Mass Measurements with Missing Energy

(A bit of spin measurement also)

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   - Endpoint Method
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Mission of the LHC: Search for new physics beyond the SM

Motivations for New Physics at the TeV Scale:

- Hierarchy Problem

  $$\delta m_H^2 \sim \frac{g^2}{8\pi^2}\Lambda_{\rm SM}^2 \sim M_Z^2 \quad \Longrightarrow \quad \Lambda_{\rm SM} \sim 1\ {\rm TeV}$$

- Dark Matter

  Thermal WIMP with $\Omega_{\rm DM}h^2 \sim \frac{0.1}{g^4}\left(\frac{m_{\rm DM}}{1\ {\rm TeV}}\right)^2 \sim 0.1$

  $$\Longrightarrow \quad m_{\rm DM} \sim 1\ {\rm TeV}$$

New physics models solving the hierarchy problem while giving a DM candidate typically involve a Z_2 symmetry under which the predicted new particles are odd, while the SM particles are even:

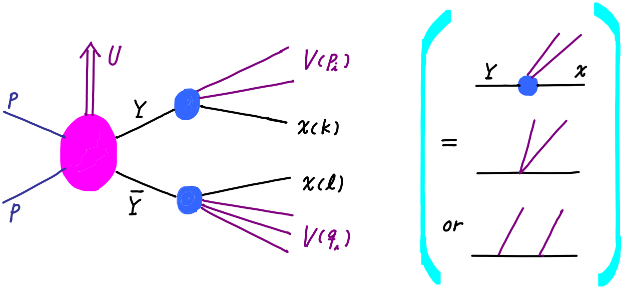
SUSY with R-parity, Little Higgs with T-parity, UED with KK-parity, ...

Z_2 Symmetry $\Rightarrow$ Lightest Z_2-odd particle χ is (quasi)stable, so it is a good candidate for a WIMP-like DM.

- LHC Signal: Multi-Jet (possibly with isolated leptons) Events with Large Missing Transverse Momentum $\not{p}_T$

Pair-produced new particle (Y) eventually decaying into visible SM particles (V) plus an invisible WIMP (χ):

$$pp \;\rightarrow\; Y + \bar{Y} \;\rightarrow\; \sum V(p_i) + \chi(k) + \sum V(q_j) + \chi(l)$$



($U \equiv$ Upstream momentum $=$ Momenta carried by the SM particles not from the decay of $Y + \bar{Y}$.)

Mass measurement of those new particles is quite non-trivial:

(i) initial parton momenta in the beam-direction are unknown,
(ii) each event involves two missing WIMPs.

Methods of mass measurement with missing energy

- Endpoint Method
- Mass Relation Method
- M_{T2}-Kink Method

May determine the new particle masses with $\mathcal{O}(\text{few})\ \%$ accuracy at the high luminosity phase of LHC if the new physics events can be identified with a rather good measurement of the visible momenta and $\not{\mathbf{p}}_T$.

Other possibilities:

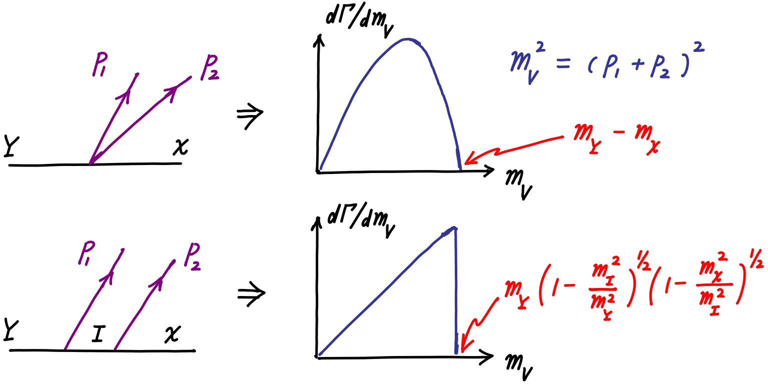
- Some Variants or Hybrids
- Production Cross Section: Too much model-dependent
- M_{eff} , M_{TGen}: Just a crude estimate

