Spin Determination with Missing Energy

Bryan Webber
Cambridge University

LHC2FC Institute
CERN, February 2009

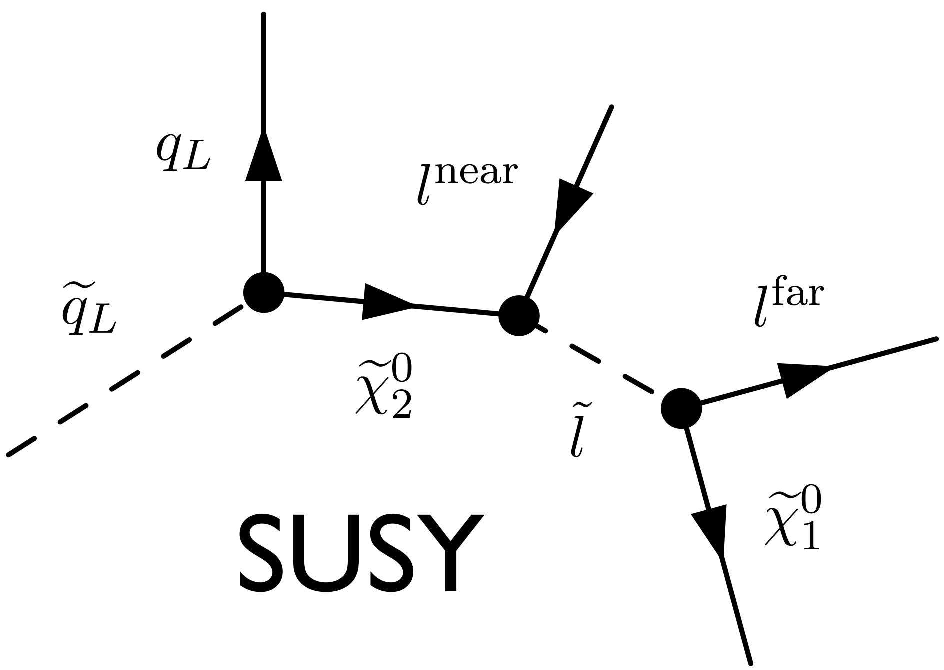
Spin Determination with ...

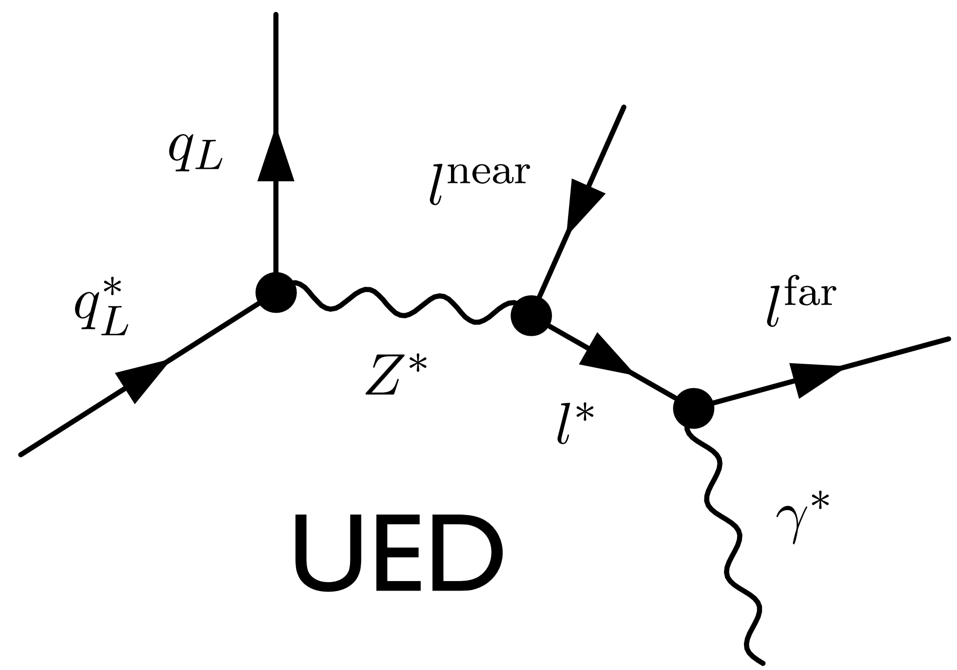
- Sequential decay chains
- Dileptons
- Gluinos
- Three-body decays
- Cross sections

See also: review by Wang & Yavin, 0802.2726

Decay chains

"Classic" decay chain

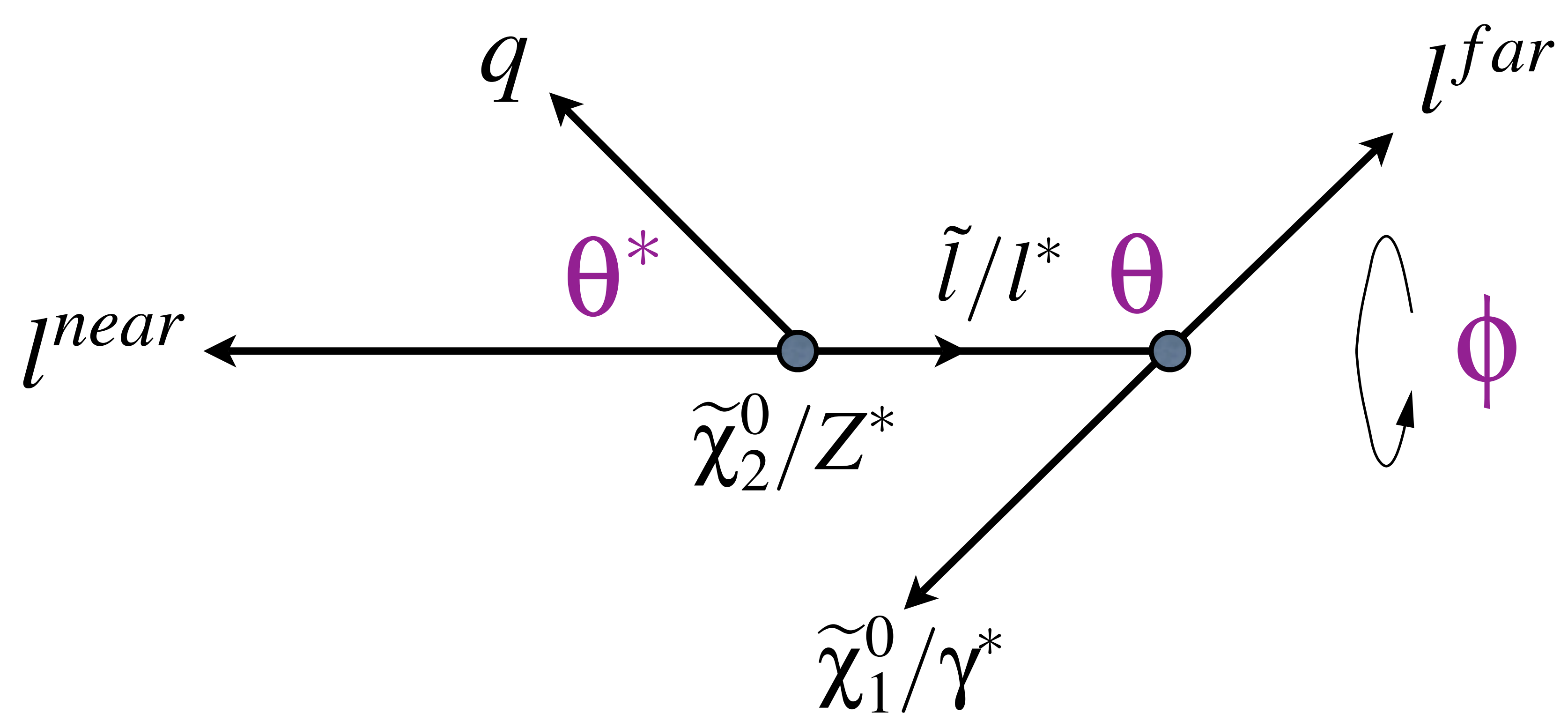




- Two distinct helicity structures, with different spin correlations:
  - Process 1: $\{q, l^{\rm near}, l^{\rm far}\} = \{q_L, l_L^-, l_L^+\}$ or $\{\bar{q}_L, l_L^+, l_L^-\}$ or $\{q_L, l_R^+, l_R^-\}$ or $\{\bar{q}_L, l_R^-, l_R^+\}$;
  - Process 2: $\{q, l^{\rm near}, l^{\rm far}\} = \{q_L, l_L^+, l_L^-\}$ or $\{\bar{q}_L, l_L^-, l_L^+\}$ or $\{q_L, l_R^-, l_R^+\}$ or $\{\bar{q}_L, l_R^+, l_R^-\}$.

Smillie, Webber, hep-ph/0507170
Datta, Kong, Matchev hep-ph/0509246

Angular variables



- θ^* defined in $\tilde{\chi}_2^0/Z^*$ rest frame
- θ, ϕ defined in $\tilde{l}/l^*$ rest frame

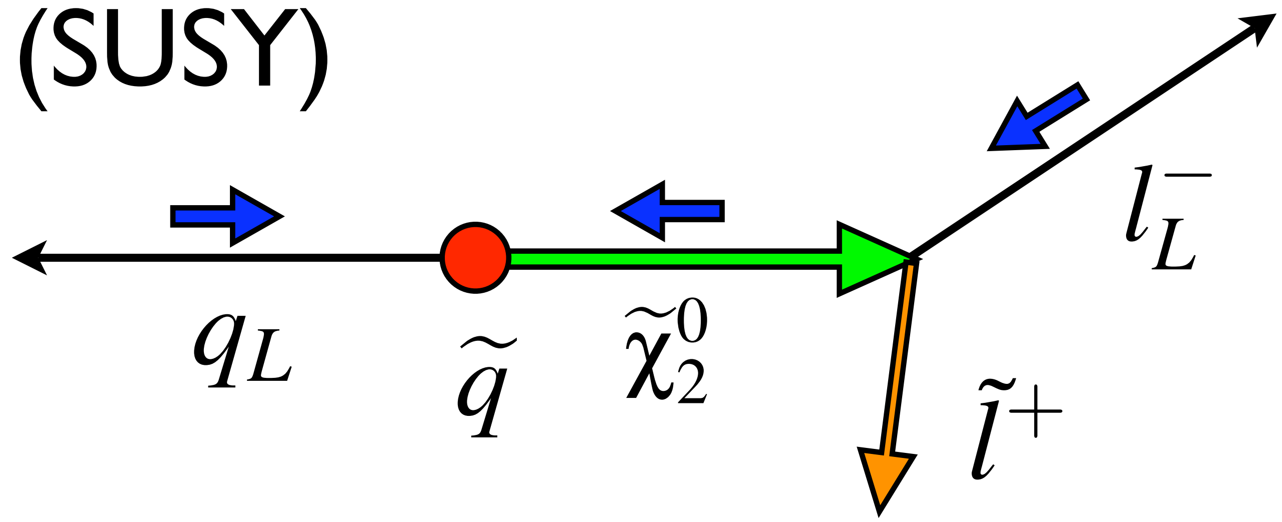
Invariant masses

- ql^{near}: $m_{ql}/(m_{ql})_{max} = \sin(\theta^*/2)$
- $l^{near}l^{far}$: $m_{ll}/(m_{ll})_{max} = \sin(\theta/2)$
- ql^{far}: $$m_{ql}/(m_{ql})_{max} = \frac{1}{2}\Big[(1-y)(1-\cos\theta^*\cos\theta)+ +(1-y)(\cos\theta^*-\cos\theta)-2\sqrt{y}\sin\theta^*\sin\theta\cos\phi\Big]^{\frac{1}{2}}$$

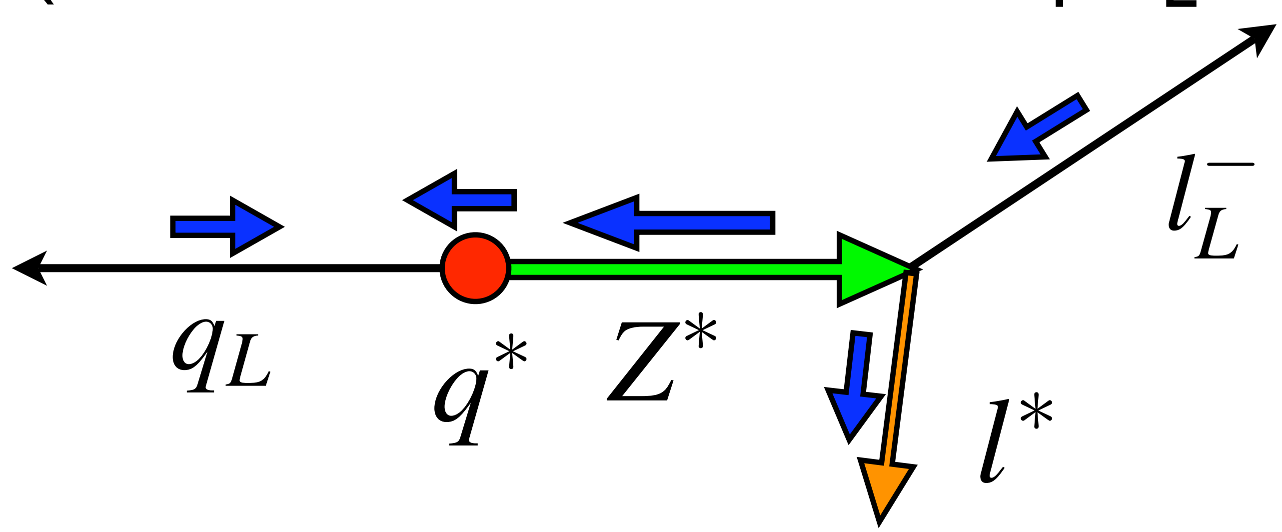
where $x = m_{Z^*}^2/m_{q^*}^2,\ y = m_{l^*}^2/m_{Z^*}^2,\ z = m_{\gamma^*}^2/m_{l^*}^2$

Helicity dependence

- Process I (SUSY)



- Process I (UED, transverse Z*: $P_T / P_L = 2x$)



➡ Both prefer high $(ql^-)^{near}$ invariant mass

UED and SUSY mass spectra

- UED models tend to have quasi-degenerate spectra

γ^*	Z^*	q_L^*	l_R^*	l_L^*
501	536	598	505	515

Table 1: UED masses in GeV, for $R^{-1} = 500$GeV, $\Lambda R = 20$, $m_h = 120$GeV, $\overline{m}_h^2 = 0$ and vanishing boundary terms at cut-off scale Λ.

($M_n \sim n/R$ broken by boundary terms and loops, with low cutoff)

- SUSY spectra typically more hierarchical

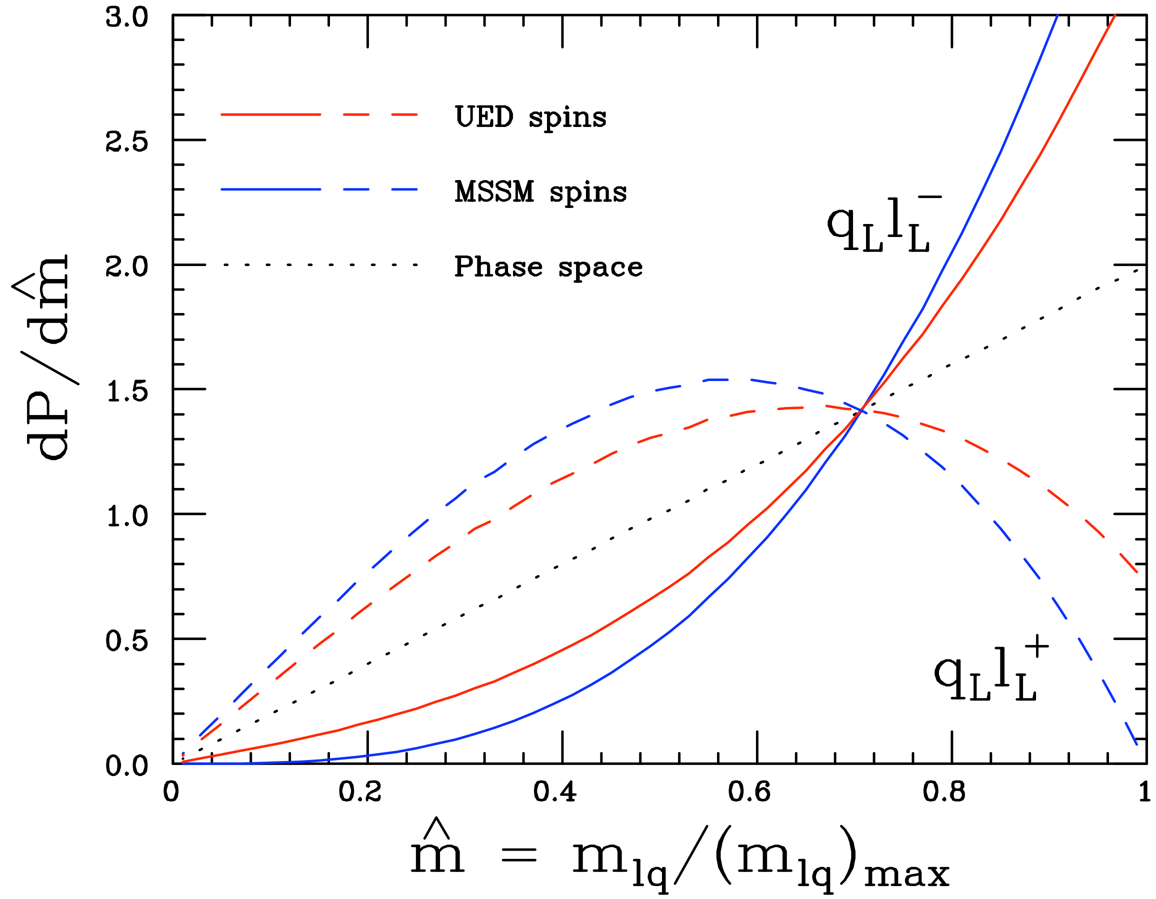
$\widetilde{\chi}_1^0$	$\widetilde{\chi}_2^0$	$\widetilde{u}_L$	$\widetilde{e}_R$	$\widetilde{e}_L$
96	177	537	143	202

Table 2: SUSY masses in GeV, for SPS point 1a.

