

Quarkonia in Nuclear Collisions

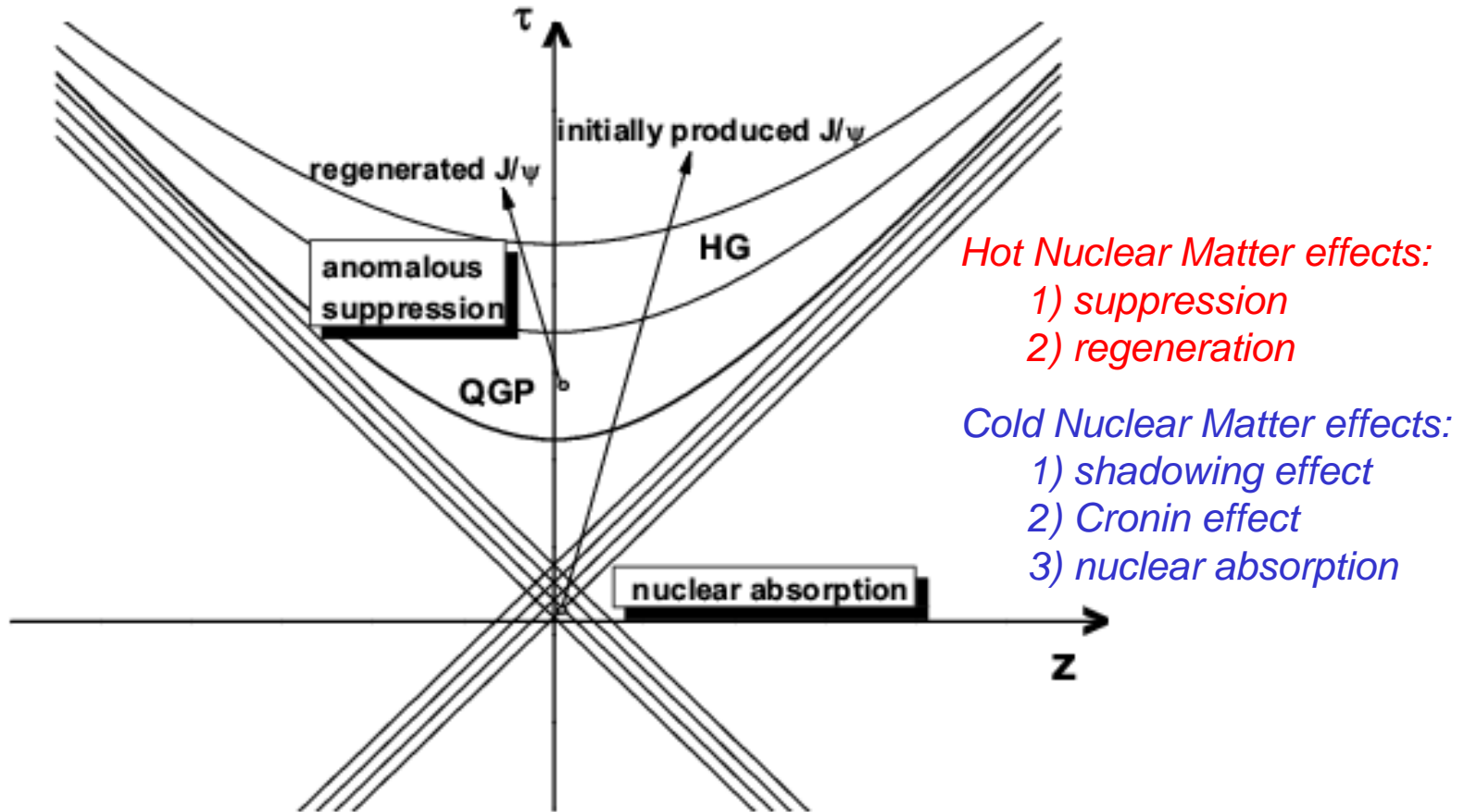
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- 1) Quarkonium TMD
- 2) Quarkonia in Strong B Field
- 3) Ω_{ccc} Production in Nuclear Collisions

reference: charmonium transverse momentum distribution in high energy nuclear collisions
Zebo Tang, Nu Xu, Kai Zhou, Pengfei Zhuang
J.Phys. G41 (2014) 12, 124006

