MSTW2008:
Parton Distributions for the LHC

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(with Alan Martin, Robert Thorne, Graeme Watt)

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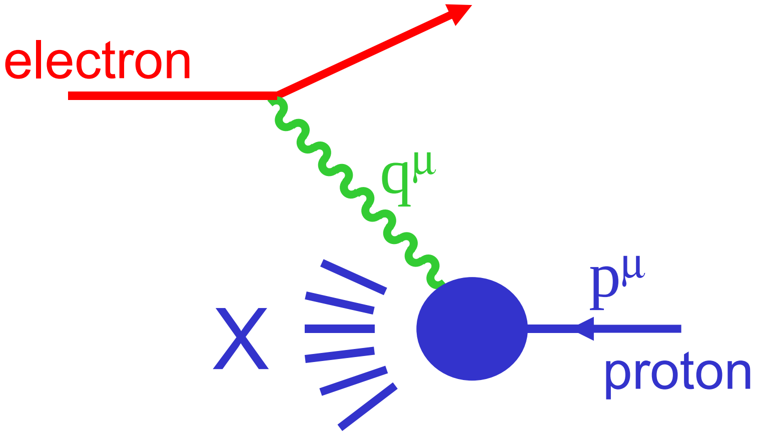
deep inelastic scattering and parton distributions

deep inelastic scattering



- variables

$Q^2 = -q^2$

$x = Q^2 / 2p \cdot q$ (Bjorken x)

$(y = Q^2 / x\, s)$

- resolution

$$\lambda = \frac{h}{Q} = \frac{2\times10^{-16}\,\mathrm{m\ GeV}}{Q}$$

at HERA, $Q^2 < 10^5\ GeV^2$

$\Rightarrow \lambda > 10^{-18}\ \mathrm{m} = r_p/1000$

- inelasticity

$$x = \frac{Q^2}{Q^2 + M_X^2 - M_p^2}$$

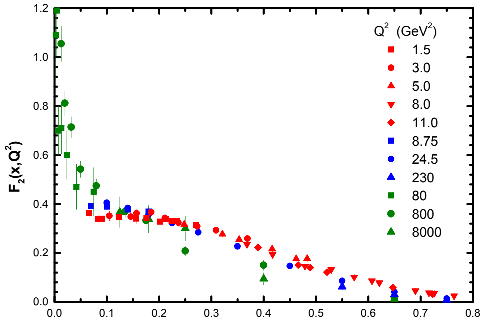
$\Rightarrow 0 < x \leq 1$

- in general, we can write

$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[y^2 F_1 + 2(1-y)x^{-1}F_2\right]$$

where the $F_i(x,Q^2)$ are called structure functions

Bjorken Scaling

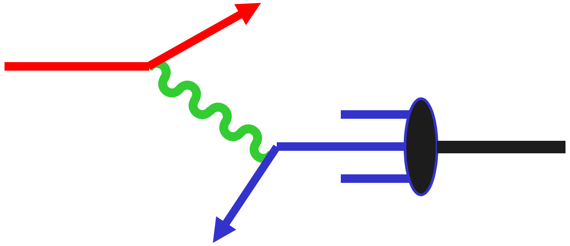


the parton model (Feynman 1969)

- photon scatters incoherently off massless, pointlike, spin-1/2 quarks

infinite momentum frame

- probability that a quark carries fraction ξ of parent proton's momentum is $q(\xi)$, $(0 < \xi < 1)$



$$F_2(x) = \sum_{q,\bar{q}} \int_0^1 d\xi \; e_q^2 \, \xi \, q(\xi) \, \delta(x-\xi) = \sum_{q,\bar{q}} e_q^2 \, x \, q(x)$$

$$= \frac{4}{9} x u(x) + \frac{1}{9} x d(x) + \frac{1}{9} x s(x) + \ldots$$

- the functions u(x), d(x), s(x), … are called parton distribution functions (pdfs) - they encode information about the proton's deep structure

extracting pdfs from experiment

- different beams (e,μ,ν,…) & targets (H,D,Fe,…) measure different combinations of quark pdfs

$$F_2^{ep} = \frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s}) + \ldots$$

$$F_2^{en} = \frac{1}{9}(u+\bar{u}) + \frac{4}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s}) + \ldots$$

$$F_2^{\nu p} = 2\left[d + s + \bar{u} + \ldots\right]$$

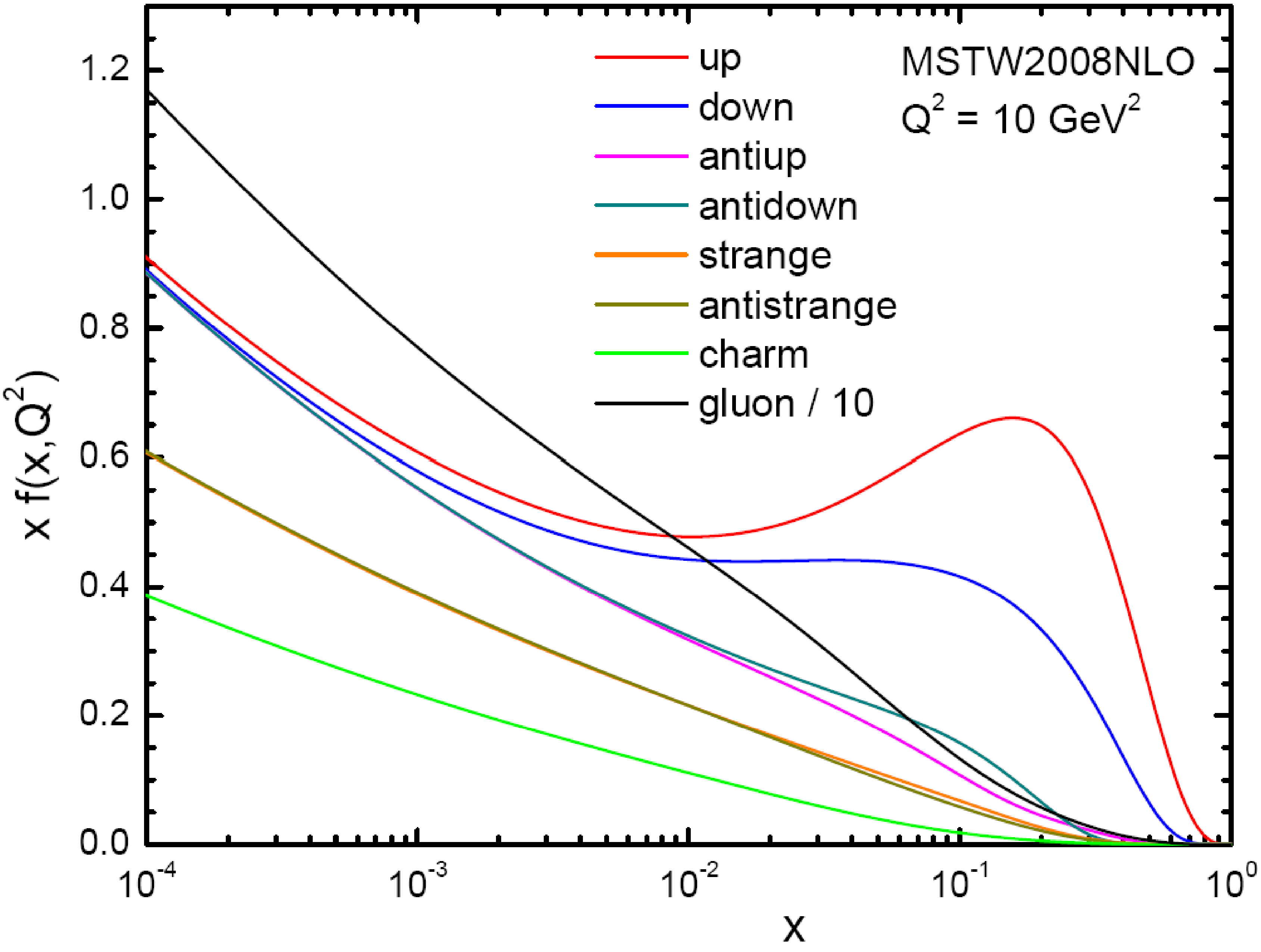
$$F_2^{\nu n} = 2\left[u + \bar{d} + \bar{s} + \ldots\right]$$

- thus the individual q(x) can be extracted from a set of structure function measurements

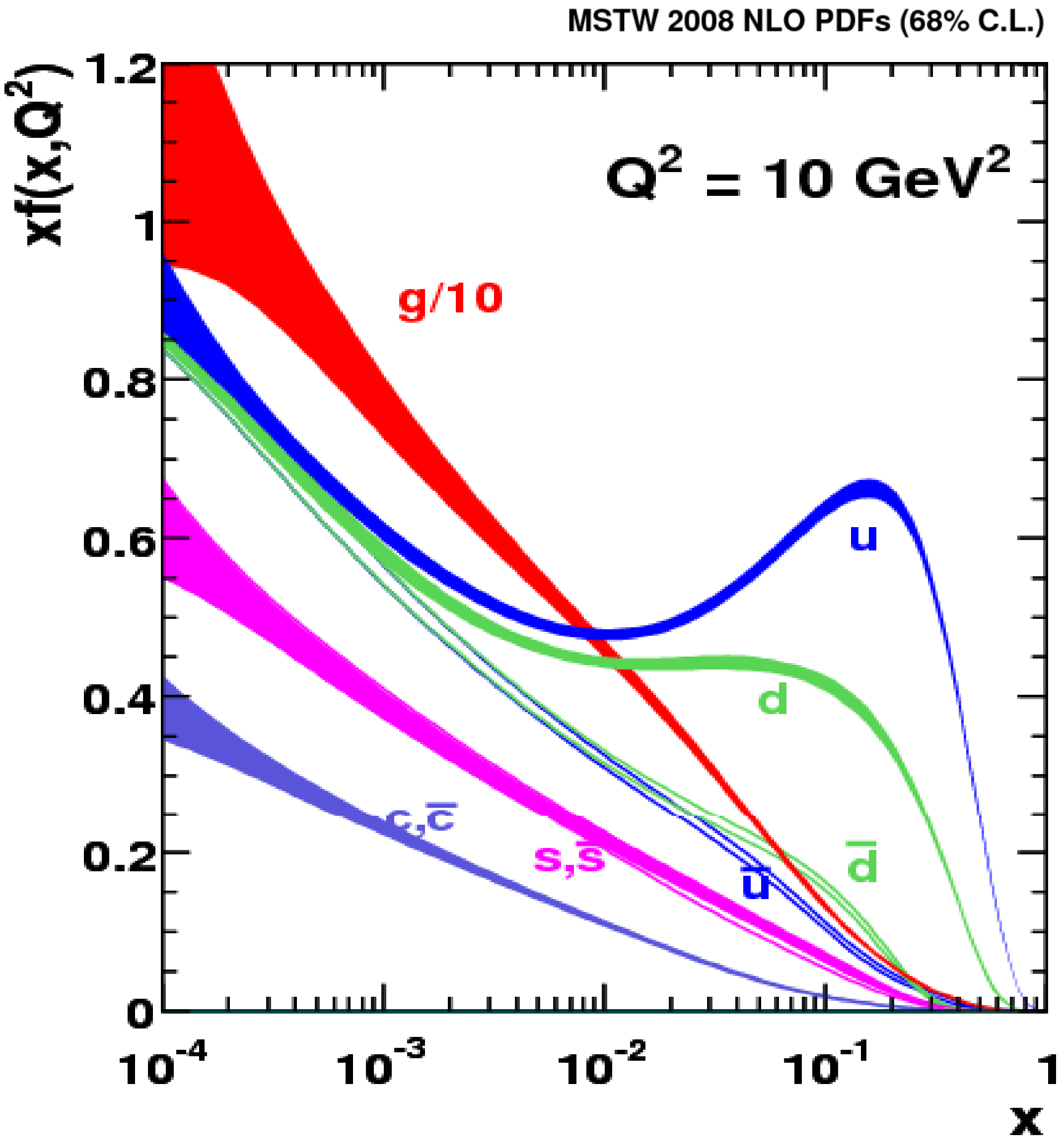
$$s = \bar{s} = \frac{5}{6}F_2^{\nu N} - 3F_2^{eN}$$

- gluon not measured directly, but carries about 1/2 the proton's momentum

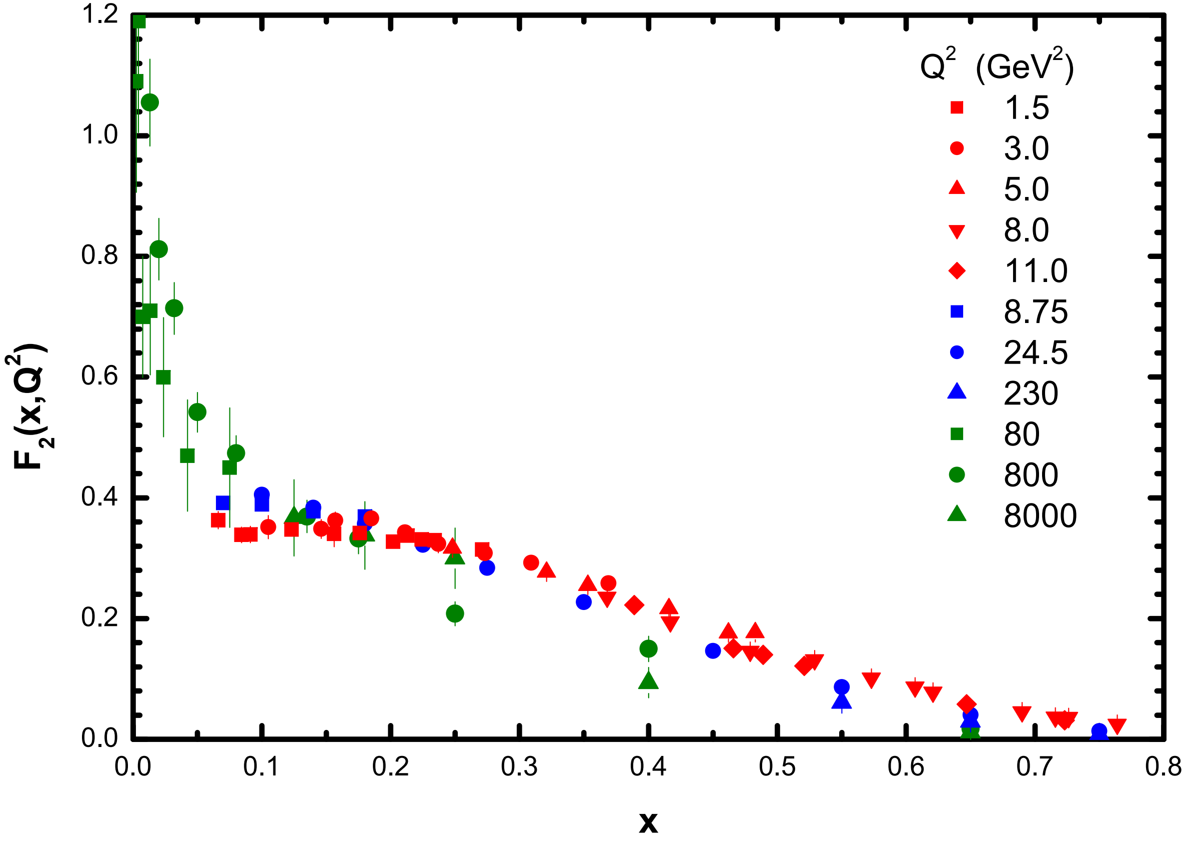
$$\sum_q \int_0^1 dx\, x\left(q(x) + \bar{q}(x)\right) = 0.55$$


up
down
antiup
antidown
strange
antistrange
charm
gluon / 10
MSTW2008NLO
Q^2 = 10 GeV^2
x f(x,Q^2)
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}

the data have errors, and therefore so do the pdfs…..

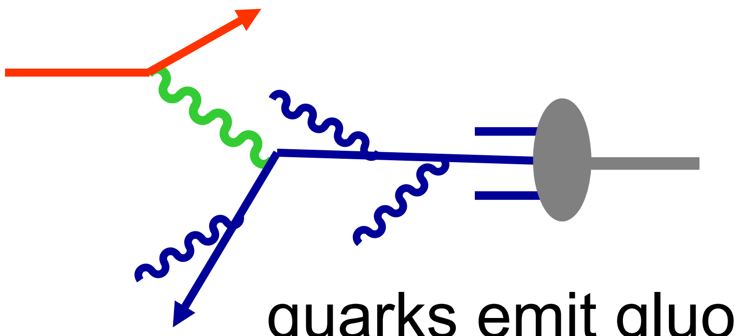


40 years of Deep Inelastic Scattering

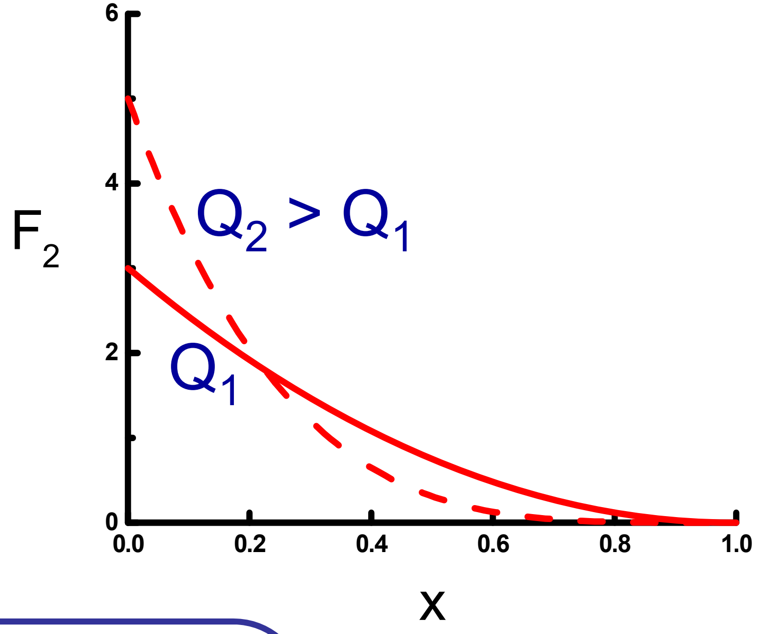


scaling violations and QCD

The structure function data exhibit systematic violations of Bjorken scaling:



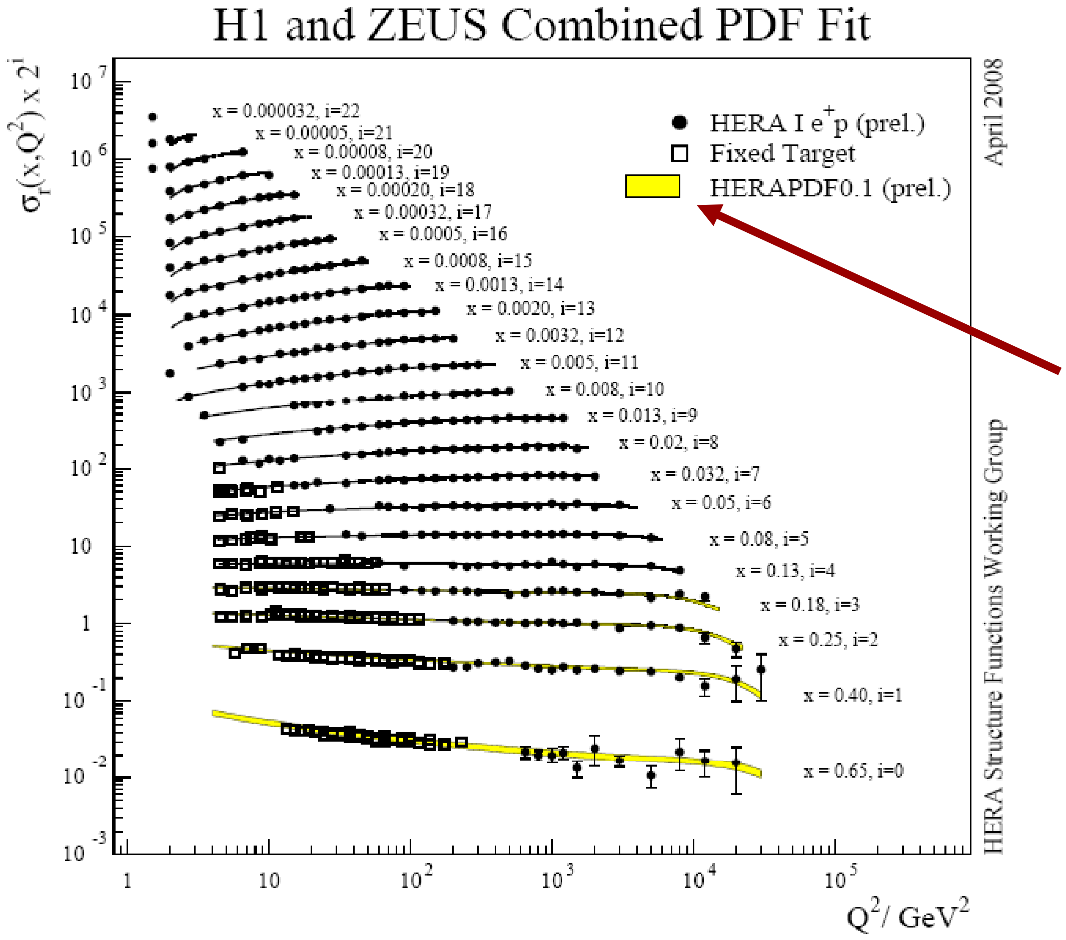
quarks emit gluons!



DGLAP equations

$$
\begin{aligned}
\frac{\partial q_i(x,Q^2)}{\partial \log Q^2} &= \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{q_iq_j}(y,\alpha_S)\, q_j(\frac{x}{y},Q^2) \\
&\quad + P_{q_ig}(y,\alpha_S)\, g(\frac{x}{y},Q^2)\Big\} \\
\frac{\partial g(x,Q^2)}{\partial \log Q^2} &= \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{gq_j}(y,\alpha_S)\, q_j(\frac{x}{y},Q^2) \\
&\quad + P_{gg}(y,\alpha_S)\, g(\frac{x}{y},Q^2)\Big\}
\end{aligned}
$$

scaling violations measured at HERA



NLO DGLAP fit

beyond lowest order in pQCD

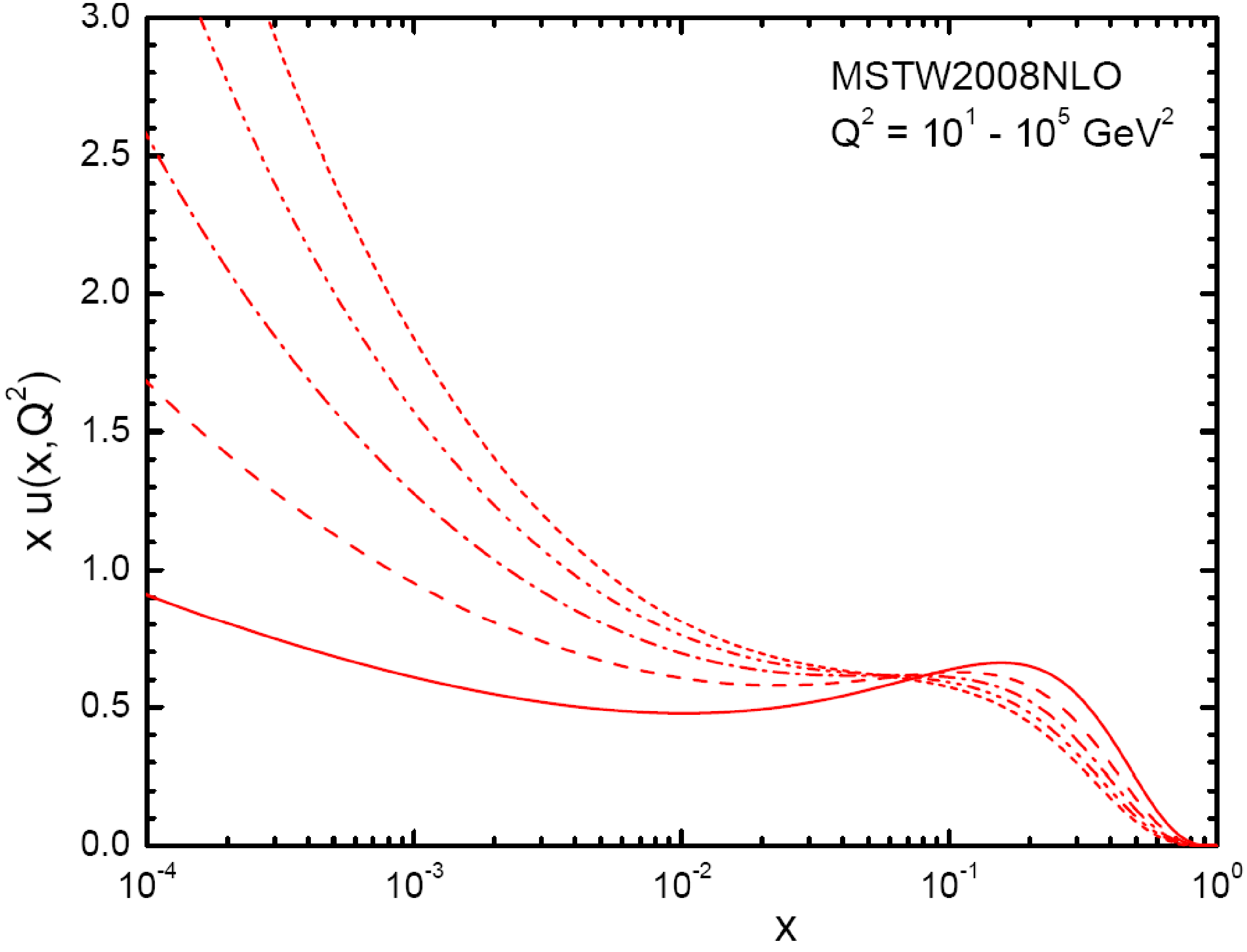
going to higher orders in pQCD is straightforward in principle, since the above structure for F_2 and for DGLAP generalises in a straightforward way:

$$
\begin{aligned}
\frac{\partial \mathbf{q_i}(\mathbf{x},\mathbf{Q}^2)}{\partial \log \mathbf{Q}^2} &= \frac{\alpha_S}{\mathbf{2}\pi}\int_{\mathbf{x}}^{1}\frac{\mathbf{dy}}{\mathbf{y}}\Big\{\mathbf{P}_{\mathbf{q_i q_j}}(\mathbf{y},\alpha_S)\ \mathbf{q_j}(\frac{\mathbf{x}}{\mathbf{y}},\mathbf{Q}^2) \\
&\quad + \mathbf{P}_{\mathbf{q_i g}}(\mathbf{y},\alpha_S)\ \mathbf{g}(\frac{\mathbf{x}}{\mathbf{y}},\mathbf{Q}^2)\Big\} \\
\frac{\partial \mathbf{g}(\mathbf{x},\mathbf{Q}^2)}{\partial \log \mathbf{Q}^2} &= \frac{\alpha_S}{\mathbf{2}\pi}\int_{\mathbf{x}}^{1}\frac{\mathbf{dy}}{\mathbf{y}}\Big\{\mathbf{P}_{\mathbf{g q_j}}(\mathbf{y},\alpha_S)\ \mathbf{q_j}(\frac{\mathbf{x}}{\mathbf{y}},\mathbf{Q}^2) \\
&\quad + \mathbf{P}_{\mathbf{gg}}(\mathbf{y},\alpha_S)\ \mathbf{g}(\frac{\mathbf{x}}{\mathbf{y}},\mathbf{Q}^2)\Big\}
\end{aligned}
$$

DGLAP:

$$P(x,\alpha_S) = \overset{\boxed{\text{1972-77}}}{\overset{\downarrow}{P^{(0)}}} + \alpha_S \overset{\boxed{\text{1977-80}}}{\overset{\downarrow}{P^{(1)}}}(x) + \alpha_S^2 \overset{\boxed{\text{2004}}}{\overset{\downarrow}{P^{(2)}}}(x) + \ldots$$

The 2004 calculation of the complete set of $P^{(2)}$ splitting functions by Moch, Vermaseren and Vogt completes the calculational tools for a consistent NNLO (massless) pQCD treatment of Tevatron & LHC hard-scattering cross sections


MSTW2008NLO
$Q^2 = 10^1 - 10^5$ GeV^2
x u(x,Q^2)
x
3.0
2.5
2.0
1.5
1.0
0.5
0.0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}

summary: how pdfs are obtained

- choose a factorisation scheme (e.g. MSbar), an order in perturbation theory (see below, e.g. LO, NLO, NNLO) and a ‘starting scale’ Q_0 where pQCD applies (e.g. 1-2 GeV)
- parametrise the quark and gluon distributions at $Q_{0,}$, e.g.