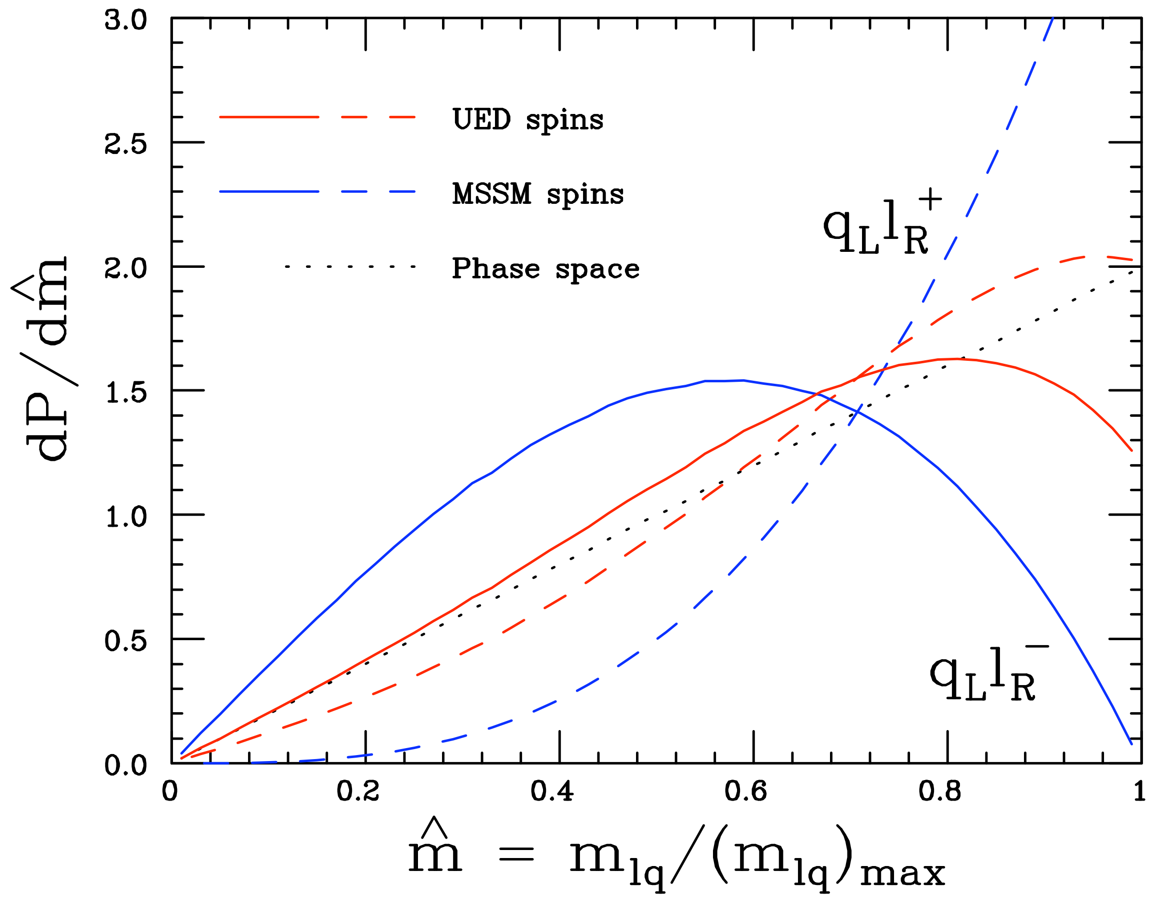
(high-scale universality)

ql^{near} mass distribution

UED masses

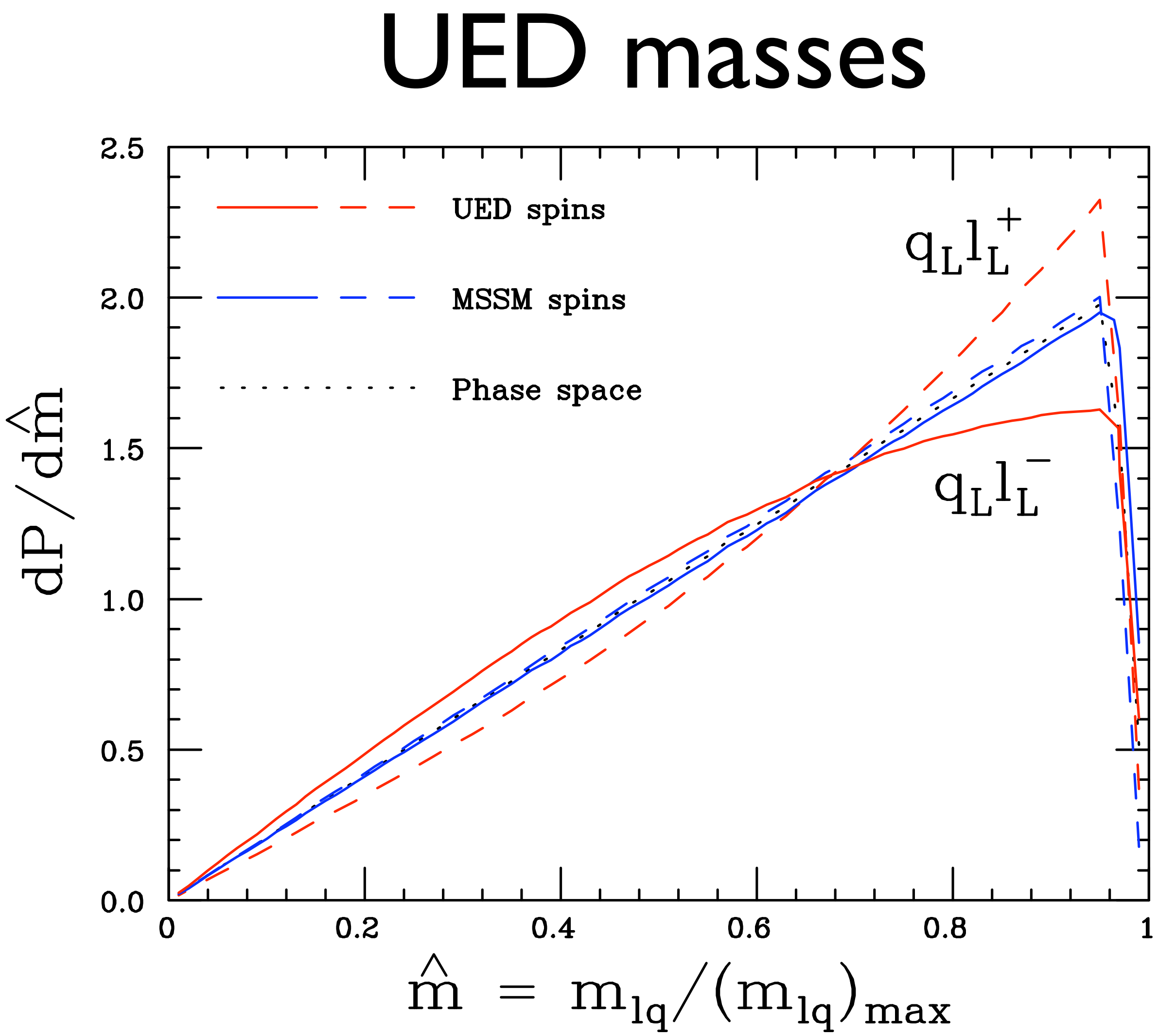


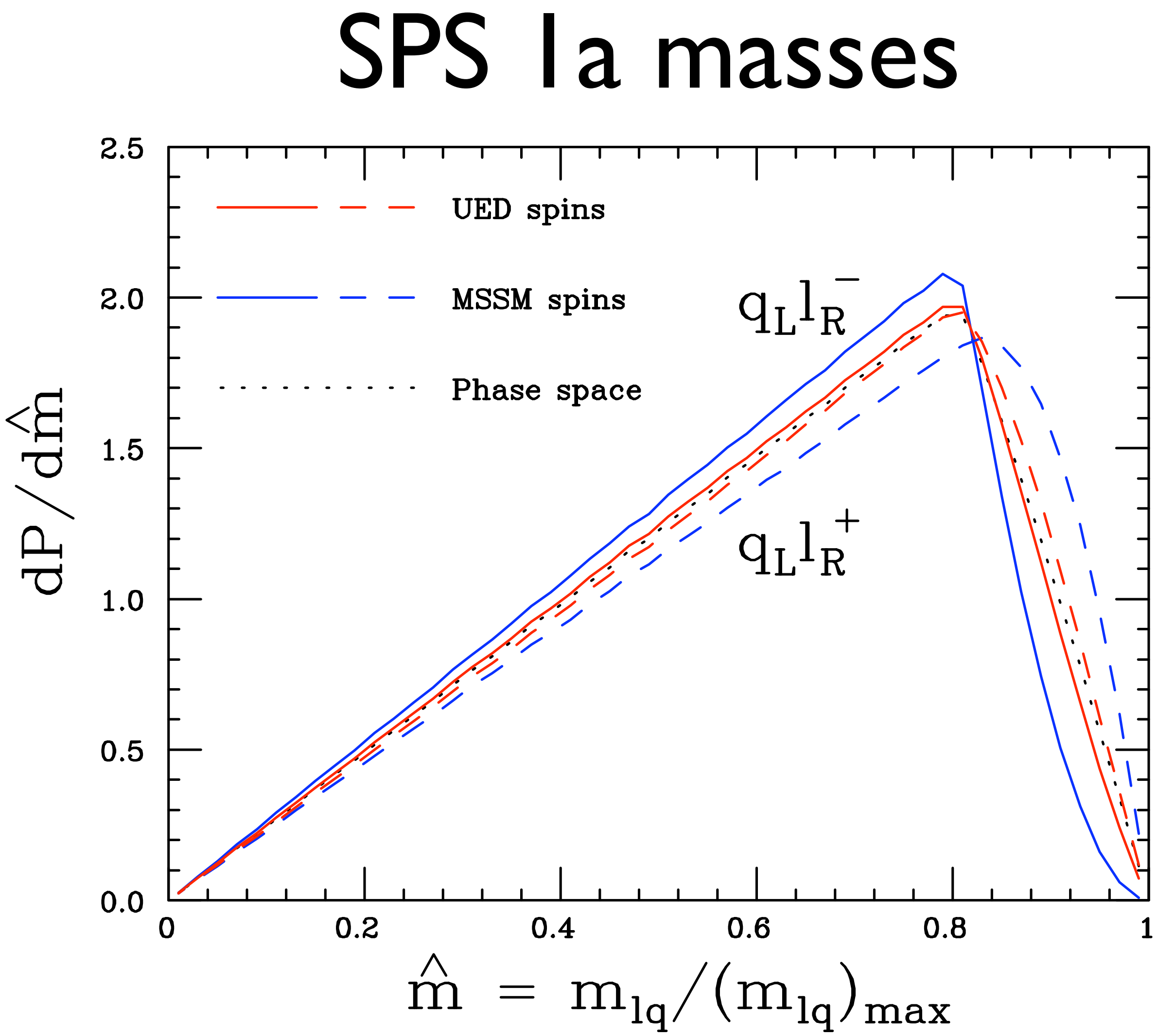
SPS 1a masses



UED and SUSY not distinguishable for UED masses

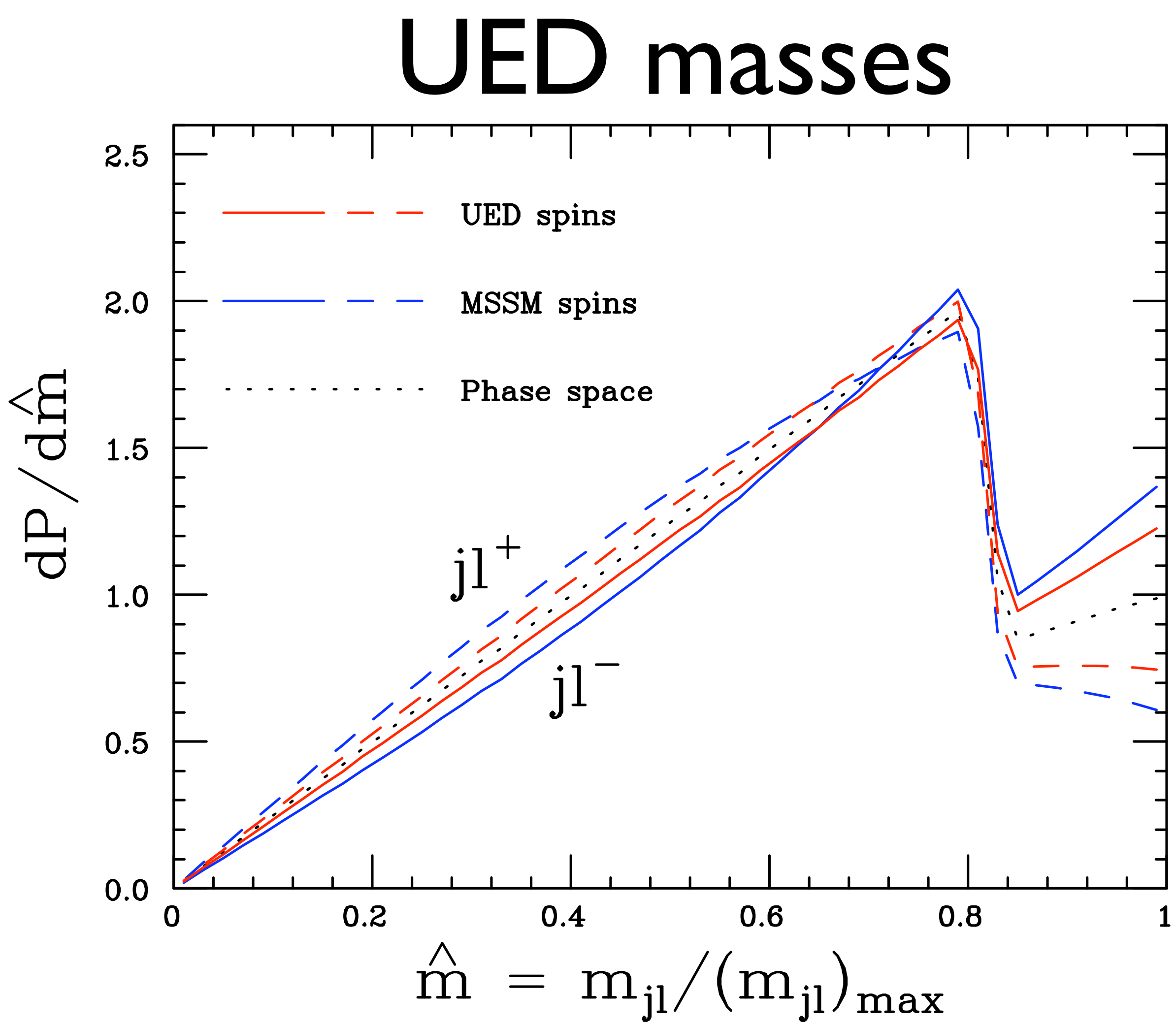
ql^{far} mass distribution

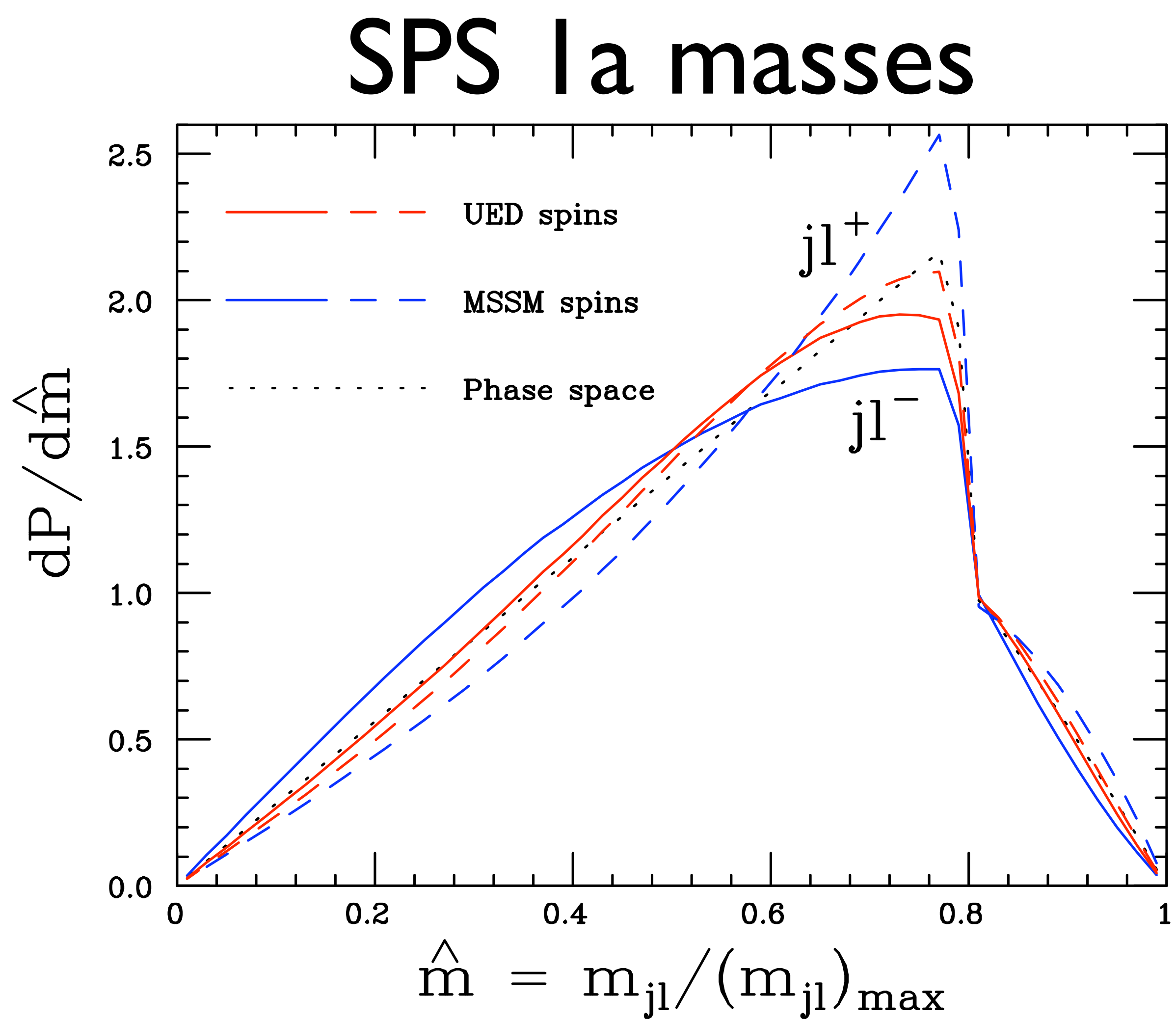




➡ Correlation weak but slightly enhances UED-SUSY difference

Jet + lepton mass distribution





- Not resolvable for UED masses, maybe for SUSY masses
- Charge asymmetry due to quark vs antiquark excess

Production cross sections (pb)

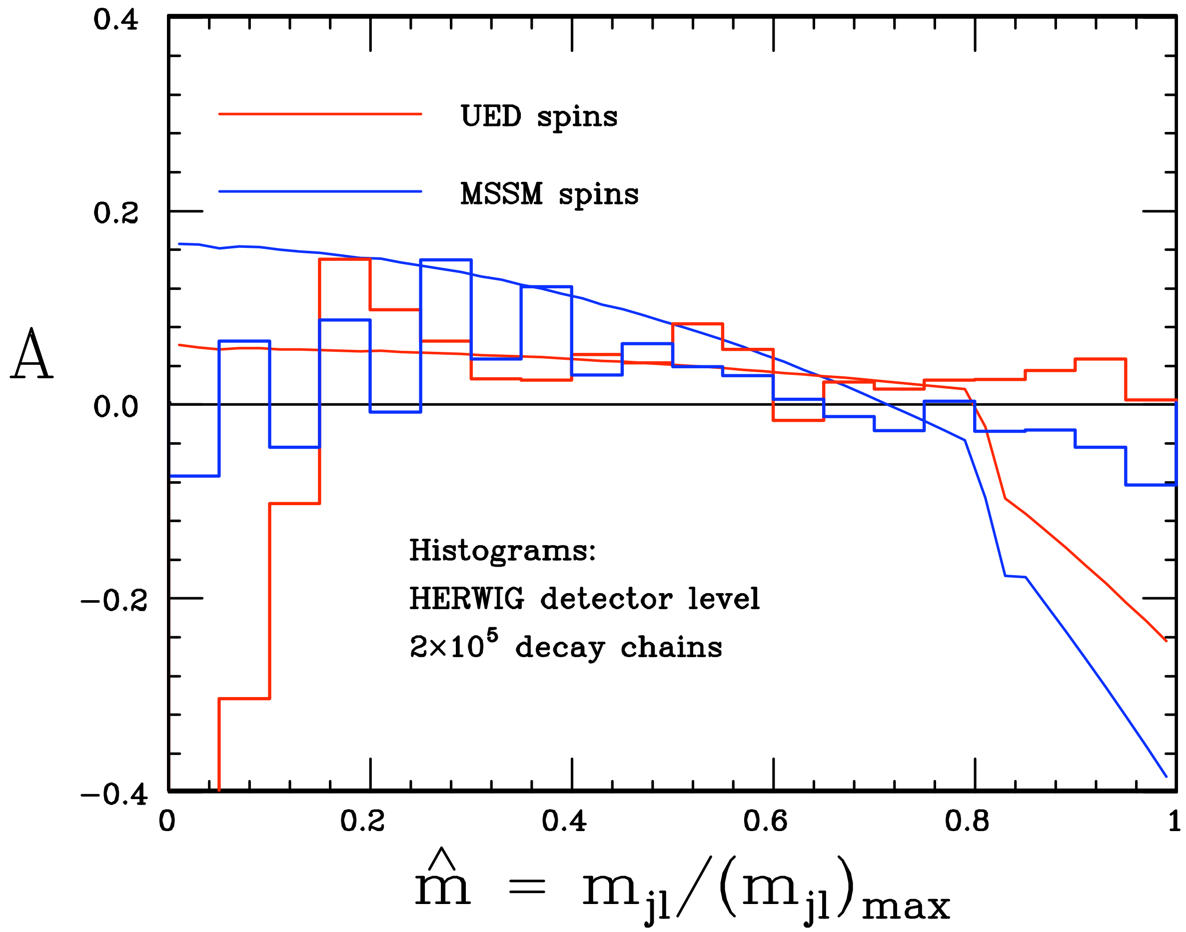
Masses	Model	σ_{all}	σ_{q^*}	$\sigma_{\bar{q}^*}$	f_q
UED	UED	253	163	84	0.66
UED	SUSY	28	18	9	0.65
SPS 1a	UED	433	224	80	0.74
SPS 1a	SUSY	55	26	11	0.70