Workshop on QCD Thermodynamics, CCNU, July 27-31, 2015

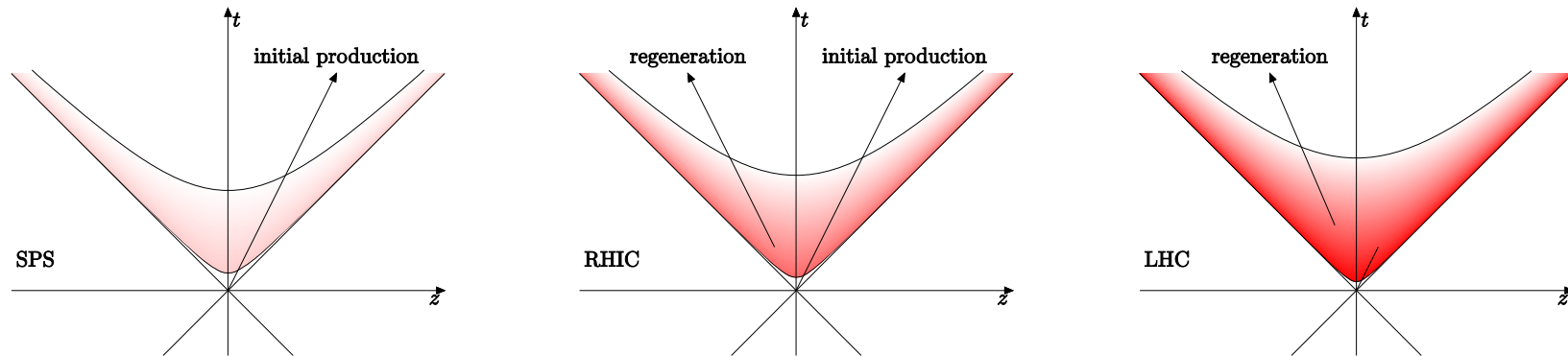
Cold and Hot Nuclear Matter Effects



CNM: R.Vogt; Hirai-Kumano-Nagai (KHN); Eskola-Kolhinen-Salgado (EKS); EPS;

HNM: Matsui, Satz, PBM, J.Stachel; R.Rapp; R.Thews;.....

Cancellation between Suppression and Regeneration



The suppression and regeneration both increase with colliding energy but work in an opposite way, the cancellation between them weakens the sensitivity of the yield to the colliding energy.

Transverse momentum distribution is created in the medium and more sensitive to the medium properties !

Initial Production and Regeneration in Different p_t Regions

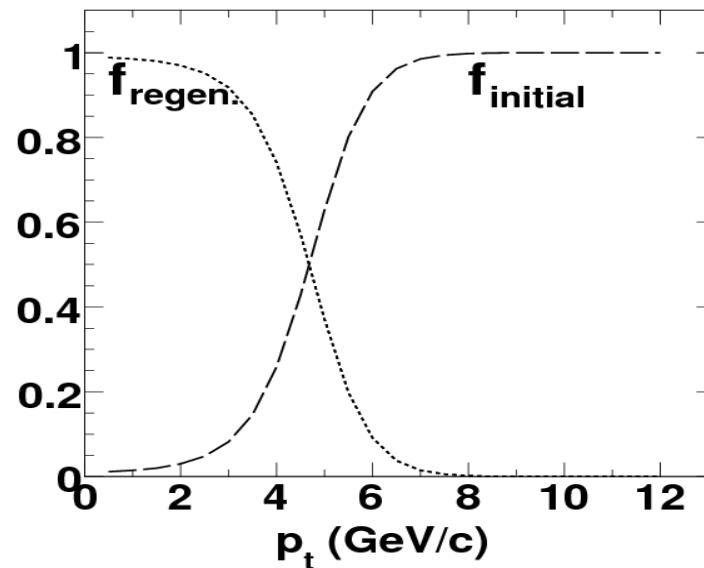
$$f(p) = f_{ini}(p) + f_{reg}(p)$$

Initially produced quarkonia:

- 1) Cronin effect leads to a p_t broadening,
- 2) Low p_t part is strongly suppressed by the hot medium, but high p_t part can survive.

Regenerated quarkonia:

produced in the later stage and carry low p_t .



5.5 TeV central Pb+Pb,
Liu, Xu, Zhuang, NPA(2009)

Conclusion:

TMD can distinguish hot mediums at SPS, RHIC and LHC !

