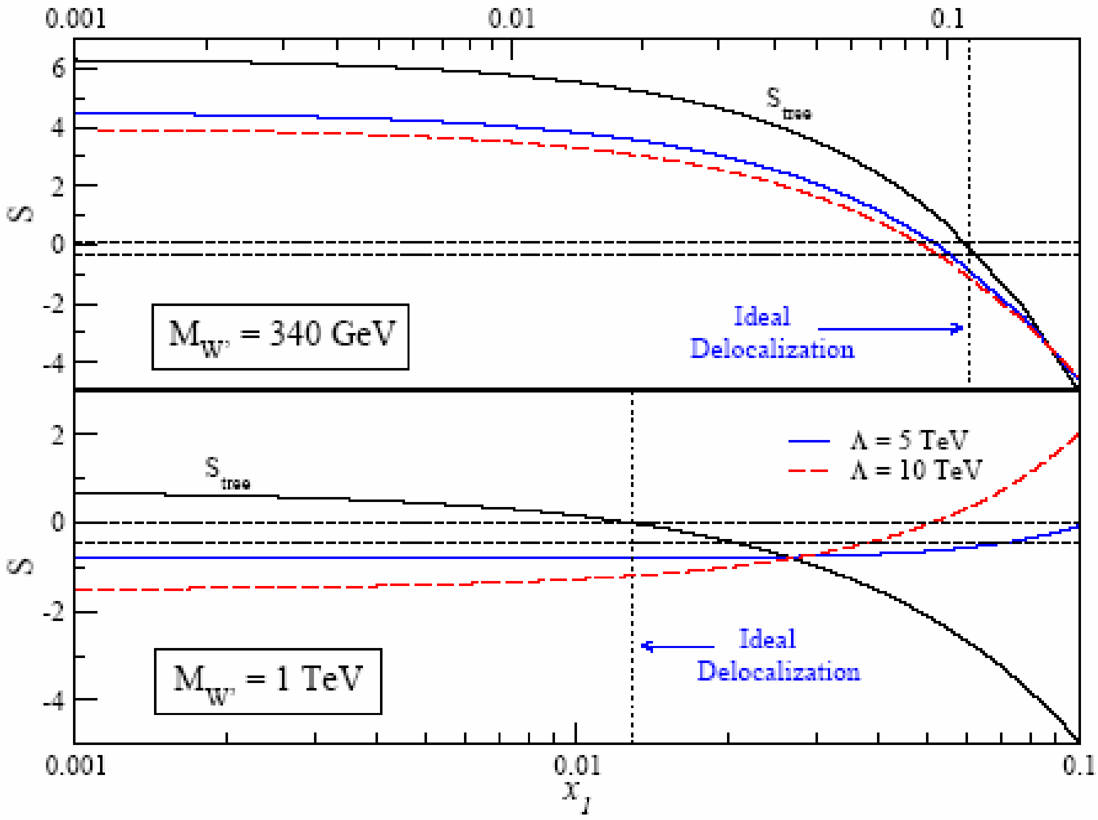
$$\alpha S_{1-loop} = \frac{\alpha}{12\pi}\ln\left(\frac{M_{W'}^2}{M_W^2}\right) - \frac{41\alpha}{24\pi}\ln\left(\frac{\Lambda^2}{M_{W'}^2}\right) + \frac{3\alpha}{4\pi}\frac{x_1}{x}\ln\left(\frac{\Lambda^2}{M_{W'}^2}\right)$$

- Unitary gauge, keeping subleading terms in x

$$S_{3-site} = S_{tree+} A_W^S \ln\left(\frac{M_{W'}^2}{M_W^2}\right) + \left(A_{W'}^S + A_W^S\right)\ln\left(\frac{M_{W'}^2}{M_W^2}\right) + S_0$$