- $\sigma_{\text{UED}} \gg \sigma_{\text{SUSY}}$ for same masses (100 pb = 1/sec)
  - $q^*/\bar{q}^* \sim 2 \;\Rightarrow$ charge asymmetry

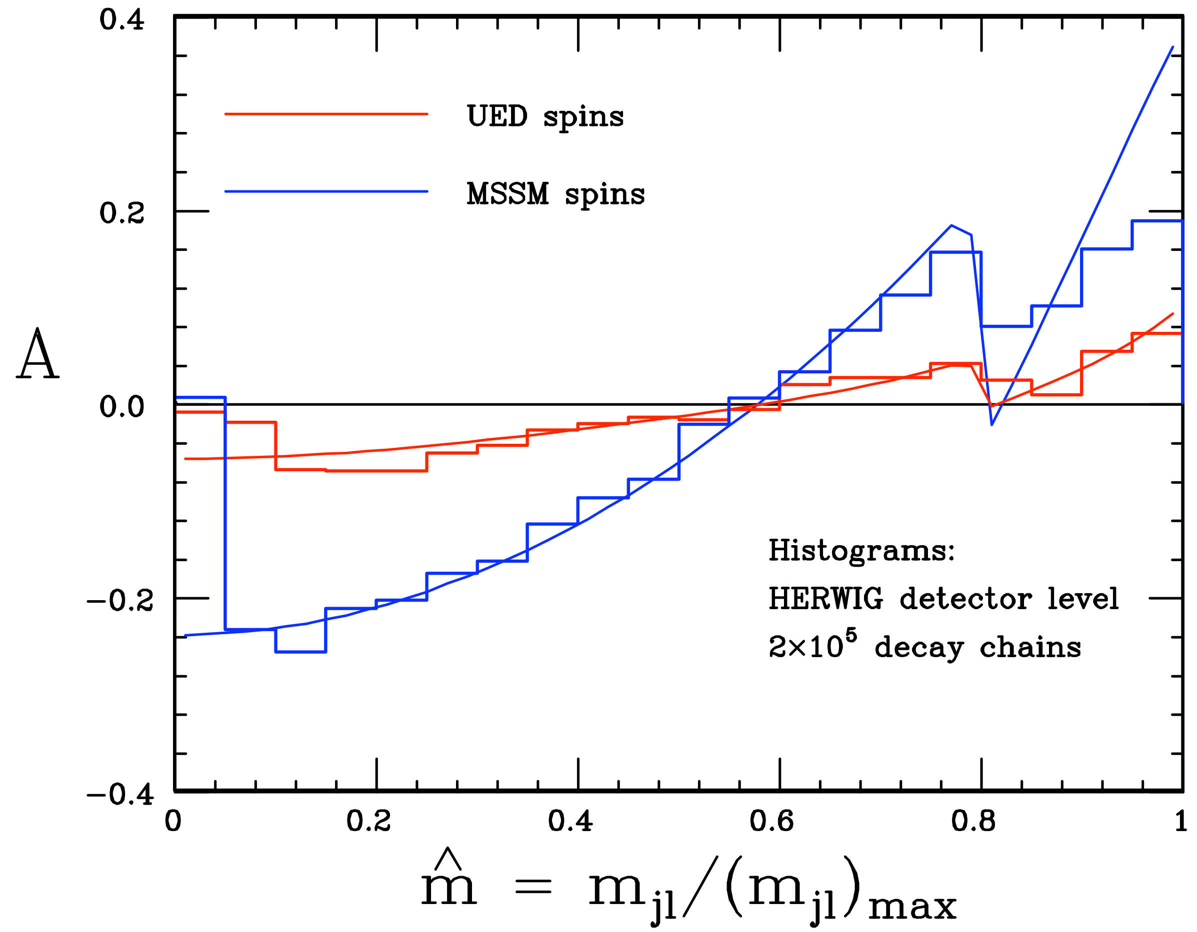
Charge asymmetry

$$A = \frac{(jl^+) - (jl^-)}{(jl^+) + (jl^-)}$$

UED masses



SPS 1a masses

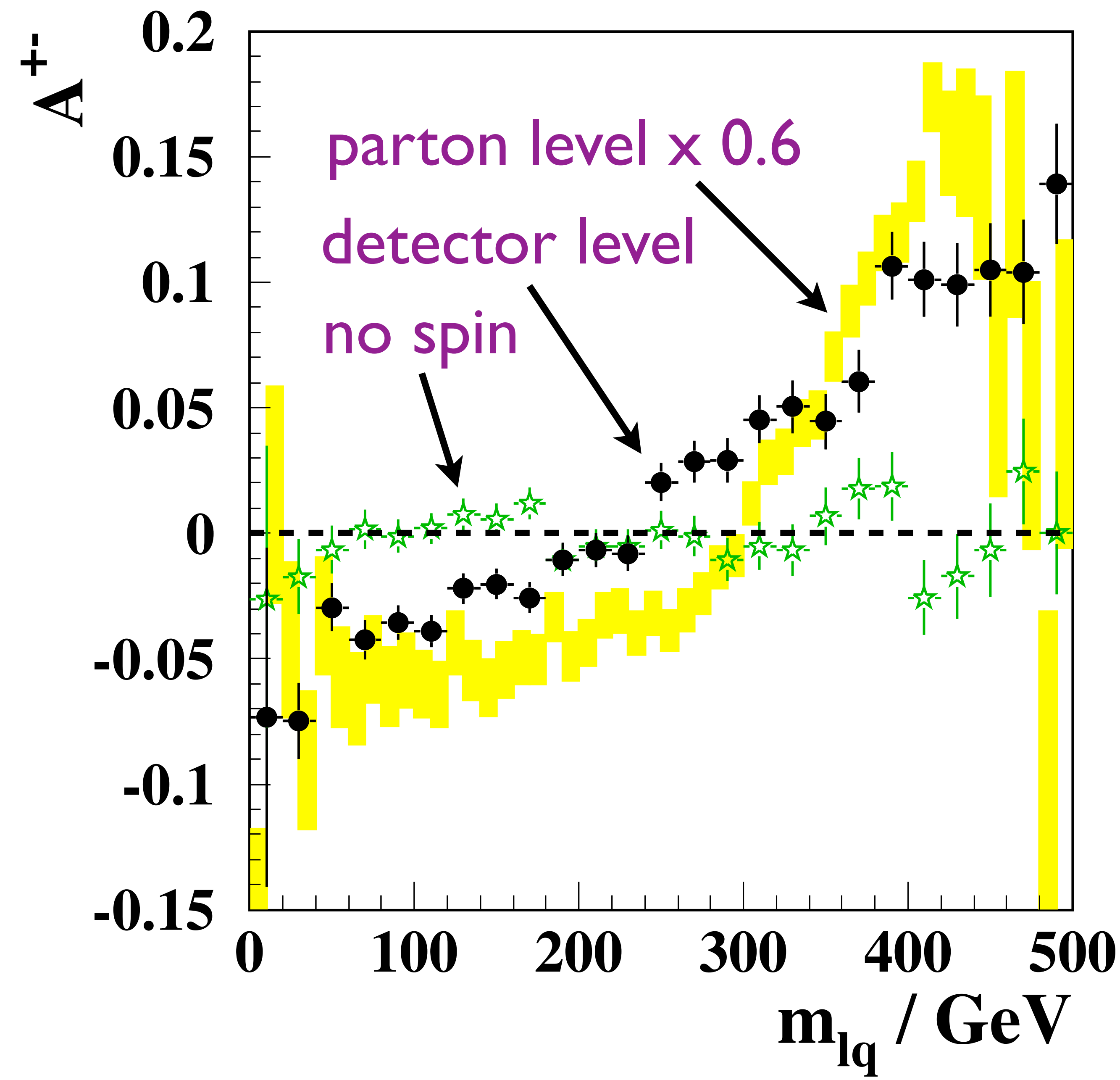


- Similar form, different magnitude
- Not detectable for UED masses

Charge asymmetry at detector level

A Barr, hep-ph/0405052

- Same decay chain:
  $\tilde{q}_L \to \tilde{\chi}_2^0 q_L \to \tilde{l}_R^{\pm} l^{\mp} q_L$
- Different MSSM point (now excluded)
- Compared with no spin (i.e. phase space) only
- More careful study of back-ground and detector effects
- Points are for 500 fb^{-1}
- Used HERWIG

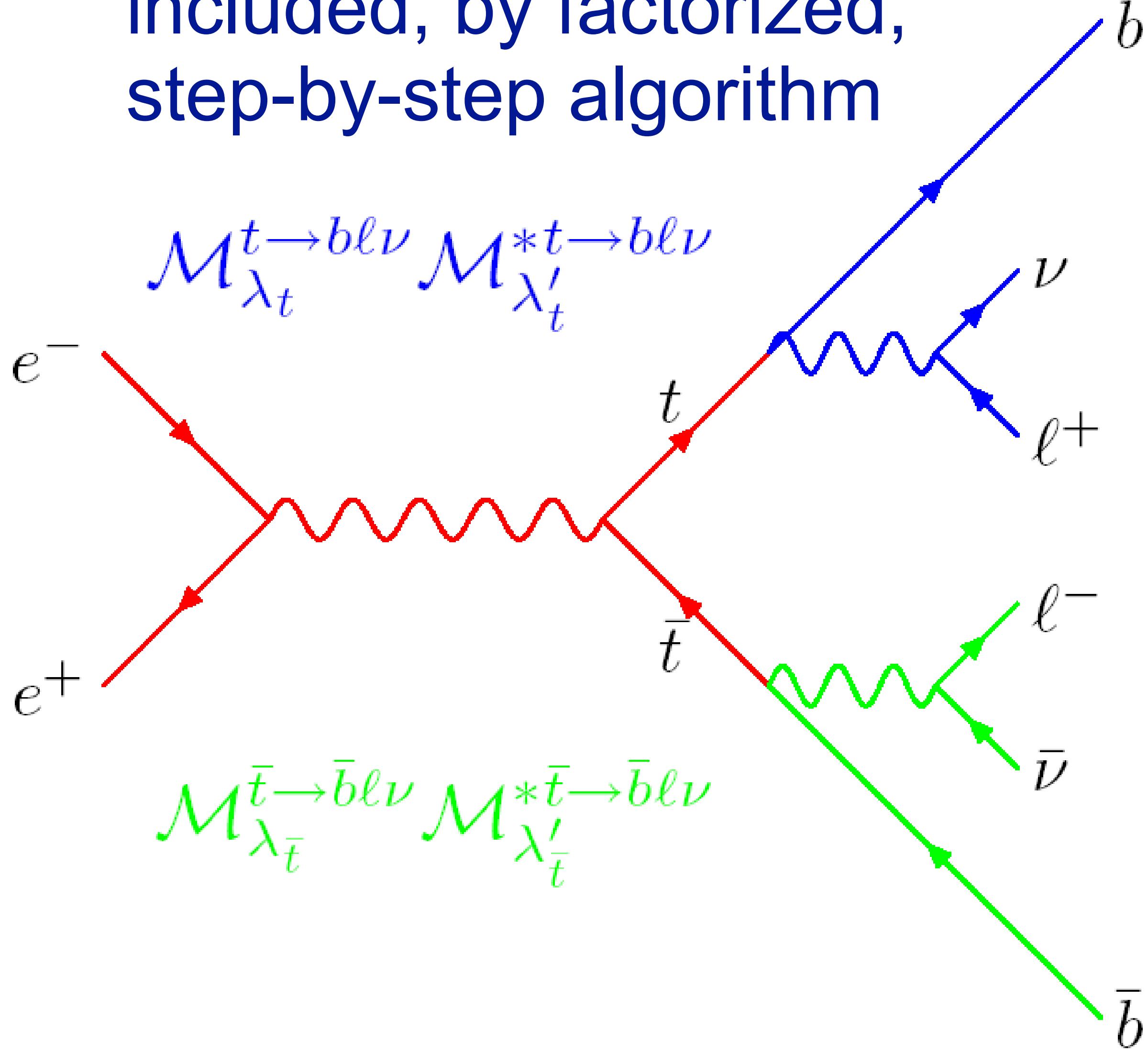


See also: Goto, Kawagoe, Nojiri, hep-ph/0406317

Production/Decay Spin Correlations in Herwig

- Example: top quark pairs in e+e- annihilation:

Full spin correlations included, by factorized, step-by-step algorithm



$$\rho_{\text{prod}}^{\lambda_c\lambda'_c\lambda_d\lambda'_d} = \mathcal{M}_{ab\to cd}^{\lambda_c\lambda_d}\mathcal{M}_{ab\to cd}^{*\lambda'_c\lambda'_d},$$

$$D_c^{\lambda_c\lambda'_c} = \mathcal{M}_{c\ \text{decay}}^{\lambda_c}\mathcal{M}_{c\ \text{decay}}^{*\lambda'_c},$$

$$|\mathcal{M}|^2 = \rho_{\text{prod}}^{\lambda_c\lambda'_c\lambda_d\lambda'_d} D_c^{\lambda_c\lambda'_c} D_d^{\lambda_d\lambda'_d}$$

$$= \rho_{\text{prod}}^{\lambda_c\lambda_c\lambda_d\lambda_d}\left(\frac{\rho_{\text{prod}}^{\lambda_c\lambda'_c\lambda_d\lambda_d} D_c^{\lambda_c\lambda'_c}}{\rho_{\text{prod}}^{\lambda_c\lambda_c\lambda_d\lambda_d}}\right)$$

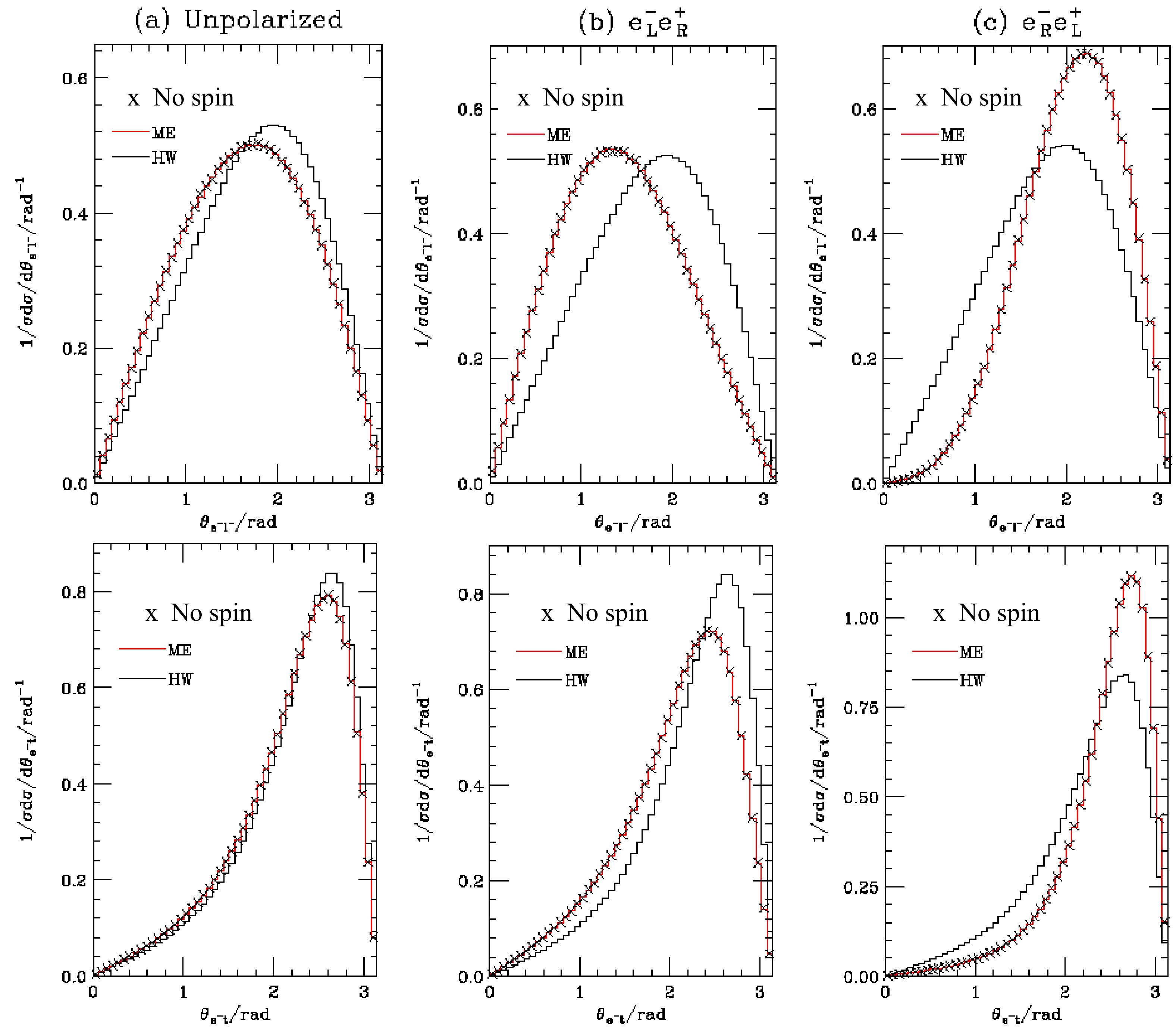
$$\times\left(\frac{\rho_{\text{prod}}^{\lambda_c\lambda'_c\lambda_d\lambda'_d} D_c^{\lambda_c\lambda'_c} D_d^{\lambda_d\lambda'_d}}{\rho_{\text{prod}}^{\lambda_c\lambda'_c\lambda_d\lambda_d} D_c^{\lambda_c\lambda'_c}}\right)$$