Basic Idea of Mass Measurement Method

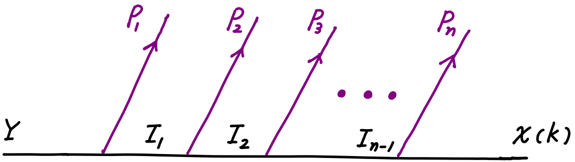
- **Endpoint Method**

Hinchliffe, Paige, Shapiro, Soderqvist, Yao; Bachacou, Hinchliffe, Paige; Allanach, Lester, Parker, Webber; Gjelsten, Miller, Osland; ...

Endpoint value of the invariant mass distribution of visible (SM) decay products depend on the new particle masses.



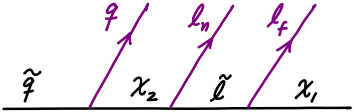
n-step cascade decay:



Number of measurable invariant mass distributions: $2^n - (n+1)$
Number of unknown new particle masses: $n+1$.

$\Longrightarrow$ For $n \geq 3$, there can be enough number of independent endpoint values to determine all masses of the produced new particles.

Squark cascade decay when $m_{\tilde{q}} > m_{\chi_2} > m_{\tilde{\ell}} > m_{\chi_1}$:



$$
\begin{aligned}
m_{\ell\ell}^{\max} &= m_{\chi_2}\sqrt{(1-m_{\tilde{\ell}}^2/m_{\chi_2}^2)(1-m_{\chi_1}^2/m_{\tilde{\ell}}^2)} \\
m_{q\ell\ell}^{\max} &= m_{\tilde{q}}\sqrt{(1-m_{\chi_2}^2/m_{\tilde{q}}^2)(1-m_{\chi_1}^2/m_{\chi_2}^2)} \\
m_{q\ell(\text{high})}^{\max} &= m_{\tilde{q}}\sqrt{(1-m_{\tilde{\chi}_2}^2/m_{\tilde{q}}^2)(1-m_{\chi_1}^2/m_{\tilde{\ell}}^2)} \\
m_{q\ell(\text{low})}^{\max} &= m_{\tilde{q}}\sqrt{(1-m_{\chi_2}^2/m_{\tilde{q}}^2)(1-m_{\tilde{\ell}}^2/m_{\chi_2}^2)} \\
\Big(m_{q\ell(\text{high})} &\equiv \max(m_{q\ell_n}, m_{q\ell_f}), \quad m_{q\ell(\text{low})} \equiv \min(m_{q\ell_n}, m_{q\ell_f}) \Big)
\end{aligned}
$$

Other relations are possible.

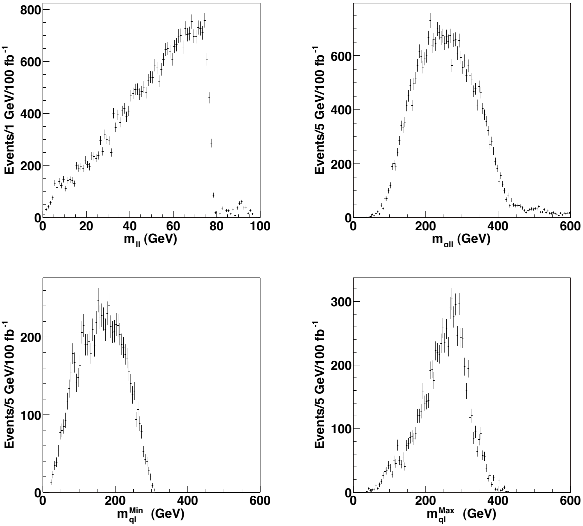
Real life is not so simple!

We have to deal with

- Combinatorics to identify the location of each particle in the event
- Energy-momentum resolution of detector
- Backgrounds
- ...