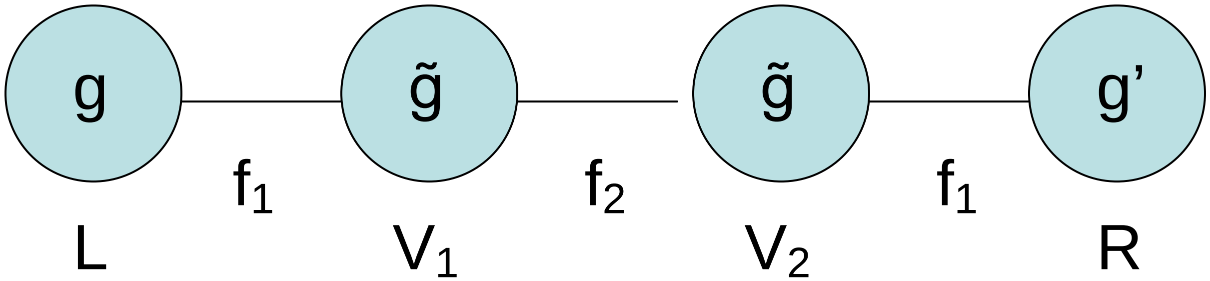
Large Corrections in 3-Site Model at 1-loop



Ideal localization only fixes S problem at tree level

See talk by de Curtis

4-Site Model



- Gauge symmetry: $SU(2)_L \times SU(2)_{V1} \times SU(2)_{V2} \times U(1)_Y$
- Why?
  - With $f_2 \neq f_1$ can "mimic" warped RS models
  - Gauge couplings approximately SM strength if:

$$x \equiv \frac{g}{\tilde{g}} << 1 \quad y \equiv \frac{g'}{\tilde{g}} << 1 \qquad g^2 \approx \frac{4\pi\alpha}{s^2}, \quad g'^2 \approx \frac{4\pi\alpha}{c^2}$$

  - Gauge sector: SM + 2 sets of heavy gauge bosons,

$\rho_i^{\pm}$, ρ^0, i=1,2

Accomando et al, arXiv:0807.5051; Chivukula, Simmons, arXiv: 0808.2017

Compute Masses & Mixings

$$L = \sum_{i=1}^{3} \frac{f_i^{\,2}}{4} Tr\left[D^{\mu}\Sigma_i D_{\mu}\Sigma_i^{+}\right]$$

$$\begin{aligned}
D_\mu\Sigma_1 &= \partial_\mu\Sigma_1 - igL_\mu\Sigma_1 + i\tilde{g}\Sigma_1 V_{1\mu} \\
D_\mu\Sigma_2 &= \partial_\mu\Sigma_2 - i\tilde{g}V_{1\mu}\Sigma_2 + i\tilde{g}\Sigma_2 V_{2\mu} \\
D_\mu\Sigma_3 &= \partial_\mu\Sigma_3 - i\tilde{g}V_{2\mu}\Sigma_3 + ig'\Sigma_3 R_\mu
\end{aligned}$$

- Compute masses as expansion in x

$$\begin{aligned}
M_W^2 &= \frac{g^2}{4}\frac{f_1^{\,2} f_2^{\,2}}{f_1^{\,2} + 2 f_2^{\,2}}\left(1 - x^2 z_w\right) \\
M_{\rho_1^\pm}^2 &= \frac{\tilde{g}^2 f_1^{\,2}}{4}\left(1 + \frac{x^2}{2}\right) \\
M_{\rho_1^\pm}^2 &= \frac{\tilde{g}^2\left(f_1^{\,2} + 2 f_2^{\,2}\right)}{4}\left(1 + \frac{x^2}{2} z^4\right)
\end{aligned}$$

$$z = \frac{f_1}{\sqrt{f_1^{\,2} + 2 f_2^{\,2}}}$$

$$z_W = \frac{1}{2}\left(1 + z^4\right)$$

- 3-site limit: $f_2 \to \infty$, $z \to 0$, ρ_2 decouples

Delocalizing Fermions in 4-Site Model

- One-Site delocalization $L = -x_1 \overline{\psi} \gamma^\mu i \left(D_\mu \Sigma_1 \right) \Sigma_1^{\ +} P_L \psi$

$$L_f = g' \overline{\psi} \gamma^\mu \left(Y_L P_L + Y_R P_R \right) B^\mu \psi + g \left(1 - x_1 \right) \overline{\psi} \gamma^\mu T^a L^{a,\mu} P_L \psi + \tilde{g} x_1 \overline{\psi} \gamma^\mu T^a V_1^{a,\mu} P_L \psi$$