Richardson, hep-ph/0110108

Top spin correlations in Herwig

Lepton-beam correlation

Top-beam correlation

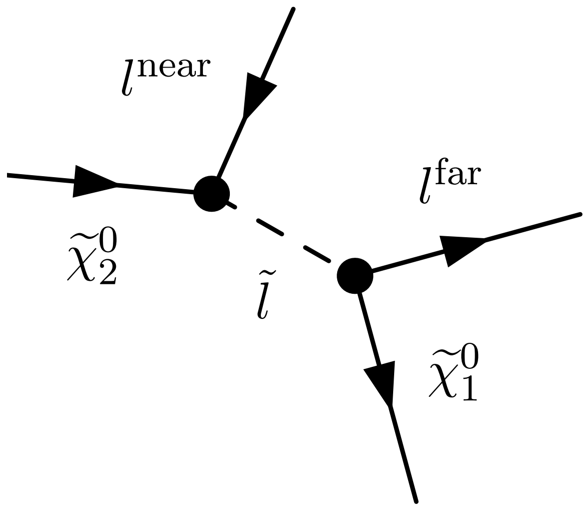


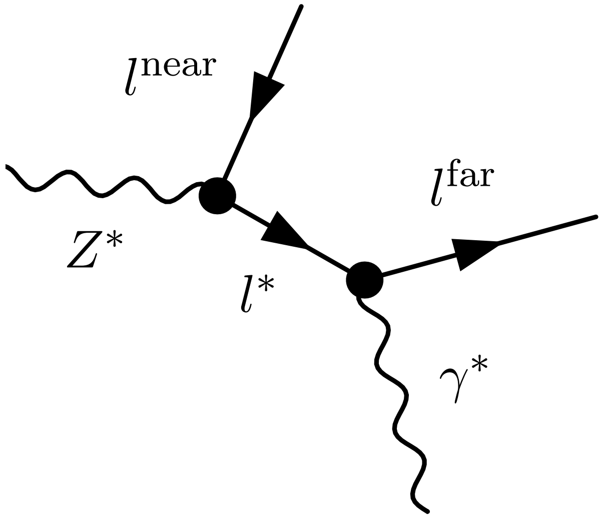
- SM, SUSY & UED in Herwig++

Hw++ manual: Bähr et al., 0803.0883

Dileptons

Dileptons in “classic” chain



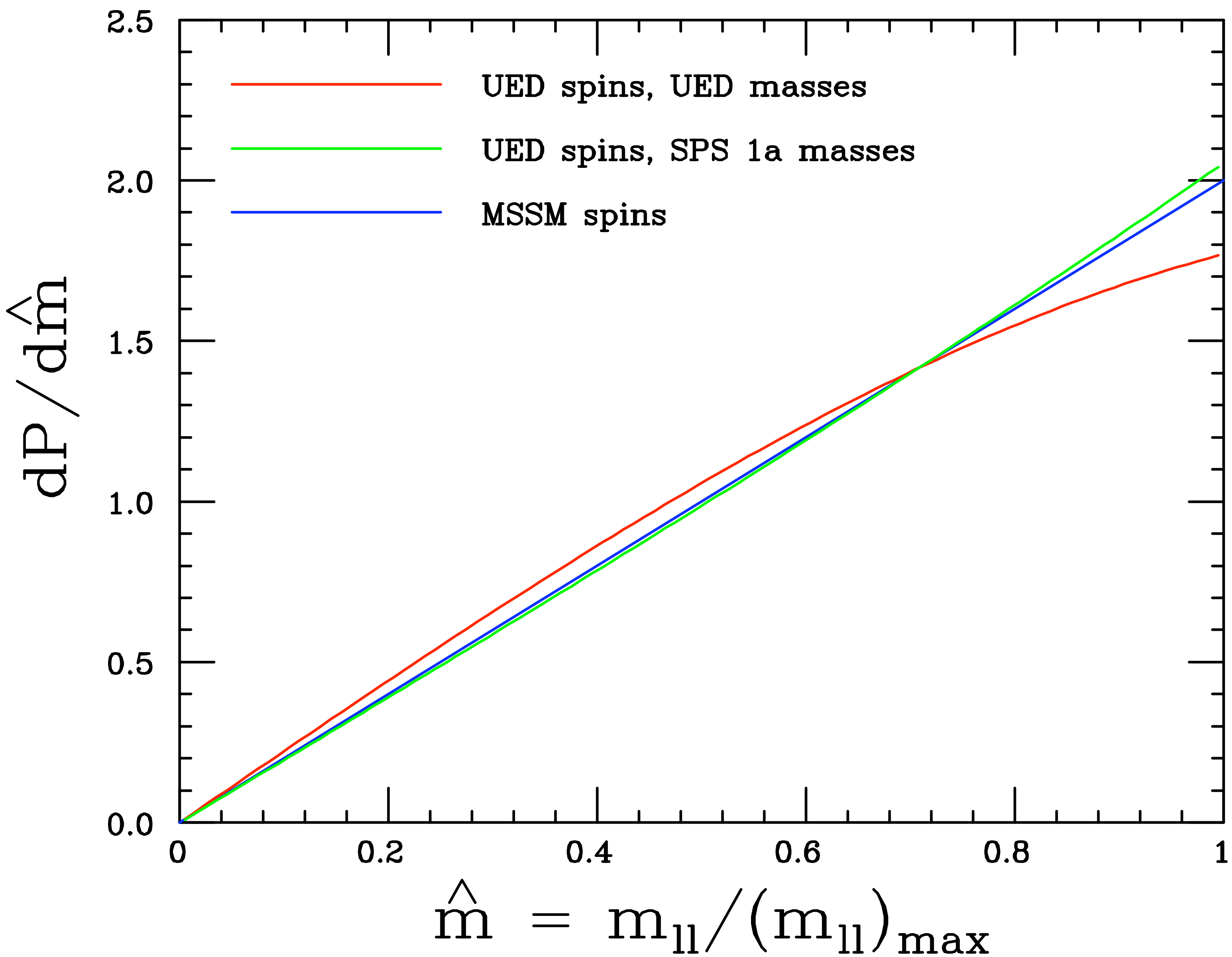


$$\frac{dP^{UED}}{d\widehat{m}_{ll}} = \frac{4\widehat{m}_{ll}}{(2+y)(1+2z)}\left[y+4z+(2-y)(1-2z)\widehat{m}_{ll}^2\right]$$

- $y = m_{l^*}^2/m_{Z^*}^2$ and $z = m_{\gamma^*}^2/m_{l^*}^2$
- UED: $y = 0.92 \quad z = 0.95$
- SPS 1a: $y = 0.65 \quad z = 0.45$

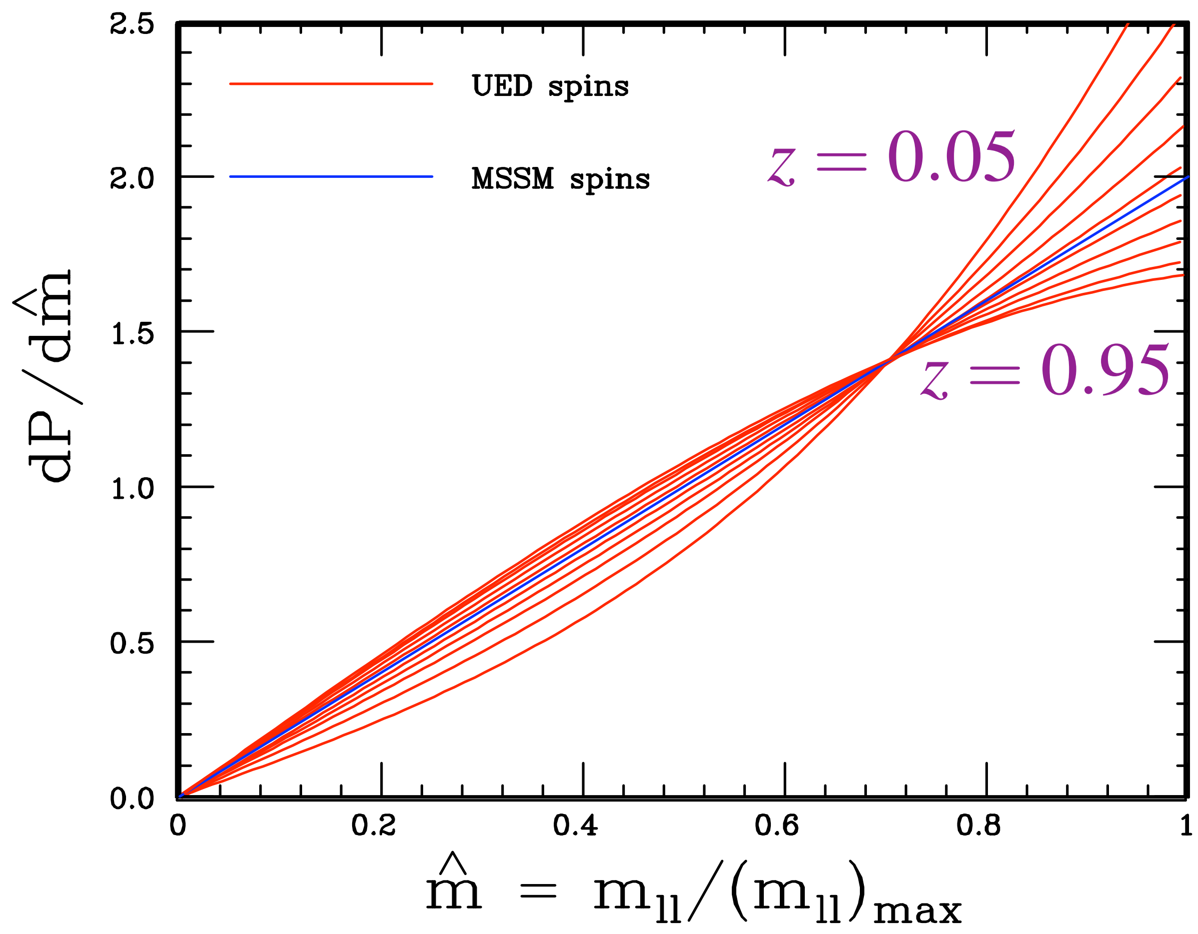
➡ Sensitivity greatest at small y and z

Dilepton mass distribution



➡ No sensitivity for these masses!

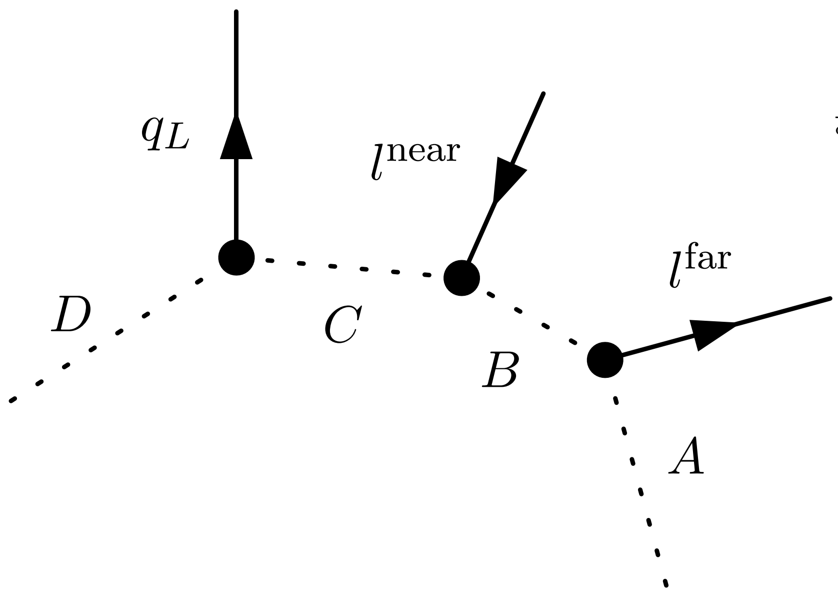
Dilepton mass distribution (2)



$$y = m_{l^*}^2 / m_{Z^*}^2 = 0.65\,,\ \ z = m_{\gamma^*}^2 / m_{l^*}^2 = 0.95 - 0.05$$

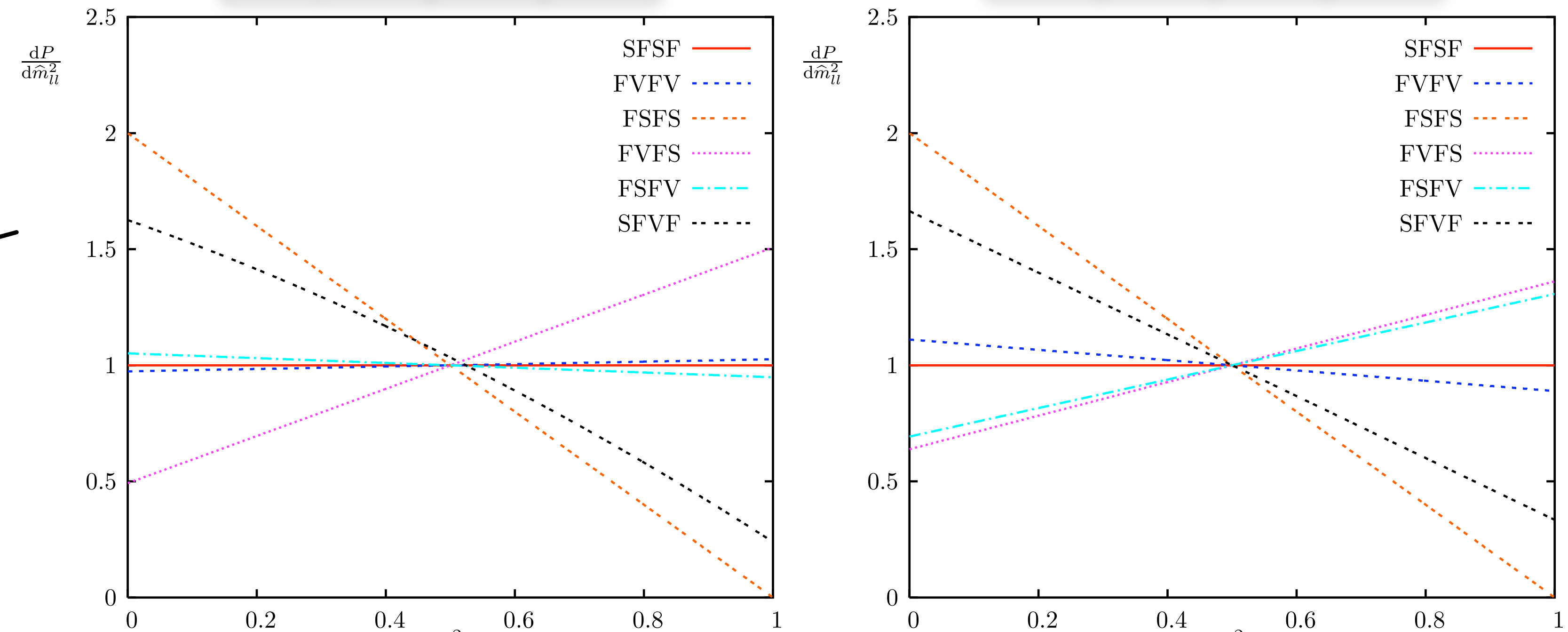
➡ Independent of masses and spins at $\widehat{m} = 1/\sqrt{2}$ $(\theta = \pi/2)$

All possible spin assignments



A	B	C	D
$\tilde{\chi}_1^0$	$\tilde{e}_R$	$\tilde{\chi}_2^0$	$\tilde{u}_L$
96	143	177	537

A	B	C	D
γ^*	l_L^*	Z^*	q_L^*
800	824	851	956



Dilepton invariant mass-squared

D	C	B	A
Scalar	Fermion	Scalar	Fermion
Fermion	Vector	Fermion	Vector
Fermion	Scalar	Fermion	Scalar
Fermion	Vector	Fermion	Scalar
Fermion	Scalar	Fermion	Vector
Scalar	Fermion	Vector	Fermion

← SUSY } not distinguishable
← UED

... but some others are.

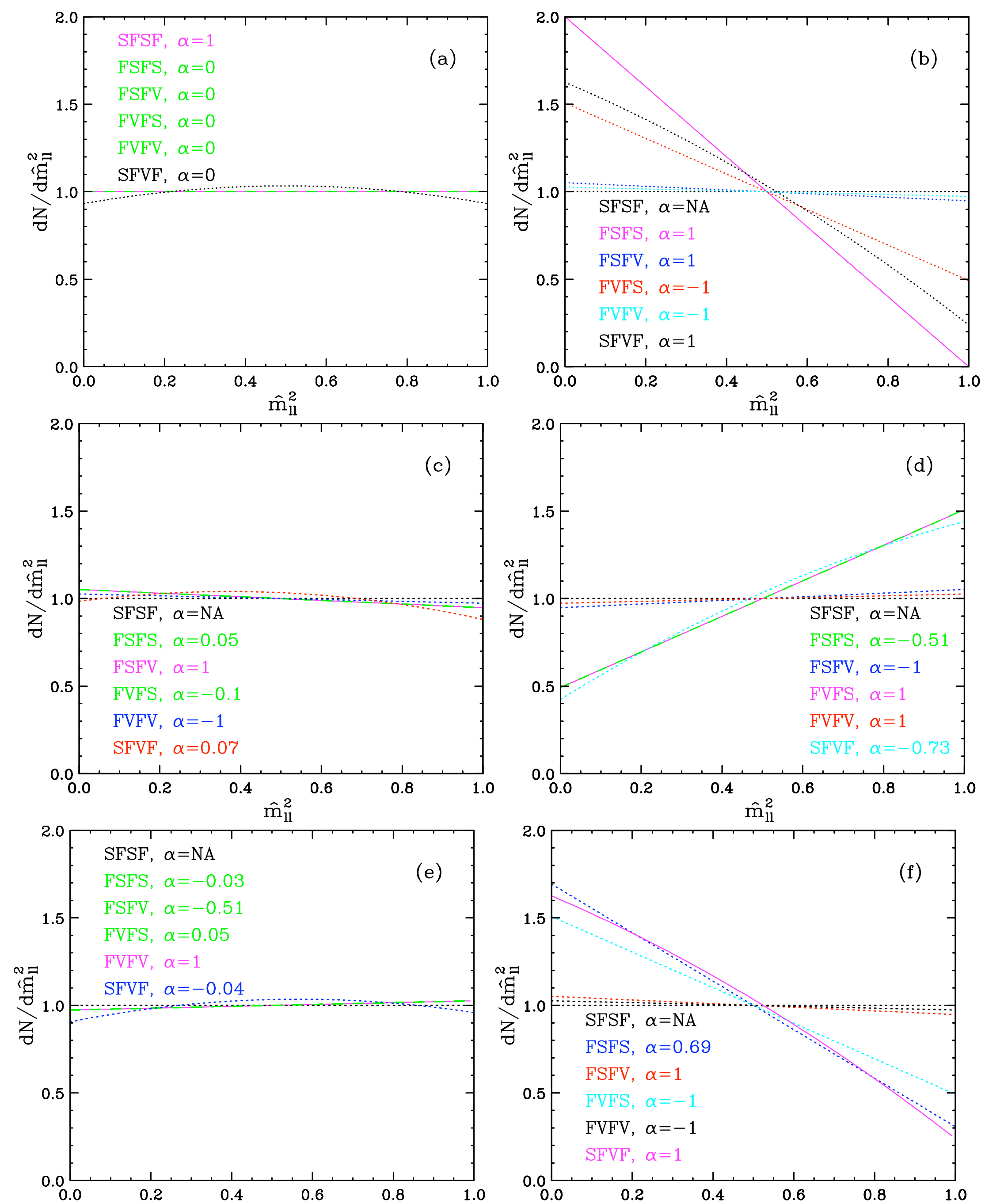
Athanasiou, Lester, Smillie, Webber, hep-ph/0605286

All possible assignments (2)

Allowing arbitrary mixtures of L and R couplings:

Processes P_{11}		Processes P_{12}	
$\{q_L, \ell^-_L, \ell^+_L\}$ $f\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_L, \ell^+_L, \ell^-_L\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{q_L, \ell^-_L, \ell^+_R\}$ $f\|c_L\|^2\|b_L\|^2\|a_R\|^2$	$\{\bar{q}_L, \ell^+_L, \ell^-_R\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_R\|^2$
$\{\bar{q}_L, \ell^-_R, \ell^+_R\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{q_L, \ell^+_R, \ell^-_R\}$ $f\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_L, \ell^-_R, \ell^+_L\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_L\|^2$	$\{q_L, \ell^+_R, \ell^-_L\}$ $f\|c_L\|^2\|b_R\|^2\|a_L\|^2$
$\{q_R, \ell^-_R, \ell^+_R\}$ $f\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_R, \ell^+_R, \ell^-_R\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{q_R, \ell^-_R, \ell^+_L\}$ $f\|c_R\|^2\|b_R\|^2\|a_L\|^2$	$\{\bar{q}_R, \ell^+_R, \ell^-_L\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_L\|^2$
$\{\bar{q}_R, \ell^-_L, \ell^+_L\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{q_R, \ell^+_L, \ell^-_L\}$ $f\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_R, \ell^-_L, \ell^+_R\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_R\|^2$	$\{q_R, \ell^+_L, \ell^-_R\}$ $f\|c_R\|^2\|b_L\|^2\|a_R\|^2$
$\{\bar{q}_L, \ell^-_L, \ell^+_L\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{q_L, \ell^+_L, \ell^-_L\}$ $f\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_L, \ell^-_L, \ell^+_R\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_R\|^2$	$\{q_L, \ell^+_L, \ell^-_R\}$ $f\|c_L\|^2\|b_L\|^2\|a_R\|^2$
$\{q_L, \ell^-_R, \ell^+_R\}$ $f\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_L, \ell^+_R, \ell^-_R\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{q_L, \ell^-_R, \ell^+_L\}$ $f\|c_L\|^2\|b_R\|^2\|a_L\|^2$	$\{\bar{q}_L, \ell^+_R, \ell^-_L\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_L\|^2$
$\{\bar{q}_R, \ell^-_R, \ell^+_R\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{q_R, \ell^+_R, \ell^-_R\}$ $f\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_R, \ell^-_R, \ell^+_L\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_L\|^2$	$\{q_R, \ell^+_R, \ell^-_L\}$ $f\|c_R\|^2\|b_R\|^2\|a_L\|^2$
$\{q_R, \ell^-_L, \ell^+_L\}$ $f\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_R, \ell^+_L, \ell^-_L\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{q_R, \ell^-_L, \ell^+_R\}$ $f\|c_R\|^2\|b_L\|^2\|a_R\|^2$	$\{\bar{q}_R, \ell^+_L, \ell^-_R\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_R\|^2$
Processes P_{21}		Processes P_{22}	