$$f_i(x,Q_0^2) = A_i x^{a_i}[1 + b_i\sqrt{x} + c_i x](1-x)^{d_i}$$

- solve DGLAP equations to obtain the pdfs at any x and scale $Q > Q_0$; fit data for parameters $\{A_i, a_i, \ldots \alpha_S\}$
- approximate the exact solutions (e.g. interpolation grids, expansions in polynomials etc) for ease of use; thus the output ‘global fits’ are available ‘off the shelf”, e.g.

```
SUBROUTINE PDF(X,Q,U,UBAR,D,DBAR,…,BBAR,GLU)
```

input | output

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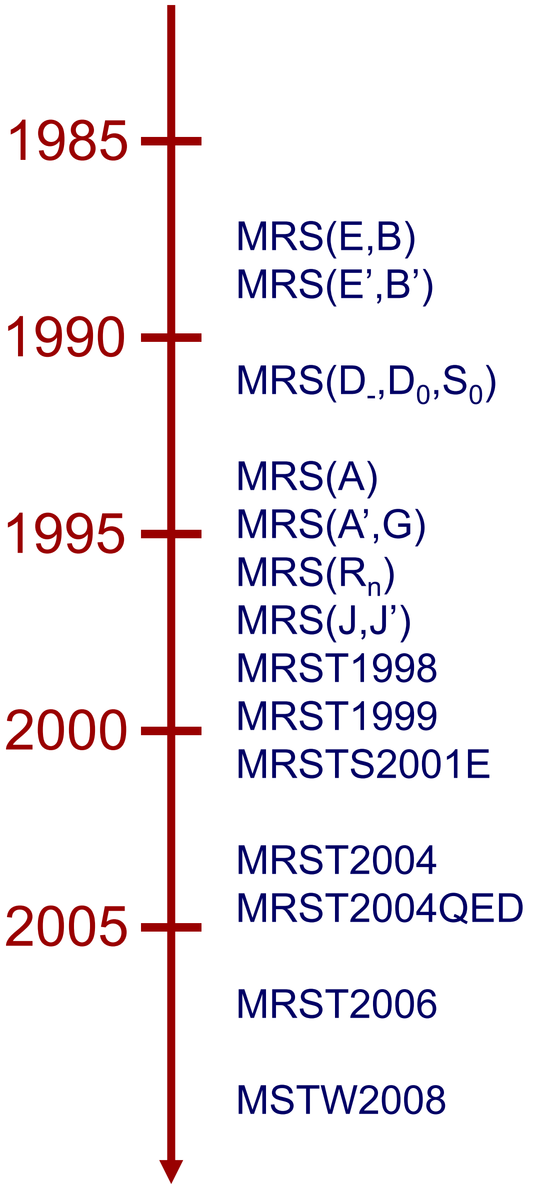
MSTW*

*Alan Martin, WJS, Robert Thorne, Graeme Watt

arXiv:0901.0002 [hep-ph])

the MRS/MRST/MSTW project



1985

MRS(E,B)
MRS(E’,B’)

1990

MRS(D_-,D_0,S_0)

MRS(A)
MRS(A’,G)

1995

MRS(R_n)
MRS(J,J’)
MRST1998
MRST1999

2000

MRSTS2001E

MRST2004
MRST2004QED

2005

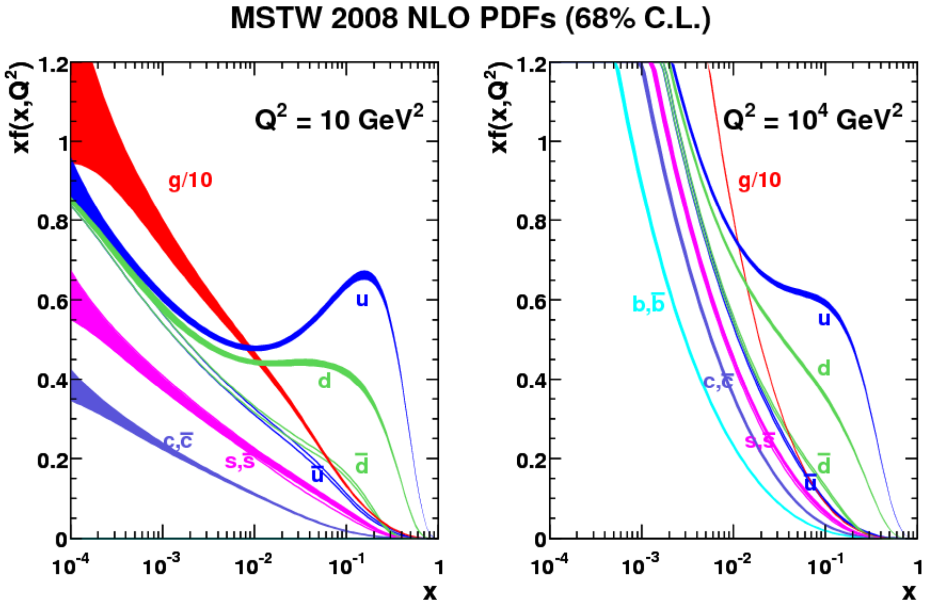
MRST2006

MSTW2008

- since 1987 (with Alan Martin, Dick Roberts, Robert Thorne, Graeme Watt), to produce ‘state-of-the-art’ pdfs
- combine experimental data with theoretical formalism to perform ‘global fits' to data to extract the pdfs in user-friendly form for the particle physics community
- currently widely used at HERA and the Fermilab Tevatron, and in physics simulations for the LHC
- currently, the only available NNLO pdf sets with rigorous treatment of heavy quark flavours

Also: CTEQ, Alekhin, NNPDF, HERAPDF, Dynamical Partons, …

MSTW 2008 update



- new data (see next slide)
- new theory/infrastructure
  - δf_i from new dynamic tolerance method: 68%cl (1σ) and 90%cl (*cf.* MRST) sets available
  - new definition of α_S (no more Λ_{QCD})
  - new GM-VFNS for c, b (see Martin et al., arXiv:0706.0459)
  - new fitting codes: FEWZ, VRAP, fastNLO
  - new grids: denser, broader coverage
  - slightly extended parameterisation at Q_0^2 :34-4=30 free parameters including α_S

code, text and figures available at: `http://projects.hepforge.org/mstwpdf/`
and in latest version 5.7.0 LHAPDF: `http://projects.hepforge.org/lhapdf/`

data sets used in fit

Data set	$N_{\mathrm{pts.}}$
H1 MB 99 e^+p NC	8
H1 MB 97 e^+p NC	64
H1 low Q^2 96–97 e^+p NC	80
H1 high Q^2 98–99 e^-p NC	126
H1 high Q^2 99–00 e^+p NC	147
ZEUS SVX 95 e^+p NC	30
ZEUS 96–97 e^+p NC	144
ZEUS 98–99 e^-p NC	92
ZEUS 99–00 e^+p NC	90
H1 99–00 e^+p CC	28
ZEUS 99–00 e^+p CC	30
H1/ZEUS $e^\pm p$ F_2^{charm}	83
H1 99–00 e^+p incl. jets	24
ZEUS 96–97 e^+p incl. jets	30
ZEUS 98–00 $e^\pm p$ incl. jets	30
DØ II $p\bar{p}$ incl. jets	110
CDF II $p\bar{p}$ incl. jets	76
CDF II $W \to l\nu$ asym.	22
DØ II $W \to l\nu$ asym.	10
DØ II Z rap.	28
CDF II Z rap.	29

Data set	$N_{\mathrm{pts.}}$
BCDMS μp F_2	163
BCDMS μd F_2	151
NMC μp F_2	123
NMC μd F_2	123
NMC $\mu n/\mu p$	148
E665 μp F_2	53
E665 μd F_2	53
SLAC ep F_2	37
SLAC ed F_2	38
NMC/BCDMS/SLAC F_L	31
E866/NuSea pp DY	184
E866/NuSea pd/pp DY	15
NuTeV νN F_2	53
CHORUS νN F_2	42
NuTeV νN xF_3	45
CHORUS νN xF_3	33
CCFR $\nu N \to \mu\mu X$	86
NuTeV $\nu N \to \mu\mu X$	84
All data sets	**2743**

- Red = New w.r.t. MRST 2006 fit.

MSTW input parametrisation

At input scale $Q_0^2 = 1\ \mathrm{GeV}^2$:

$$
\begin{aligned}
xu_v &= A_u\, x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\sqrt{x}+\gamma_u\, x)\\
xd_v &= A_d\, x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\sqrt{x}+\gamma_d\, x)\\
xS &= A_S\, x^{\delta_S}(1-x)^{\eta_S}(1+\epsilon_S\sqrt{x}+\gamma_S\, x)\\
x\bar{d}-x\bar{u} &= A_\Delta\, x^{\eta_\Delta}(1-x)^{\eta_S+2}(1+\gamma_\Delta\, x+\delta_\Delta\, x^2)\\
xg &= A_g\, x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g\, x)+A_{g'}\, x^{\delta_{g'}}(1-x)^{\eta_{g'}}\\
xs+x\bar{s} &= A_+\, x^{\delta_S}\,(1-x)^{\eta_+}(1+\epsilon_S\sqrt{x}+\gamma_S\, x)\\
xs-x\bar{s} &= A_-\, x^{\delta_-}(1-x)^{\eta_-}(1-x/x_0)
\end{aligned}
$$

Note: 20 parameters allowed to go free for eigenvector PDF sets, *cf.* 15 for MRST sets

which data sets determine which partons?

Process	Subprocess	Partons	x range
$\ell^\pm \{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu})\, N \to \mu^-(\mu^+)\, X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu\, N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu}\, N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+ \{d, s\} \to \{u, c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm c\bar{c} X$	$\gamma^* c \to c,\ \gamma^* g \to c\bar{c}$	$c,\ g$	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm \nu)\, X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+\ell^-)\, X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

pdf uncertainties

$$\Delta\chi^2_{\text{global}} \equiv \chi^2_{\text{global}} - \chi^2_{\text{min}} = \sum_{i,j=1}^{n} H_{ij}(a_i - a_i^0)(a_j - a_j^0)$$

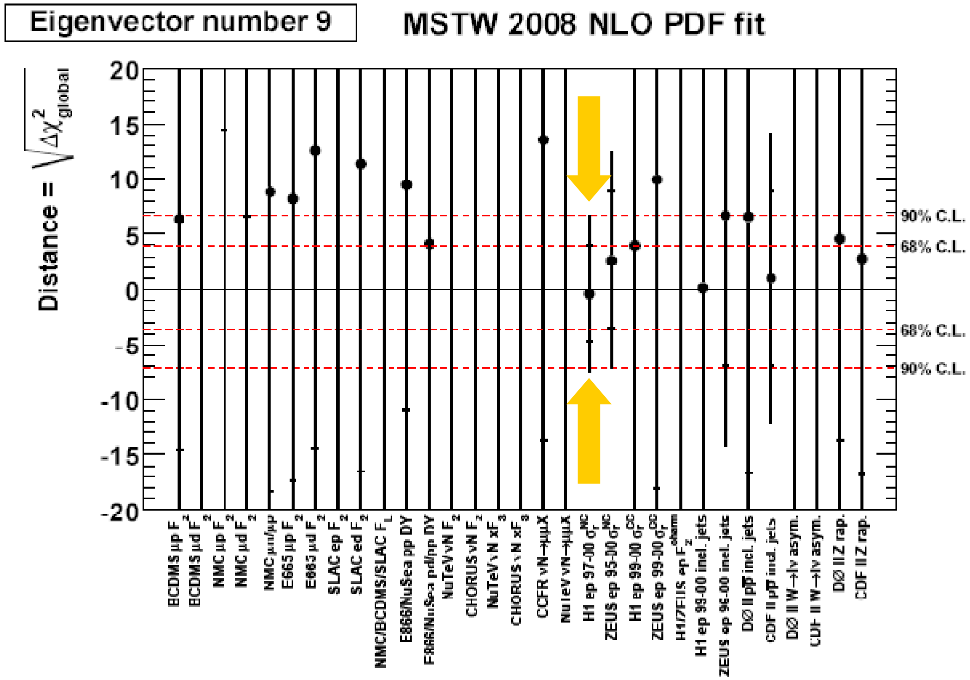
$$\vec{a} - \vec{a}^0 = \sum_{k=1,n} z_k \vec{e}_k \text{ where } (H^{-1})\cdot\vec{e}_k = \lambda_k \vec{e}_k,\ \vec{e}_k\cdot\vec{e}_l = \lambda_k \delta_{kl}$$

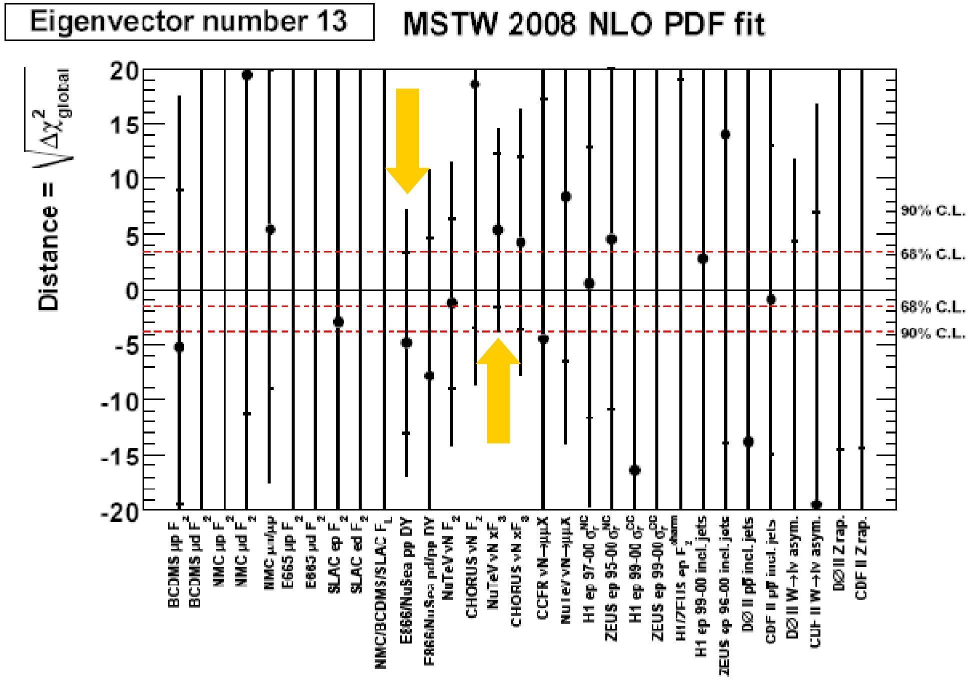
$$\text{then} \quad \Delta\chi^2_{\text{global}} = \sum_{k=1,n} z_k^2 \le T^2 \quad (T = \text{tolerance})$$

this defines a set of n 'error' pdfs, spanning the allowed variation in the parameters, as determined by T:

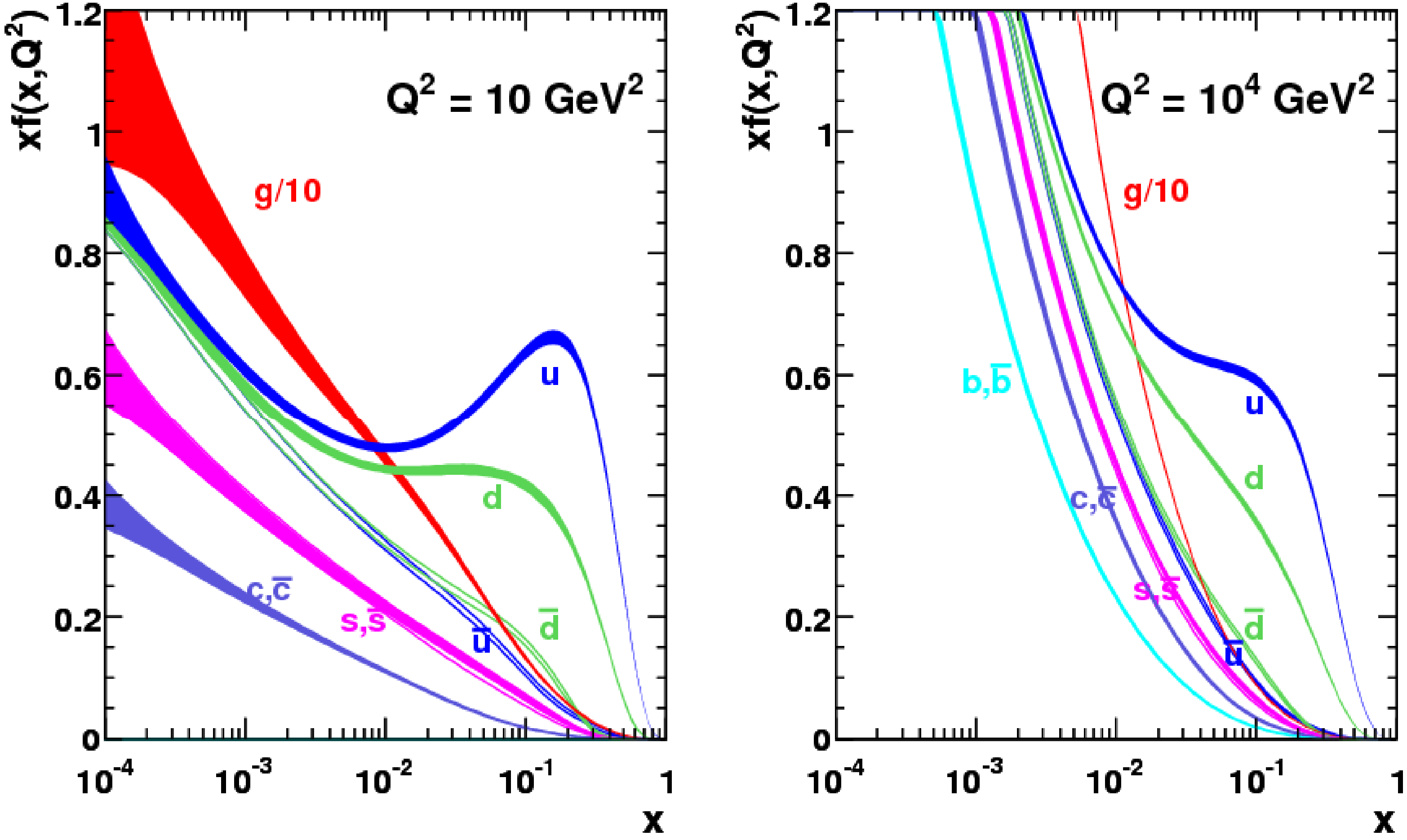
$$\vec{a}(S_k^{\pm}) = \vec{a}^0 \pm T\vec{e}_k$$

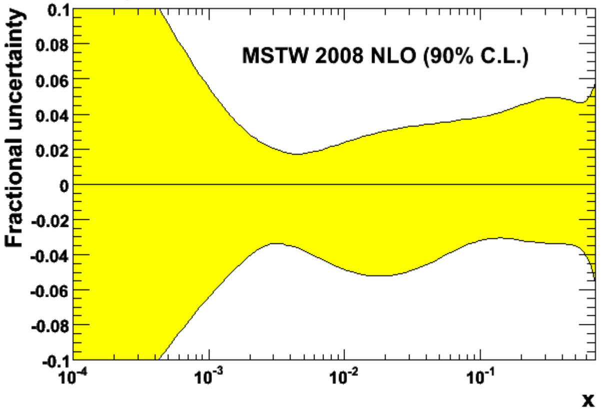
rather than using a fixed value of T (*cf*. MRST, CTEQ), we determine the 'dynamic' tolerance for each eigenvector from the condition that all data sets should be described within their 68% or 90% or … confidence limit


Eigenvector number 9
MSTW 2008 NLO PDF fit
Distance = $\sqrt{\Delta\chi^2_{\mathrm{global}}}$
20
15
10
5
0
-5
-10
-15
-20
90% C.L.
68% C.L.
68% C.L.
90% C.L.
BCDMS μp F_2
BCDMS μd F_2
NMC μp F_2
NMC μd F_2
NMC μn/μp
E665 μp F_2
E665 μd F_2
SLAC ep F_2
SLAC ed F_2
NMC/BCDMS/SLAC F_L
E866/NuSea pp DY
E866/NuSea pd/pp DY
NuTeV νN F_2
CHORUS νN F_2
NuTeV νN xF_3
CHORUS νN xF_3
CCFR νN→μμX
NuTeV νN→μμX
H1 ep 97-00 σ_r^{NC}
ZEUS ep 95-00 σ_r^{NC}
H1 ep 99-00 σ_r^{CC}
ZEUS ep 99-00 σ_r^{CC}
H1/ZEUS ep F_2^{charm}
H1 ep 99-00 incl. jets
ZEUS ep 96-00 incl. jets
DØ II pp̄ incl. jets
CDF II pp̄ incl. jets
DØ II W→lν asym.
CDF II W→lν asym.
DØ II Z rap.
CDF II Z rap.

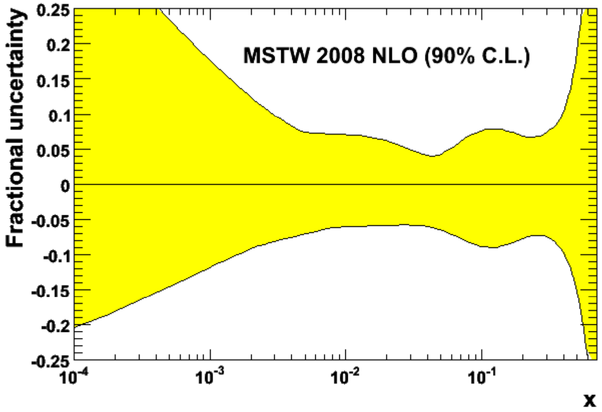

Eigenvector number 13
MSTW 2008 NLO PDF fit
Distance = $\sqrt{\Delta\chi^2_{\mathrm{global}}}$
20
15
10
5
0
-5
-10
-15
-20
90% C.L.
68% C.L.
68% C.L.
90% C.L.
BCDMS μp F_2
BCDMS μd F_2
NMC μp F_2
NMC μd F_2
NMC μn/μp
E665 μp F_2
E665 μd F_2
SLAC ep F_2
SLAC ed F_2
NMC/BCDMS/SLAC F_L
E866/NuSea pp DY
E866/NuSea pd/pp DY
NuTeV νN F_2
CHORUS νN F_2
NuTeV νN xF_3
CHORUS νN xF_3
CCFR νN→μμX
NuTeV νN→μμX
H1 ep 97-00 σ_r^{NC}
ZEUS ep 95-00 σ_r^{NC}
H1 ep 99-00 σ_r^{CC}
ZEUS ep 99-00 σ_r^{CC}
H1/ZEUS ep F_2^{charm}
H1 ep 99-00 incl. jets
ZEUS ep 96-00 incl. jets
DØ II pp̄ incl. jets
CDF II pp̄ incl. jets
DØ II W→lν asym.
CDF II W→lν asym.
DØ II Z rap.
CDF II Z rap.

MSTW 2008 NLO PDFs (68% C.L.)

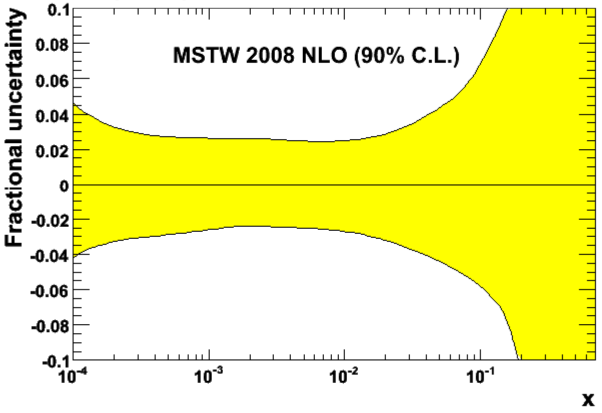




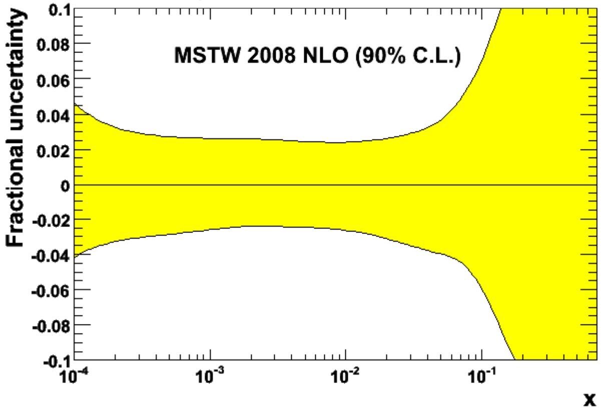
Up valence distribution at $Q^2 = 10^4$ GeV^2



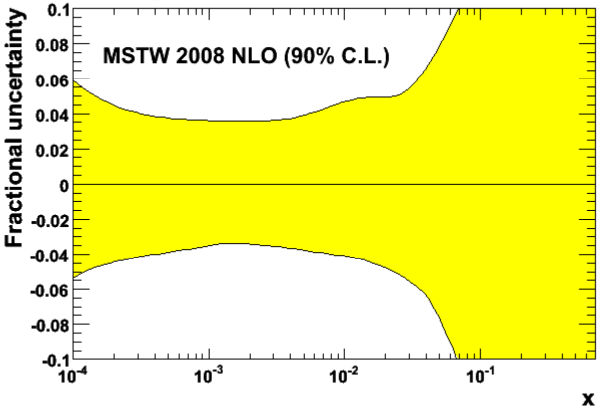
Down valence distribution at $Q^2 = 10^4$ GeV^2



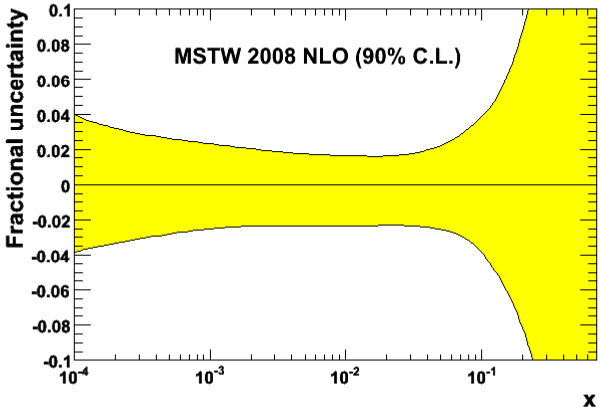
Up antiquark distribution at $Q^2 = 10^4$ GeV^2



Down antiquark distribution at $Q^2 = 10^4$ GeV^2

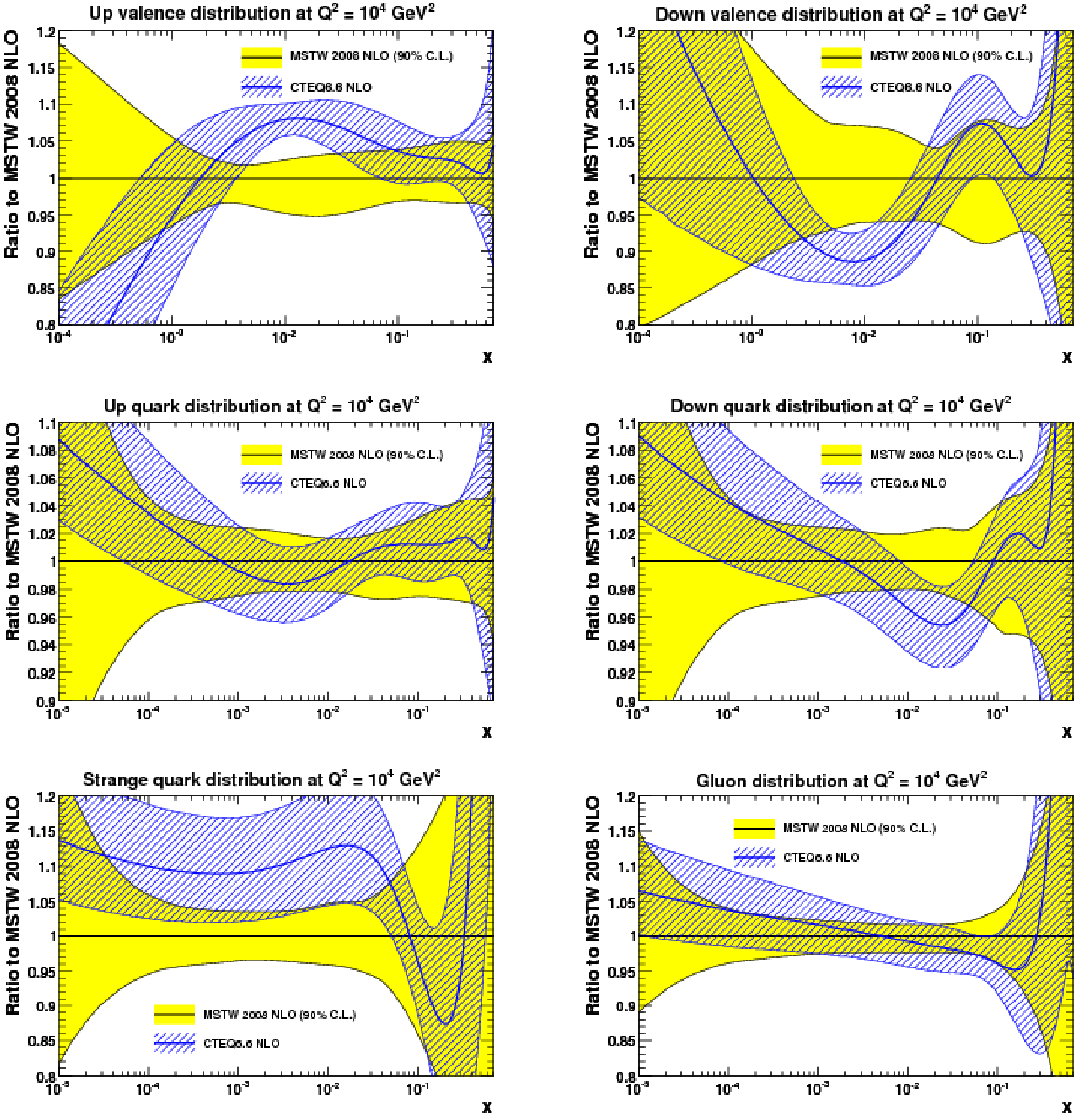


Strange quark distribution at $Q^2 = 10^4$ GeV^2



Gluon distribution at $Q^2 = 10^4$ GeV^2

MSTW2008(NLO) vs. CTEQ6.6



Note:

CTEQ error bands slightly larger ‘by construction’ (different definition of tolerance)

CTEQ light quarks and gluons slightly larger at small x because of imposition of positivity on gluon at Q_0^2

a note on α_S

- world average value (PDG 2008):

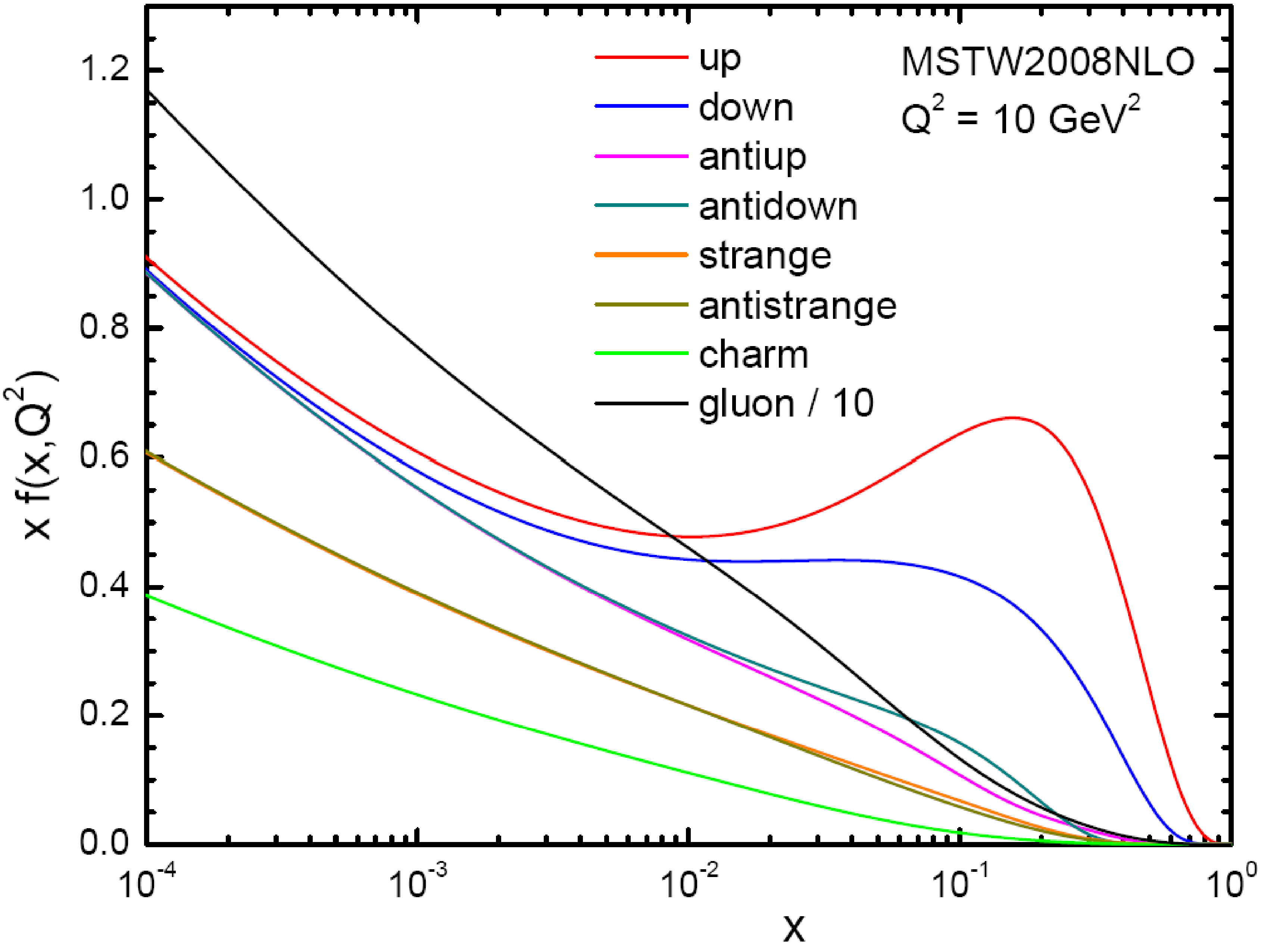
$$\alpha_S^{\overline{MS},NNLO}(M_Z^2) = 0.1176 \pm 0.002$$

- MSTW global fit value (minimum χ^2):

$$\alpha_S^{\overline{MS},NNLO}(M_Z^2) = 0.1171$$

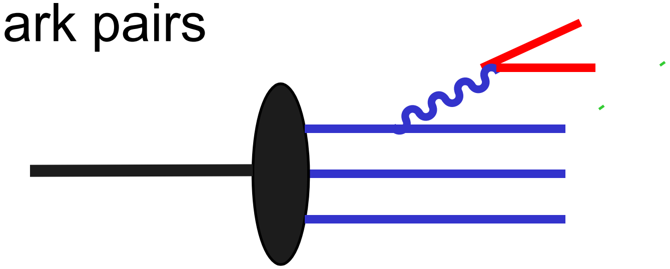
- the pdf error sets are generated with α_S fixed at its 'best fit' value, therefore variation of (e.g. jets, top, etc at LHC) cross sections with α_S is not explicitly included in the 'pdf error'

Note: $\alpha_S^{\overline{MS},NLO}(M_Z^2) = 0.1202$


up
down
antiup
antidown
strange
antistrange
charm
gluon / 10
MSTW2008NLO
Q^2 = 10 GeV^2
x f(x,Q^2)
x
0.0
0.2
0.4
0.6
0.8
1.0
1.2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}

the asymmetric sea

- the sea presumably arises when ‘primordial‘ valence quarks emit gluons which in turn split into quark-antiquark pairs, with suppressed splitting into heavier quark pairs

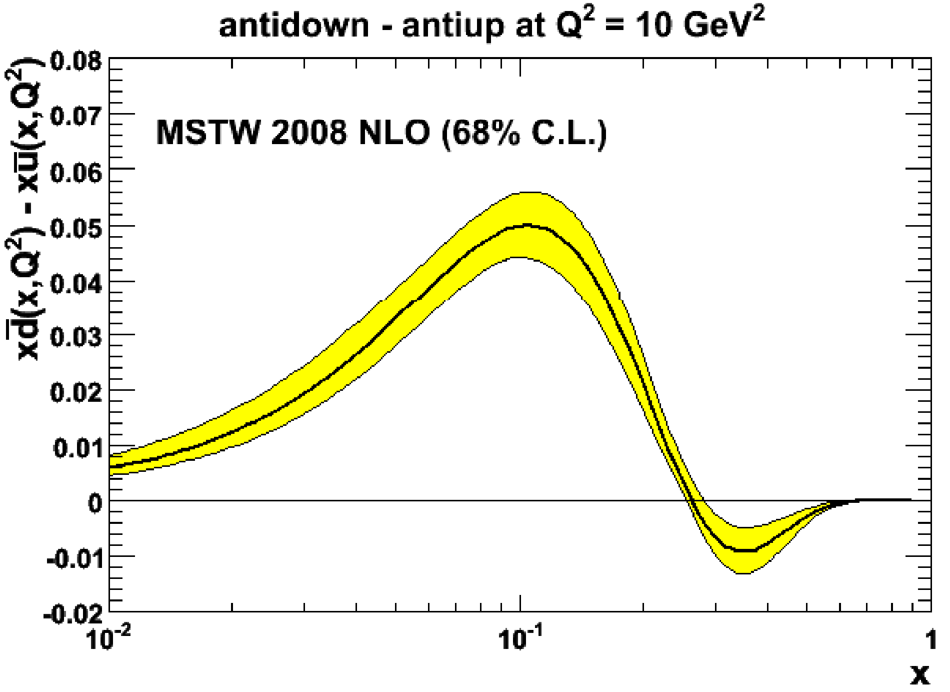


- so we naively expect

$$\bar{u} \approx \bar{d} > \bar{s} > \bar{c} > \ldots$$

- but why such a big d-u asymmetry? Meson cloud, Pauli exclusion, …?

The ratio of Drell-Yan cross sections for *pp,pn* → $\mu^+\mu^-$ + *X* provides a measure of the difference between the *u* and *d* sea quark distributions



strange

earliest pdf fits had SU(3) symmetry: $s(x,Q_0^2) = \bar{s}(x,Q_0^2) = \bar{u}(x,Q_0^2) = \bar{d}(x,Q_0^2)$

later relaxed to include (constant) strange suppression (cf. fragmentation):

$$s(x,Q_0^2) = \bar{s}(x,Q_0^2) = \frac{\kappa}{2}\left[\bar{u}(x,Q_0^2) + \bar{d}(x,Q_0^2)\right]$$

with κ = 0.4 – 0.5

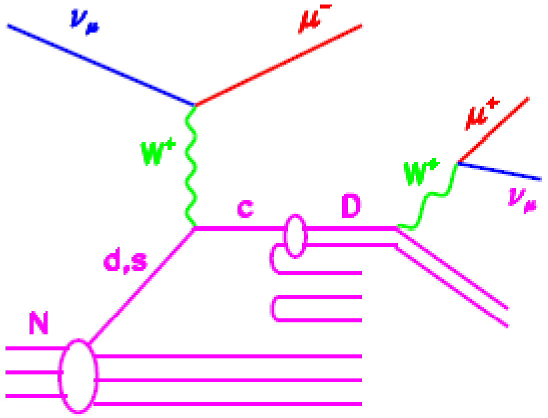
nowadays, dimuon production in νN DIS (CCFR, NuTeV) allows ‘direct’ determination:

$$\frac{d\sigma}{dxdy}\left(\nu_\mu(\bar{\nu}_\mu)N \to \mu^+\mu^- X\right) = B_c\,\mathcal{N}\mathcal{A}\,\frac{d\sigma}{dxdy}\left(\nu_\mu s(\bar{\nu}_\mu \bar{s}) \to c\mu^-(\bar{c}\mu^+)X\right)$$

in the range 0.01 < x < 0.4

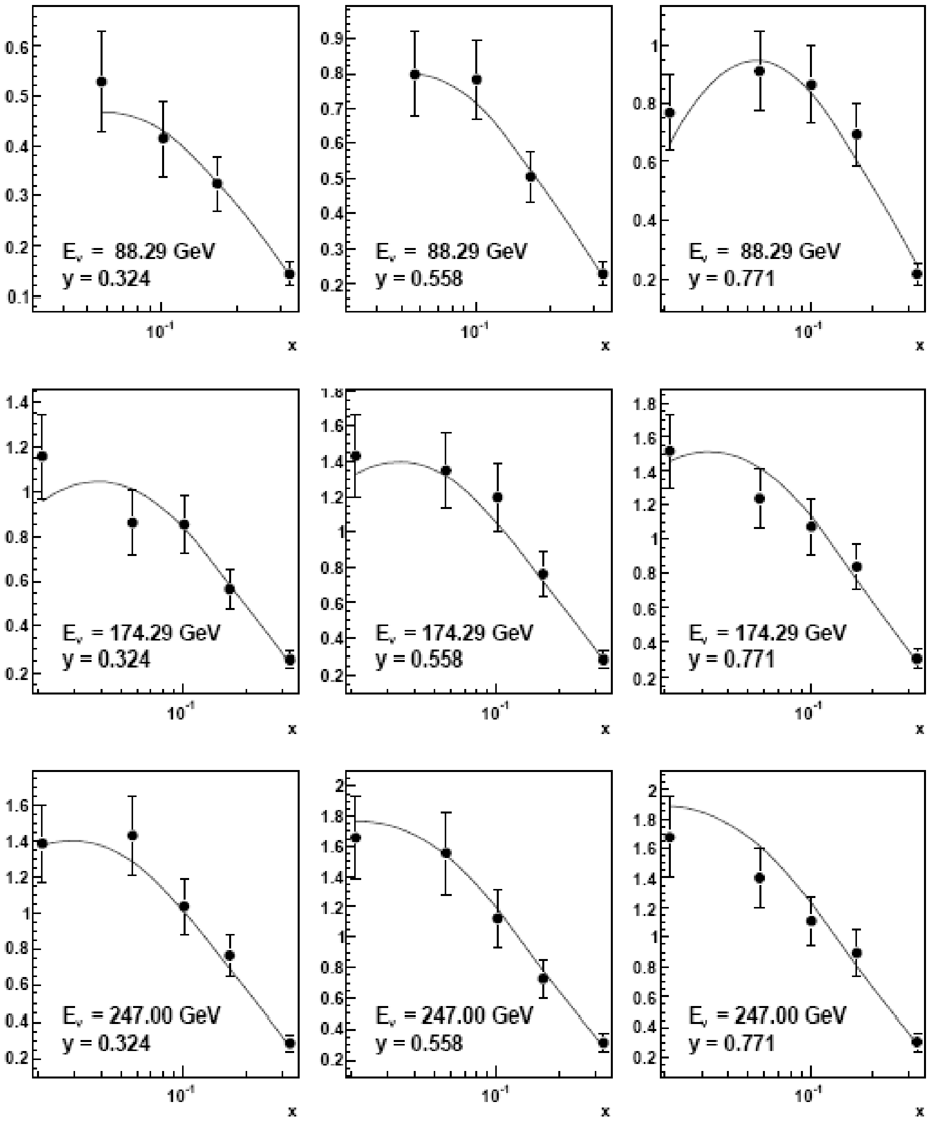
data seem to prefer $s(x,Q_0^2) - \bar{s}(x,Q_0^2) \neq 0$

theoretical explanation?!

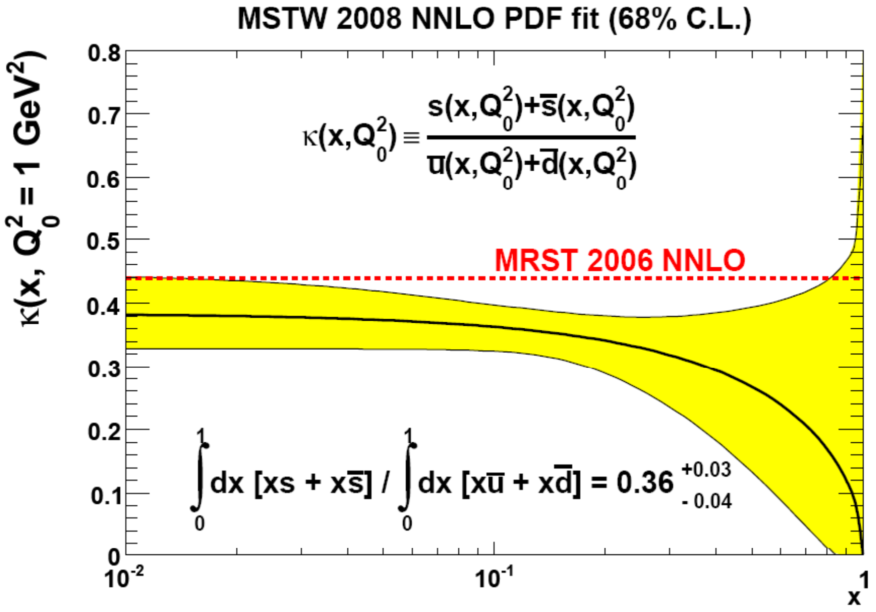


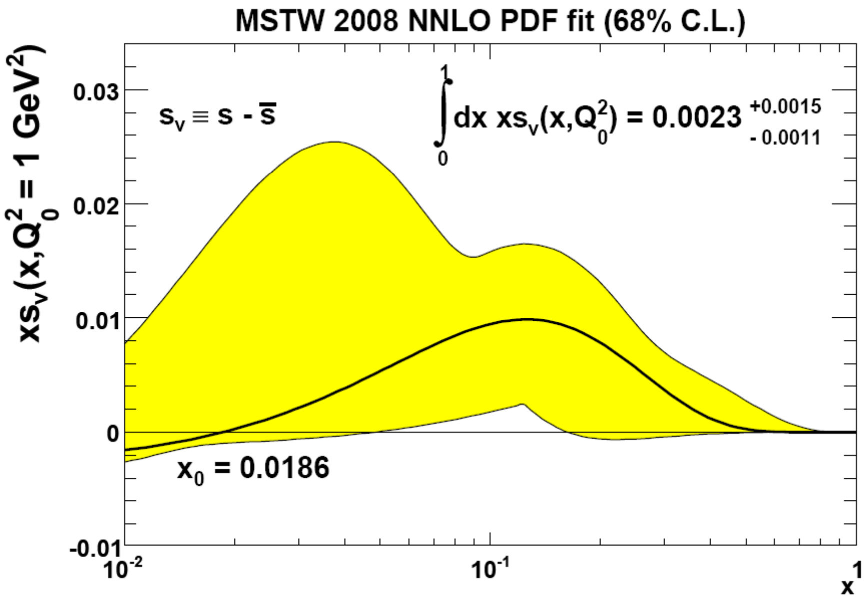
NuTeV $\frac{100\pi}{G_F^2 M_N E_\nu}\frac{d\sigma}{dxdy}(\nu_\mu N \to \mu^+\mu^- X)$ in GeV^{-2}

MSTW 2008 NNLO PDF fit, χ^2 = 13 for 21 DOF



MSTW


MSTW 2008 NNLO PDF fit (68% C.L.)


MSTW 2008 NNLO PDF fit (68% C.L.)

MSTW

charm, bottom

considered sufficiently massive to allow pQCD treatment: $g \to Q\overline{Q}$

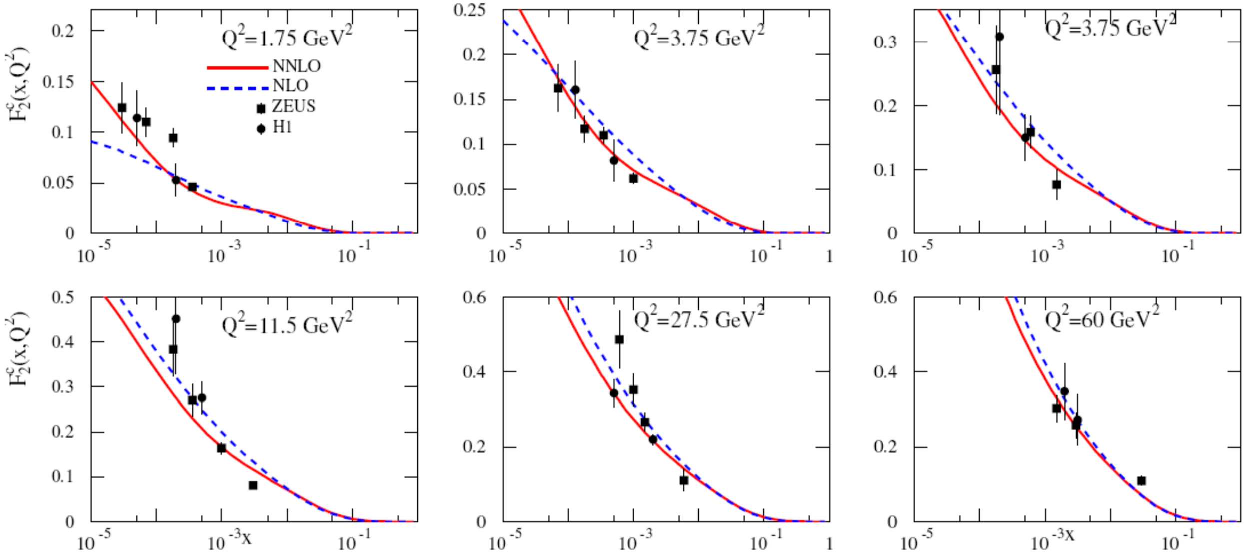
distinguish two regimes:
(i) $Q^2 \sim m_H^2$ include full m_H dependence to get correct threshold behaviour
(ii) $Q^2 \gg m_H^2$ treat as ~massless partons to resum $\alpha_S^n \log^n(Q^2/m_H^2)$ via DGLAP

FFNS: OK for (i) only **ZM-VFNS:** OK for (ii) only

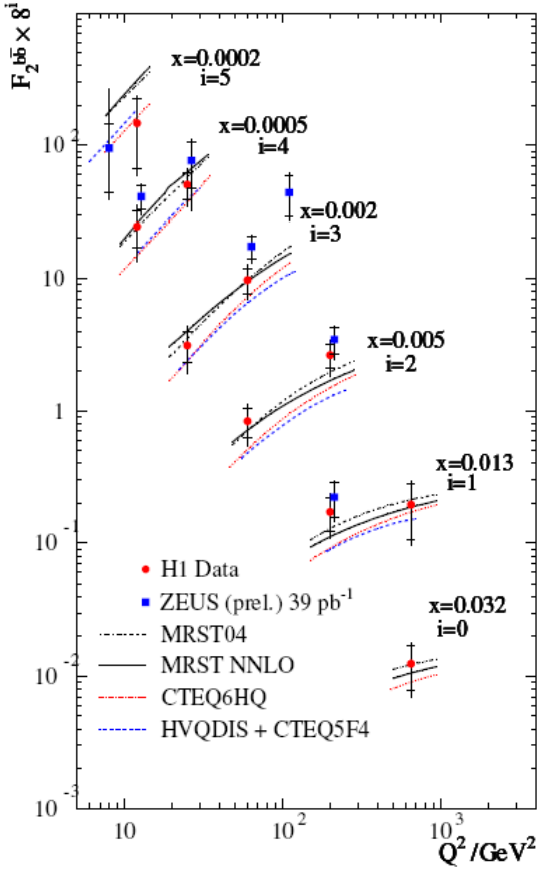
consistent **GM**(=general mass)-**VFNS** now available (e.g. ACOT(χ), Roberts-Thorne) which interpolates smoothly between the two regimes

Note: definition of these is tricky and non-unique (ambiguity in assignment of $O(m_H^2/Q^2)$ contributions), and the implementation of improved treatment (e.g. in going from MRST2006 to MSTW 2008) can have a big effect on light partons

charm and bottom structure functions



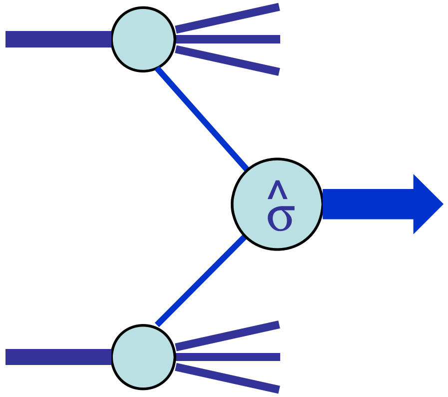
MSTW 2008



3

impact of pdfs on precision phenomenology at LHC

the QCD **factorization theorem** for hard-scattering (short-distance) inclusive processes



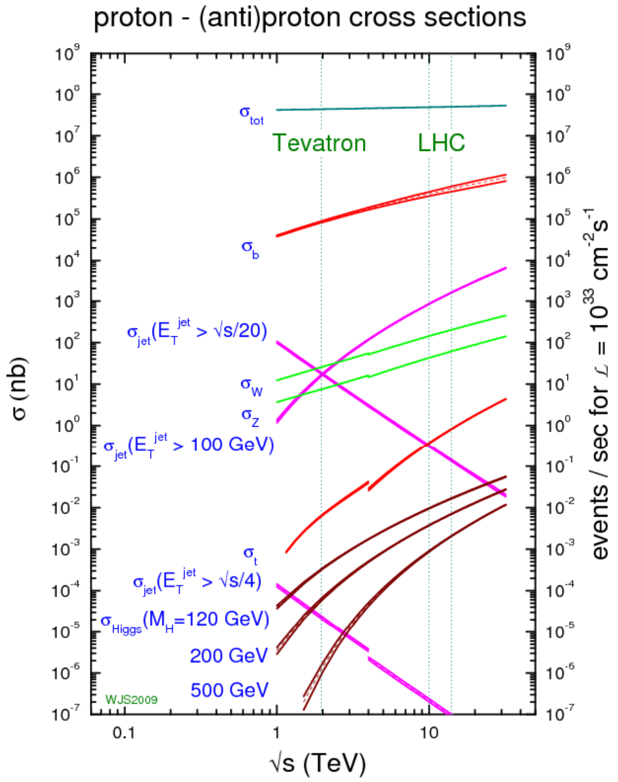
$$\begin{aligned}\sigma_{\mathrm{X}} &= \sum_{\mathbf{a,b}} \int_0^1 \mathbf{dx_1 dx_2}\ \mathbf{f_a}(\mathbf{x_1}, \mu_{\mathrm{F}}^2)\ \mathbf{f_b}(\mathbf{x_2}, \mu_{\mathrm{F}}^2) \\ &\times\ \hat{\sigma}_{\mathbf{ab}\to\mathrm{X}}\left(\mathbf{x_1}, \mathbf{x_2}, \{\mathbf{p}_{\mathbf{i}}^{\mu}\}; \alpha_{\mathrm{S}}(\mu_{\mathrm{R}}^2), \alpha(\mu_{\mathrm{R}}^2), \frac{\mathbf{Q}^2}{\mu_{\mathrm{R}}^2}, \frac{\mathbf{Q}^2}{\mu_{\mathrm{F}}^2}\right)\end{aligned}$$