$\Longrightarrow$ Errors

Result for SUSY SPS1a Point: Weiglein et. al. hep-ph/0410364



Input masses: $(m_{\tilde{q}}, m_{\chi_2}, m_{\tilde{\ell}}, m_{\chi_1}) = (540, 177, 143, 96)$ GeV

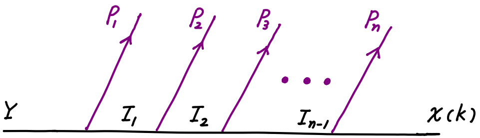
Fitted masses: $(543 \pm 13,\ 180 \pm 9,\ 146 \pm 11,\ 98 \pm 9)$ $\quad (\int \mathcal{L} = 100\,\text{fb}^{-1})$

- **<u>Mass Relation Method</u>**

Nojiri, Polesello, Tovey; Kawagoe, Nojiri, Polesello; Cheng, Engelhardt, Gunion, Han, McElrath; ...

Reconstruct the missing momentum with on-shell constraints.

***n*-step cascade decay:**



Number of on-shell constraints for N-events: $(n+1)N$

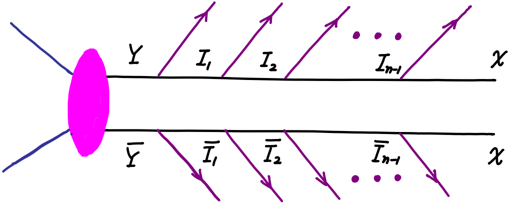
$$k^2 = m_\chi^2, \quad (k+p_n)^2 = m_{I_{n-1}}^2, \ldots \quad (k+p_1+\ldots+p_n)^2 = m_Y^2$$

Number of unknowns: $4N + (n+1)$

(N-missing momenta and $(n+1)$-unknown masses)

$\Longrightarrow$ For $n \geq 4$, on-shell mass relations provide more constraints than those necessary for reconstructing the missing momenta, and thus can give non-trivial constraints on the new particle masses.

Symmetric cascade decays with on-shell and $\not{p}_T$ constraints:



Number of constraints for N-events: $[2(n+1)+2]\,N$

(mass relations + $\not{p}_T$ constraints)

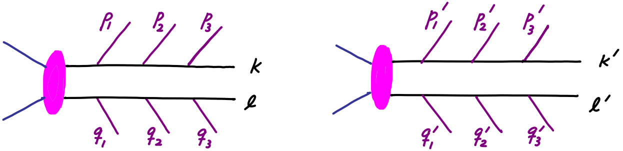
Number of unknowns: $8N + (n+1)$

($2N$-missing momenta $+(n+1)$-unknown masses)

$\Longrightarrow$ For $n \geq 3$, on-shell mass relations and $\not{p}_T$ constraints provide more constraints than those necessary for reconstructing the missing momenta.

For $n = 3$, all the four new particle masses might be determine by combining the constraints from two events.

Cheng, Engelhardt, Gunion, Han, McElrath



- 16 unknowns: k^μ, l^μ, k'^μ, l'^μ
- 12 mass-shell constraints: $k^2 = l^2 = k'^2 = l'^2$,

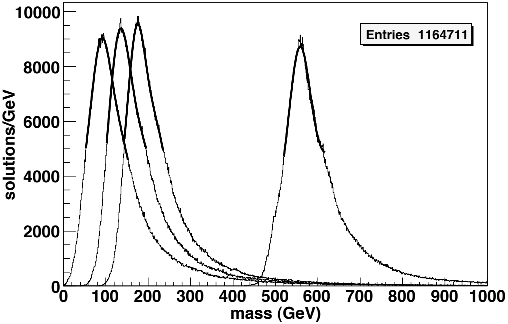
$(k + p_3)^2 = (l + q_3)^2 = (k' + p'_3)^2 = (l' + q'_3)^2$,
$(k + p_2 + p_3)^2 = (l + q_2 + q_3)^2 = (k' + p'_2 + p'_3)^2 = (l' + q'_2 + q'_3)^2$,
$(k + p_1 + p_2 + p_3)^2 = (l + q_1 + q_2 + q_3)^2 = (k' + p'_1 + p'_2 + p'_3)^2$
$= (l' + q'_1 + q'_2 + q'_3)^2$,

- 4 $\not{\mathbf{p}}_T$-constraints: $\mathbf{k}_T + \mathbf{l}_T = \not{\mathbf{p}}_T$, $\quad \mathbf{k}'_T + \mathbf{l}'_T = \not{\mathbf{p}}'_T$

8 complex solutions for each event-pair, of which more than one can be real, and many wrong solutions from wrong combinatorics.