Data from	Can this data be fitted by model SFSF	FSFS	FSFV	FVFS	FVFV	SFVF
SFSF	yes	no	no	no	no	no
FSFS	no	yes	maybe	no	no	no
FSFV	no	yes	yes	no	no	no
FVFS	no	no	no	yes	maybe	no
FVFV	no	no	no	yes	yes	no
SFVF	no	no	no	no	no	yes

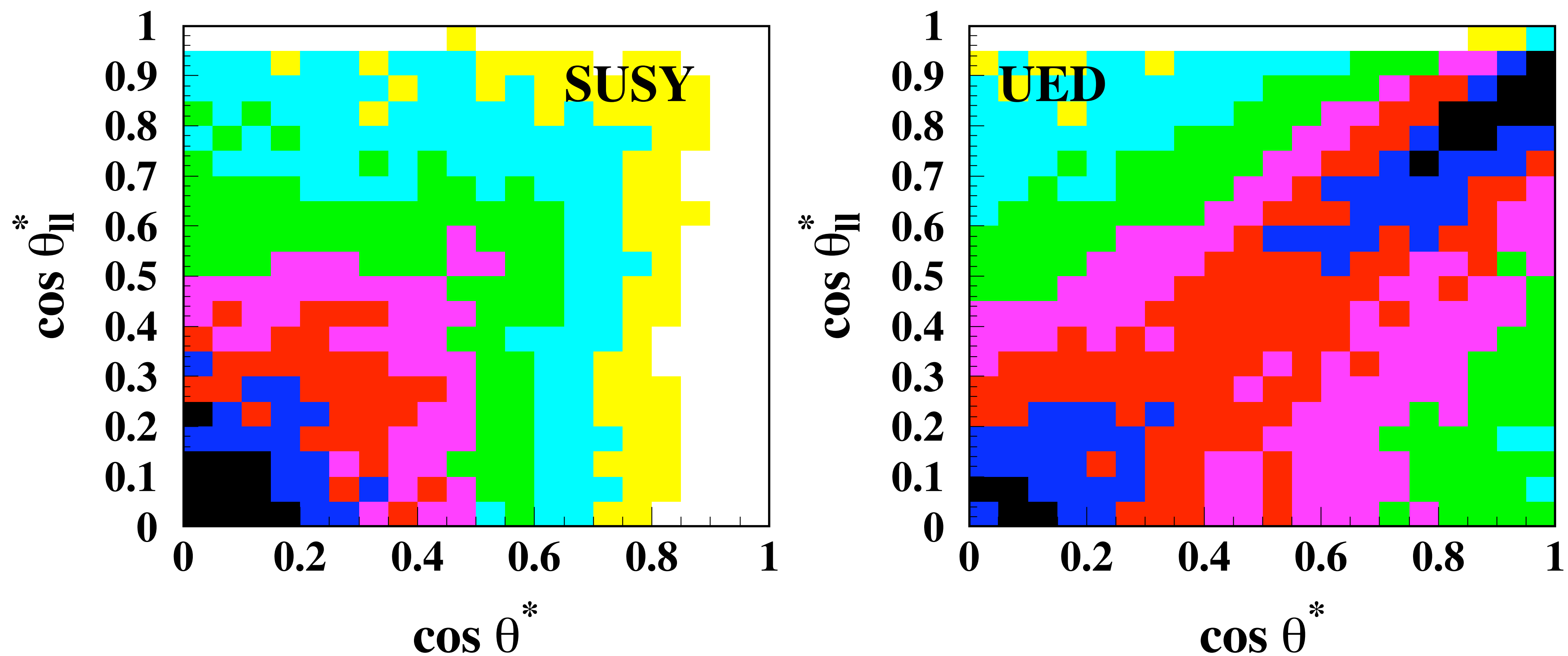


Burns, Kong, Matchev, Park, 0808.2472

Dilepton invariant mass-squared

Dislepton production

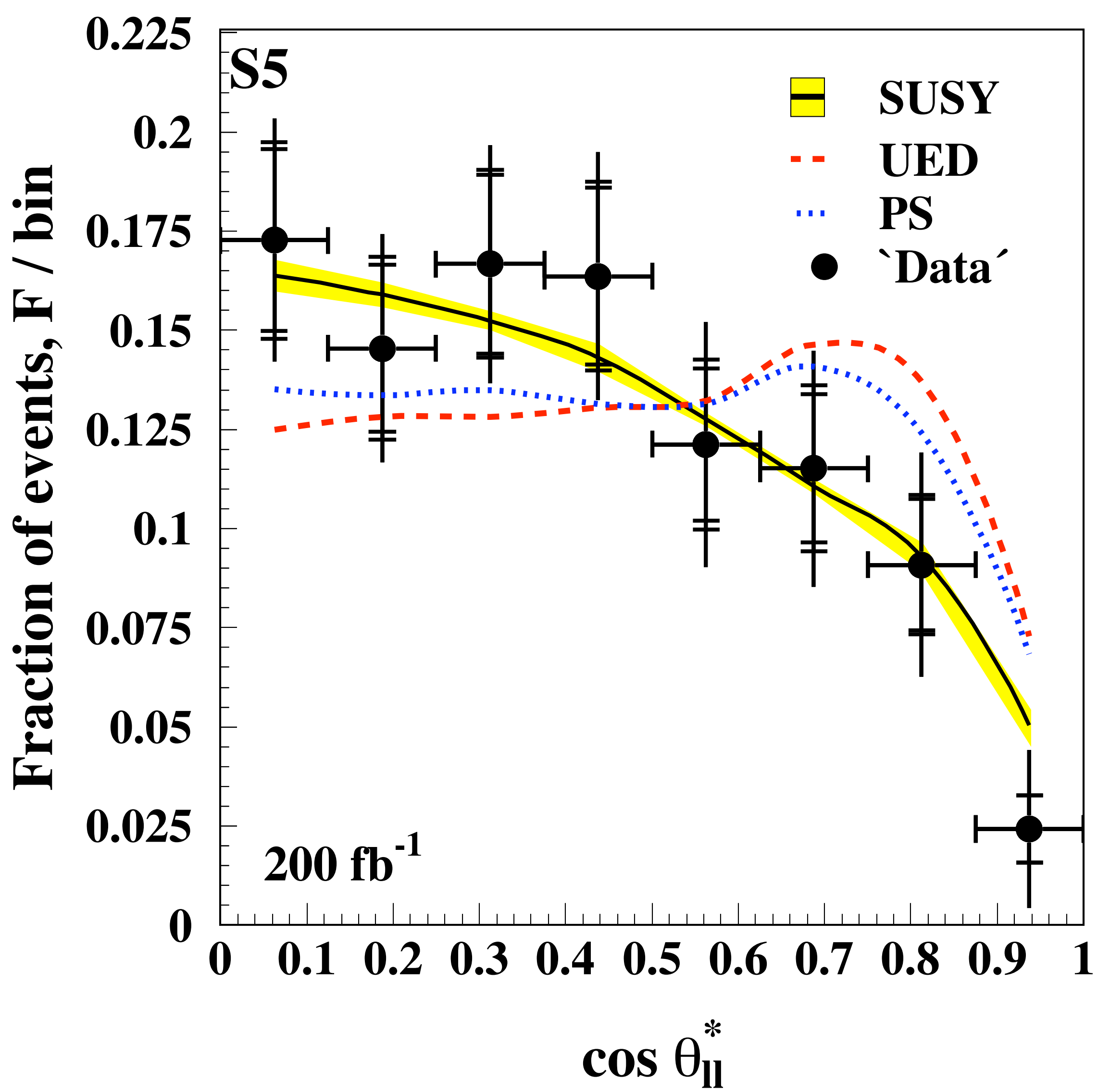
- $q\bar{q} \to Z^0/\gamma \to \tilde{\ell}^+\tilde{\ell}^- \to \tilde{\chi}_1^0\ell^+ \; \tilde{\chi}_1^0\ell^-$
- Distribution of $\cos\theta^*_{ll} \equiv \tanh(\Delta\eta_{\ell^+\ell^-}/2)$ is correlated with Z^0/γ decay angle θ^*

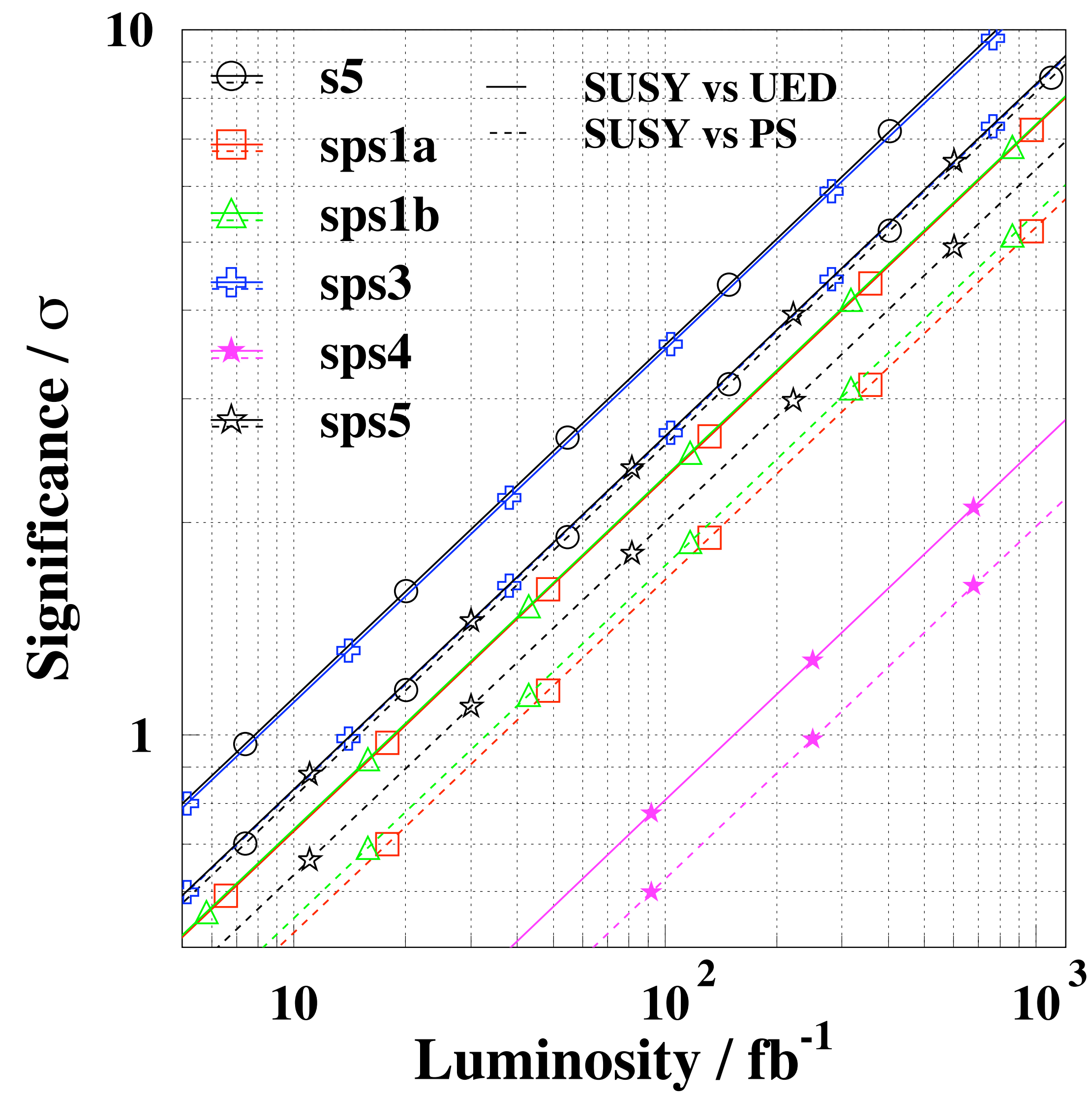


(neglects KKlepton polarisation)

A Barr, hep-ph/0511115

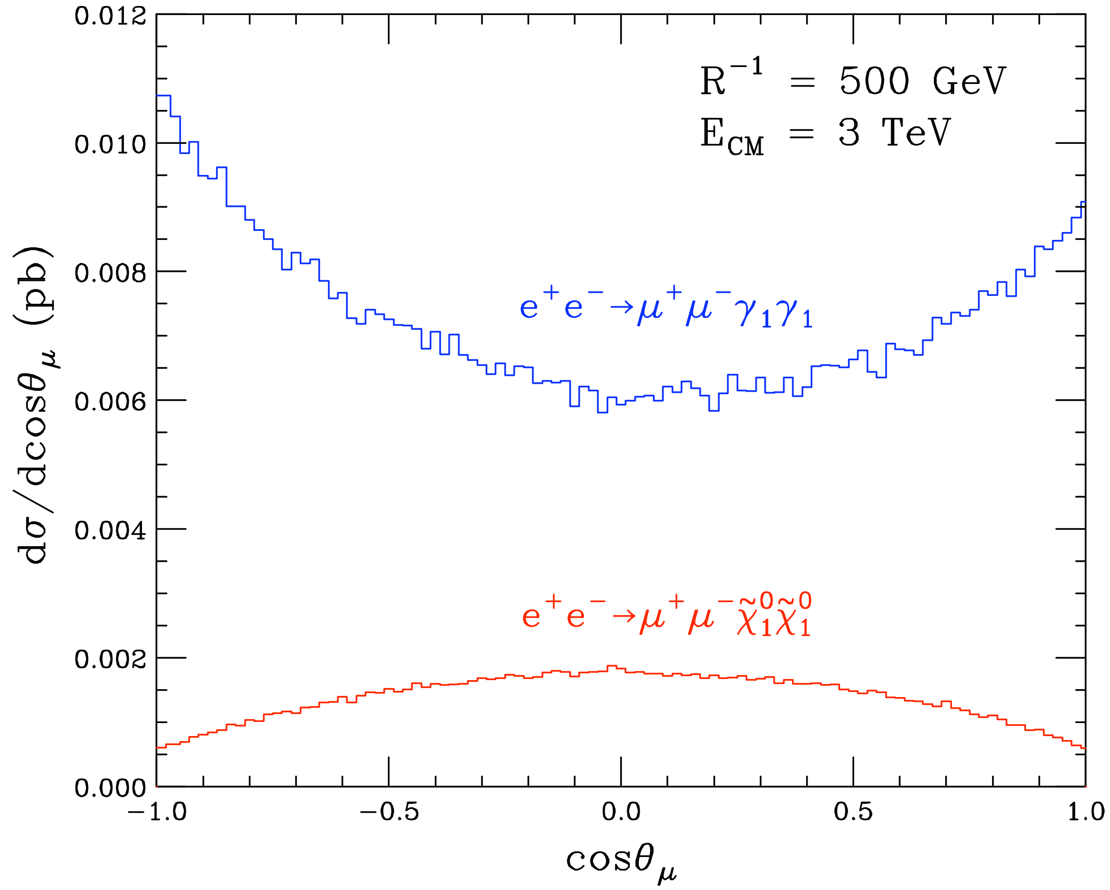
Dislepton production (2)

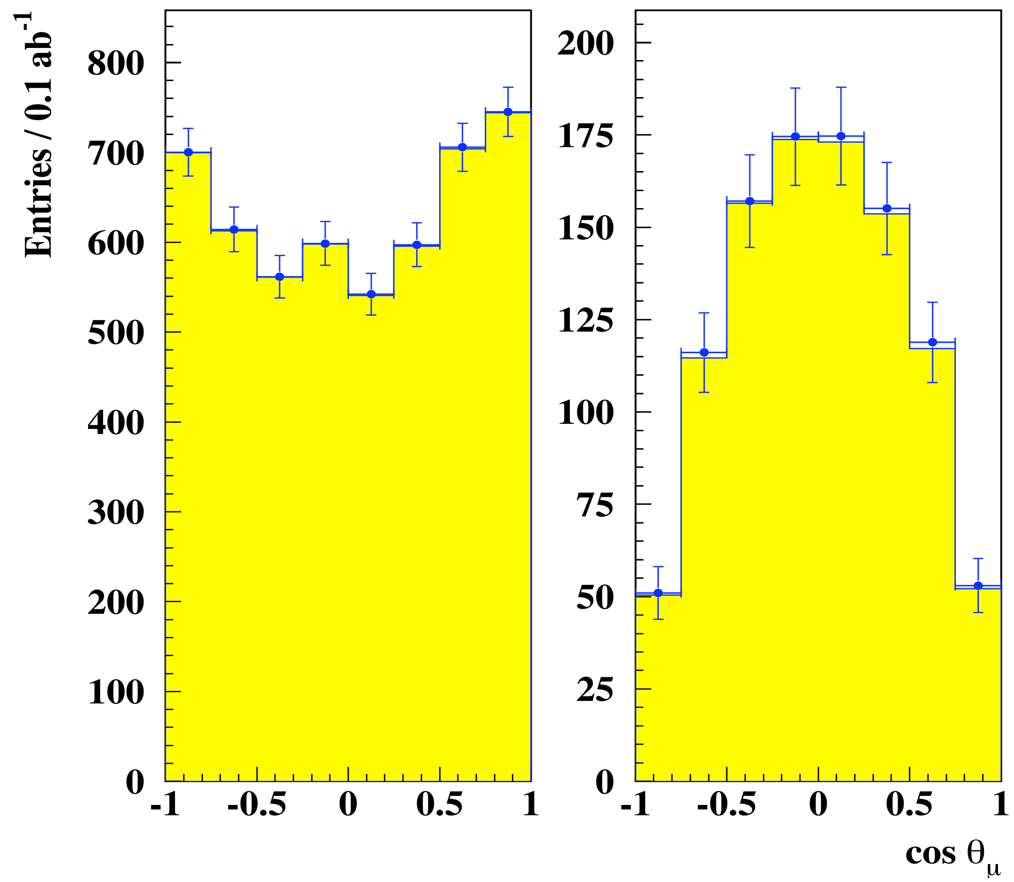




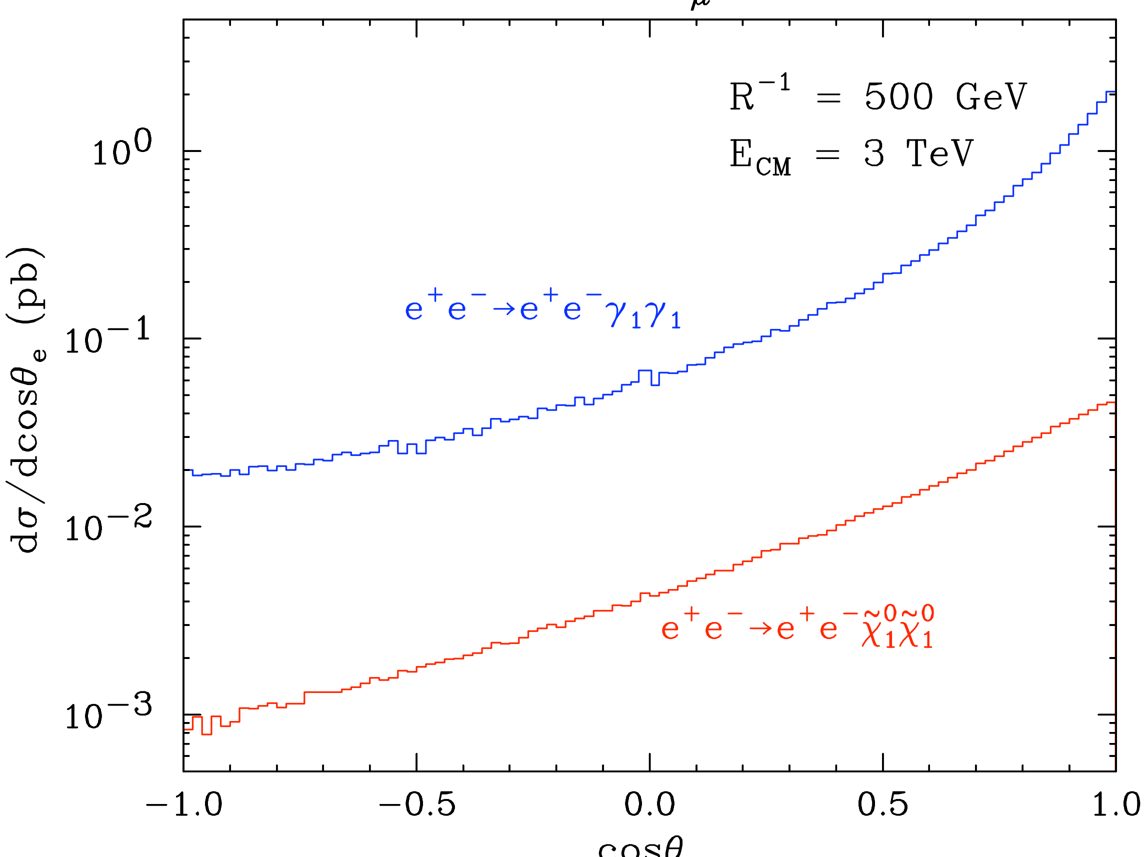
- Outer error bars: after SUSY & SM background subtraction
- Significance strongly dependent on mass spectrum