A Dynamic Transport Approach for Quarkonium Motion

Yan, Xu, Zhuang, PRL(2006)

Liu, Qu, Xu, Zhuang, PLB(2009)

- QGP evolution

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu n^\mu = 0 \quad + \text{Lattice QCD equation of state}$$

- quarkonium motion ($\Psi = J/\psi, \psi', \chi_c$)

$$\partial f_\Psi / \partial \tau + \mathbf{v}_\Psi \cdot \nabla f_\Psi = -\alpha_\Psi f_\Psi + \beta_\Psi. \quad \text{hot nuclear matter effects}$$

$$\alpha_\Psi(\mathbf{p}_t, \mathbf{x}_t, \tau | \mathbf{b}) = \frac{1}{2E_\Psi} \int \frac{d^3 \mathbf{p}_g}{(2\pi)^3 2E_g} W_{g\Psi}^{c\bar{c}}(s) f_g(\mathbf{p}_g, \mathbf{x}_t, \tau) \Theta(T(\mathbf{x}_t, \tau | \mathbf{b}) - T_c),$$

$$\beta_\Psi(\mathbf{p}_t, \mathbf{x}_t, \tau | \mathbf{b}) = \frac{1}{2E_\Psi} \int \frac{d^3 \mathbf{p}_g}{(2\pi)^3 2E_g} \frac{d^3 \mathbf{p}_c}{(2\pi)^3 2E_c} \frac{d^3 \mathbf{p}_{\bar{c}}}{(2\pi)^3 2E_{\bar{c}}} W_{c\bar{c}}^{g\Psi}(s) f_c(\mathbf{p}_c, \mathbf{x}_t, \tau | \mathbf{b}) f_{\bar{c}}(\mathbf{p}_{\bar{c}}, \mathbf{x}_t, \tau | \mathbf{b}) \times (2\pi)^4 \delta^{(4)}(p + p_g - p_c - p_{\bar{c}}) \Theta(T(\mathbf{x}_t, \tau | \mathbf{b}) - T_c),$$

- assuming thermalized charm quark distribution:

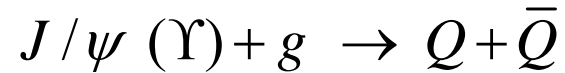
$$f_c(x, q) \sim \frac{1}{e^{q^\mu u_\mu / T} + 1} \quad u_\mu(x) \text{ and } T(x) \text{ determined by hydrodynamics}$$

- analytic solution

$$f_\Psi(\mathbf{p}_t, \mathbf{x}_t, \tau | \mathbf{b}) = f_\Psi(\mathbf{p}_t, \mathbf{x}_t - \mathbf{v}_\Psi(\tau - \tau_0), \tau_0 | \mathbf{b}) e^{-\int_{\tau_0}^{\tau} d\tau' \alpha_\Psi(\mathbf{p}_t, \mathbf{x}_t - \mathbf{v}_\Psi(\tau - \tau'), \tau' | \mathbf{b})} + \int_{\tau_0}^{\tau} d\tau' \beta_\Psi(\mathbf{p}_t, \mathbf{x}_t - \mathbf{v}_\Psi(\tau - \tau'), \tau' | \mathbf{b}) e^{-\int_{\tau'}^{\tau} d\tau'' \alpha_\Psi(\mathbf{p}_t, \mathbf{x}_t - \mathbf{v}_\Psi(\tau - \tau''), \tau'' | \mathbf{b})}.$$