Correct masses have better chance to give a real solution.

Number of mass solutions for multi-event-pairs, including the errors in real detector simulation and employing the cut reducing wrong combinatorics: Cheng, Engelhardt, Gunion, Han, McElrath



Input masses: $(m_{\tilde{q}}, m_{\chi_2}, m_{\tilde{\ell}}, m_{\chi_1}) = (568, 180, 143, 97)$ GeV

Fitted masses: $(562 \pm 4,\ 179 \pm 3,\ 139 \pm 3,\ 94 \pm 3) \quad (\int \mathcal{L} = 300\,\mathrm{fb}^{-1})$

M_{T2}-Kink Method

Cho, Choi, Kim, Park; Gripaios; Barr, Gripaios, Lester; Nojiri, Sakurai, Shimizu, Takeuchi; Barr, Ross, Serna; Burns, Kong, Matchev, Park; ...

Previous methods require a long cascade decay ($n \geq 3$) to determine the full new particle spectrum.

However, there are many well-motivated new physics models which do not give a long cascade decay: SUSY with $m_{sfermion} \gg m_{gaugino}$ (Focus point scenario, String moduli-mediation, Loop-split SUSY, ...)

- Mass relation method simply can not be applied.
- Endpoint methods can determine only the gaugino mass differences.
- M_{T2}-kink method can determine the full gaugino mass spectrum.

- **Transverse mass of decay products for $Y \to V(p) + \chi(k)$:**

$$M_T^2 = m_V^2 + m_\chi^2 + 2\sqrt{m_V^2 + |\mathbf{p}_T|^2}\sqrt{m_\chi^2 + |\mathbf{k}_T|^2} - 2\mathbf{p}_T \cdot \mathbf{k}_T$$

An analogue of the invariant mass $M^2 = (p+k)^2$, but independent of the momentum components in the beam-direction.

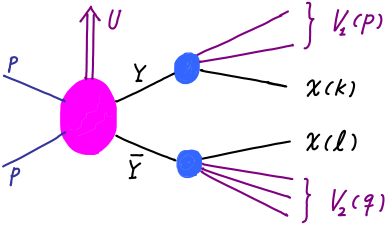
One may use an arbitrary trial WIMP mass m_χ to define M_T:
(True WIMP mass $= m_\chi^{\text{true}}$)

$$M_T(m_\chi = m_\chi^{\text{true}}) \leq m_Y^{\text{true}}$$

If m_χ^{true} is known, and $\mathbf{k}_T$ can be read off from $\not{\mathbf{p}}_T$, m_Y^{true} can be determined without knowing k_L by the endpoint of the transverse mass distribution. (Example: $W \to \ell(p) + \nu(k)$.)

M_{T2} is a generalization of M_T applied to generic new physics event with two missing particles: Lester and Summers

$$p + p \to Y + \bar{Y} \to V_1(p) + \chi(k) + V_2(q) + \chi(l)$$



$$M_{T2}(\text{event}; m_\chi) \qquad \Big(\{\text{event}\} = \{m_{V_1}, \mathbf{p}_T, m_{V_2}, \mathbf{q}_T, \not{\mathbf{p}}_T\}\Big)$$

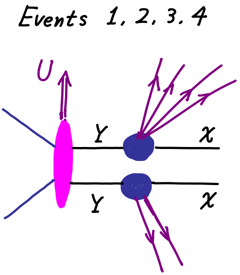
$$= \min_{\mathbf{k}_T + \mathbf{l}_T = \not{\mathbf{p}}_T} \left[\max\Big(M_T(\mathbf{p}_T, m_{V_1}, \mathbf{k}_T, m_\chi), M_T(\mathbf{q}_T, m_{V_2}, \mathbf{l}_T, m_\chi) \Big) \right]$$