Disleptons at CLIC





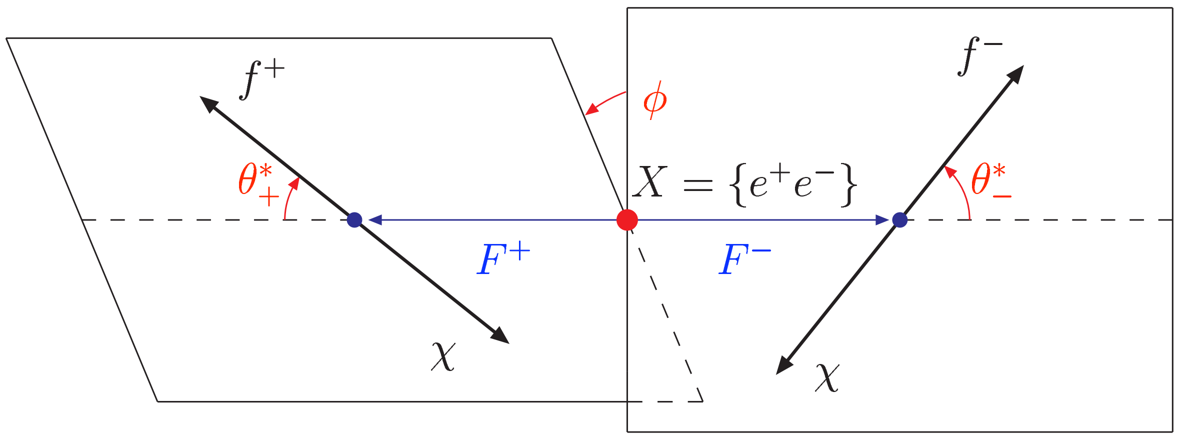
Detector level

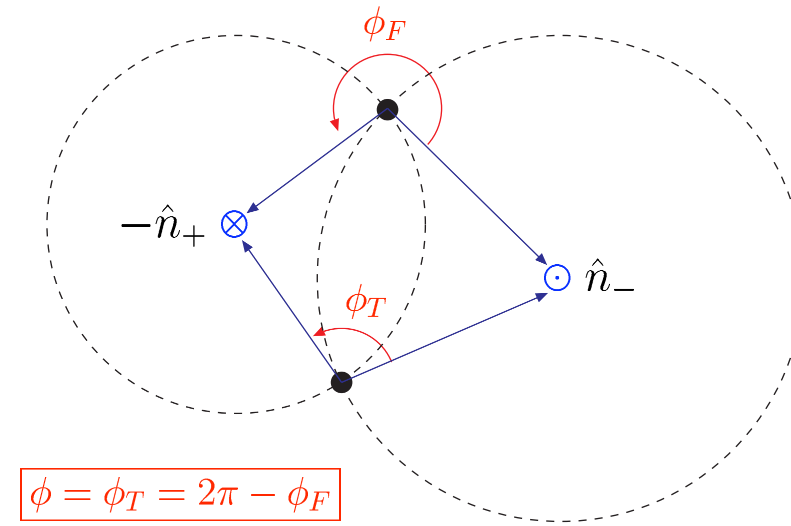


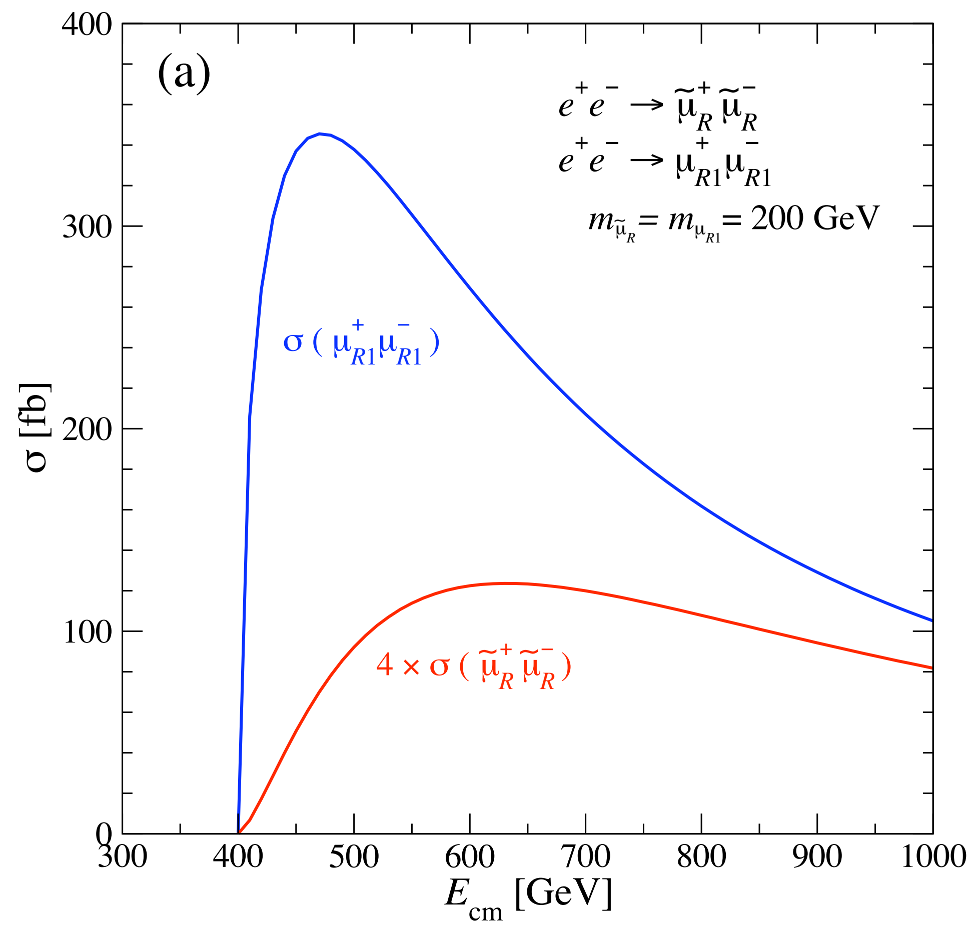
Battaglia, Datta, DeRoeck, Kong, Matchev, hep-ph/0502041, 0507084

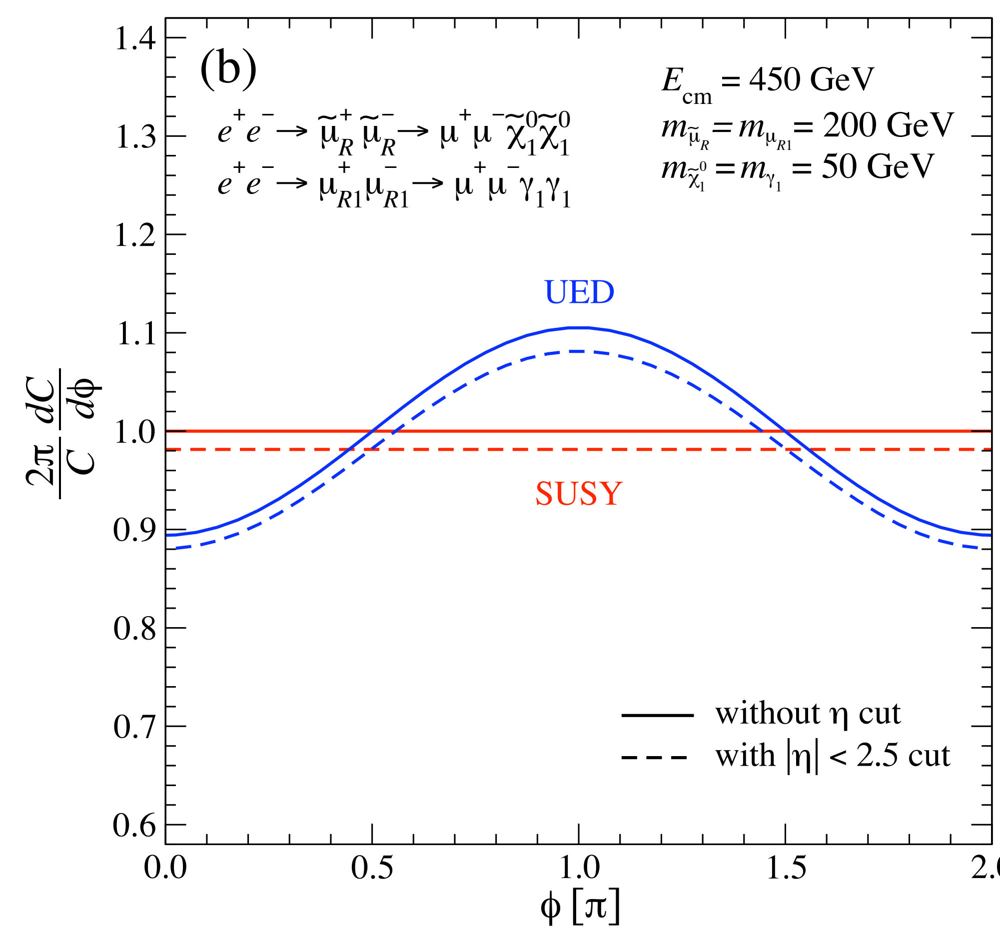
UED: Bhattacharya, Dey, Kundu, Raychaudhuri, hep-ph/0502031

Azimuthal correlations in e^+e^-





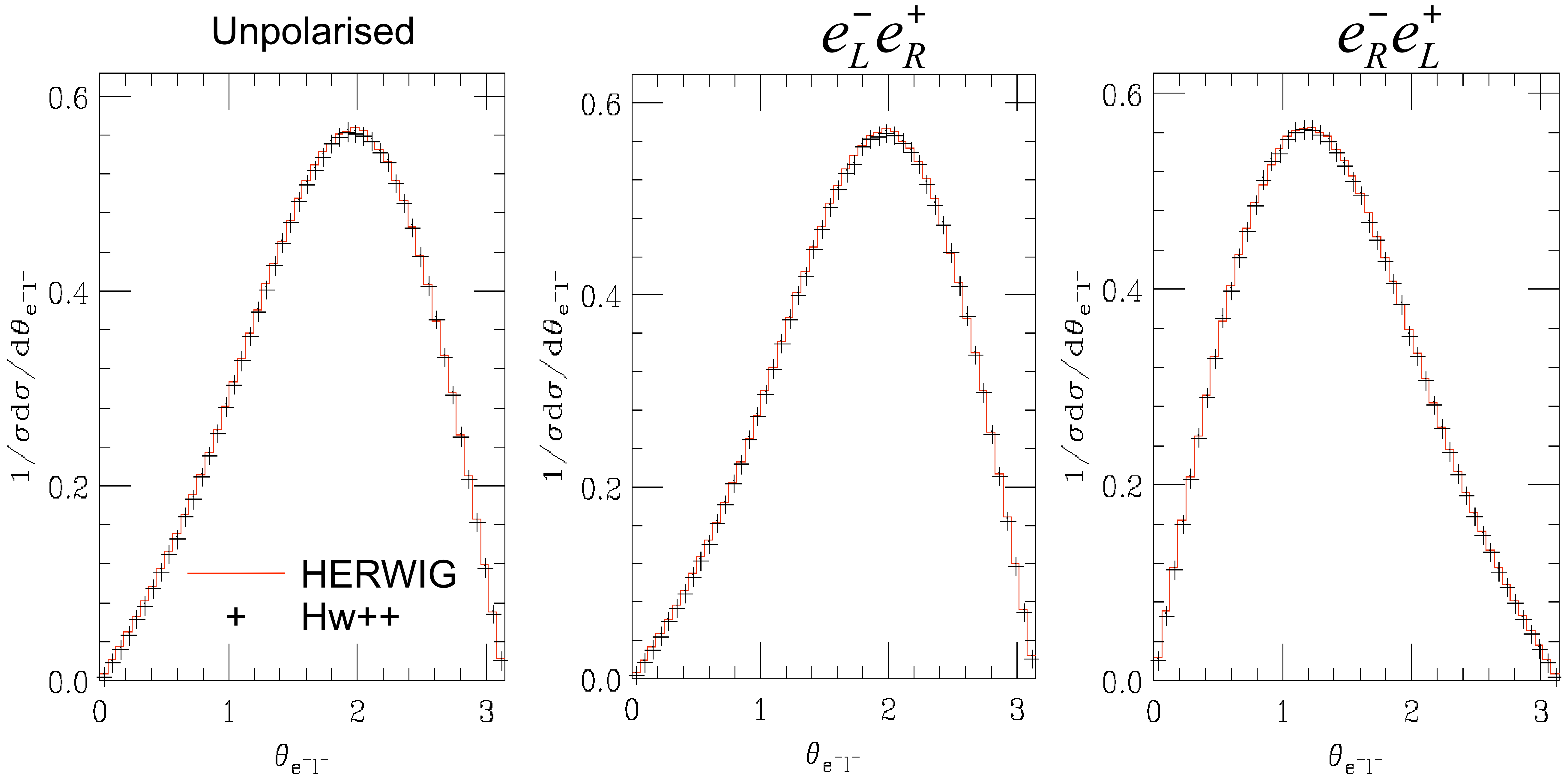




Buckley, Choi, Mawatari, Murayama, 0811.3030

Spin Correlations in HERWIG

$$e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_1^0 \to \tilde{l}_R^+ l^- \tilde{\chi}_1^0 \to l^+l^-\tilde{\chi}_1^0\tilde{\chi}_1^0$$

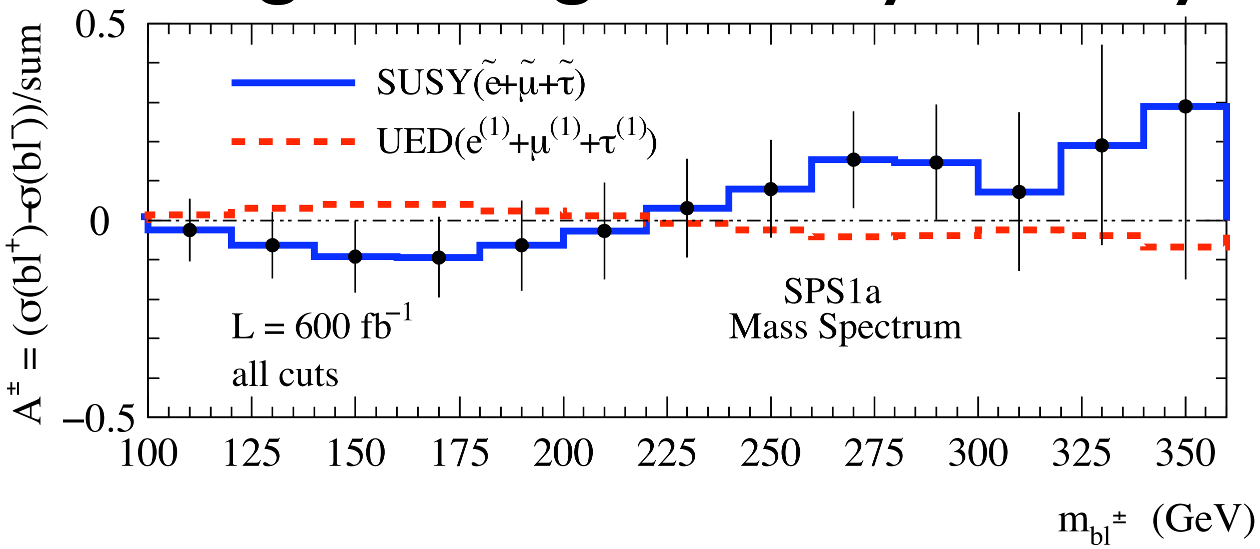


Gigg, Richardson, hep-ph/0703199

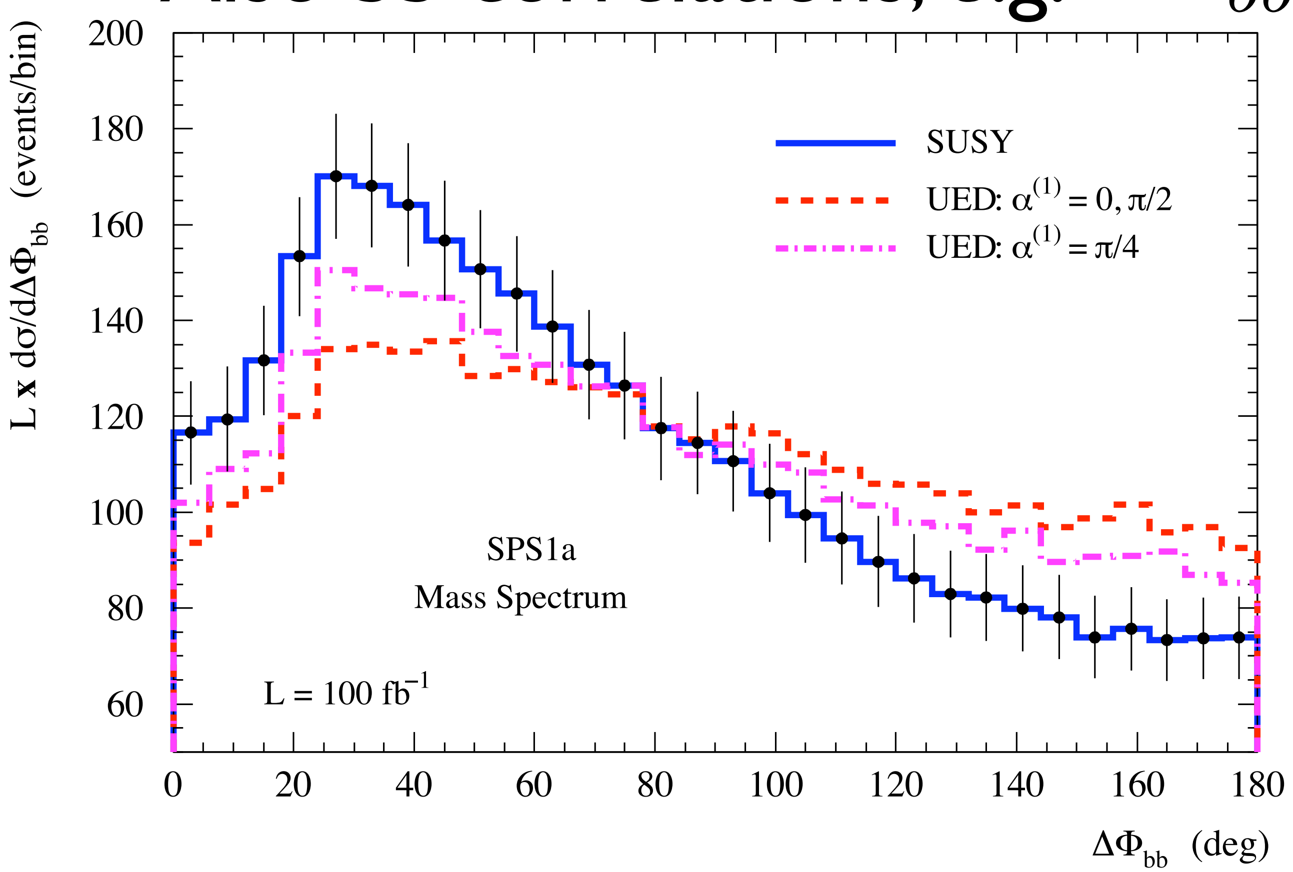
Gluinos

Gluino spin determination

Tag b charge → asymmetry



Also $b\bar{b}$ correlations, e.g. $\Delta\Phi_{bb}$



- $\tilde{g} \to b\tilde{b}_1^*/\bar{b}\tilde{b}_1$
  $\tilde{b}_1 \to \tilde{\chi}_2^0 \to \tilde{\ell} \to \tilde{\chi}_1^0$
- $pp \to \tilde{g}\tilde{g} \to jjb\bar{b}\ell^+\ell^- + \not{p}_T$
  $pp \to \tilde{q}\tilde{g} \to jb\bar{b}\ell^+\ell^- + \not{p}_T$
- Cuts:

$p_{T,b} > 50$ GeV $\quad p_{T,\ell} > 10$ GeV

$p_{T,j}^{\min} > 40$ GeV $\quad p_{T,j}^{\max} > 150$ GeV

$|\eta_i| < 2.4 \quad \Delta R_{ik} > 0.4 \quad (i,k = b,j,\ell)$

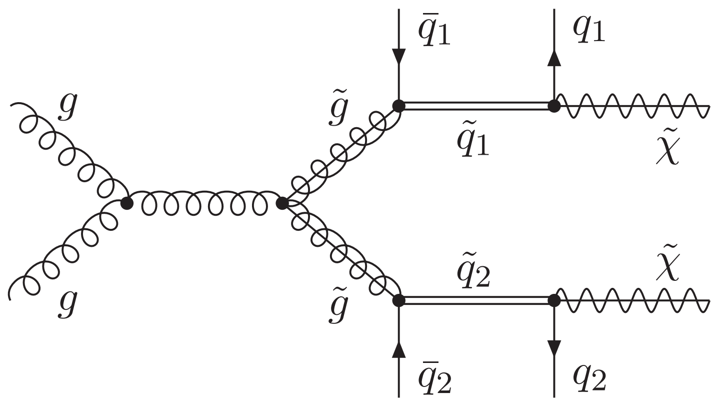
$m_{\ell\ell} < 80$ GeV $\quad M_{\text{eff}} > 450$ GeV

$m_{jj} < 300$ GeV

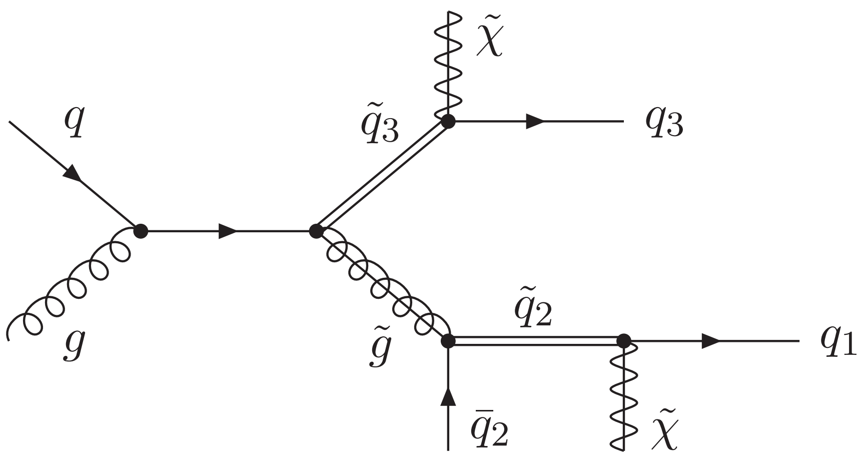
- Varied UED mixing angle

Alves, Eboli, Plehn, hep-ph/0605067

Gluino spin correlations

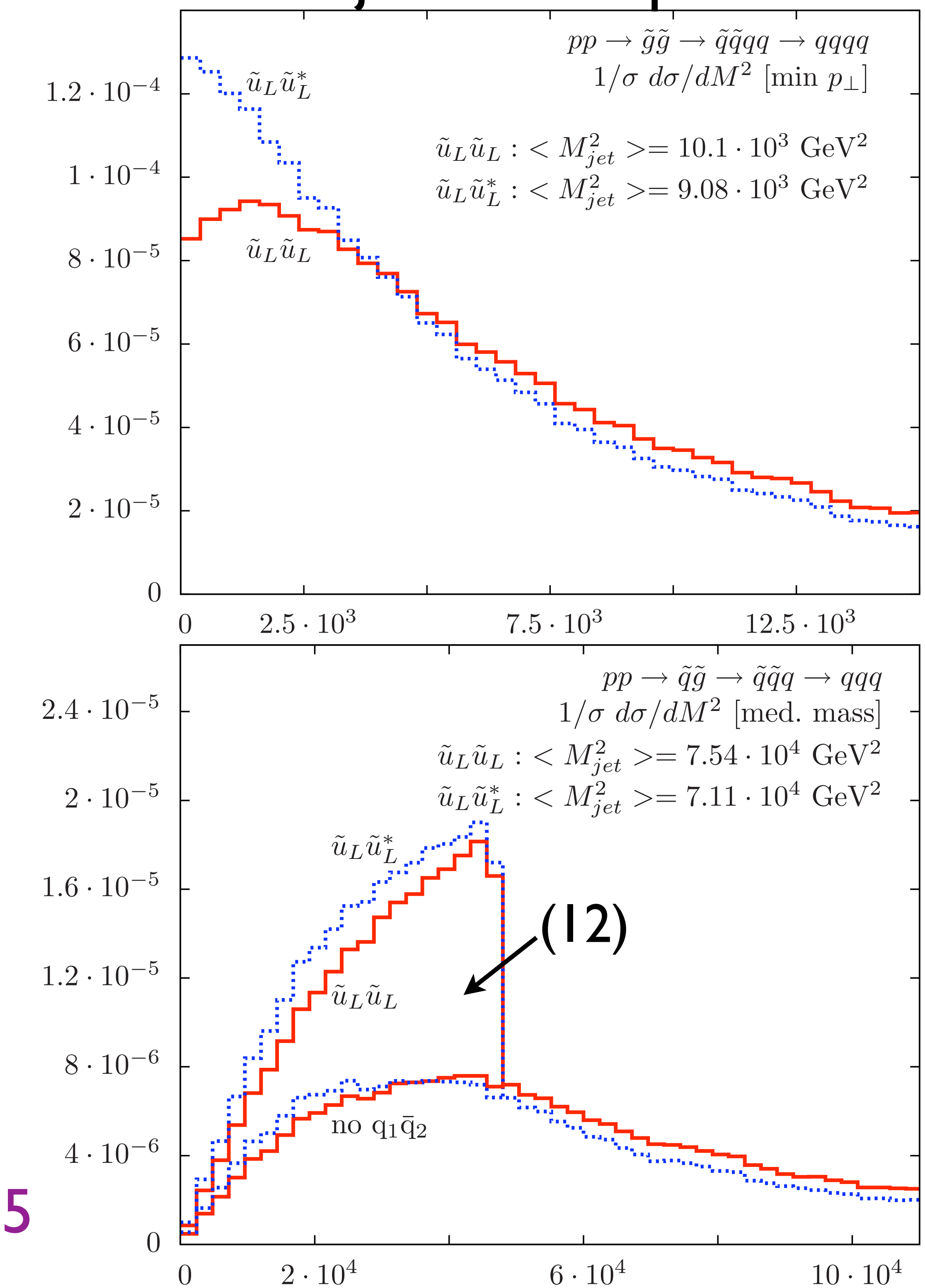


- Lowest mass dijet ~ (12)



- Medium mass dijet ~ (23)

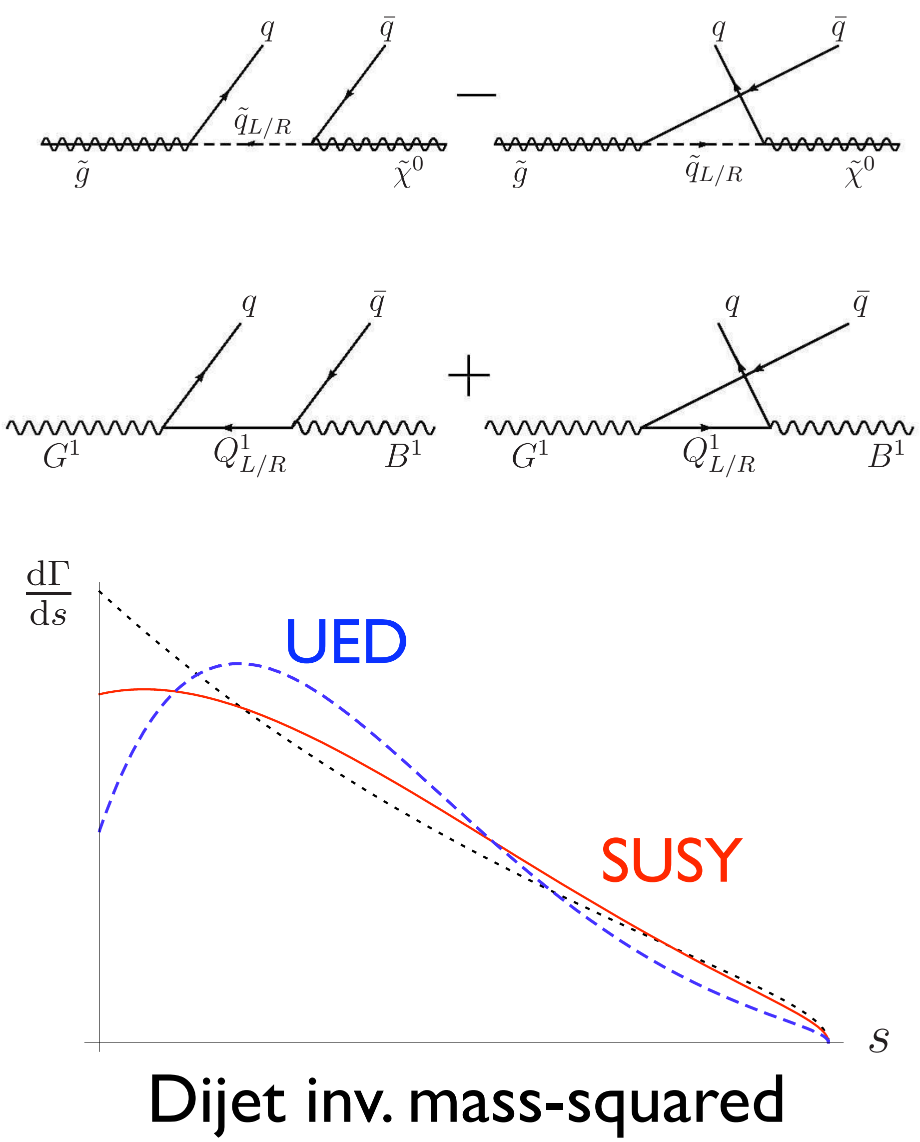
Dijet mass-squared



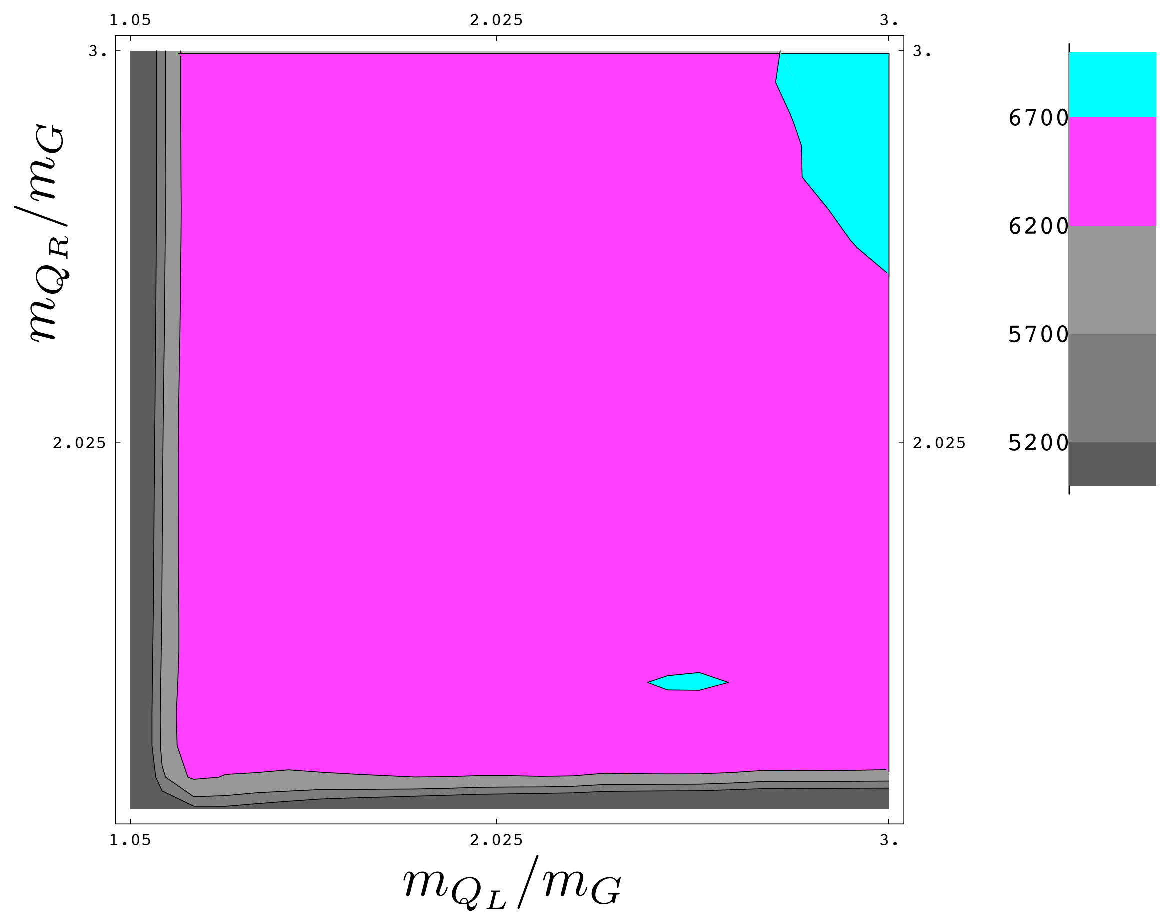
Krämer, Popenda, Spira, Zerwas, 0902.3795

Three-body decays

Three-body gluino decays



Number of events needed to discriminate



Kullback-Leibler measure:

$$N \sim \log R / \mathrm{KL}(T, S)$$

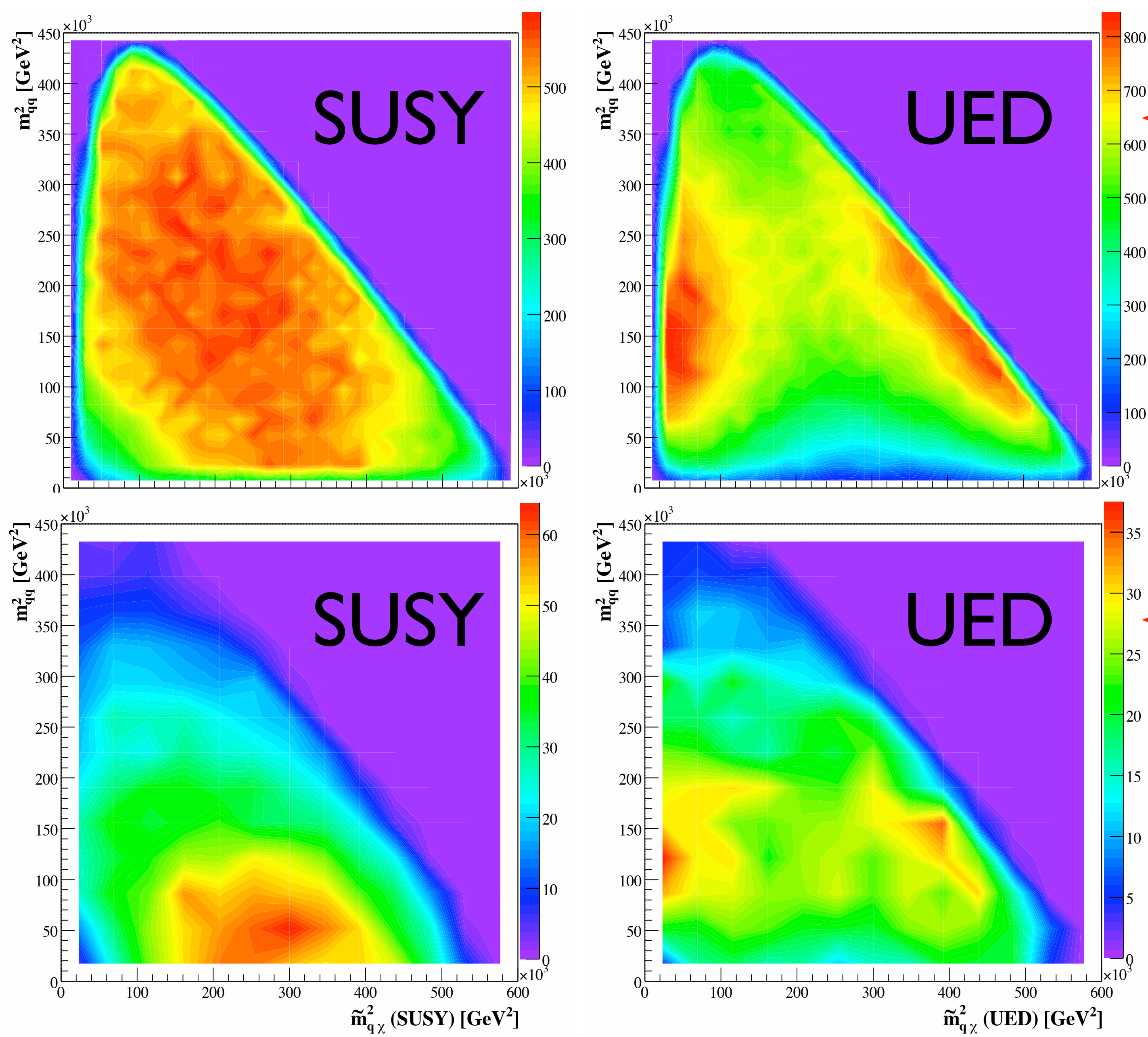
$$\mathrm{KL}(T, S) = \int_m \log \left(\frac{p(m|T)}{p(m|S)} \right) p(m|T)\, dm$$

Csaki, Heinonen, Perelstein, 0707.0014

M_{T2}-assisted spin determination

$$pp \to Y(1) + \bar{Y}(2) \to V(p_1)\chi(k_1) + V(p_2)\chi(k_2), \quad Y \to q(p_q)\bar{q}(p_{\bar{q}})\chi(k)$$

$$M_{T2}(p_i, m_\chi) \equiv \min_{\mathbf{k}_{1T}+\mathbf{k}_{2T}=\mathbf{p}_T^{\text{miss}}} \left[\max\{M_T^{(1)}, M_T^{(2)}\}\right]$$ → **assign 4-momenta**



← $m_{\chi,Y} = m^{\text{true}}_{\chi,Y}$

$\mathcal{L} = \infty$

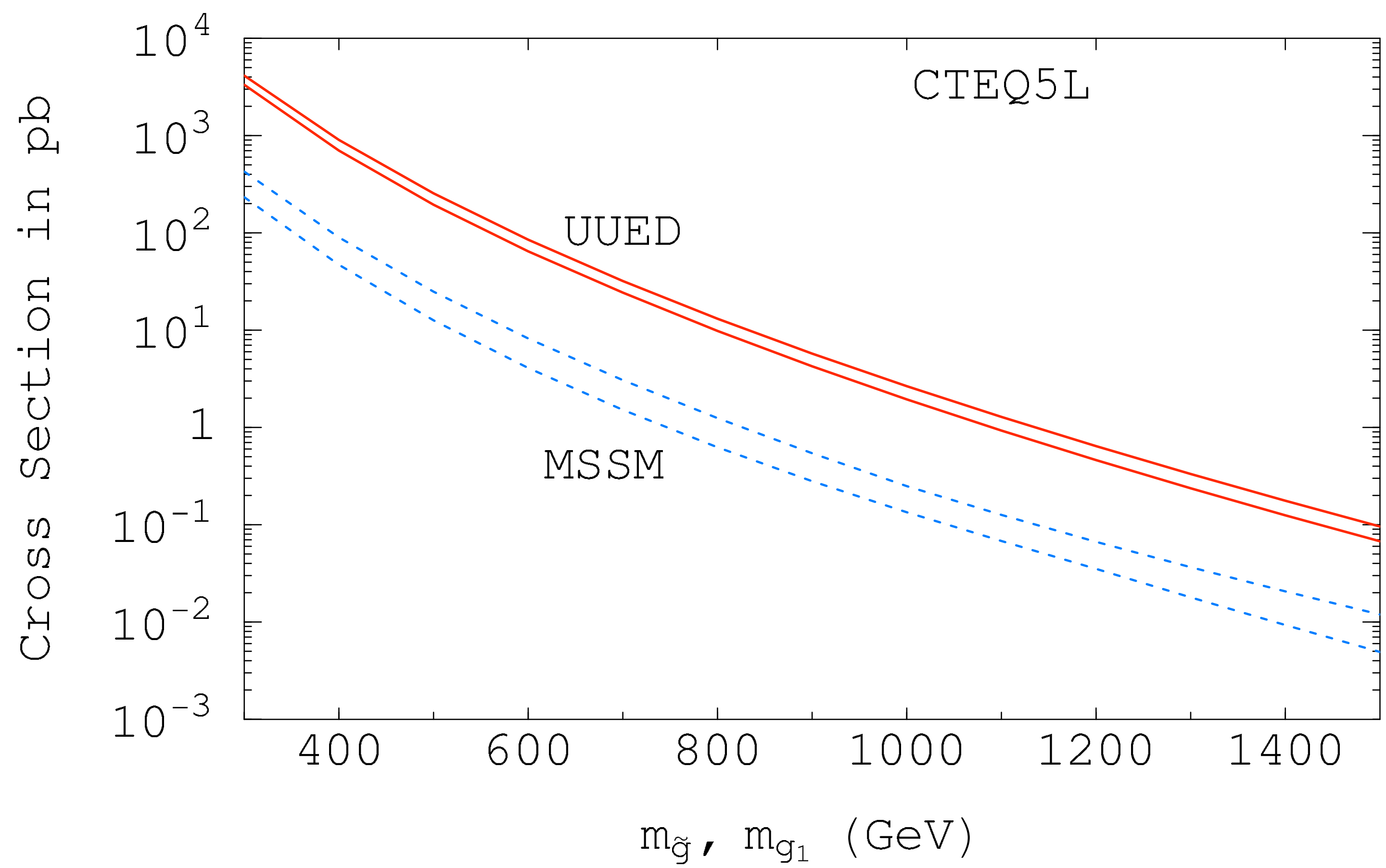
← $m_\chi = 0,\ m_Y = M_{T2}^{\max}(m_\chi = 0)$