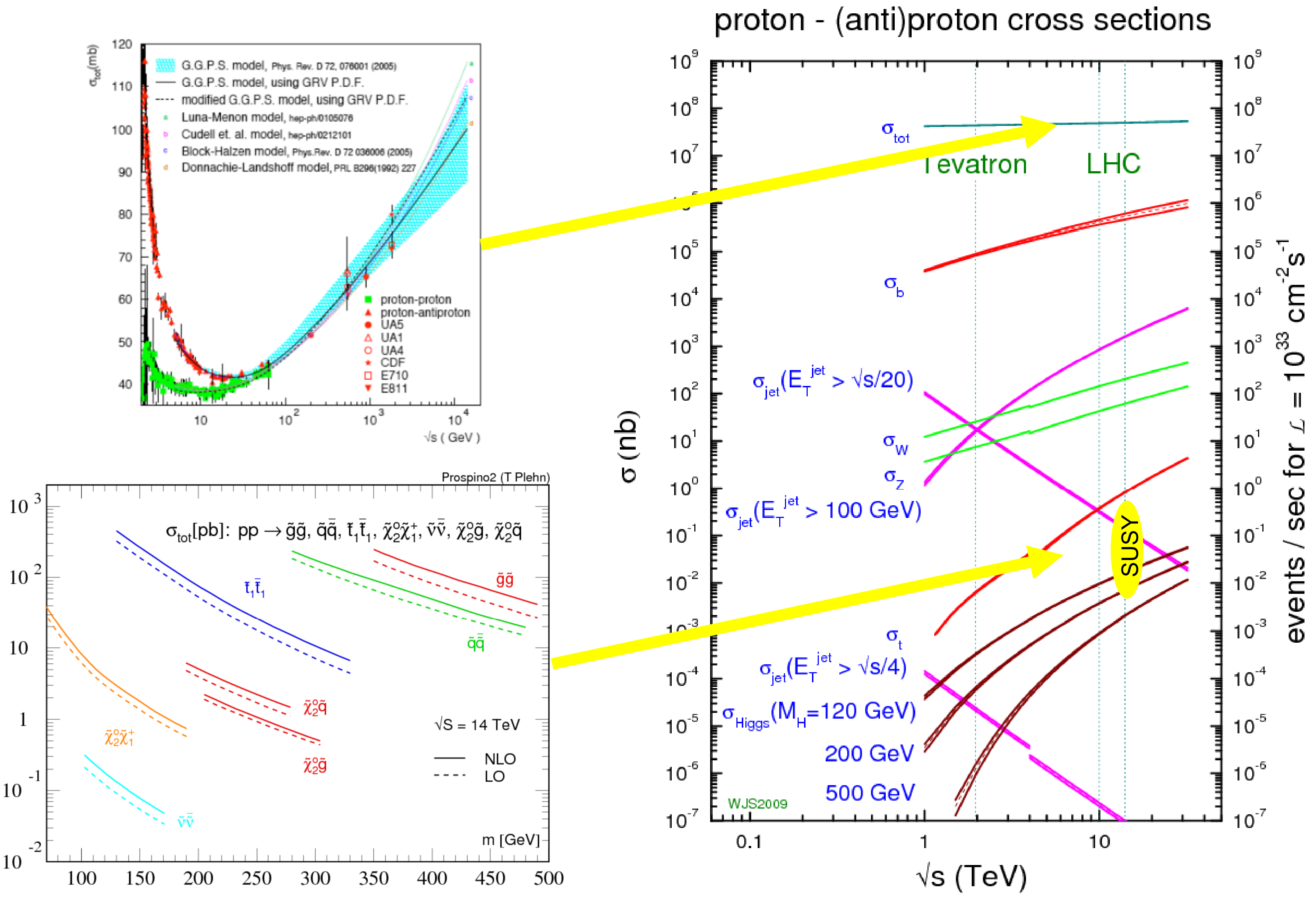
where X=W, Z, H, high-E_T jets, SUSY sparticles, black hole, …, and Q is the ‘hard scale’ (e.g. = M_X), usually $\mu_F = \mu_R = Q$, and $\hat{\sigma}$ is known …

- to some fixed order in pQCD, e.g. high-E_T jets

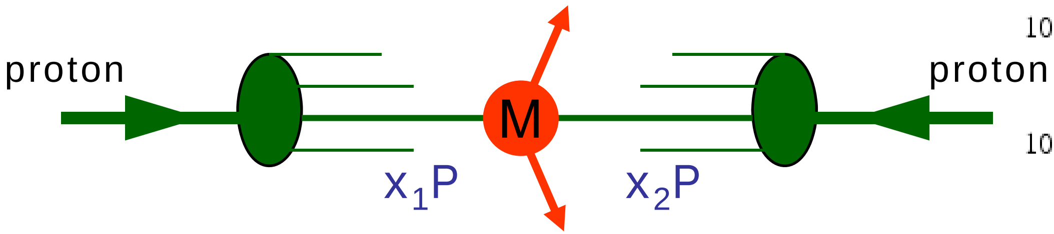
$$\hat{\sigma} = A\alpha_S^2 + B\alpha_s^3$$

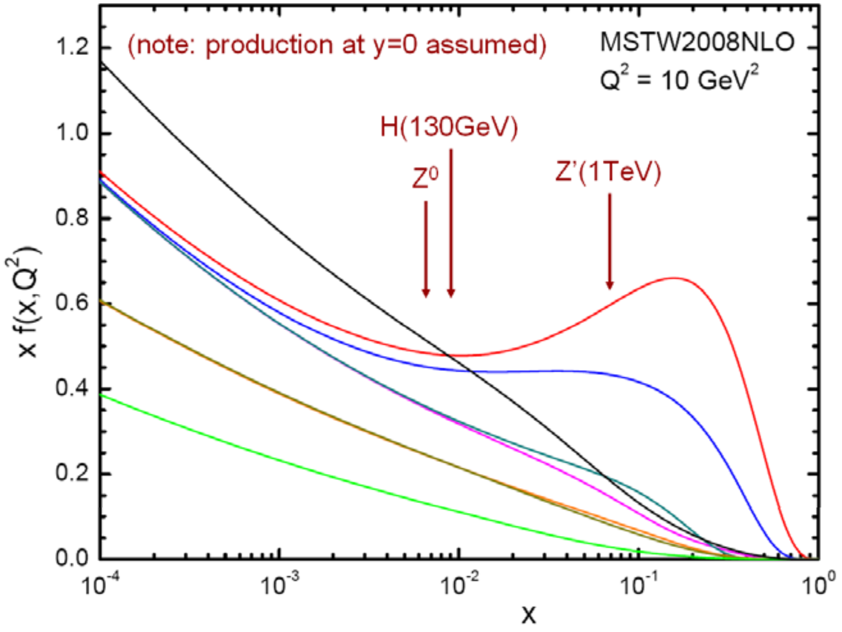
- or ‘improved’ by some leading logarithm approximation (LL, NLL, …) to all orders via resummation

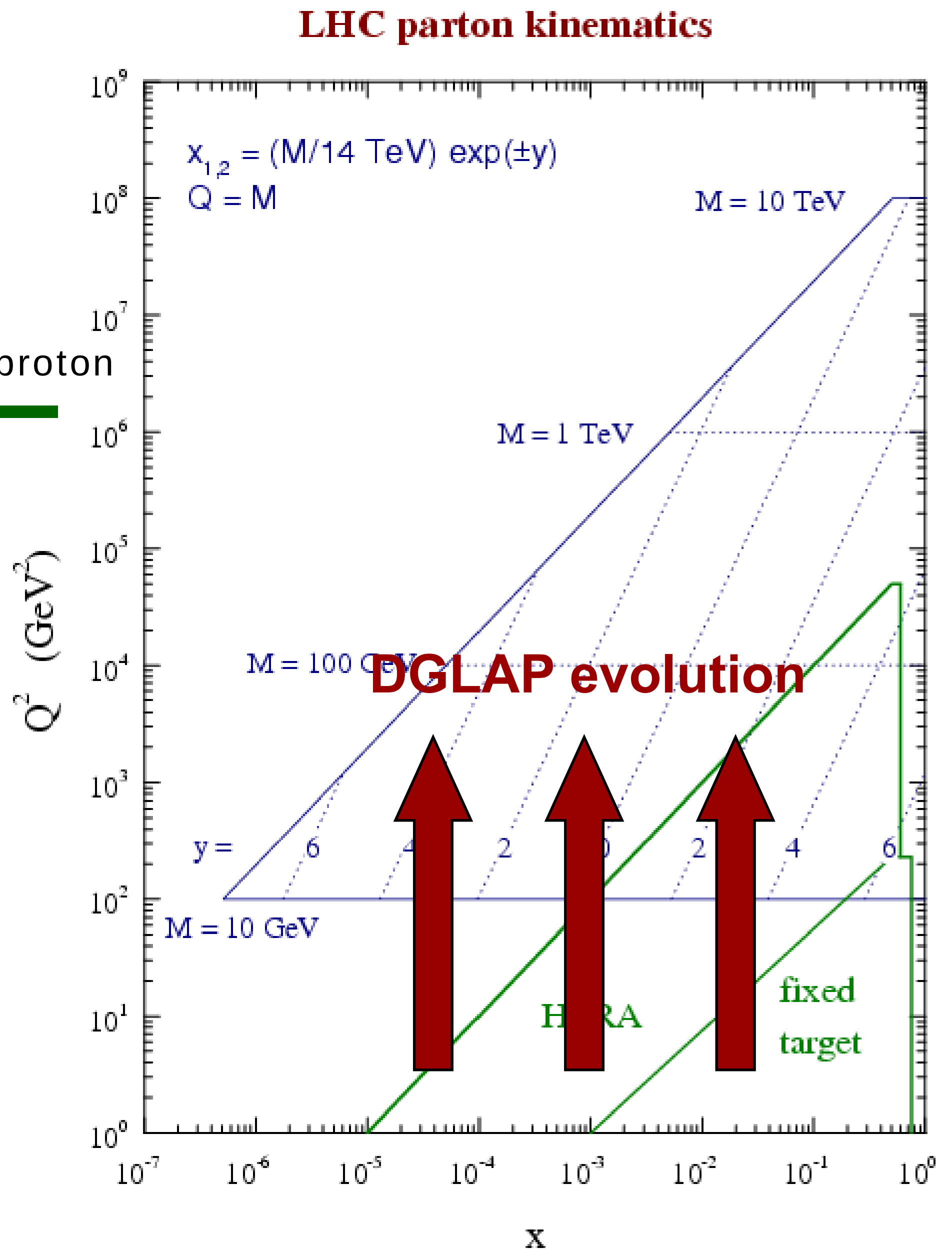



σ_tot(mb)
120
110
100
90
80
70
60
50
40
G.G.P.S. model, Phys. Rev. D 72, 076001 (2005)
G.G.P.S. model, using GRV P.D.F.
modified G.G.P.S. model, using GRV P.D.F.
Luna-Menon model, hep-ph/0105076
Cudell et. al. model, hep-ph/0212101
Block-Halzen model, Phys.Rev. D 72 036006 (2005)
Donnachie-Landshoff model, PRL B296(1992) 227
proton-proton
proton-antiproton
UA5
UA1
UA4
CDF
E710
E811
10
10^2
10^3
10^4
√s (GeV)
Prospino2 (T Plehn)
σ_tot[pb]: pp → g̃g̃, q̃q̄̃, t̃₁t̄̃₁, χ̃⁰₂χ̃⁺₁, ν̃ν̄̃, χ̃⁰₂g̃, χ̃⁰₂q̃
10^3
10^2
10
1
10^-1
10^-2
t̃₁t̄̃₁
g̃g̃
q̃q̄̃
χ̃⁰₂q̃
χ̃⁰₂g̃
χ̃⁰₂χ̃⁺₁
ν̃ν̄̃
√S = 14 TeV
NLO
LO
m [GeV]
100
150
200
250
300
350
400
450
500
proton - (anti)proton cross sections
σ (nb)
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
σ_tot
Tevatron
LHC
σ_b
σ_jet(E_T^jet > √s/20)
σ_W
σ_Z
σ_jet(E_T^jet > 100 GeV)
SUSY
σ_t
σ_jet(E_T^jet > √s/4)
σ_Higgs(M_H=120 GeV)
200 GeV
500 GeV
WJS2009
0.1
1
10
√s (TeV)
events / sec for ℒ = 10^33 cm^-2 s^-1

momentum fractions x_1 and x_2
determined by mass and rapidity of **X**







pdfs at LHC – the issues

- high precision cross section predictions require accurate knowledge of pdfs: $\delta\sigma_{th} = \delta\sigma_{pdf} + \ldots$

  → improved signal and background predictions

  → easier to spot new physics

- ‘standard candle’ processes (e.g. σ_Z) to
  - check formalism (factorisation, DGLAP, …)
  - measure machine luminosity?

- learning more about pdfs from LHC measurements. e.g.
  - high-E_T jets → gluon?
  - W^+, W^-, Z^0 → quarks?
  - forward Drell-Yan → small x?
  - …

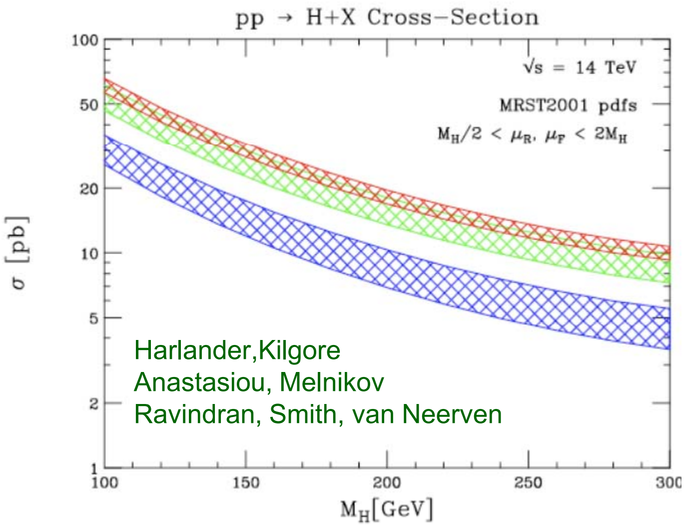
how important is pdf precision?

- Example 1: $\sigma(M_H=120\ \mathrm{GeV})$ @ LHC

  $\delta\sigma_{pdf} \approx \pm 2\%$, $\delta\sigma_{ptNNL0} \approx \pm 10\%$

  $\delta\sigma_{ptNNLL} \approx \pm 8\%$

→ $\delta\sigma_{theory} \approx \pm 10\%$

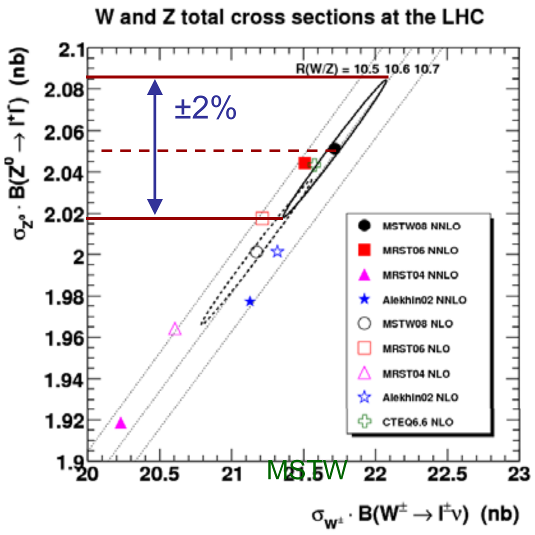


Harlander,Kilgore
Anastasiou, Melnikov
Ravindran, Smith, van Neerven

- Example 2: $\sigma(Z^0)$ @ LHC

  $\delta\sigma_{pdf} \approx \pm 2\%$, $\delta\sigma_{ptNNL0} \approx \pm 2\%$

→ $\delta\sigma_{theory} \approx \pm 3\%$

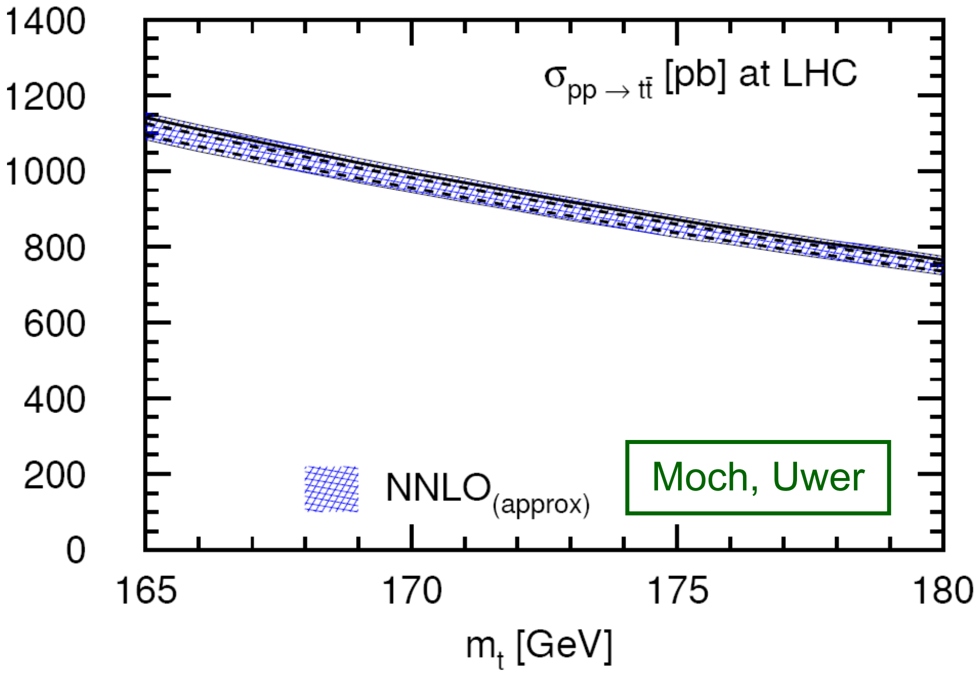


MSTW

- Example 3: σ(tt) @ LHC

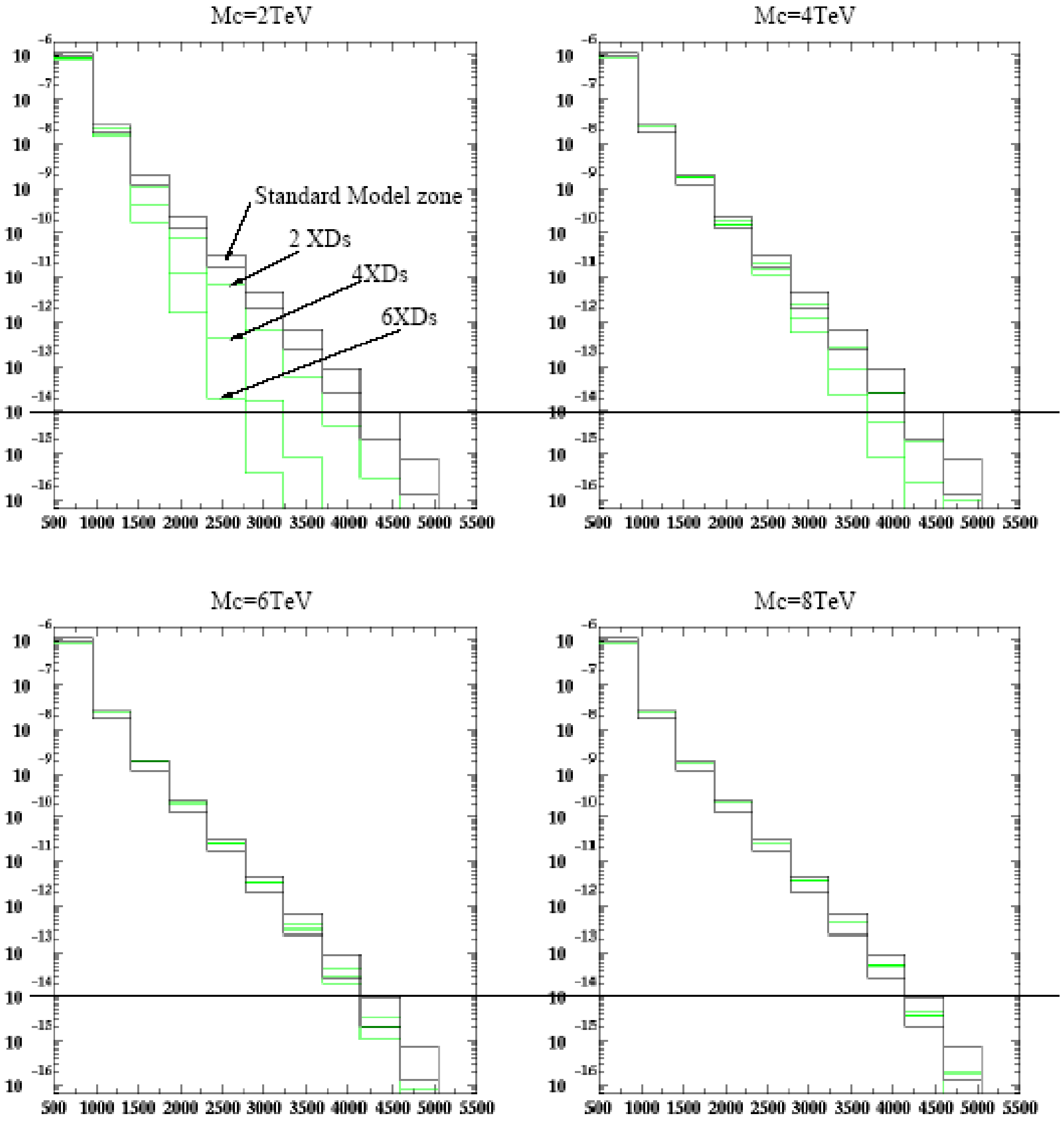
  $\delta\sigma_{pdf} \approx \pm 2\%, \quad \delta\sigma_{ptNNL0approx} \approx \pm 3\%$

→ $\delta\sigma_{theory} \approx \pm 4\%$



- Example 4: quantitative limits on New Physics depend on pdfs

sensitivity of dijet cross section at LHC to large extra dimensions



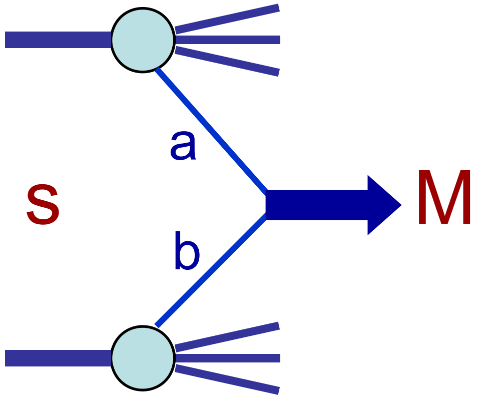
- LED accelerate the running of α_S as the compactification scale M_c is approached

- sensitivity attentuated by pdf uncertainties in SM prediction

Ferrag (ATLAS), hep-ph/0407303

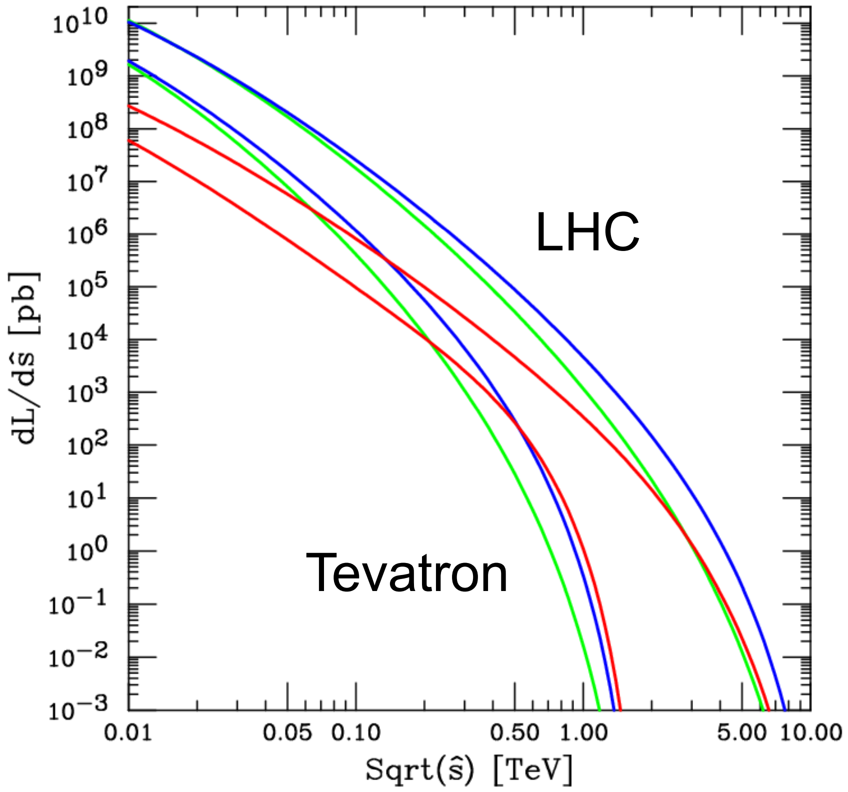
parton luminosity functions

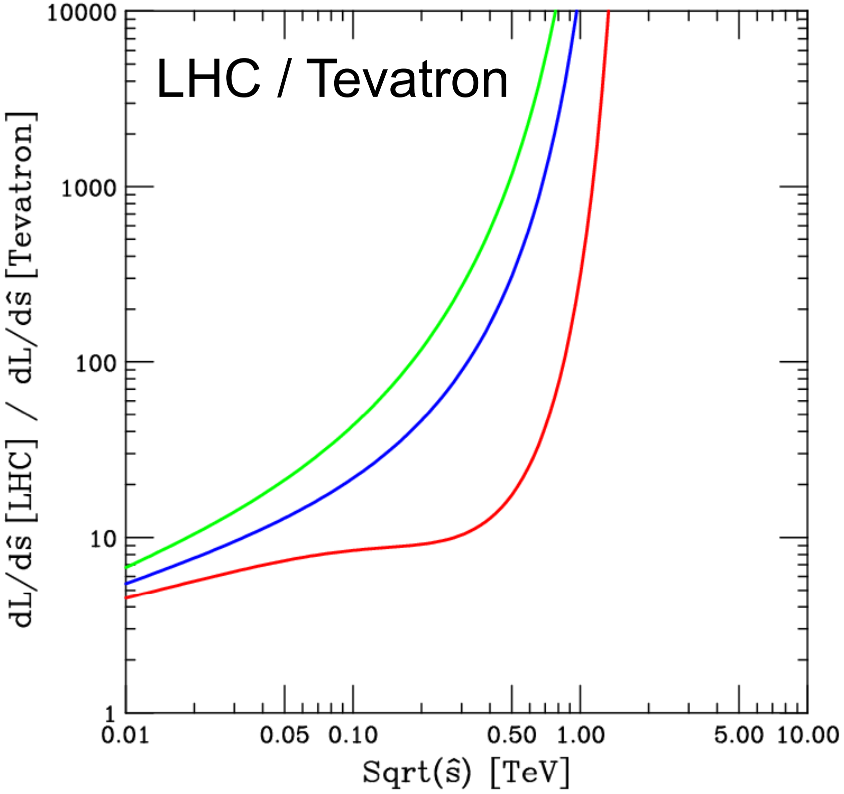
- a quick and easy way to assess the mass and collider energy dependence of production cross sections



$$\begin{aligned}\hat{\sigma}_{ab\to X} &= C_X \delta(\hat{s} - M^2) \\ \sigma_X &= \int_0^1 dx_a dx_b\, f_a(x_a, M^2) f_b(x_b, M^2)\, C_X\, \delta(x_a x_b - \tau) \\ &\equiv C_X \left[\frac{1}{s}\frac{\partial \mathcal{L}_{ab}}{\partial \tau}\right] \qquad\qquad (\tau = M^2/s) \\ \frac{\partial \mathcal{L}_{ab}}{\partial \tau} &= \int_0^1 dx_a dx_b\, f_a(x_a, M^2) f_b(x_b, M^2)\, \delta(x_a x_b - \tau)\end{aligned}$$

- i.e. all the mass and energy dependence is contained in the X-independent parton luminosity function in [ ]
- useful combinations are $ab = gg, \sum_q q\bar{q}, \ldots$
- and also useful for assessing the uncertainty on cross sections due to uncertainties in the pdfs