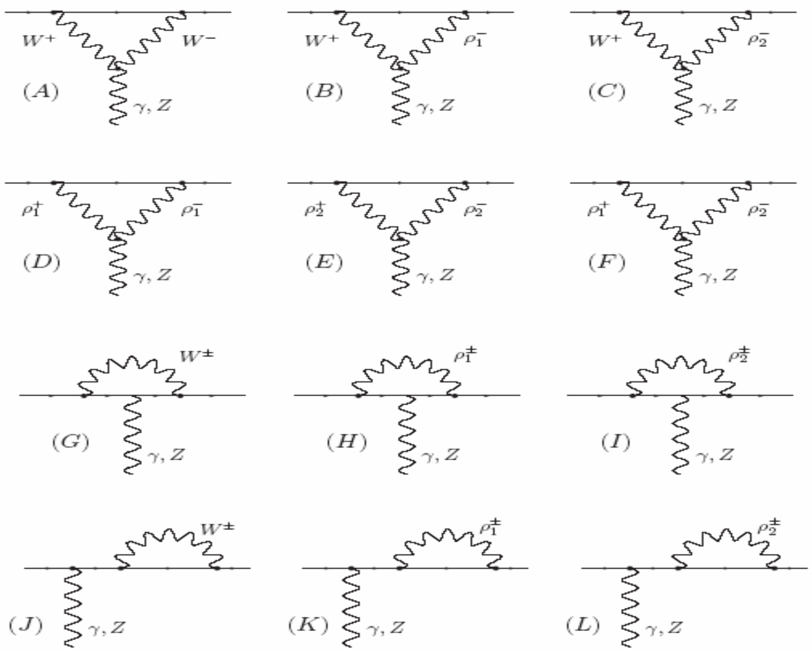
- Contribution to S at tree level

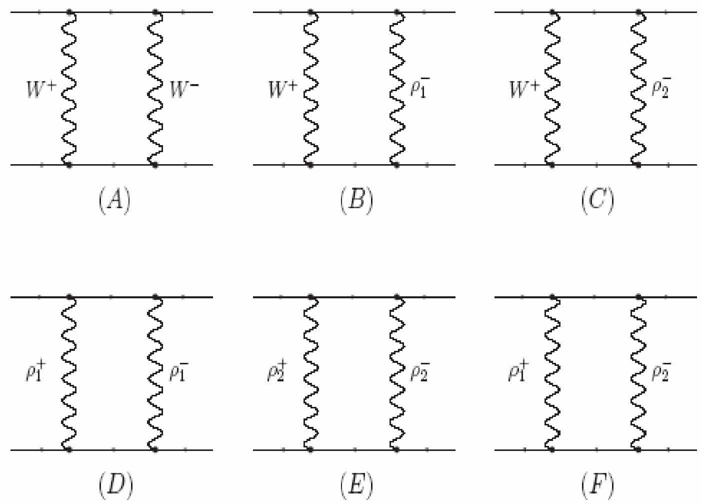
$$\alpha S_{tree} = \frac{4 s_W^2 M_W^2}{M_{\rho_1^0}^2} \left[1 + \frac{M_{\rho_1^0}^2}{M_{\rho_2^0}^2} - x_1 \left(\frac{4 M_{\rho_1^0}^2}{g^2 f_1^2} \right) \right]$$

$$\approx \frac{4 s_W^2 M_W^2}{M_{\rho_1^\pm}^2} \left[1 + z^2 - \frac{x_1 M_{\rho_1^\pm}^2}{2 M_W^2} \left(1 - z^2 \right) \right] + O(x^2)$$

- Pick x_1 to minimize S_{tree}

Calculate S Using Pinch Technique





Just like 3-Site calculation, but more of it….