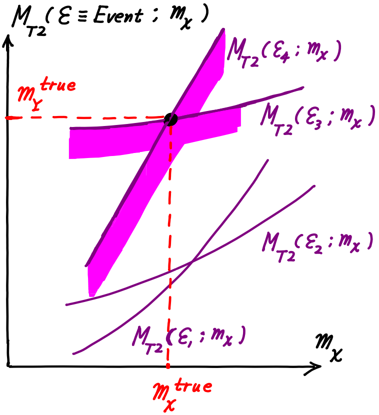
- For each event, $M_{T2}(\text{event}; m_\chi)$ is an increasing function of m_χ.
- $M_{T2}(\text{event}; m_\chi = m_\chi^{\text{true}}) \leq m_Y^{\text{true}}$ for all events.

M_{T2}**-Kink**: If the event set has an enough variety,

$$M_{T2}^{\max}(m_\chi) = \max_{\{\text{all events}\}} \left[M_{T2}(\text{event}; m_\chi) \right]$$

has a kink-structure at $m_\chi = m_\chi^{\text{true}}$ with $M_{T2}^{\max}(m_\chi = m_\chi^{\text{true}}) = m_Y^{\text{true}}$.





More Inclusive $\Longrightarrow$ Sharper Kink

What kind of variety ?

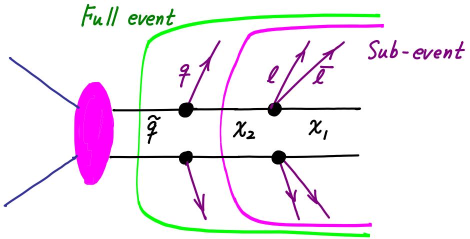
- The visible decay products of $Y \to V + \chi$ can have significantly different invariant masses: Cho, Choi, Kim, Park

  V is a multi-particle state.

- The event can have a large upstream transverse momentum U_T: Gripaios; Barr, Gripaios, Lester

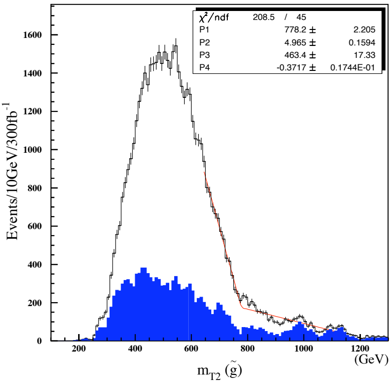
  Y is produced from the decay of heavier particle.

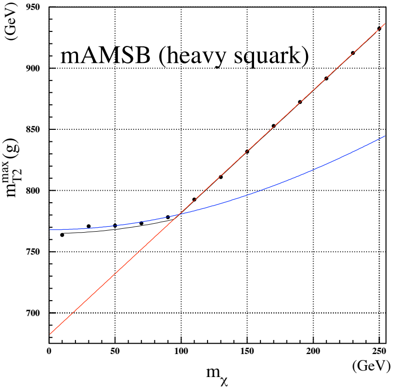
For cascade decays, M_{T2}-kink method can be applied to generic sub-event:



Gluino M_{T2}-Kink in heavy sfermion scenario:

Cho, Choi, Kim, Park





Input masses: $(m_{\tilde{g}}, m_{\chi_1}) = (780\,\text{GeV},\ 98\,\text{GeV})$ (Wino-like χ_1)

Fitted masses: $(776 \pm \text{few},\ 97 \pm \text{few}))$ $(\int \mathcal{L} = 300\,\text{fb}^{-1})$

First Application of M_{T2} to Real Data

CDF (Feb. 2009)