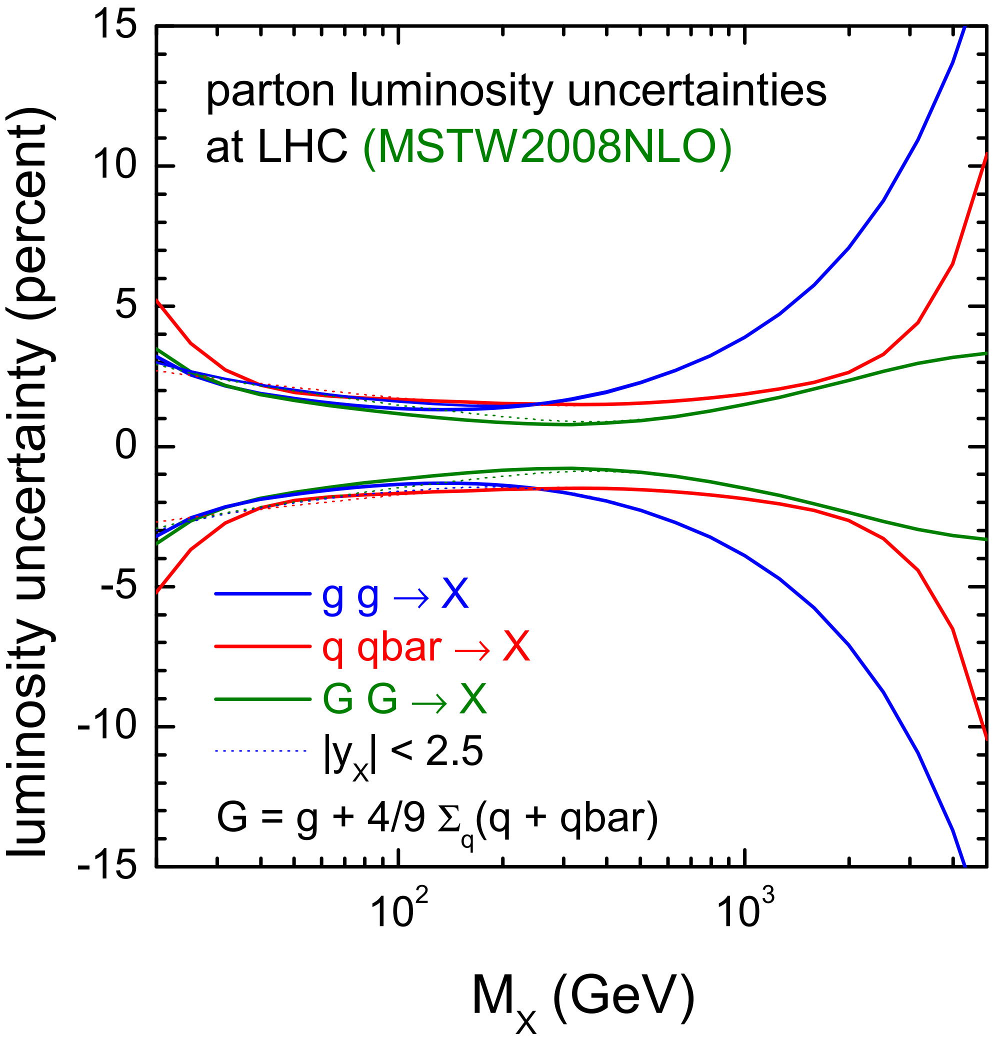
🟩 $= \; gg$

🟦 $= \sum_i (gq_i + g\bar{q}_i + q_i g + \bar{q}_i g)$

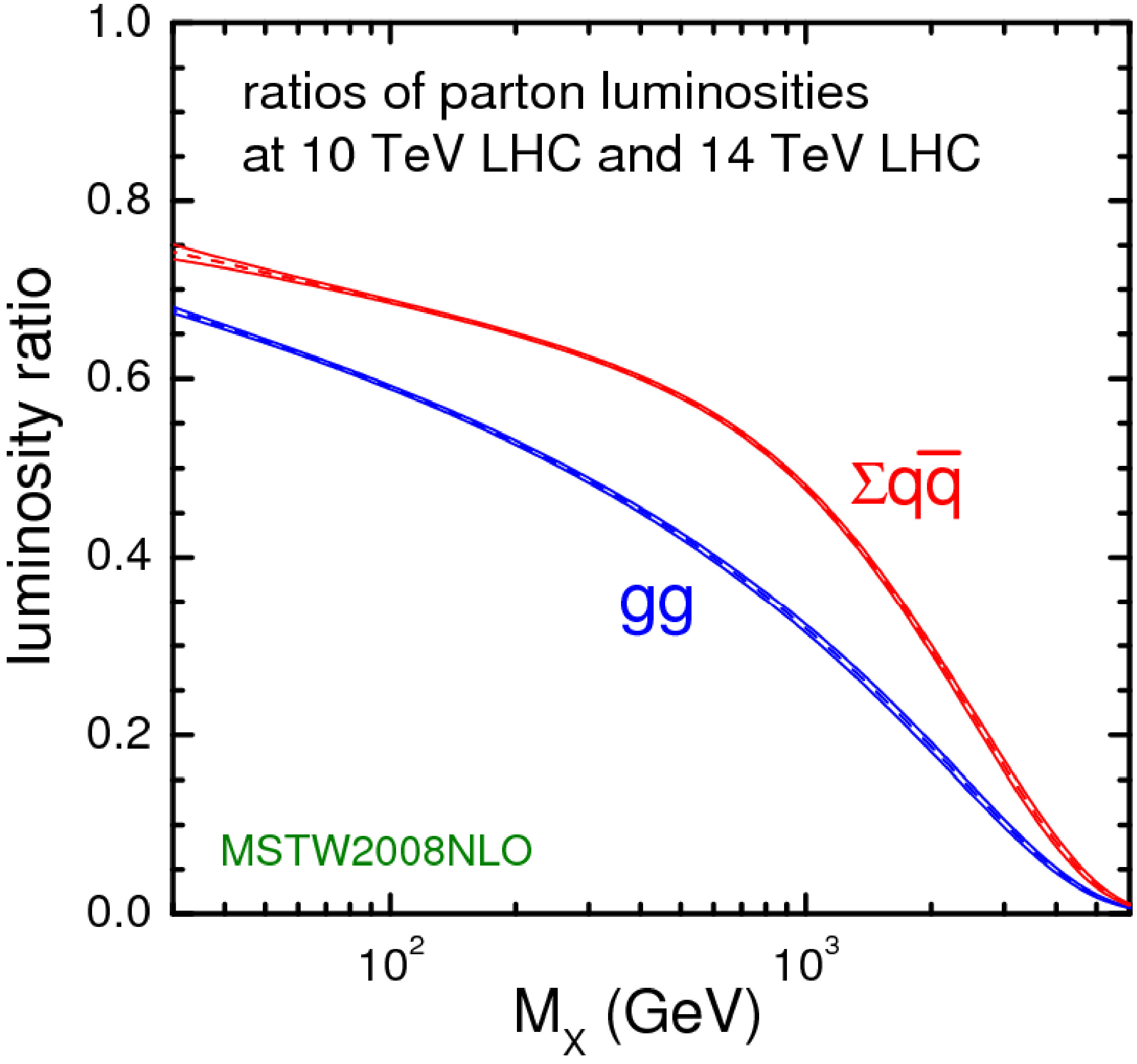
🟥 $= \sum_i (q_i\bar{q}_i + \bar{q}_i q_i)$

Huston, Campbell, S (2007)

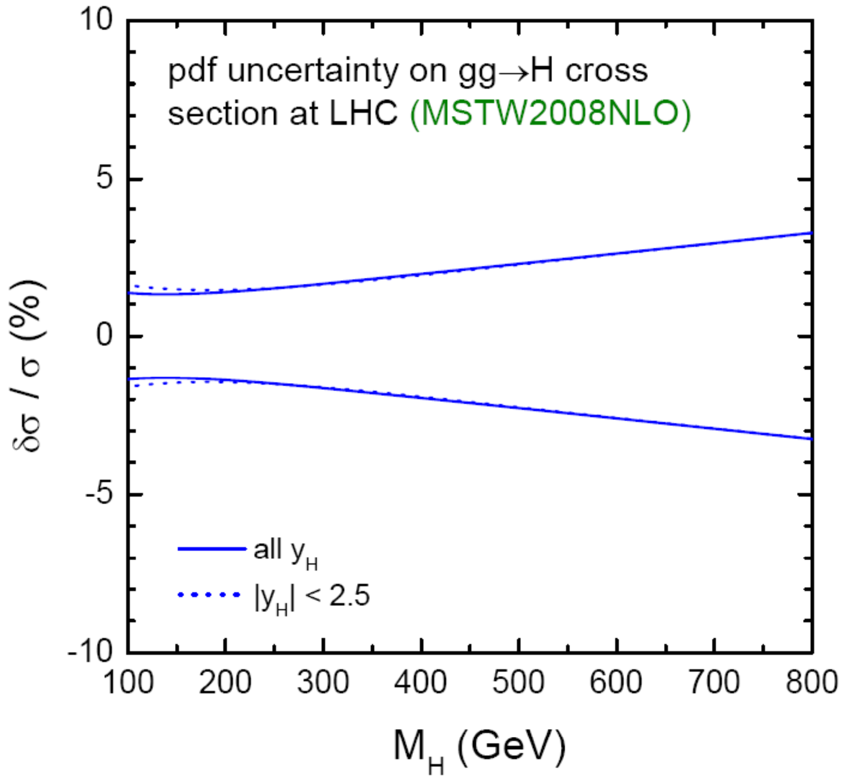
parton luminosity uncertainties at LHC

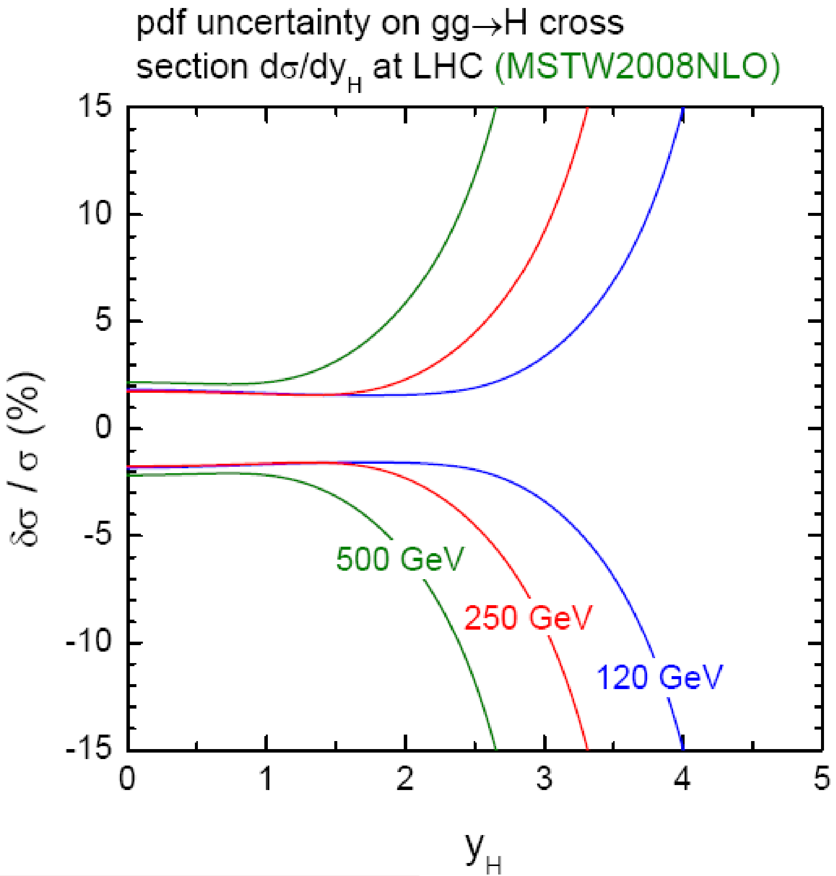


LHC at 10 TeV

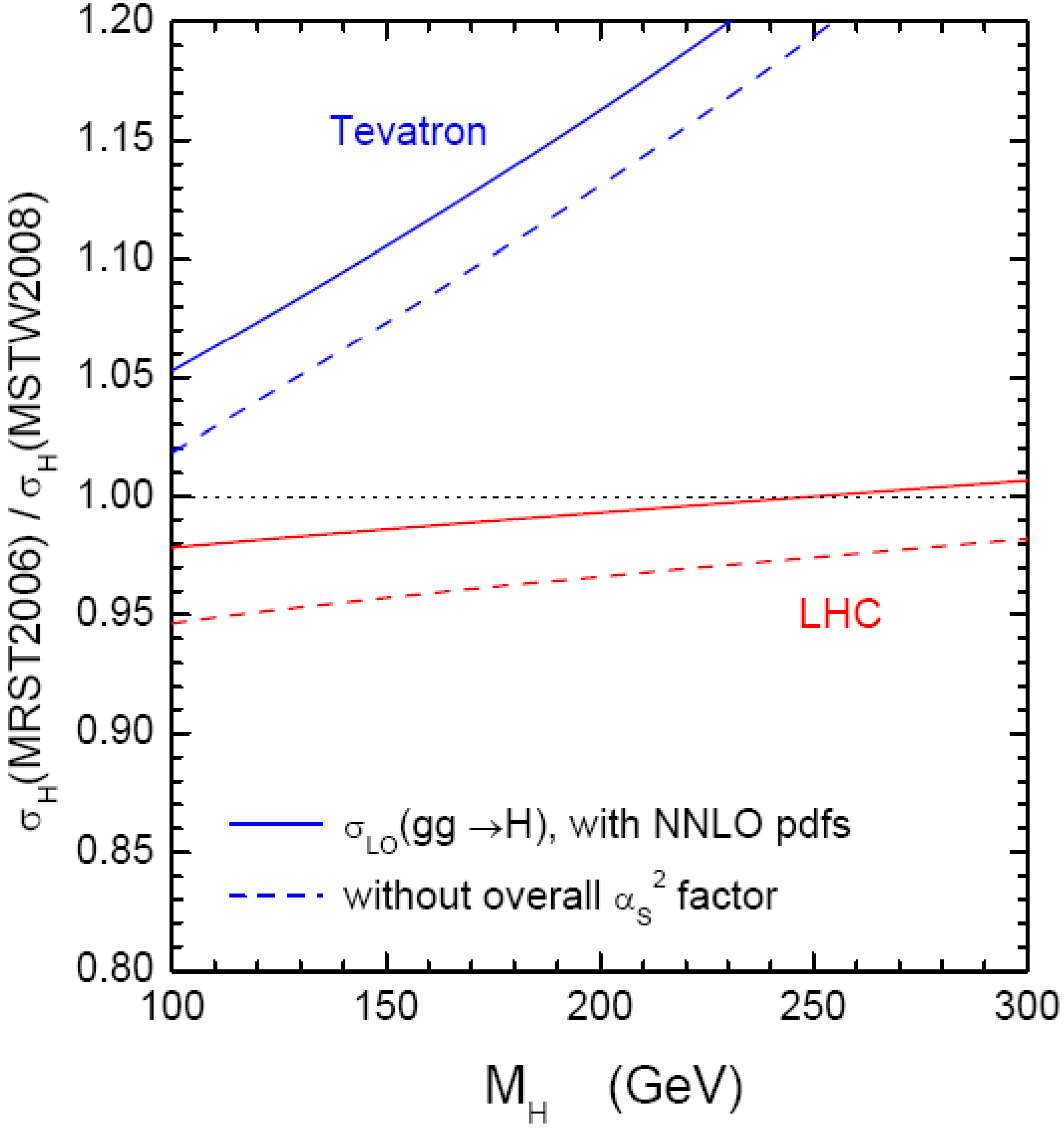

ratios of parton luminosities
at 10 TeV LHC and 14 TeV LHC
$\Sigma q\bar{q}$
gg
MSTW2008NLO
luminosity ratio
1.0
0.8
0.6
0.4
0.2
0.0
10^2
10^3
M_X (GeV)

pdf uncertainty on σ(gg→H)





→ typically ± 2-3% pdf uncertainty, except near edges of phase space

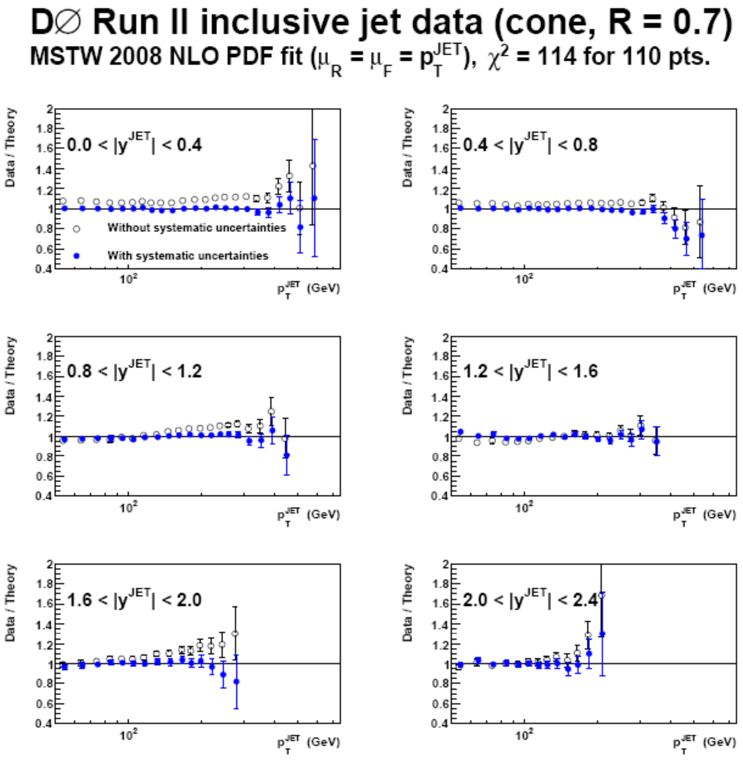

1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
100
150
200
250
300
σ_H(MRST2006) / σ_H(MSTW2008)
M_H (GeV)
Tevatron
LHC
$\sigma_{LO}(gg \rightarrow H)$, with NNLO pdfs
without overall α_S^2 factor

impact of Tevatron jet data on fits

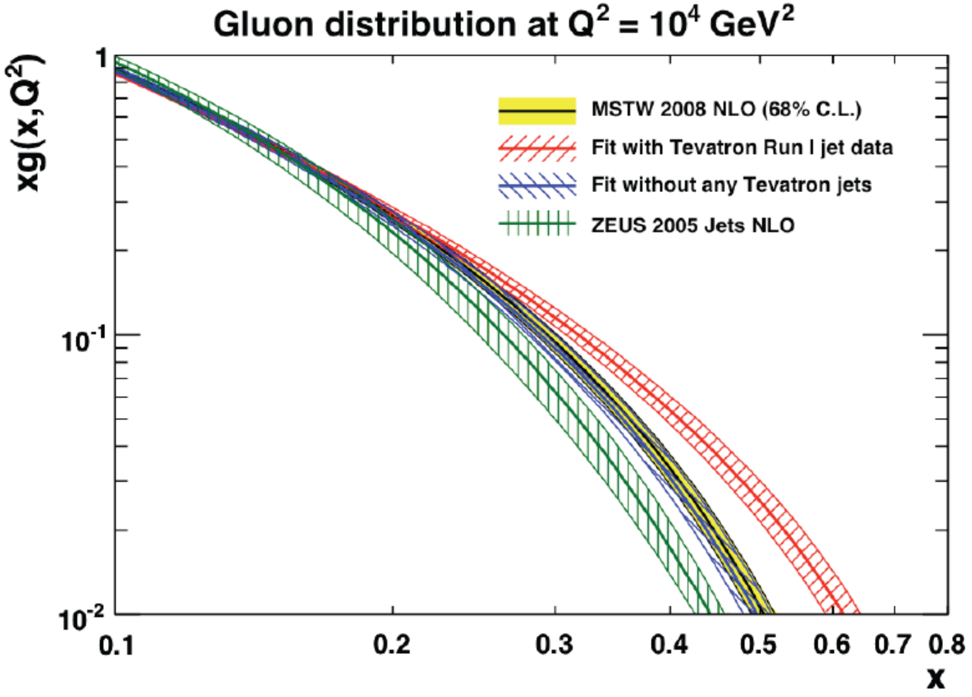
- a distinguishing feature of pdf sets is whether they use (MRST/MSTW, CTEQ,…) or do not use (H1, ZEUS, Alekhin, NNPDF,…) Tevatron jet data in the fit: the impact is on the *high-x gluon*

  (Note: Run II data requires slightly softer gluon than Run I data)

- the (still) missing ingredient is the full NNLO pQCD correction to the cross section, but not expected to have much impact in practice


DØ Run II inclusive jet data (cone, R = 0.7)

MSTW 2008 NLO PDF fit ($\mu_R = \mu_F = p_T^{JET}$), χ^2 = 114 for 110 pts.


Gluon distribution at $Q^2 = 10^4$ GeV^2

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- ‘standard candle’ processes (e.g. σ_Z) to
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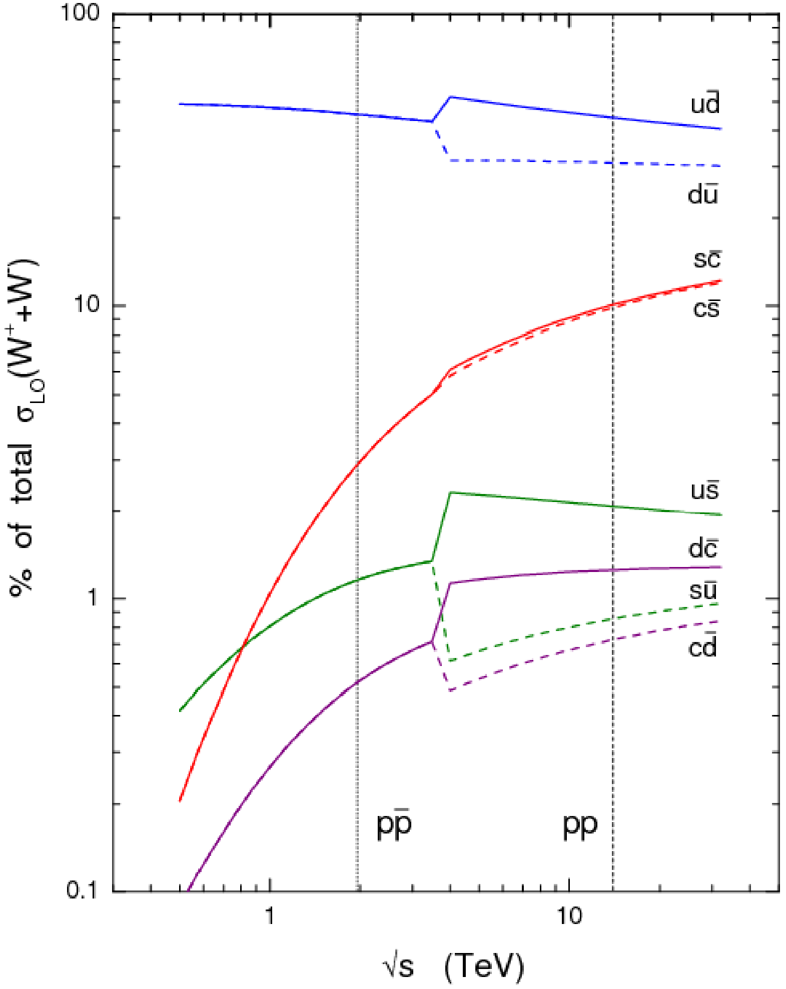
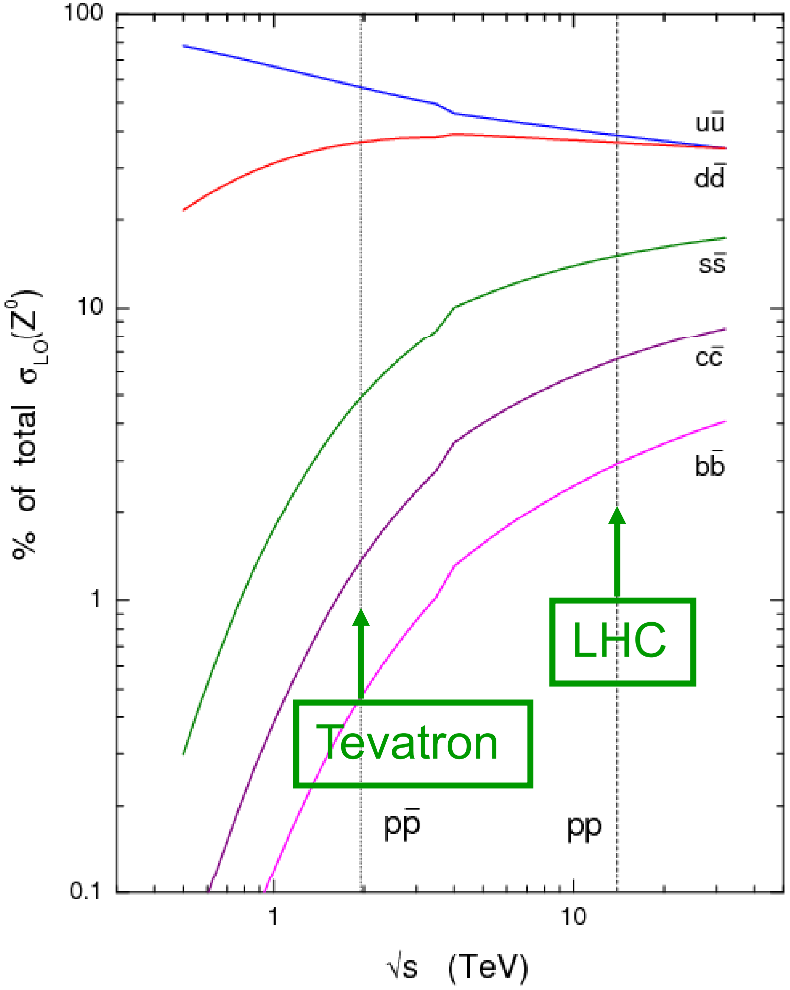
- learning more about pdfs from LHC measurements. e.g.
  - high-E_T jets → gluon?
  - W^+, W^-, Z^0 → quarks?
  - forward DY → small x?
  - …

standard candles: $\sigma(W,Z)$ @ LHC

- cross sections (total and rapidity distributions) known to NNLO pQCD and NLO EW; perturbation series seems to be converging quickly
- EW parameters well measured at LEP
- samples pdfs where they are well measured (in x) in DIS
- … although the mix of quark flavours is different: F_2 and $\sigma(W,Z)$ probe *different* combinations of u,d,s,c,b → sea quark distributions important
- precise measurement of cross section *ratios* at LHC (e.g. $\sigma(W^+)/\sigma(W^-)$, $\sigma(W^\pm)/\sigma(Z)$) will allow these subtle effects to be explored further

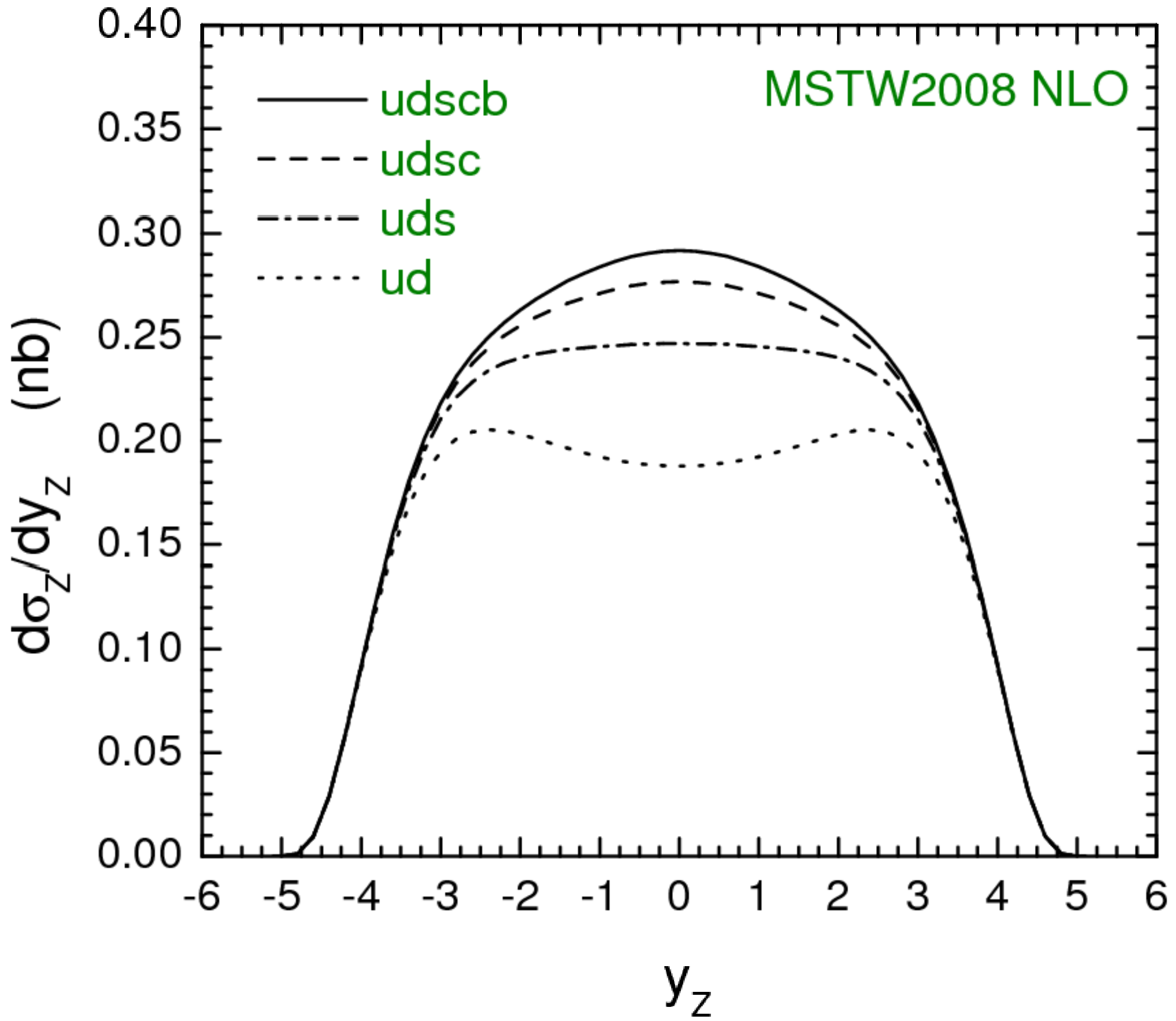


flavour decomposition of W cross sections



flavour decomposition of Z^0 cross sections

at LHC, ~30% of W and Z total cross sections involves s,c,b quarks


MSTW2008 NLO
udscb
udsc
uds
ud
$d\sigma_Z/dy_Z$ (nb)
y_Z
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6