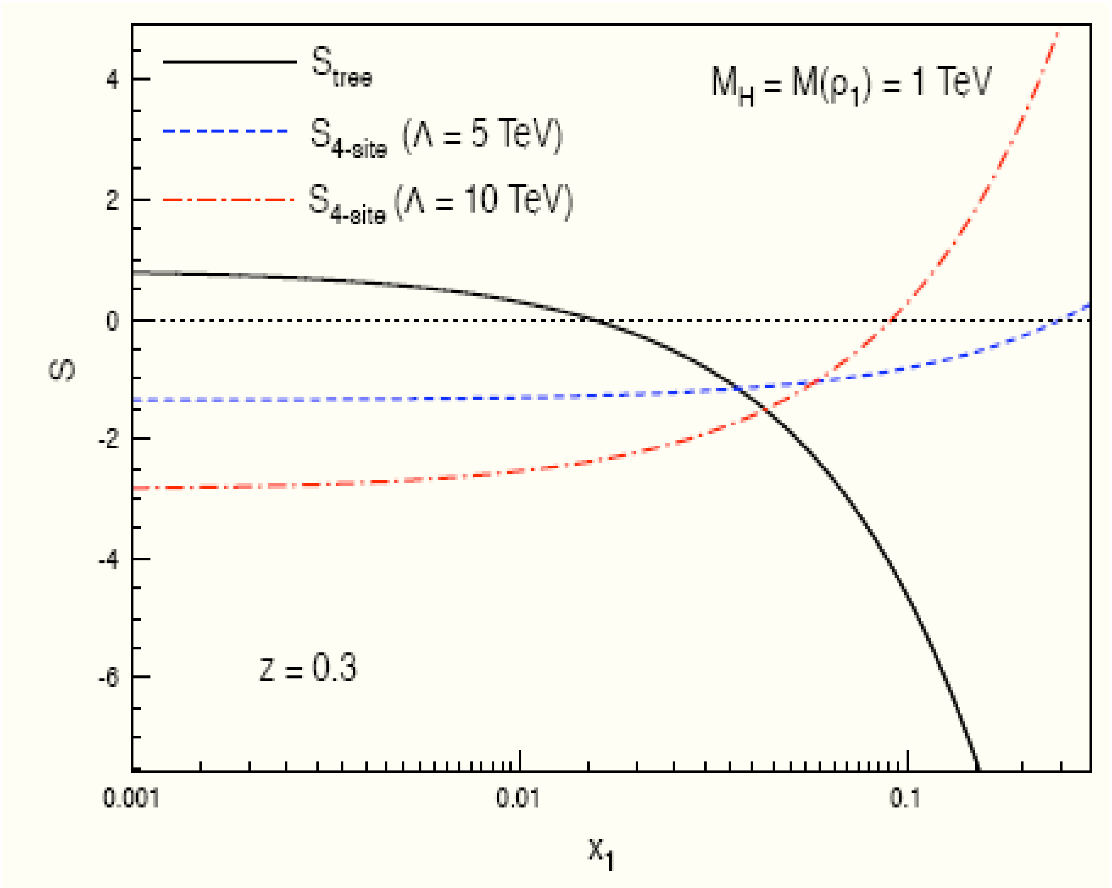
S at 1-Loop in 4-Site Model

$$\alpha S_{4-Site} = \frac{4 s_W^2 M_W^2}{M_{\rho_1^\pm}^2}\left(1+z^2-\frac{x_1 M_{\rho_1^\pm}^2}{2M_W^2}\left(1-z^2\right)\right)+\frac{\alpha}{12\pi}\ln\left(\frac{M_{\rho_1^\pm}^2}{M_{H,ref}^2}\right)$$
$$-\frac{\alpha}{\pi}\left[\frac{41+z^2+17z^4+17z^6}{24}-\frac{3x_1}{4x^2}\left(1+z^2\right)\right]\ln\left(\frac{M_{\rho_2^\pm}^2}{M_{\rho_1^\pm}^2}\right)$$
$$-\frac{\alpha}{\pi}\left[\frac{83-16z^2-33z^4}{24}-\frac{3x_1}{4x^2}\left(1+z^2\right)\right]\ln\left(\frac{\Lambda^2}{M_{\rho_2^\pm}^2}\right)$$
$$-8\pi\alpha\left[\left(1-z^2\right)\left(c_1(\Lambda)+c_2(\Lambda)\right)+\left(1+z^4\right)c_3(\Lambda)\right]$$

Note scaling between different energy regimes

Dawson & Jackson, arXIv:0810.5068

One-Loop Results in 4-Site Model



$$z = \frac{M_{\rho_1^\pm}}{M_{\rho_2^\pm}}$$

Large fine tuning needed at 1-loop

The Moral of the Story is….

- *Triplet models*
  - Can't use STU approach in triplet models
  - Triplet models can fit EW data if heavy scalars roughly degenerate
  - Minimizing the potential requires small mixing in neutral and charged *sectors*

- *2, 3, 4 Site Higgsless models*
  - Ideal delocalization at tree level doesn't solve problem at 1-loop
  - Unknown coefficients make predictions problematic