Using Only M_{T2} for the CDF Dilepton $t\bar{t}$ Data

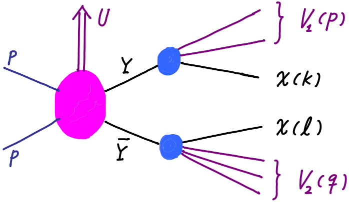
(3 fb^{-1})

$$m_t = 167.9^{+4.8}_{-4.1}(\mathrm{stat}) \pm 2.9(\mathrm{sys})\ \mathrm{GeV}$$

New Collider Variable for Spin Measurement

M_{T2}-Assisted-On-Shell (MAOS) Reconstruction of WIMP Momentum: Cho, Choi, Kim, Park, arXiv:0810.4853 [hep-ph]

The main difficulty of spin measurement arises from that the WIMP momenta k^μ and l^μ can not be reconstructed event-by-event.



If m_χ^{true} and m_Y^{true} are known, correct WIMP momenta can be reconstructed for the M_{T2}-endpoint events:

$$M_{T2}(\text{event}, m_\chi^{\text{true}}) = m_Y^{\text{true}}, \quad \mathbf{k}_T + \mathbf{l}_T = \not{\mathbf{p}}_T,$$

$$k^2 = l^2 = (m_\chi^{\text{true}})^2, \ (k+p)^2 = (l+q)^2 = (m_Y^{\text{true}})^2,$$

$$\Longrightarrow \quad \mathbf{k}^\mu = \mathbf{k}^\mu_{\text{true}}, \quad \mathbf{l}^\mu = \mathbf{l}^\mu_{\text{true}}$$

Even for generic new physics events, and even when $m_\chi^{\rm true}$ and $m_Y^{\rm true}$ are unknown, one can do a similar reconstruction of WIMP momenta.

Introduce trial WIMP and mother particle masses, $(m_\chi,\, m_Y)$, and impose the constraints:

$$k^2 = l^2 = m_\chi^2, \quad (k+p)^2 = (l+q)^2 = m_Y^2, \quad \mathbf{k}_T + \mathbf{l}_T = \not{\mathbf{p}}_T,$$
$$M_{T2}(p, q, \not{\mathbf{p}}, m_\chi) = M_T(p, \mathbf{k}_T, m_\chi) = M_T(q, \mathbf{l}_T, m_\chi)$$

$$\Longrightarrow \quad k^\mu = k^{\mu(\pm)}_{\rm maos}(p, q, \not{\mathbf{p}}_T, m_\chi, m_Y), \quad l^\mu = l^{\mu(\pm)}_{\rm maos}(p, q, \not{\mathbf{p}}_T, m_\chi, m_Y)$$

- If $m_\chi^{\rm true}$ and $m_Y^{\rm true}$ are known, use $m_\chi = m_\chi^{\rm true}$ and $m_Y = m_Y^{\rm true}$.
- Unless, one can simply use $m_\chi = 0$ and $m_Y = M_{T2}^{\rm max}(m_\chi = 0)$.

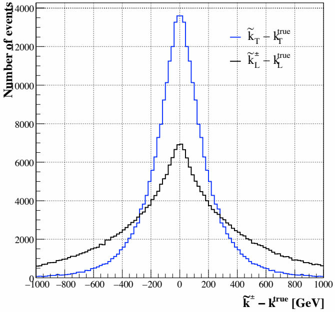
$\Longrightarrow$ **Event by event, MAOS momentum of each WIMP is determined (with two-fold ambiguity) in terms of the visible momenta and $\not{\mathbf{p}}_T$.**

For the purpose of spin measurement, MAOS momenta provide a good approximation for the unmeasurable true WIMP momenta.

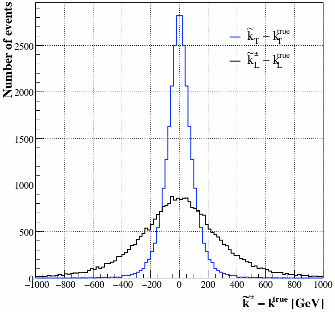
Example: 3-body decay of gluino pair for mSUGRA SPS2 point

$$\tilde{g}\tilde{g} \to q\bar{q}\chi_1 q\bar{q}\chi_1 \quad (m_{\tilde{g}} = 780\,\text{GeV},\ m_{\chi_1} = 122\,\text{GeV})$$

- Distribution of $\mathbf{k}_{\text{maos}} - \mathbf{k}_{\text{true}}$ for $m_\chi = 0$ and $m_Y = M_{T2}^{\max}(0)$.