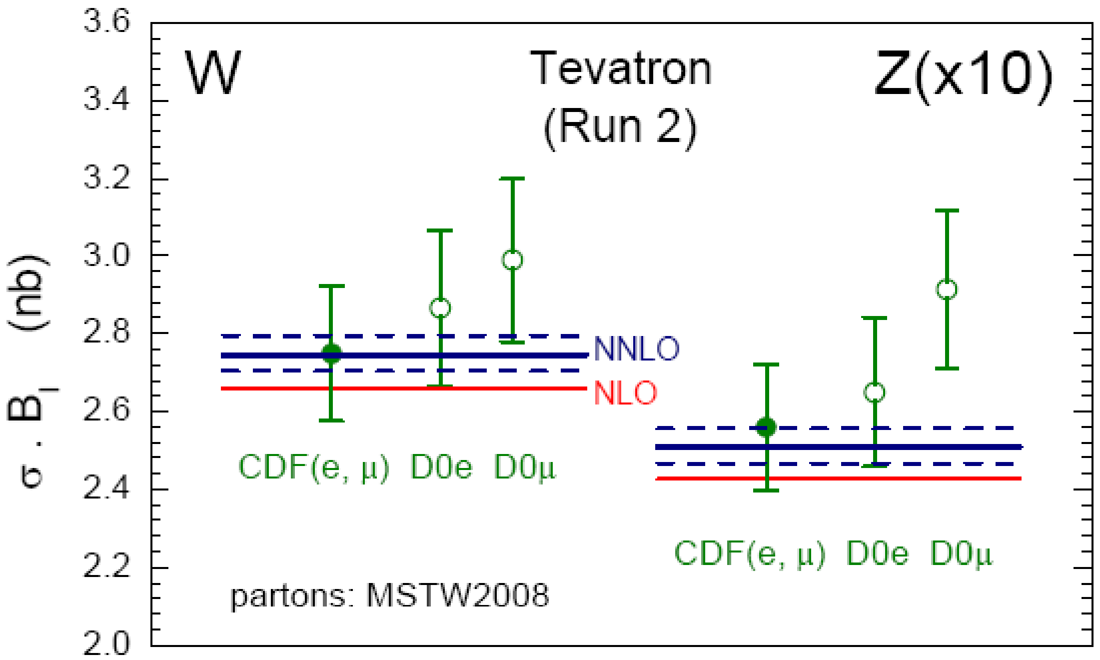
Tevatron, $\sqrt{s} = 1.96$ TeV	$B_{l\nu} \cdot \sigma_W$ (nb)	$B_{l^+l^-} \cdot \sigma_Z$ (nb)	R_{WZ}
MSTW 2008 LO	$1.963^{+0.025}_{-0.028}$ $\left(^{+1.2\%}_{-1.4\%}\right)$	$0.1788^{+0.0023}_{-0.0025}$ $\left(^{+1.3\%}_{-1.4\%}\right)$	$10.98^{+0.02}_{-0.03}$ $\left(^{+0.2\%}_{-0.3\%}\right)$
MSTW 2008 NLO	$2.659^{+0.057}_{-0.045}$ $\left(^{+2.1\%}_{-1.7\%}\right)$	$0.2426^{+0.0054}_{-0.0043}$ $\left(^{+2.2\%}_{-1.8\%}\right)$	$10.96^{+0.03}_{-0.02}$ $\left(^{+0.3\%}_{-0.2\%}\right)$
MSTW 2008 NNLO	$2.747^{+0.049}_{-0.042}$ $\left(^{+1.8\%}_{-1.5\%}\right)$	$0.2507^{+0.0048}_{-0.0041}$ $\left(^{+1.9\%}_{-1.6\%}\right)$	$10.96^{+0.03}_{-0.03}$ $\left(^{+0.2\%}_{-0.2\%}\right)$

LHC, $\sqrt{s} = 10$ TeV	$B_{l\nu} \cdot \sigma_W$ (nb)	$B_{l^+l^-} \cdot \sigma_Z$ (nb)	R_{WZ}
MSTW 2008 LO	$12.57^{+0.13}_{-0.19}$ $\left(^{+1.1\%}_{-1.5\%}\right)$	$1.163^{+0.011}_{-0.017}$ $\left(^{+1.0\%}_{-1.5\%}\right)$	$10.81^{+0.02}_{-0.02}$ $\left(^{+0.2\%}_{-0.2\%}\right)$
MSTW 2008 NLO	$14.92^{+0.31}_{-0.24}$ $\left(^{+2.1\%}_{-1.6\%}\right)$	$1.390^{+0.029}_{-0.022}$ $\left(^{+2.1\%}_{-1.5\%}\right)$	$10.73^{+0.02}_{-0.02}$ $\left(^{+0.2\%}_{-0.2\%}\right)$
MSTW 2008 NNLO	$15.35^{+0.26}_{-0.25}$ $\left(^{+1.7\%}_{-1.6\%}\right)$	$1.429^{+0.024}_{-0.022}$ $\left(^{+1.7\%}_{-1.6\%}\right)$	$10.74^{+0.02}_{-0.02}$ $\left(^{+0.2\%}_{-0.2\%}\right)$

LHC, $\sqrt{s} = 14$ TeV	$B_{l\nu} \cdot \sigma_W$ (nb)	$B_{l^+l^-} \cdot \sigma_Z$ (nb)	R_{WZ}
MSTW 2008 LO	$18.51^{+0.22}_{-0.32}$ $\left(^{+1.2\%}_{-1.7\%}\right)$	$1.736^{+0.019}_{-0.028}$ $\left(^{+1.1\%}_{-1.6\%}\right)$	$10.66^{+0.02}_{-0.02}$ $\left(^{+0.2\%}_{-0.2\%}\right)$
MSTW 2008 NLO	$21.17^{+0.42}_{-0.36}$ $\left(^{+2.0\%}_{-1.7\%}\right)$	$2.001^{+0.040}_{-0.032}$ $\left(^{+2.0\%}_{-1.6\%}\right)$	$10.58^{+0.02}_{-0.02}$ $\left(^{+0.2\%}_{-0.2\%}\right)$
MSTW 2008 NNLO	$21.72^{+0.36}_{-0.36}$ $\left(^{+1.7\%}_{-1.7\%}\right)$	$2.051^{+0.035}_{-0.033}$ $\left(^{+1.7\%}_{-1.6\%}\right)$	$10.59^{+0.02}_{-0.03}$ $\left(^{+0.2\%}_{-0.3\%}\right)$

Note: at NNLO, factorisation and renormalisation scale variation M/2 → 2M gives an additional ± 2% change in the LHC cross sections

comparison with measured Tevatron cross sections



data errors dominated by ±6% systematic
error from luminosity uncertainty!

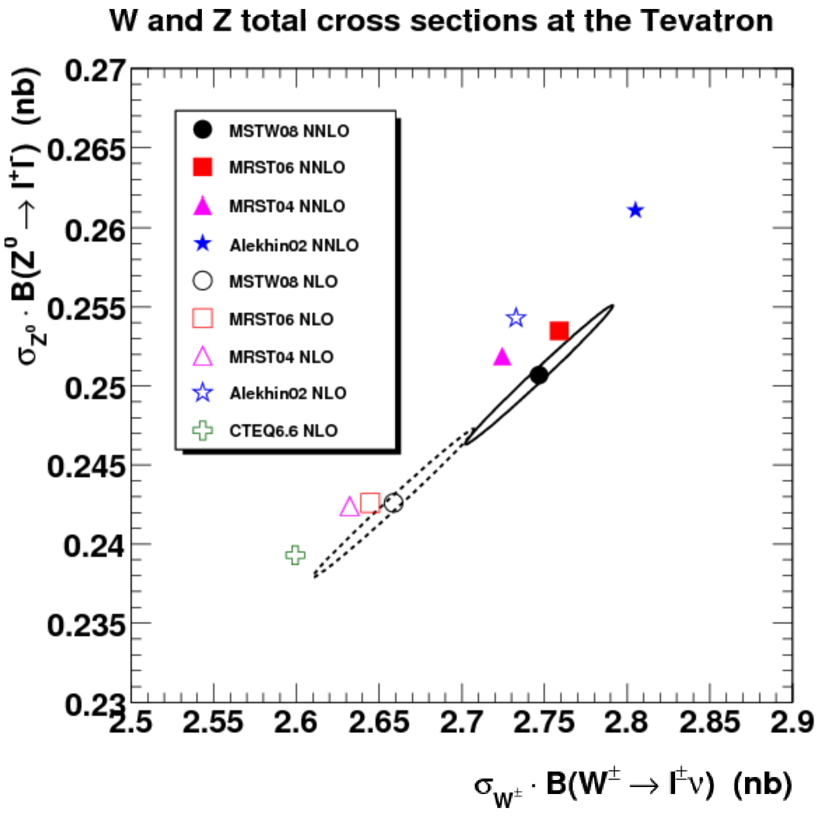
predictions for $\sigma(W,Z)$ @ LHC (Tevatron)

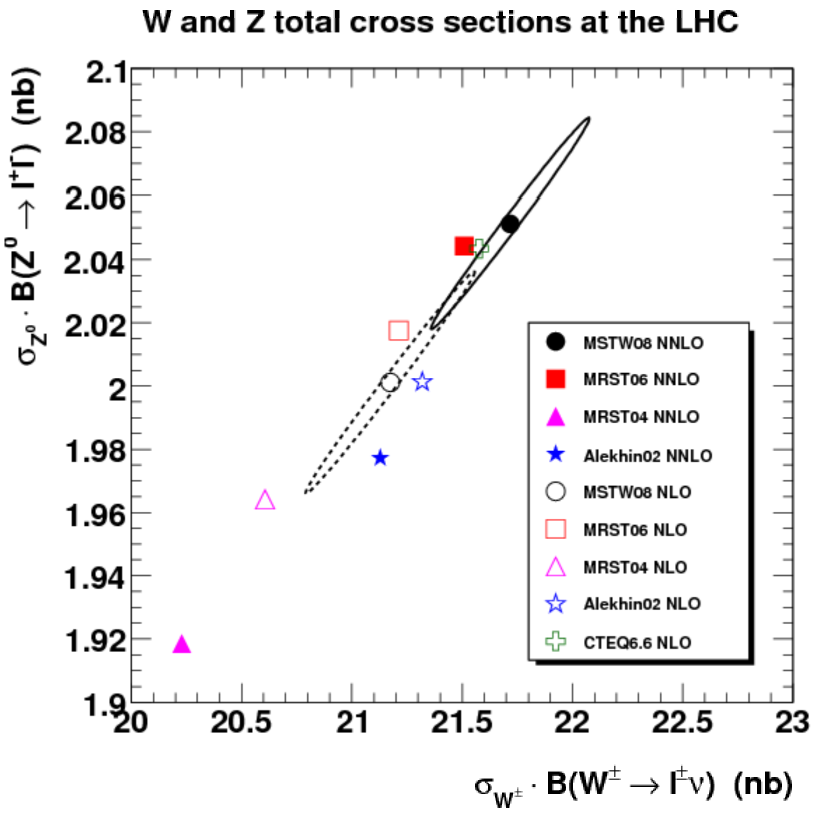
	$B_{l\upsilon}.\sigma_W$ (nb)	$B_{ll}.\sigma_Z$ (nb)
MSTW 2008 NLO	21.17 (2.659)	2.001 (0.2426)
MSTW 2008 NNLO	21.72 (2.747)	2.051 (0.2507)

MRST 2006 NLO	21.21 (2.645)	2.018 (0.2426)
MRST 2006 NNLO	21.51 (2.759)	2.044 (0.2535)
MRST 2004 NLO	20.61 (2.632)	1.964 (0.2424)
MRST 2004 NNLO	20.23 (2.724)	1.917 (0.2519)
CTEQ6.6 NLO	21.58 (2.599)	2.043 (0.2393)
Alekhin 2002 NLO	21.32 (2.733)	2.001 (0.2543)
Alekhin 2002 NNLO	21.13 (2.805)	1.977 (0.2611)

MSTW

predictions for σ(W,Z) @ Tevatron, LHC





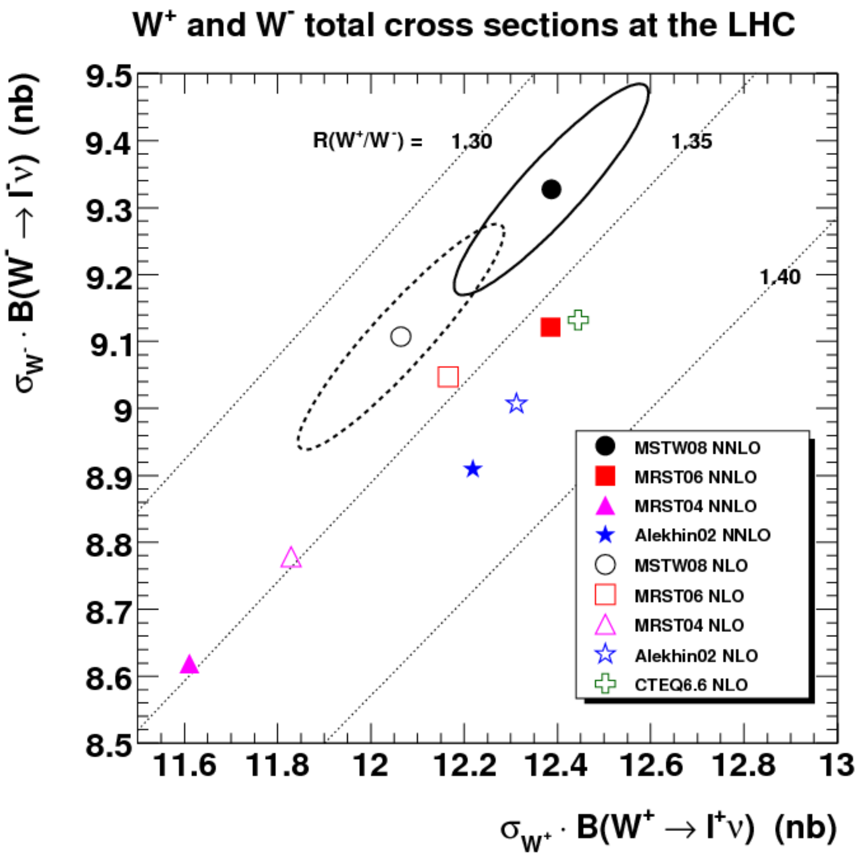
- MRST/MSTW NNLO: 2008 ~ 2006 > 2004 mainly due to changes in treatment of *charm*
- CTEQ: 6.6 ~ 6.5 > 6.1 due to changes in treatment of *s,c,b*
- NLO: CTEQ6.6 2% higher than MSTW 2008 at LHC, because of slight differences in quark (u,d,s,c) pdfs, difference within quoted uncertainty

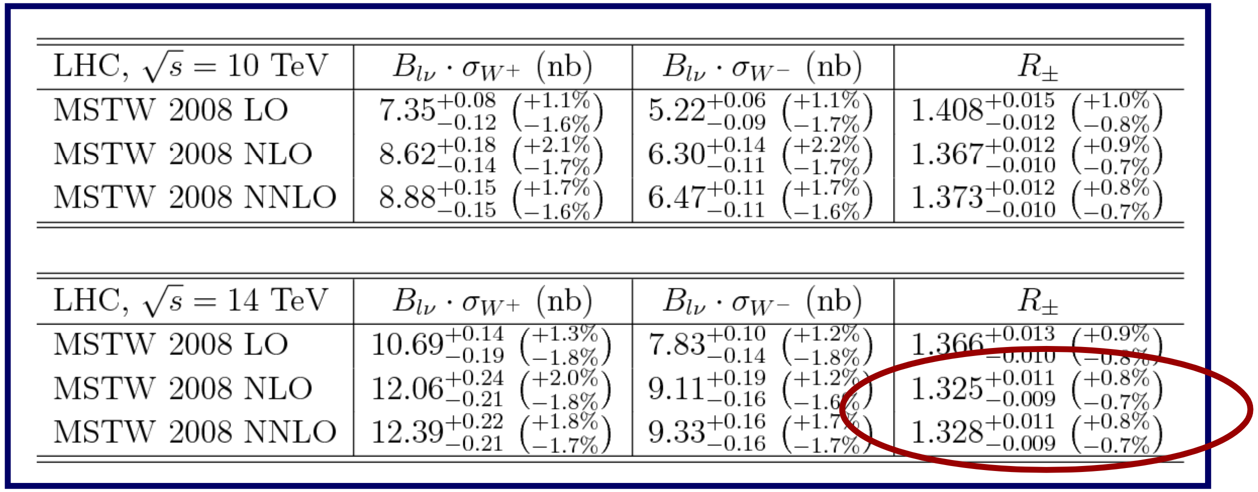
$$\boxed{R_\pm = \sigma(W^+\to l^+\nu) \,/\, \sigma(W^-\to l^-\nu)}$$

$$R_\pm \approx \frac{u(x_1)\bar{d}(x_2) + c(x_1)\bar{s}(x_2) + (1\leftrightarrow 2)}{d(x_1)\bar{u}(x_2) + s(x_1)\bar{c}(x_2) + (1\leftrightarrow 2)}$$

$\delta\sigma_{th} \approx \delta\sigma_{pdf} \approx \pm 1\%$,

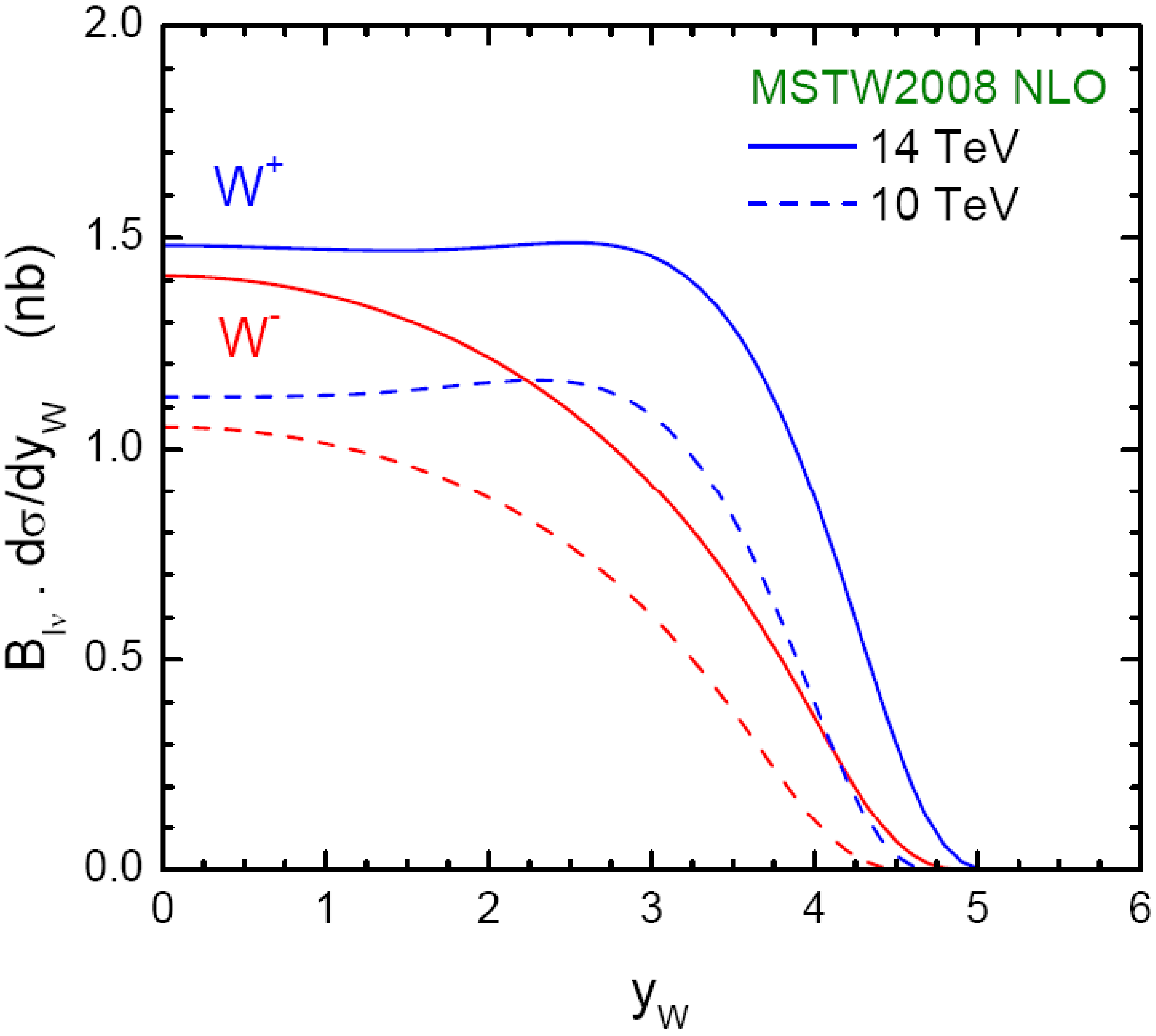
$\delta\sigma_{expt} \approx$???





LHC, $\sqrt{s} = 10$ TeV	$B_{l\nu}\cdot\sigma_{W^+}$ (nb)	$B_{l\nu}\cdot\sigma_{W^-}$ (nb)	$R_\pm$
MSTW 2008 LO	$7.35^{+0.08}_{-0.12}$ $\left({+1.1\% \atop -1.6\%}\right)$	$5.22^{+0.06}_{-0.09}$ $\left({+1.1\% \atop -1.7\%}\right)$	$1.408^{+0.015}_{-0.012}$ $\left({+1.0\% \atop -0.8\%}\right)$
MSTW 2008 NLO	$8.62^{+0.18}_{-0.14}$ $\left({+2.1\% \atop -1.7\%}\right)$	$6.30^{+0.14}_{-0.11}$ $\left({+2.2\% \atop -1.7\%}\right)$	$1.367^{+0.012}_{-0.010}$ $\left({+0.9\% \atop -0.7\%}\right)$
MSTW 2008 NNLO	$8.88^{+0.15}_{-0.15}$ $\left({+1.7\% \atop -1.6\%}\right)$	$6.47^{+0.11}_{-0.11}$ $\left({+1.7\% \atop -1.6\%}\right)$	$1.373^{+0.012}_{-0.010}$ $\left({+0.8\% \atop -0.7\%}\right)$

LHC, $\sqrt{s} = 14$ TeV	$B_{l\nu}\cdot\sigma_{W^+}$ (nb)	$B_{l\nu}\cdot\sigma_{W^-}$ (nb)	$R_\pm$
MSTW 2008 LO	$10.69^{+0.14}_{-0.19}$ $\left({+1.3\% \atop -1.8\%}\right)$	$7.83^{+0.10}_{-0.14}$ $\left({+1.2\% \atop -1.8\%}\right)$	$1.366^{+0.013}_{-0.010}$ $\left({+0.9\% \atop -0.8\%}\right)$
MSTW 2008 NLO	$12.06^{+0.24}_{-0.21}$ $\left({+2.0\% \atop -1.8\%}\right)$	$9.11^{+0.19}_{-0.16}$ $\left({+1.2\% \atop -1.6\%}\right)$	$1.325^{+0.011}_{-0.009}$ $\left({+0.8\% \atop -0.7\%}\right)$
MSTW 2008 NNLO	$12.39^{+0.22}_{-0.21}$ $\left({+1.8\% \atop -1.7\%}\right)$	$9.33^{+0.16}_{-0.16}$ $\left({+1.7\% \atop -1.7\%}\right)$	$1.328^{+0.011}_{-0.009}$ $\left({+0.8\% \atop -0.7\%}\right)$


MSTW2008 NLO
14 TeV
10 TeV
W^+
W^-
$B_{l\nu} \cdot d\sigma/dy_W$ (nb)
y_W
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6

using the W^{+-} charge asymmetry at the LHC

- at the Tevatron $\sigma(W^+) = \sigma(W^-)$, whereas at LHC $\sigma(W^+) \sim 1.3\sigma(W^-)$
- can use this asymmetry to calibrate backgrounds to new physics, since typically $\sigma_{NP}(X \rightarrow W^+ + \ldots) = \sigma_{NP}(X \rightarrow W^- + \ldots)$
- example:

$$gg \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b} \rightarrow W^{\pm}(\rightarrow l^{\pm} + \nu) + 4\text{jets}$$

in this case

$$\sigma_{\text{signal}}(W^+ + 4\text{jets}) = \sigma_{\text{signal}}(W^- + 4\text{jets})$$

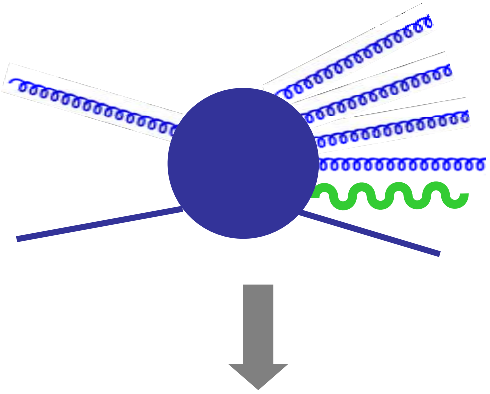
whereas…

$$\sigma_{\text{QCDbkgd}}(W^+ + 4\text{jets}) \neq \sigma_{\text{QCDbkgd}}(W^- + 4\text{jets})$$

which can in principle help distinguish signal and background

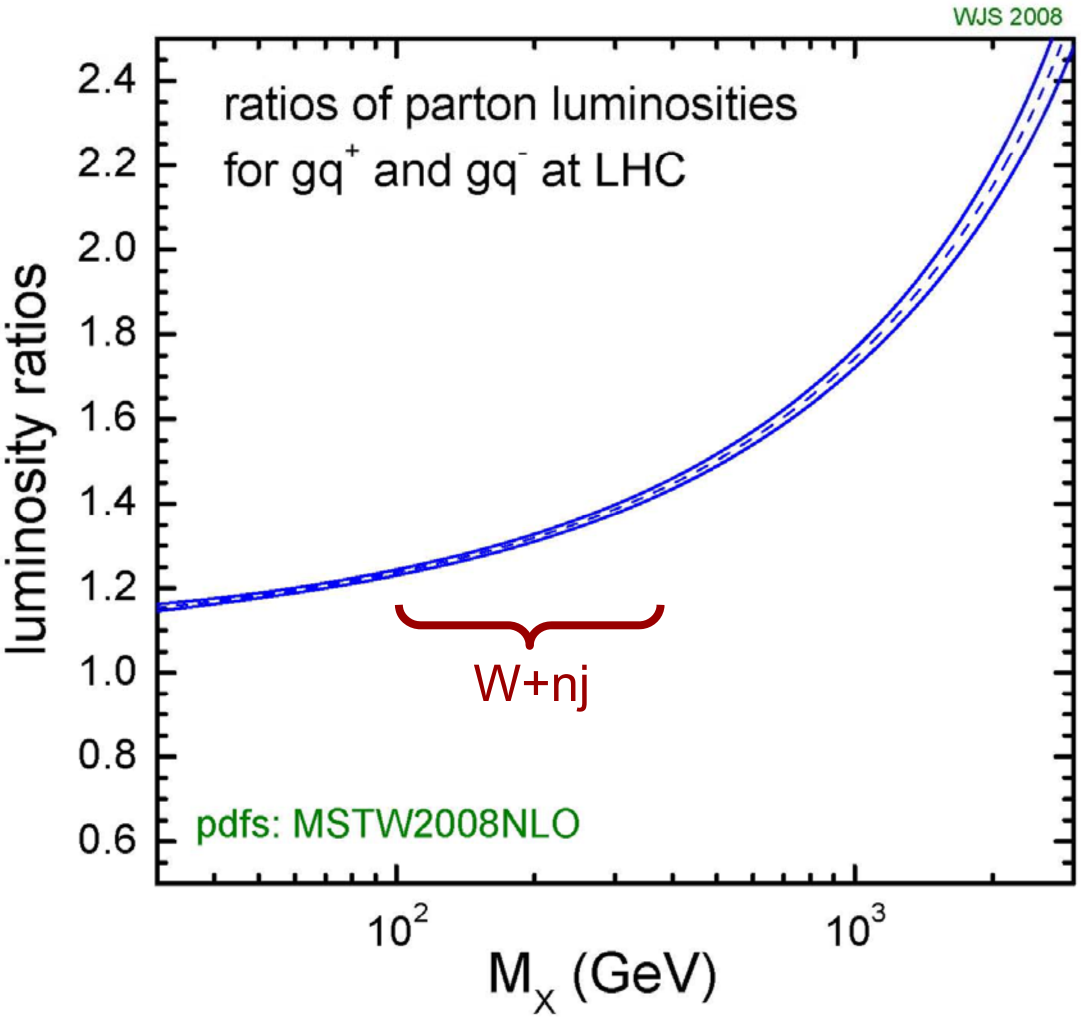
W^{+-} + n jets @ LHC

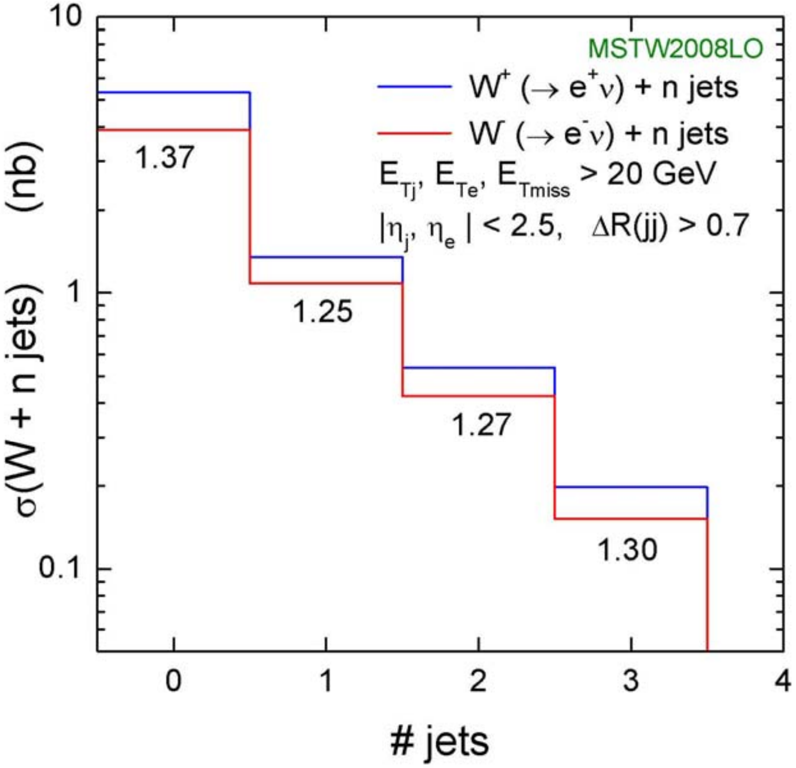
for $n_{jet} > 1$ dominant subprocess is:

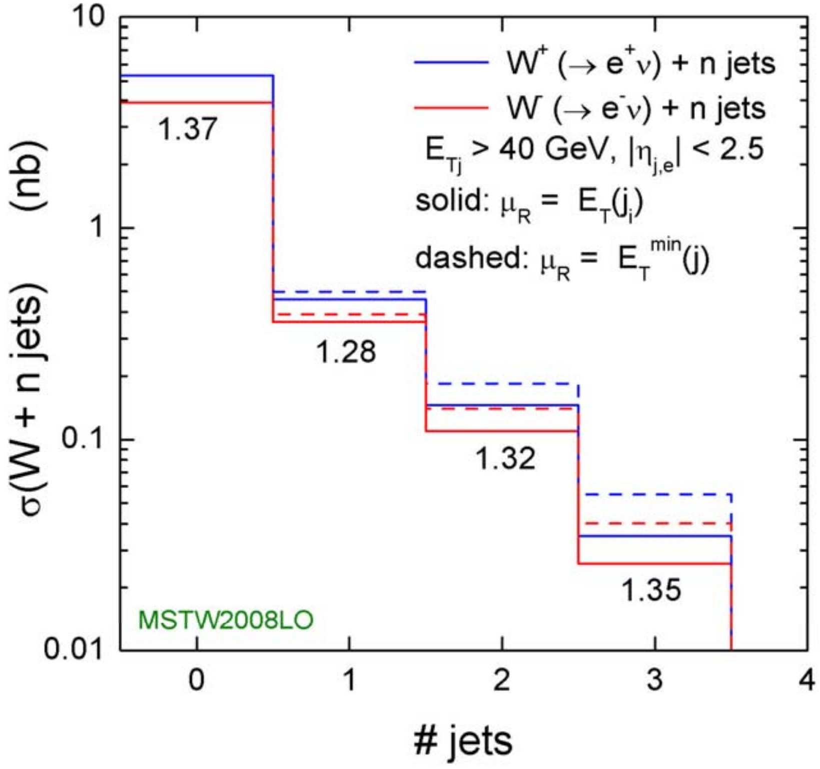


$$g + q^{+} \rightarrow W^{+} + q^{-} + ng$$
$$g + q^{-} \rightarrow W^{-} + q^{+} + ng$$

	%qq	%qg	%gg
W+0j	100	0	0
W+1j	75	25	0
W+2j	18	75	7
W+3j	18	72	10

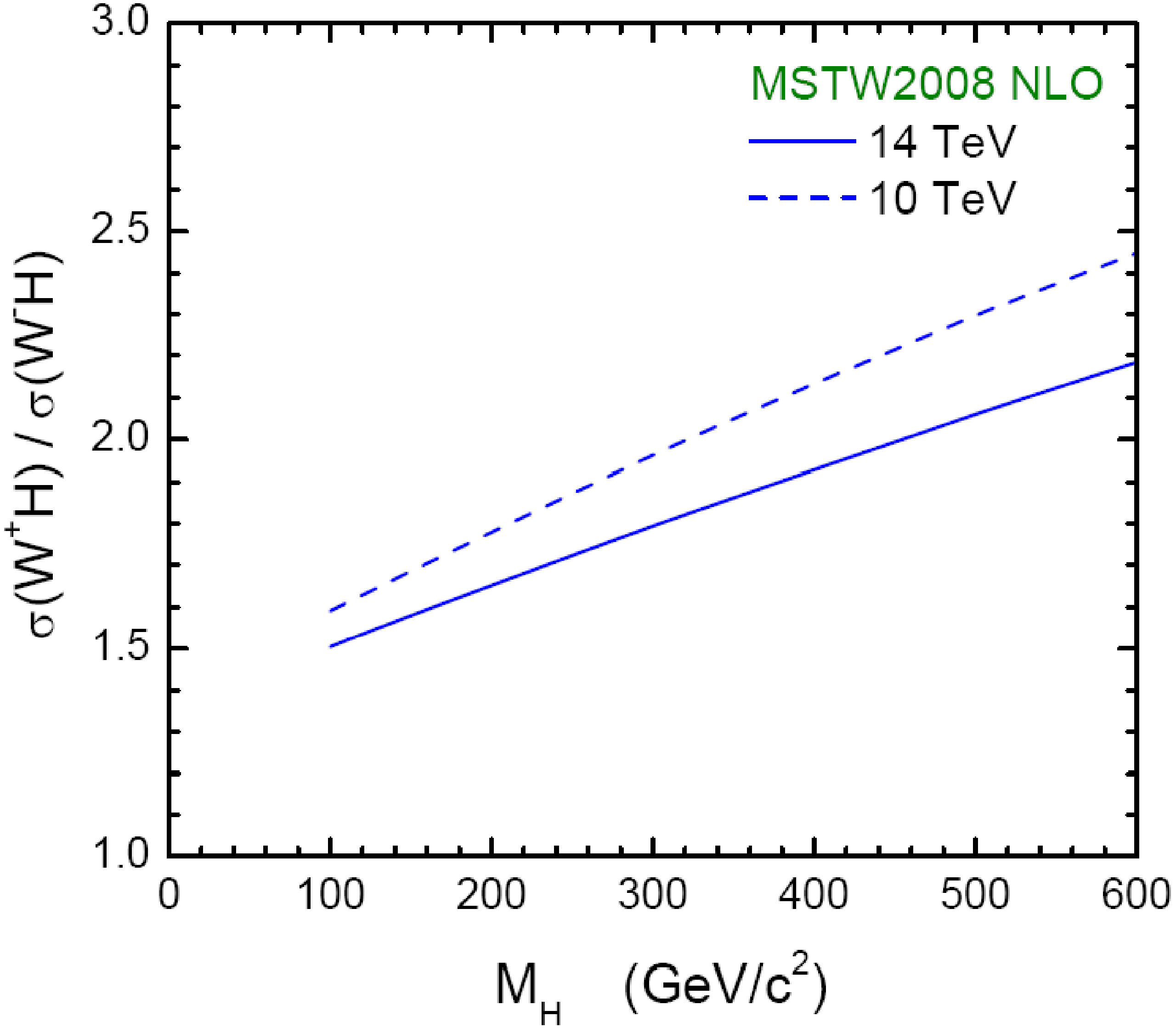


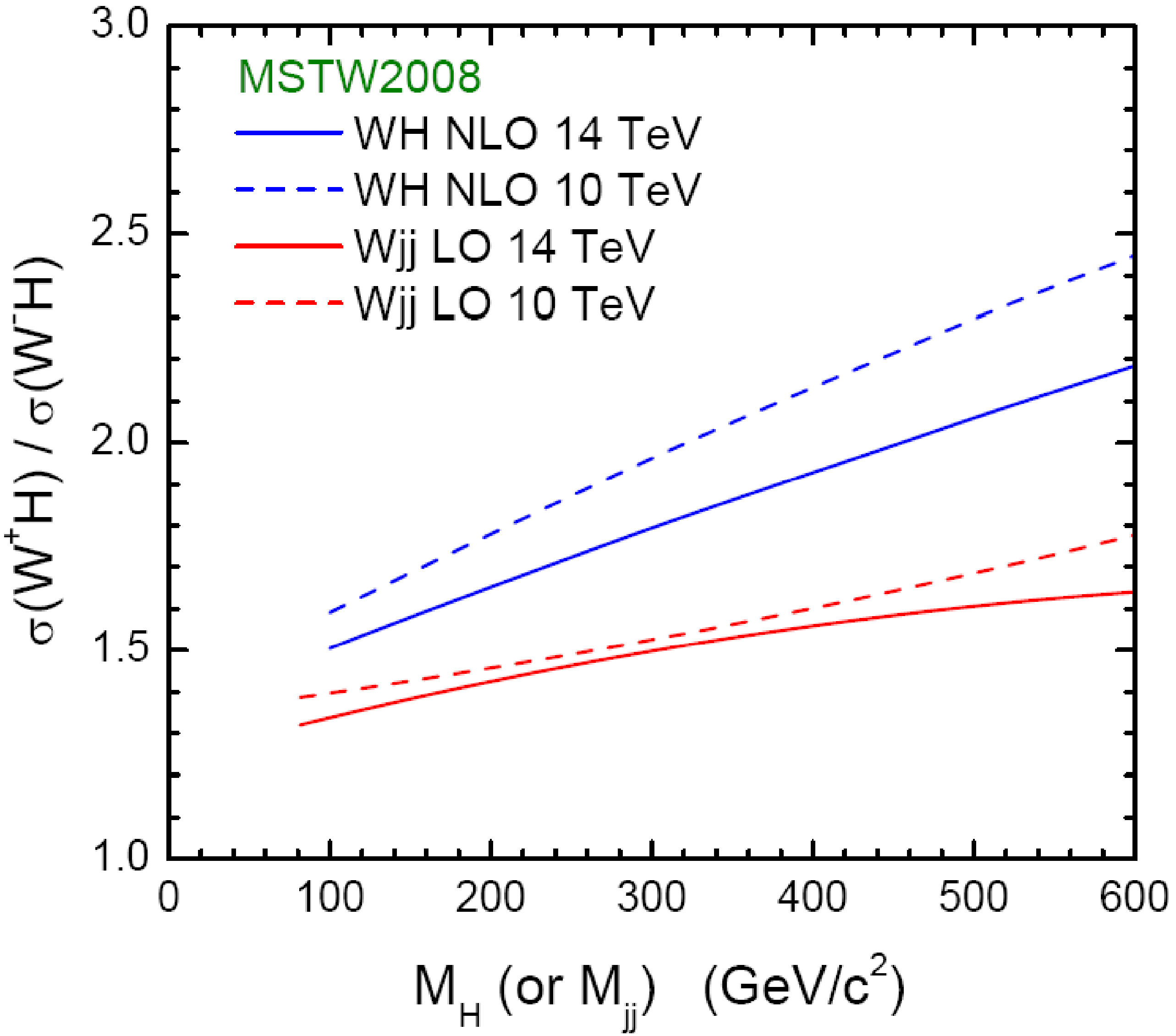




W^+/W^- ratio:

- very sensitive to u/d pdf ratio
- varies with y_W
- depends slightly on n_{jet} and E_{Tj}(min)
- fairly independent of scale choice etc


MSTW2008 NLO
14 TeV
10 TeV
$\sigma(W^+H) / \sigma(W^-H)$
M_H (GeV/c^2)
3.0
2.5
2.0
1.5
1.0
0
100
200
300
400
500
600


MSTW2008
WH NLO 14 TeV
WH NLO 10 TeV
Wjj LO 14 TeV
Wjj LO 10 TeV
$\sigma(W^+H) / \sigma(W^-H)$
3.0
2.5
2.0
1.5
1.0
0
100
200
300
400
500
600
M_H (or M_{jj}) (GeV/c^2)

pdfs at LHC – the issues

- high precision cross section predictions require accurate knowledge of pdfs: $\delta\sigma_{th} = \delta\sigma_{pdf} + \ldots$

  → improved signal and background predictions

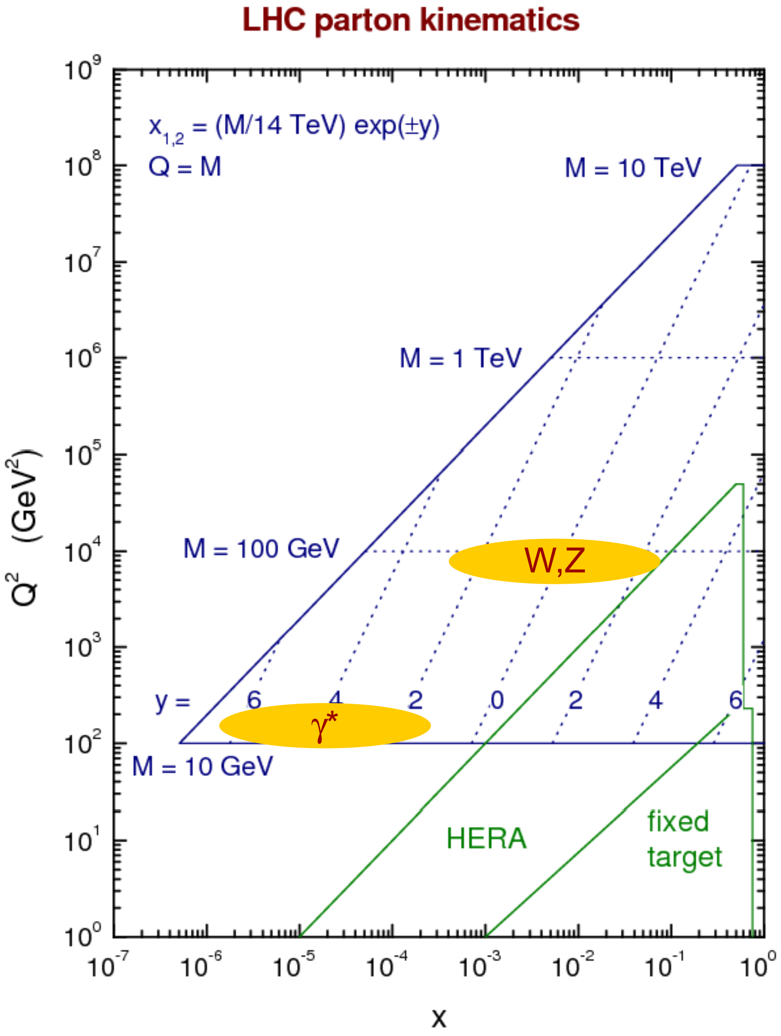
  → easier to spot new physics

- 'standard candle' processes (e.g. σ_Z) to
  - check formalism (factorisation, DGLAP, …)
  - measure machine luminosity?

- learning more about pdfs from LHC measurements. e.g.
  - high-E_T jets → gluon?
  - W^+, W^-, Z^0 → quarks?
  - forward DY → small x?
  - …

impact of LHC measurements on pdfs

- the standard candles: central $\sigma(W,Z,tt,jets)$ as a probe and test of pdfs in the $x \sim 10^{-2\pm1}$, $Q^2 \sim 10^{4-6}$ GeV^2 range where most New Physics is expected (H, SUSY, ….)
- forward production of (relatively) low-mass states (e.g. γ^*,dijets,…) to access partons at $x<<1$ (and $x\sim1$)



LHCb

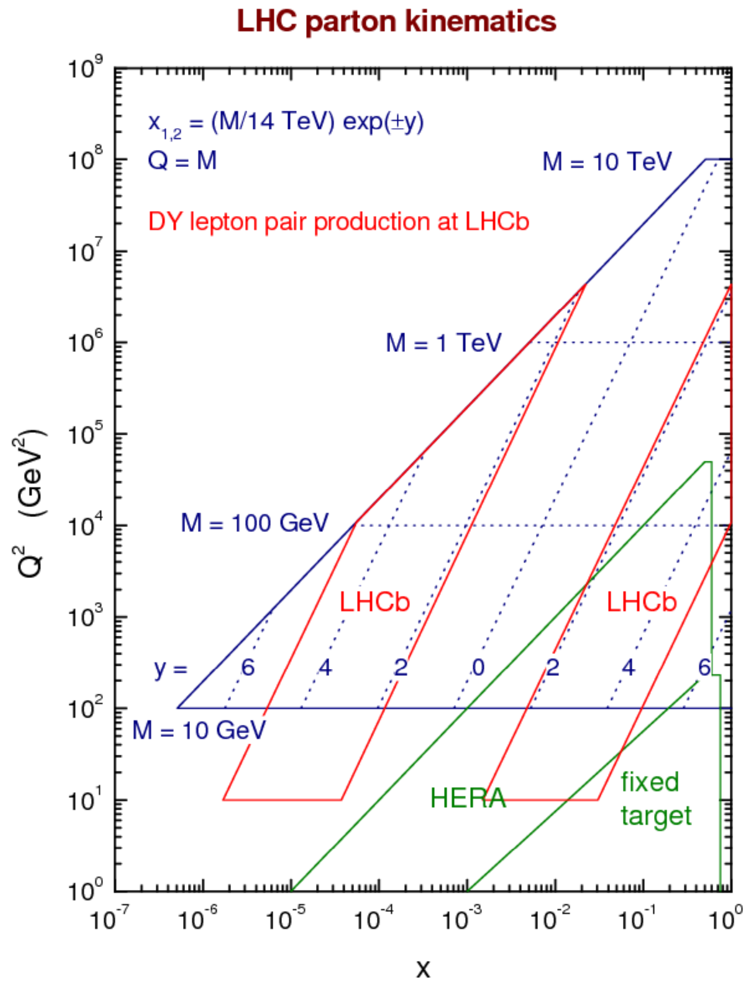
Unique features

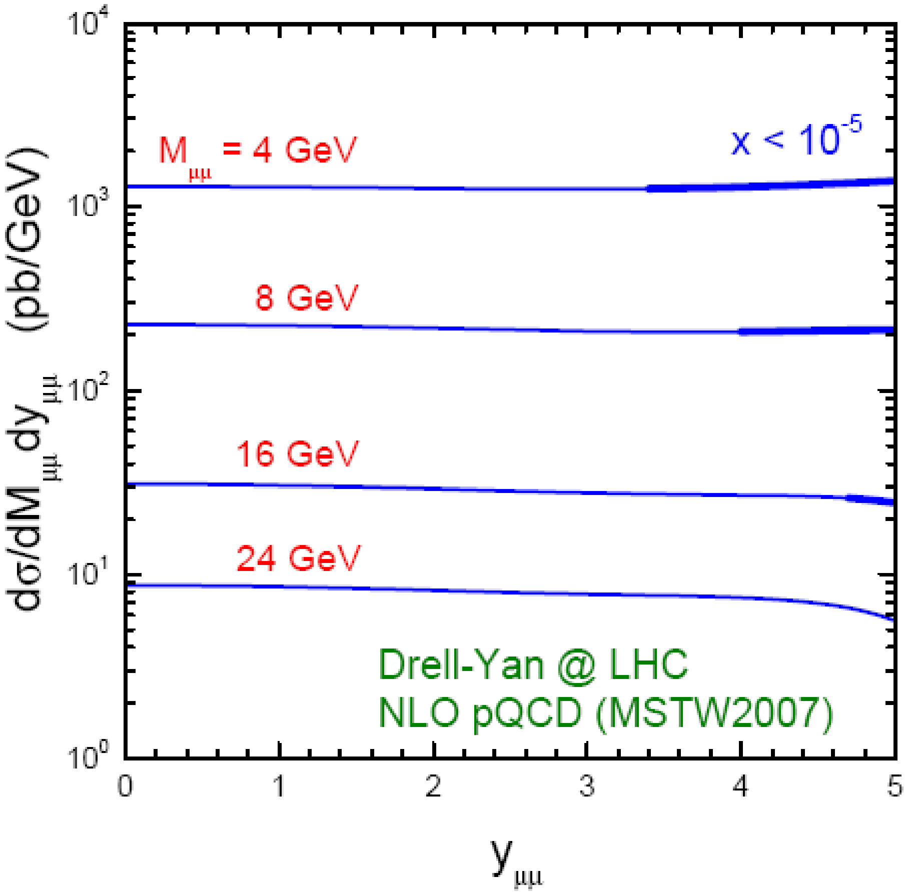
- pseudo-rapidity range 1.9 - 4.9
  - 1.9 - 2.5 complementary to ATLAS/CMS
  - > 2.5 unique to LHCb
- beam defocused at LHCb: 1 year of running = 2 fb^{-1}
- trigger on low momentum muons: $p > 8$ GeV, $p_T > 1$ GeV

⇒ access to unique range of (x,Q^2)

LHCb

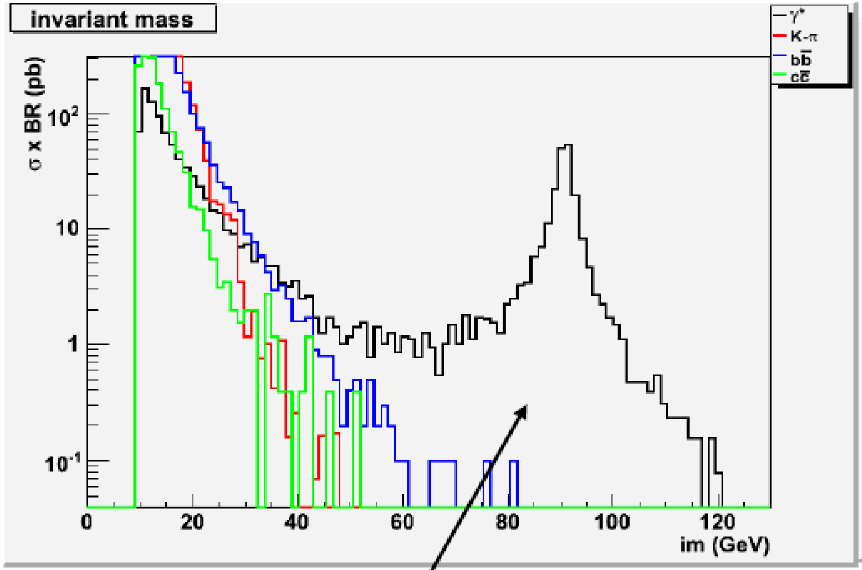
→ detect forward, low p_T muons from $q\bar{q} \rightarrow \mu^+\mu^-$

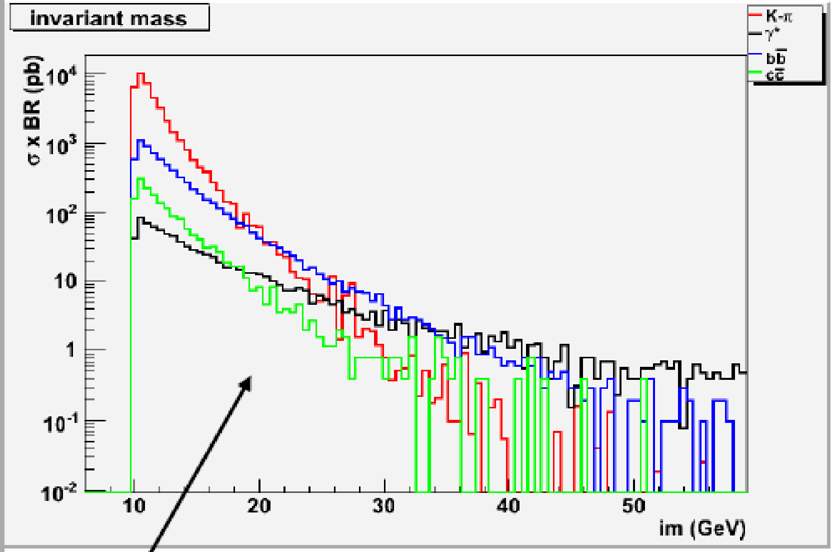




γ*->μμ selection

- 4-vector Pythia + LHCb acceptance

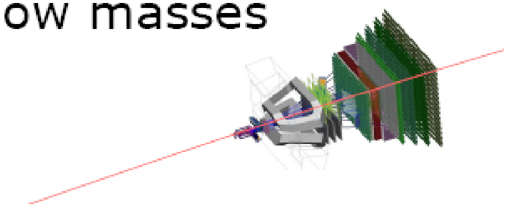




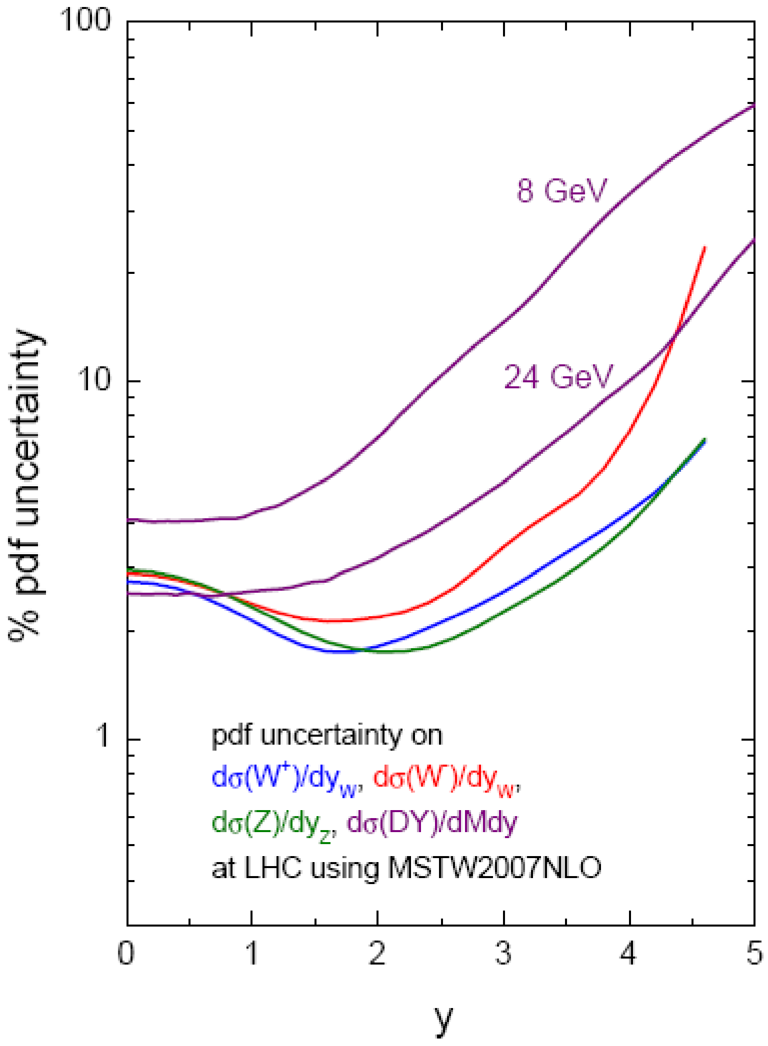
Dominant at Z but large backgrounds at low masses