$\mathcal{L} = 300\,\text{fb}^{-1}$

Cho, Choi, Kim, Park, 0810.4853

Cross sections

Cross sections imply spins

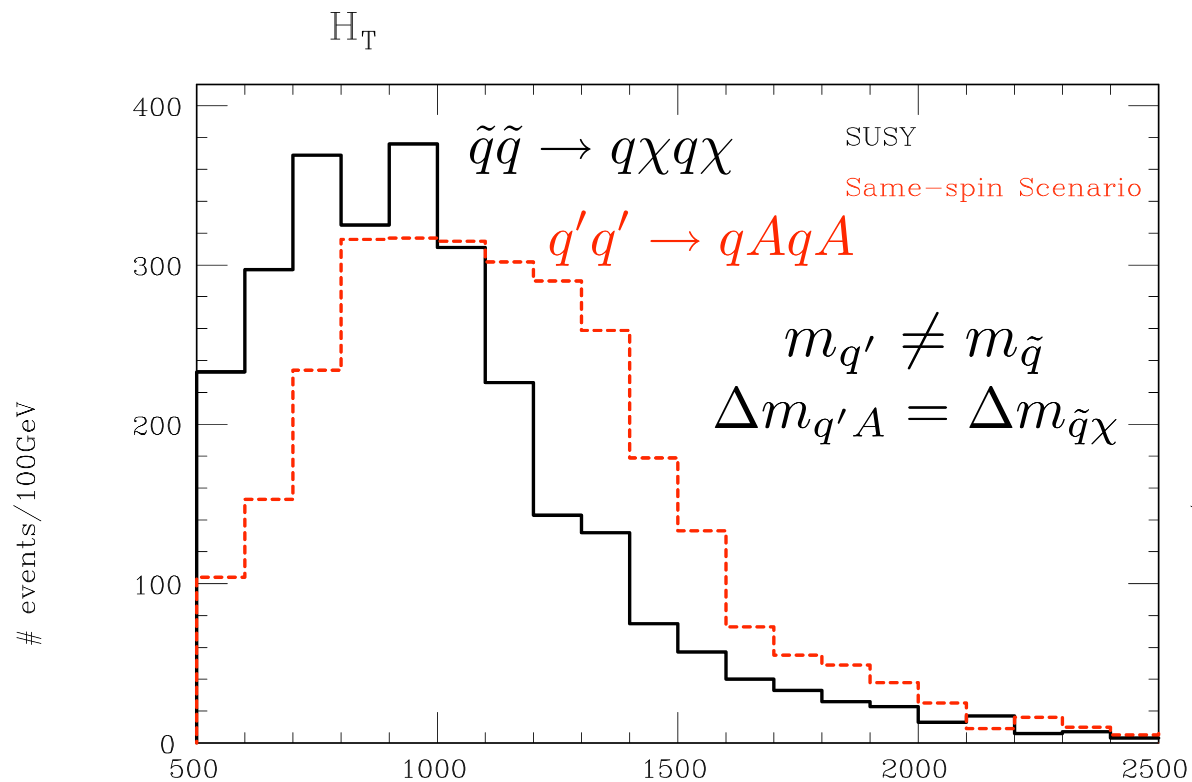


	MSSM	U-UED
Production Cross sections	$\sigma_{\tilde{g}\tilde{g}} = 4.51$ pb	$\sigma_{g_1 g_1} = 65.95$ pb
Branching Fractions	$\tilde{g} \to q\bar{q}'\chi_1^\pm = 0.45$ $\tilde{g} \to q\bar{q}\chi_2^0 = 0.28$ $\tilde{g} \to q\bar{q}\chi_1^0 = 0.27$	$g_1 \to q\bar{q}'W_1^\pm = 0.45$ $g_1 \to q\bar{q}'Z_1 = 0.28$ $g_1 \to q\bar{q}'B_1 = 0.27$
	$\chi_1^\pm \to q\bar{q}'\chi_1^0 = 0.67$ $\chi_1^\pm \to \ell\nu\chi_1^0 = 0.33$ $\chi_2^0 \to q\bar{q}\chi_1^0 = 0.94$ $\chi_2^0 \to \ell\bar{\ell}\chi_1^0 = 0.04$ $\chi_2^0 \to \nu\bar{\nu}\chi_1^0 = 0.01$	$W_1^\pm \to q\bar{q}'B_1 = 0.18$ $W_1^\pm \to \ell\nu B_1 = 0.82$ $Z_1^\pm \to q\bar{q}B_1 = 0.22$ $Z_1^\pm \to \ell\bar{\ell}B_1 = 0.39$ $Z_1^\pm \to \nu\bar{\nu}B_1 = 0.39$
Cascade Fractions		
1-lepton	0.248	0.385
OS 2-lepton	0.030	0.183
SS 2-lepton	0.011	0.068
3-lepton	0.003	0.081
Cascade Rates		
1-lepton	1.12 pb	25.39 pb
OS 2-lepton	0.13 pb	12.06 pb
SS 2-lepton	0.05 pb	4.48 pb
3-lepton	0.014 pb	5.34 pb

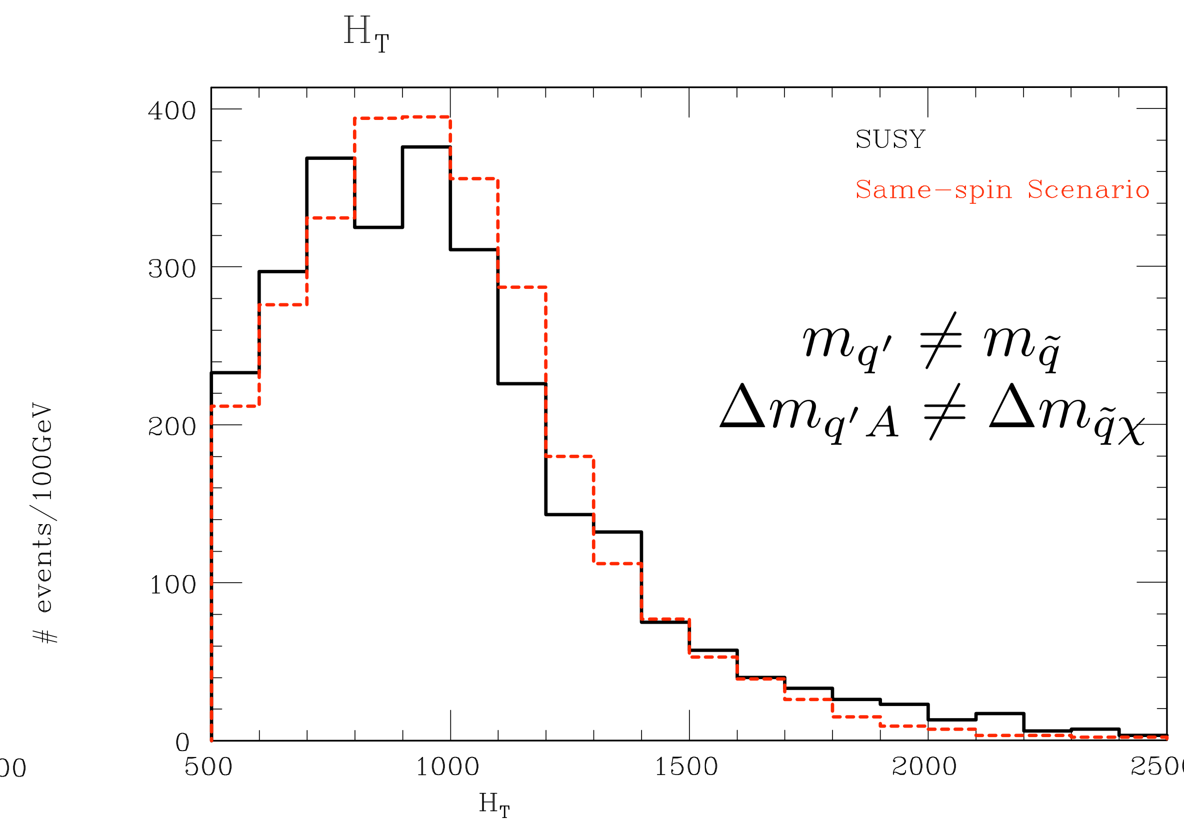
➡ Higher spins mean higher cross sections (for given masses)

Datta, Kane, Toharia hep-ph/0510204

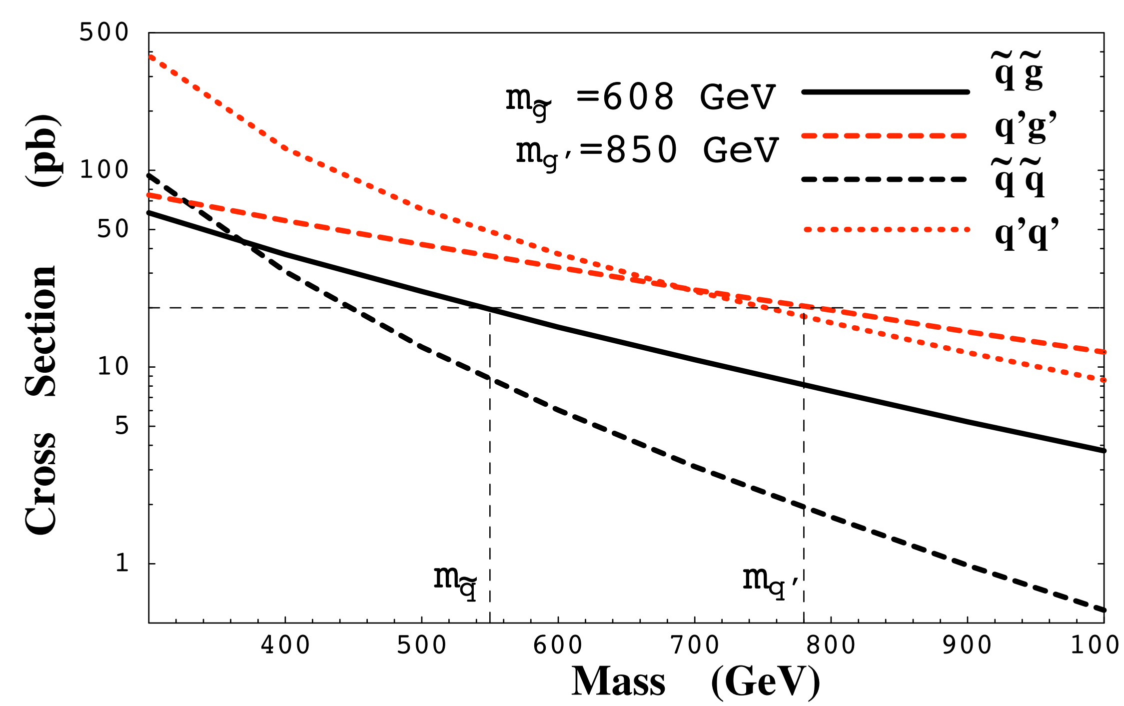
Cross sections imply spins (2)



$$H_T = \sum E_T + \not{E}_T$$

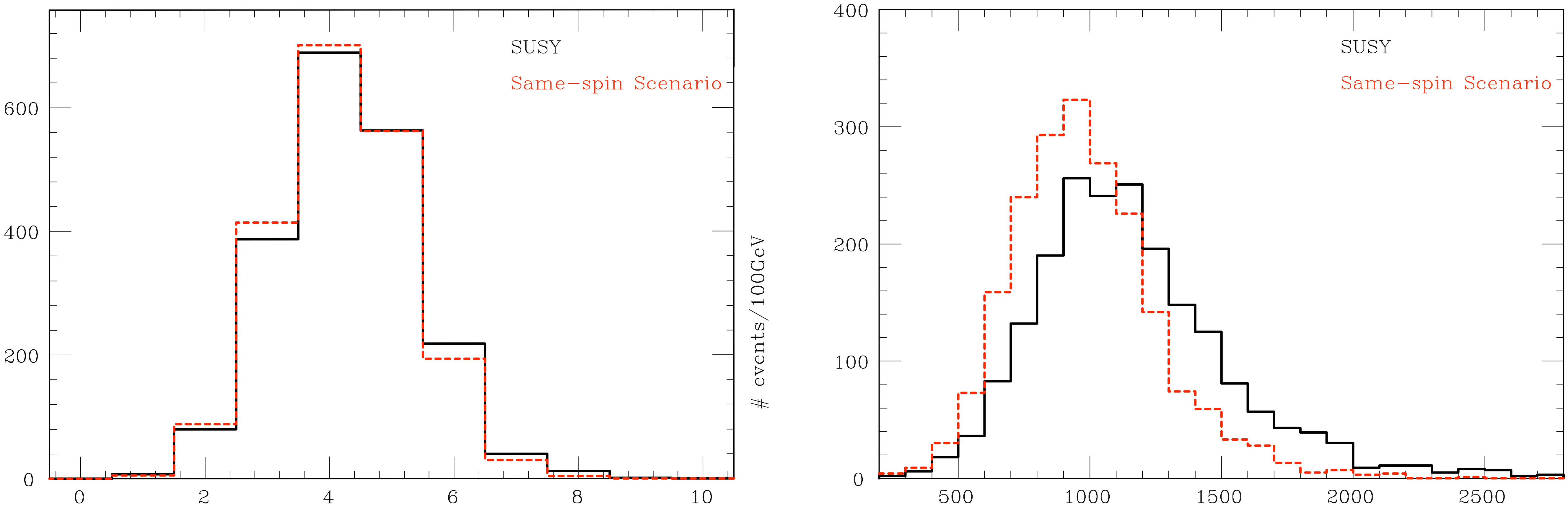


- Can match cross section and one distribution by adjusting masses
- Cannot match several cross sections or distributions ...



Kane, Petrov, Shao, Wang, 0805.1397

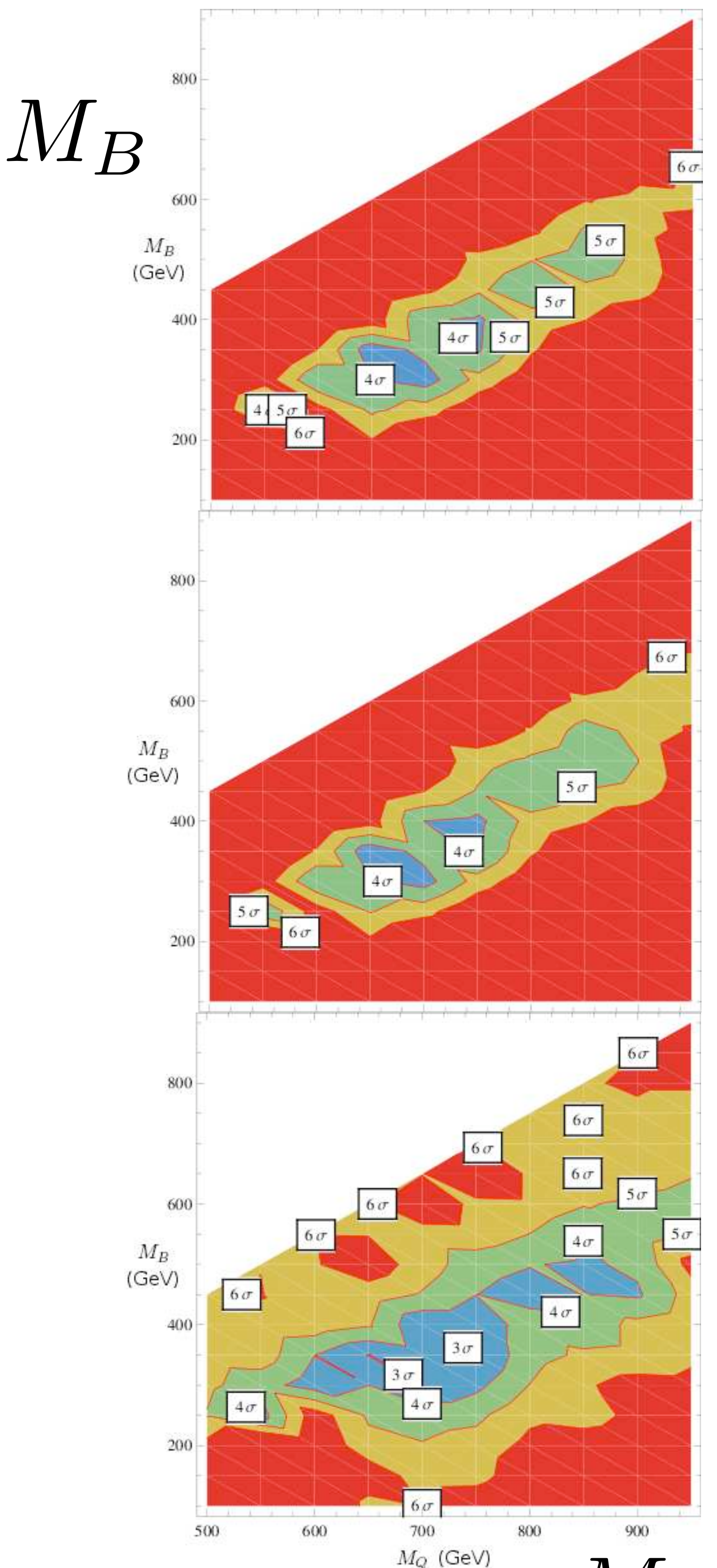
Cross sections imply spins (3)



Jet multiplicity $\qquad H_T$

- Can vary masses to fit cross section and one distribution
- E.g. match jet counts → H_T doesn't match → ambiguity resolved

SUSY vs Littlest Higgs



- Fit $pp \to \tilde{q}\tilde{q}^*$, $\tilde{q} \to q\tilde{\chi}$
  with $pp \to Q\bar{Q}$, $Q \to qB$ ← LTP (J=1)
- $M_{\tilde{q}} = 500\,\mathrm{GeV}$, $M_{\tilde{\chi}} = 100\,\mathrm{GeV}$
  Vary M_Q, M_B
- 10 observables, 2 fb^{-1}

← No $\sigma_{\rm obs}$ (surprising!)

← No $\langle \not{E}_T \rangle$, $\langle H_T \rangle$ (not surprising)

Hallenbeck, Perelstein, Spethmann, Thom, Vaughan, 0812.3135

Conclusions

- Sequential decay chains
  - Possibilities -- but difficult for degenerate masses
- Dileptons
  - SUSY vs UED difficult at LHC -- other cases possible
- Gluinos
  - Some ideas -- just starting
- Three-body decays
  - M_{T2} assistance looks useful here (and elsewhere?)
- Cross sections
  - Should be included

→ Full simulations (and data) needed!