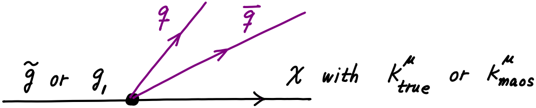


Full Events



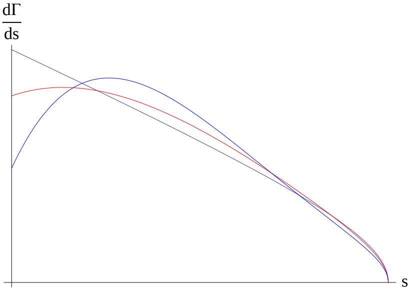
Near M_{T2}-Endpoint Events (10 %)

Invariant mass distributions:



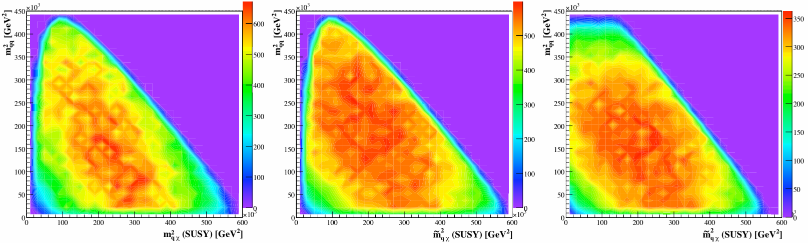
$$\mathbf{s} = (p_q + p_{\bar{q}})^2, \;\; \mathbf{t}_{\rm true} = (p_{q,\bar{q}} + k_{\rm true})^2, \;\; \mathbf{t}_{\rm maos} = (p_{q,\bar{q}} + k_{\rm maos})^2$$

Without $k^\mu_{\rm maos}$, one may consider **the s-distribution** to distinguish SUSY from UED: Csaki, Heinonen, Perelstein

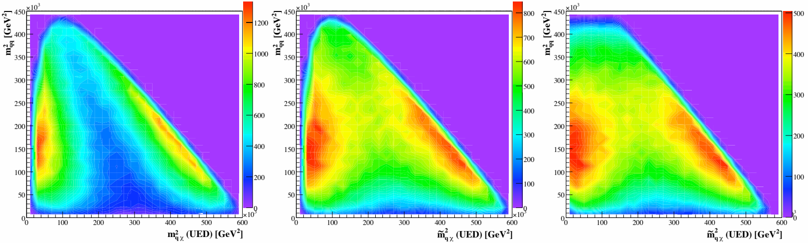


With $k^{\mu}_{\rm maos}$, one can use **the s-t$_{\rm maos}$ distribution**: Cho, Choi, Kim, Park

$$\frac{d\Gamma}{ds\,dt_{\rm true}} \qquad \frac{d\Gamma}{ds\,dt_{\rm maos}} \quad \text{for } (m_\chi^{\rm true}, m_Y^{\rm true}) \text{ and } (0, M_{T2}^{\max}(0))$$



gluino 3-body decay



KK-gluon 3-body decay

Summary

- There are several methods to determine new particle masses from missing energy events, (i) endpoint method, (ii) mass-relation method, (iii) M_{T2}-kink method, and also their variants or hybrids.

  These methods may determine new particle masses with $\mathcal{O}(\text{few})\ \%$ accuracy at the high luminosity phase ($\int \mathcal{L}_{\text{LHC}} \sim 100\ \text{fb}^{-1}$), while the efficiency of each method differs from case by case.

- A new collider variable, the MAOS momentum, has been introduced, which approximates the true WIMP momentum quite well, so can provide a powerful tool for spin measurement.