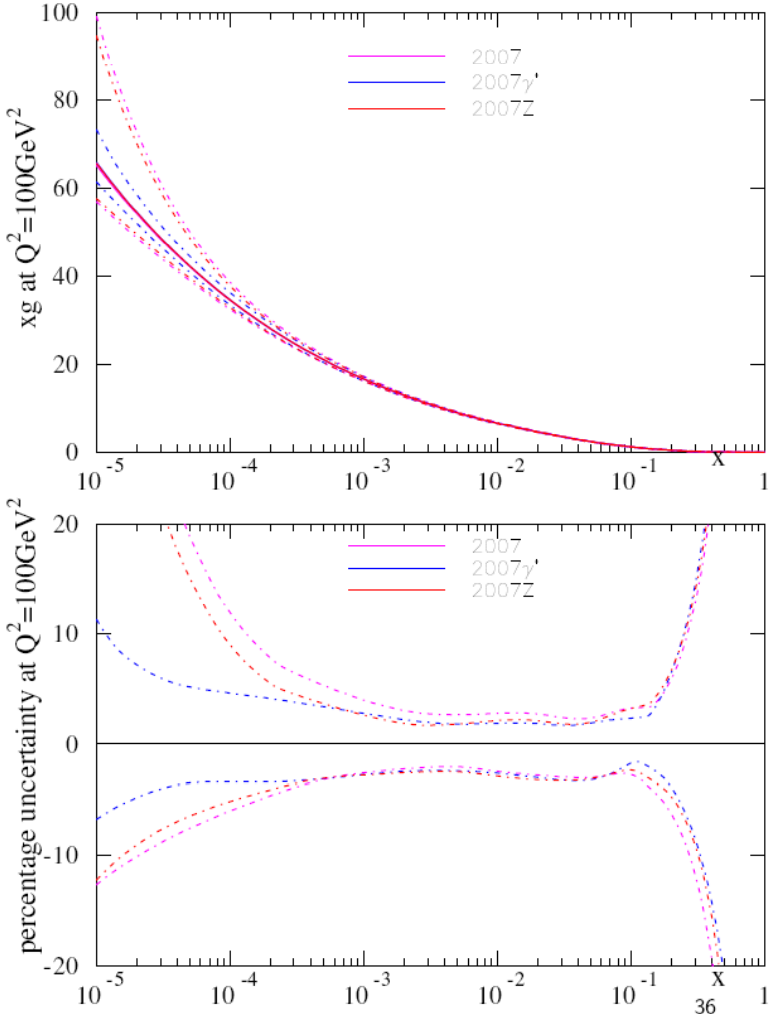
8th April 2008

Ronan McNulty et al, at DIS08



Impact of 1 fb^{-1} LHCb data for forward Z and γ^* (M = 14 GeV) production on the gluon distribution uncertainty

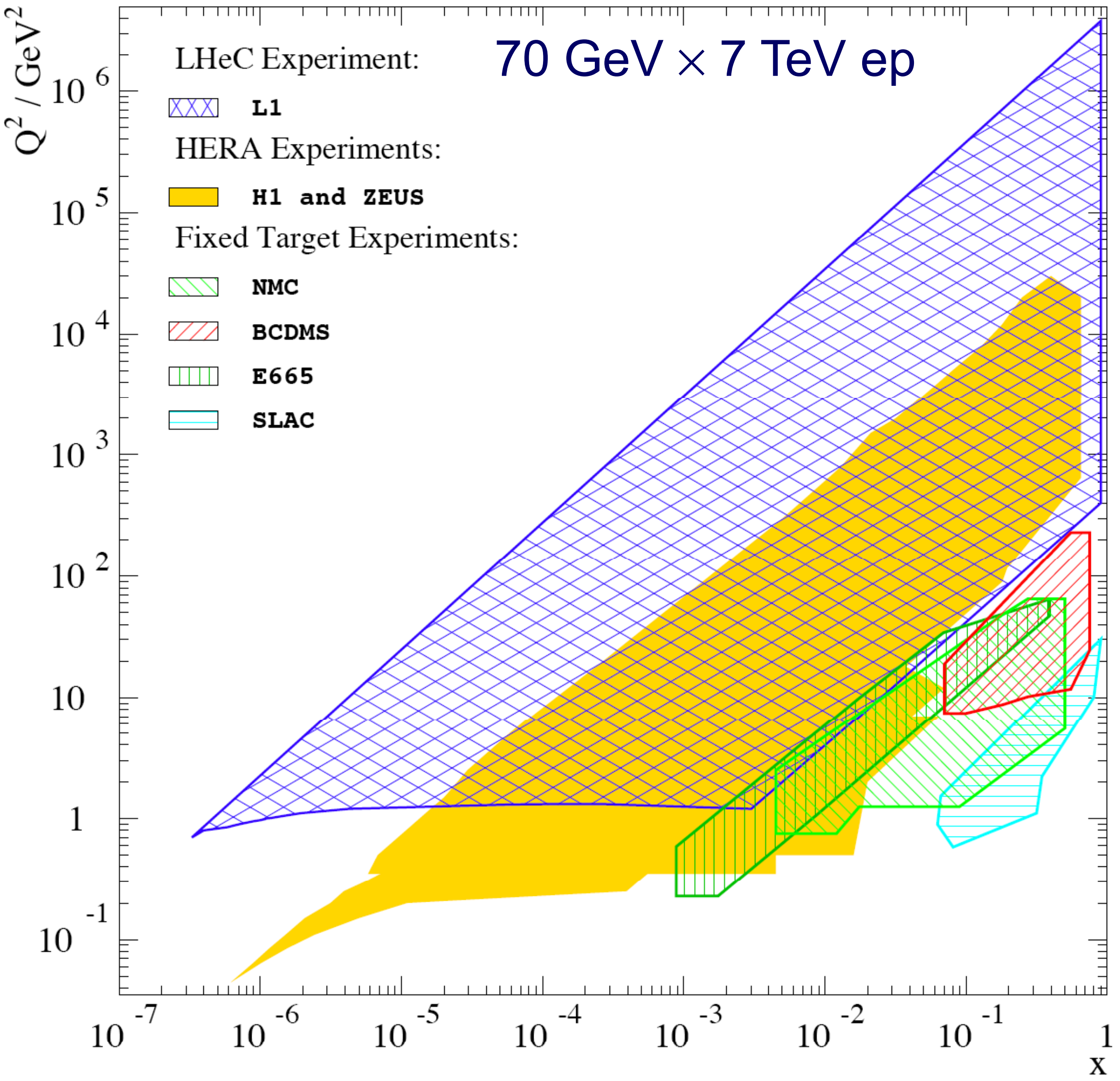




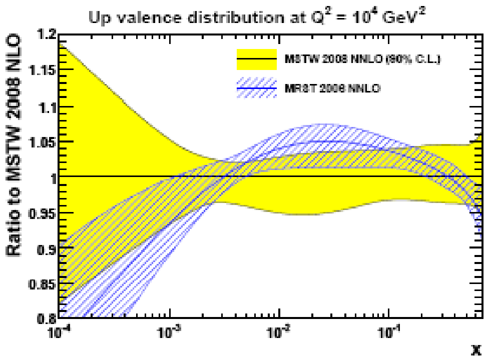
summary

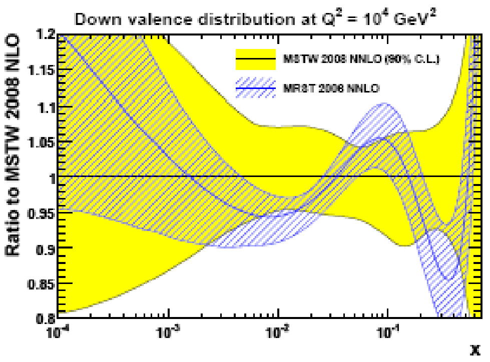
- precision phenomenology at high-energy colliders such as the LHC requires an accurate knowledge of the distribution functions of partons in hadrons
- determining pdfs from global fits to data is now a major industry… the MSTW collaboration has last month released its latest (2008) LO, NLO, NNLO sets
- pdf uncertainty for ‘new physics’ cross sections not expected to be too important (few % level), apart from at very high mass
- ongoing high-precision studies of standard candle cross sections and ratios
- potential of LHCb to access very small x via low-mass Drell-Yan lepton pair production

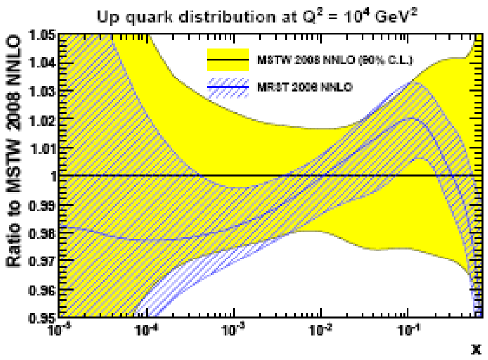
extra slides

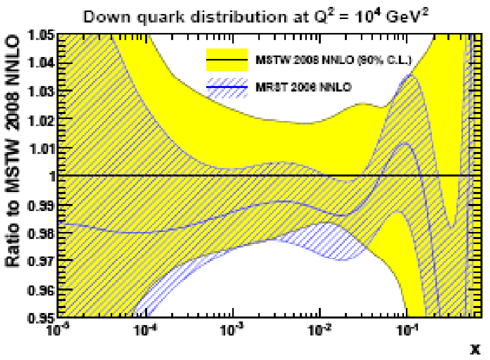

70 GeV × 7 TeV ep
LHeC Experiment:
L1
HERA Experiments:
H1 and ZEUS
Fixed Target Experiments:
NMC
BCDMS
E665
SLAC
Q^2 / GeV^2
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

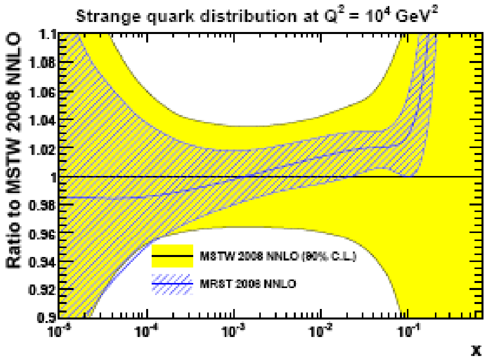
MSTW2008 *vs* MRST2006

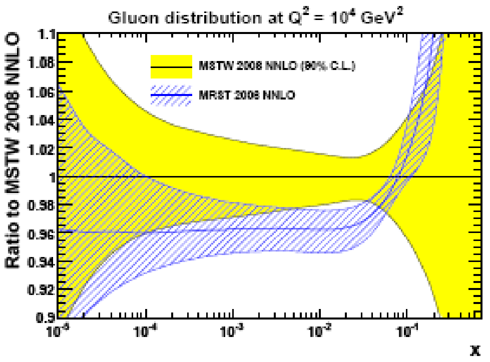




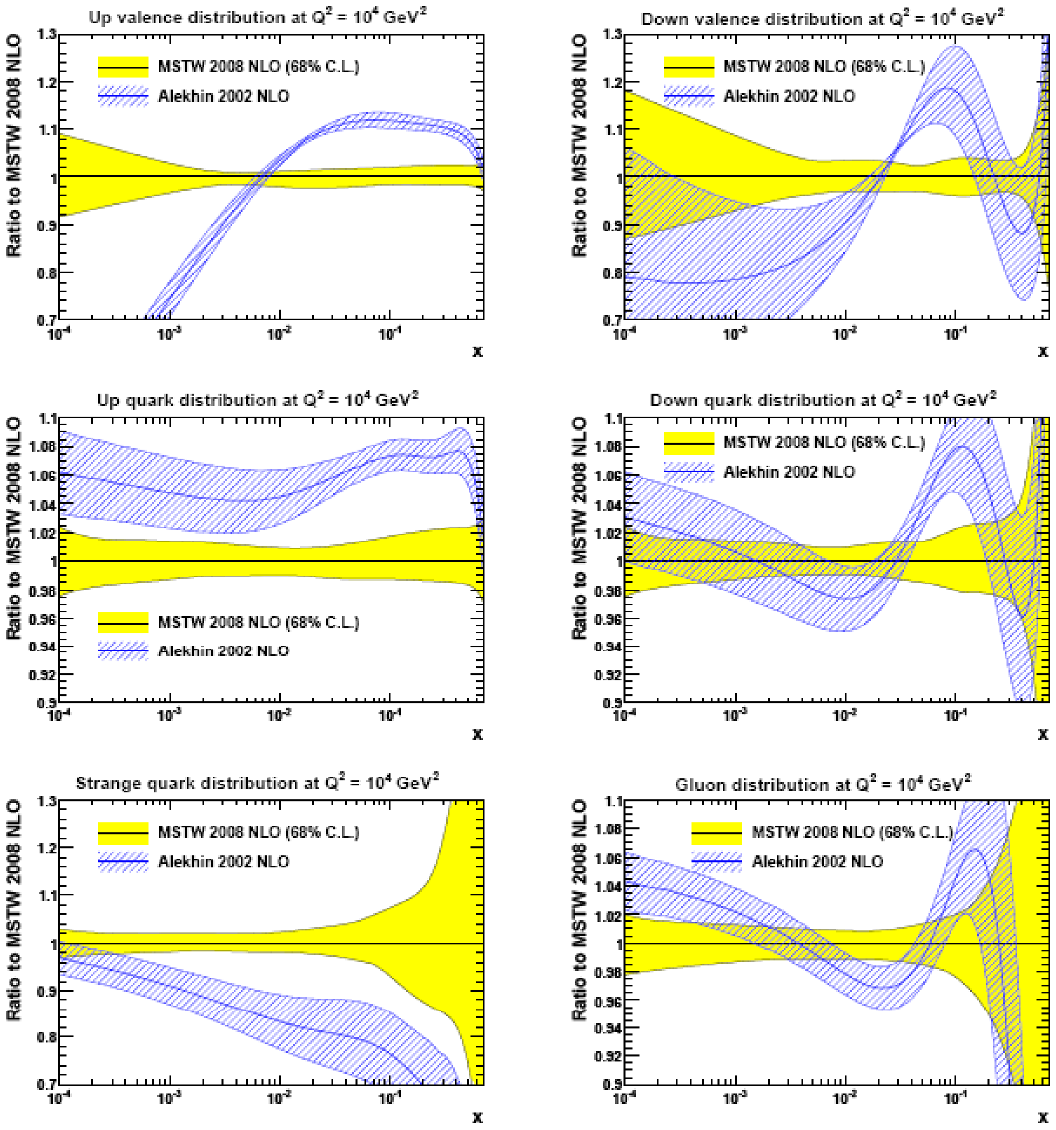




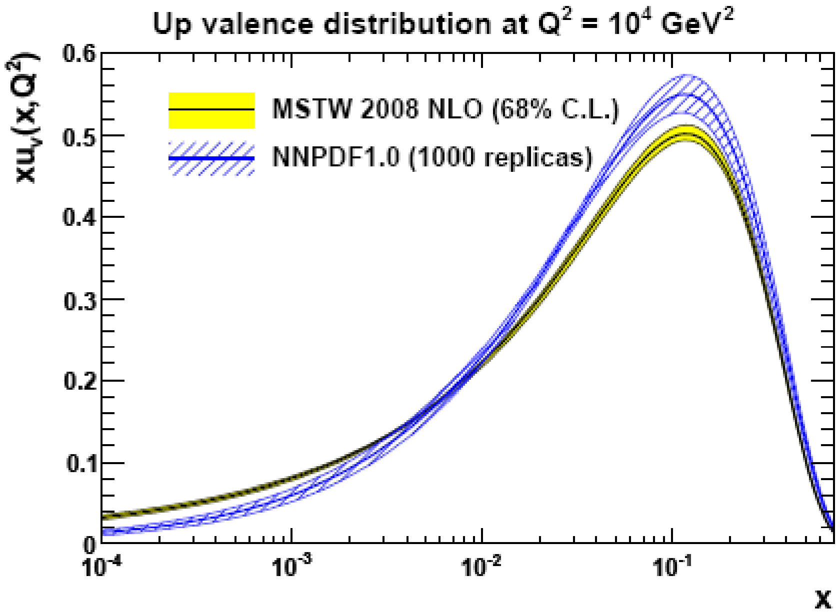


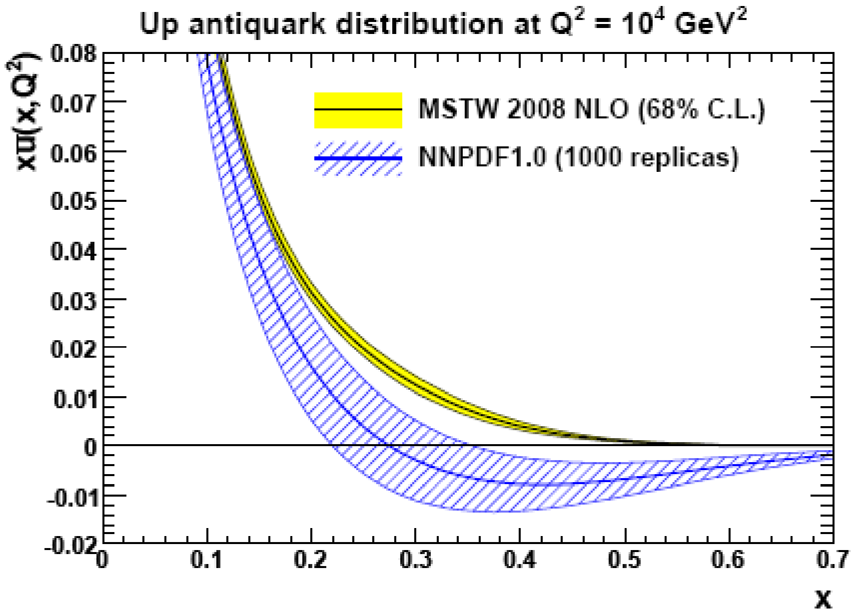


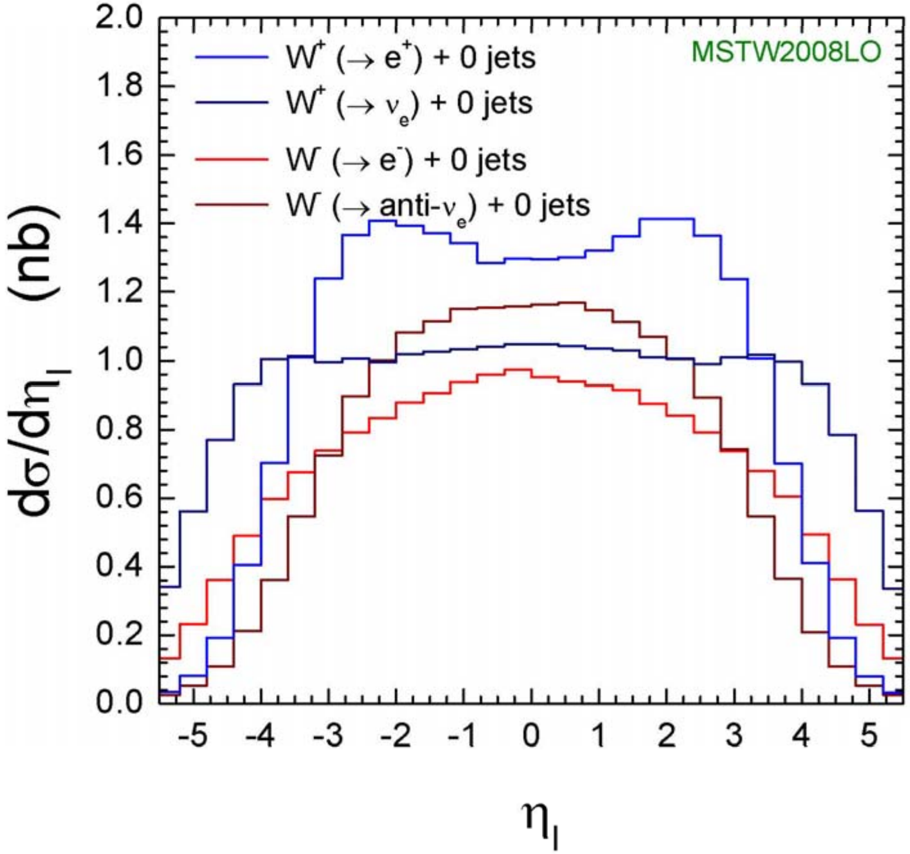
MSTW2008 *vs* Alekhin2002

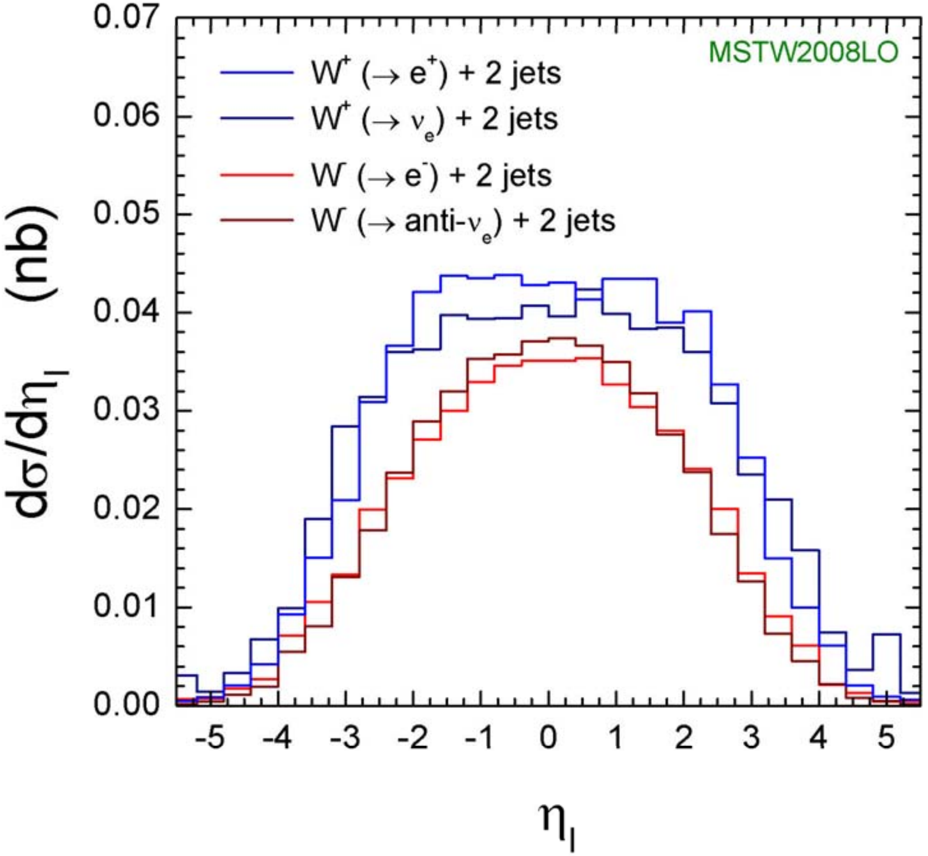


MSTW2008 *vs* NNPDF1.0

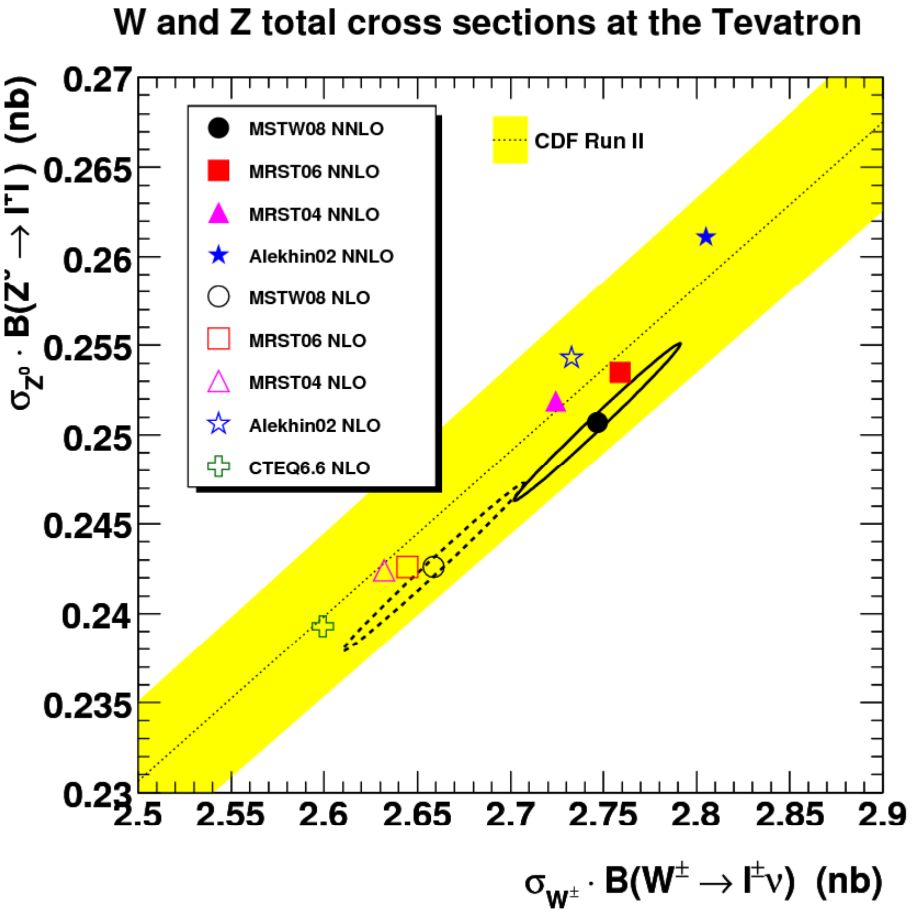


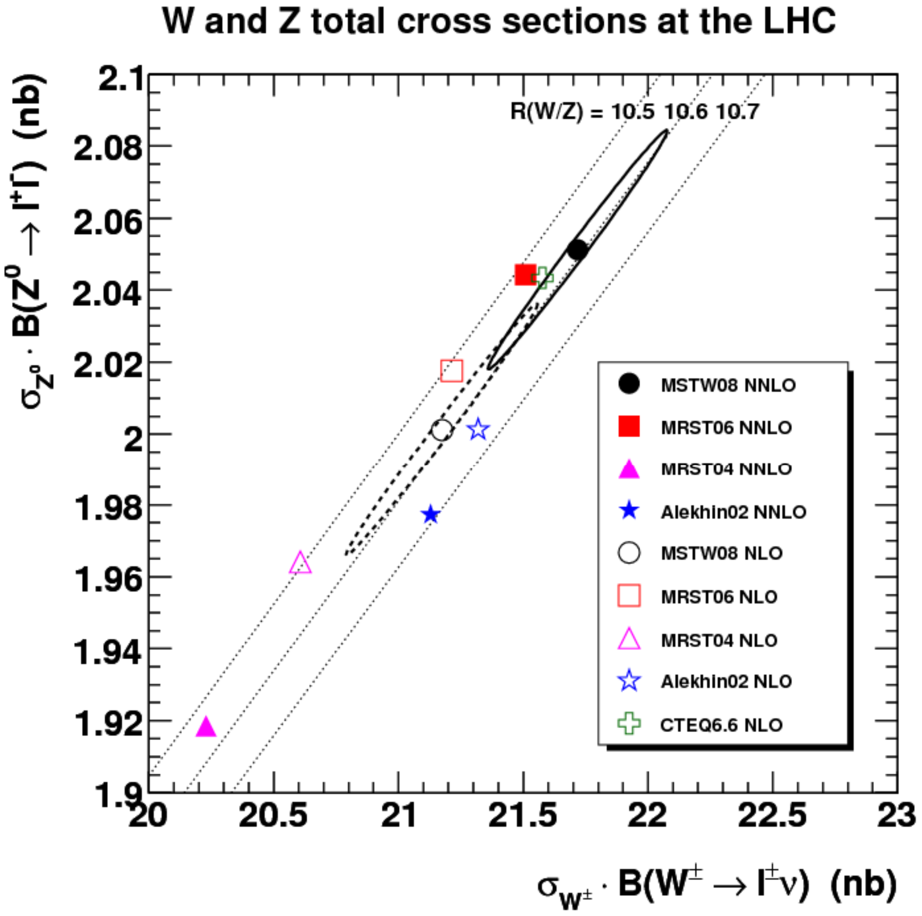



MSTW2008LO
W⁺ (→ e⁺) + 0 jets
W⁺ (→ ν_e) + 0 jets
W⁻ (→ e⁻) + 0 jets
W⁻ (→ anti-ν_e) + 0 jets
dσ/dη_l (nb)
η_l
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
-5
-4
-3
-2
-1
0
1
2
3
4
5


MSTW2008LO
W⁺ (→ e⁺) + 2 jets
W⁺ (→ ν_e) + 2 jets
W⁻ (→ e⁻) + 2 jets
W⁻ (→ anti-ν_e) + 2 jets
dσ/dη_l (nb)
η_l
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
-5
-4
-3
-2
-1
0
1
2
3
4
5

R(W/Z)=σ(W)/σ(Z) @ Tevatron & LHC





CDF 2007: R = 10.84 ± 0.15 (stat) ± 0.14 (sys)