cold nuclear matter effects

Dissociation and Regeneration Rates



- *gluon dissociation cross section calculated by OPE (Bhanot, Peskin, 1999):*

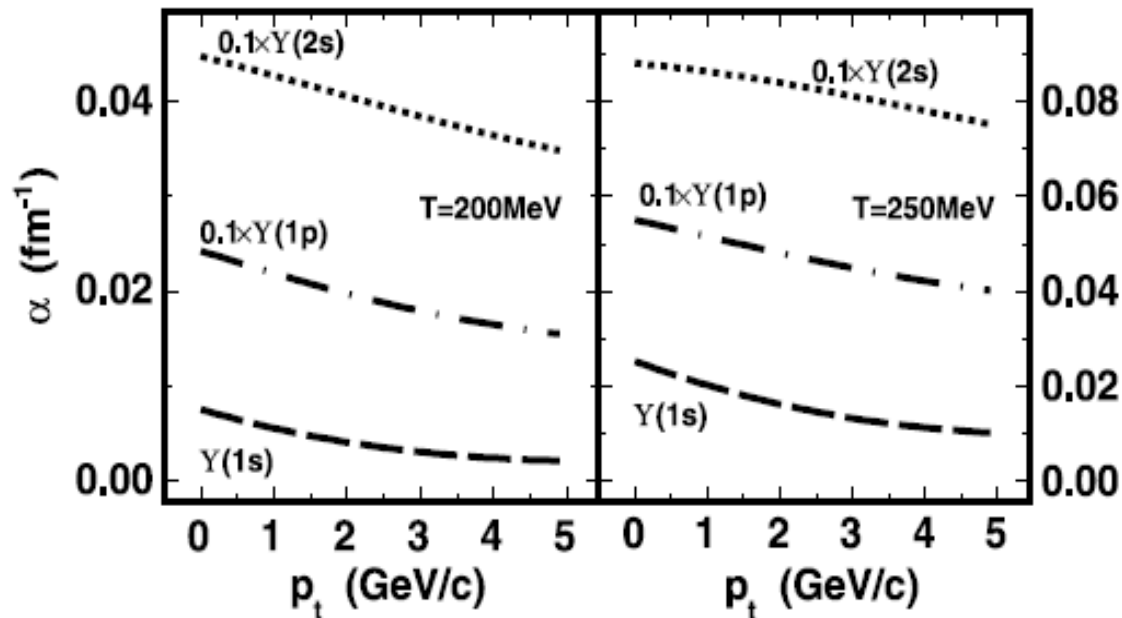
$$\sigma(p_\psi, p_g)$$

- *at finite temperature, we use the classical relation*

$$\sigma(p_\psi, p_g, T) = \frac{\langle r^2 \rangle(T)}{\langle r^2 \rangle(0)} \sigma(p_\psi, p_g) \quad \langle r^2 \rangle(T) \text{ potential model}$$

Digal, Kaczmarek, Karsch, Satz, 2005

- *Υ dissociation rate*



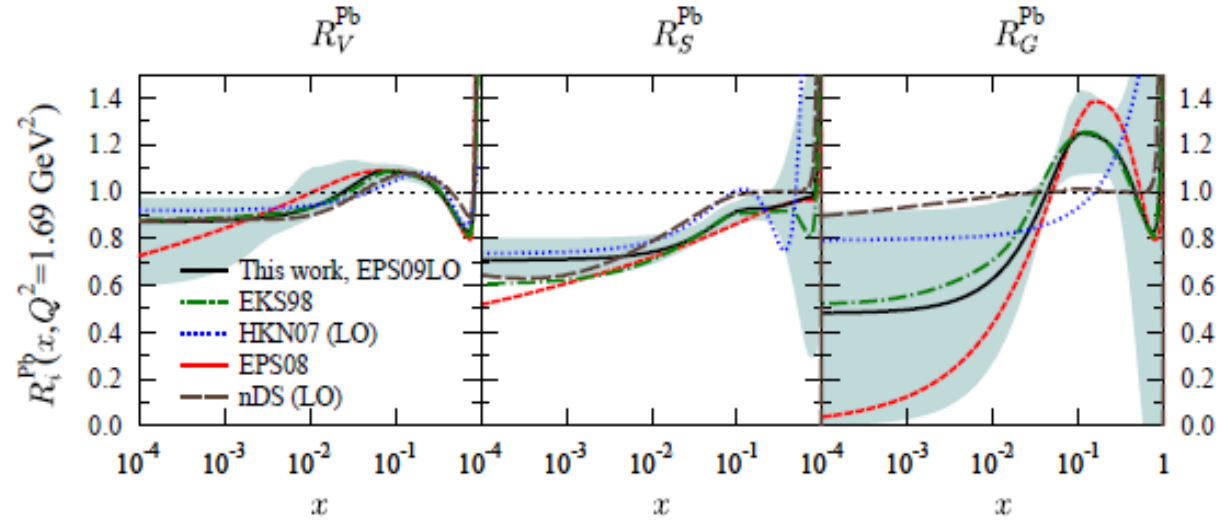
Liu, Chen, Xu, Zhuang, PLB(2011)

- *regeneration rate is determined by the detailed balance*

Shadowing and Cronin Effects

nuclear shadowing factor

$$R_i^A(x, \mu_F) = \frac{f_i^A(x, \mu_F)}{A f_i(x, \mu_F)}, i = q, \bar{q}, g.$$



for $g + g \rightarrow \Psi + g$

$$x_{1,2} = \frac{\sqrt{m_\Psi^2 + p_t^2}}{\sqrt{S_{NN}}} e^{\pm y} \sim \begin{cases} 0.25 & \text{SPS antishadowing} \\ 0.02 & \text{RHIC weak shadowing} \\ 0.0015 & \text{LHC strong shadowing} \end{cases}$$

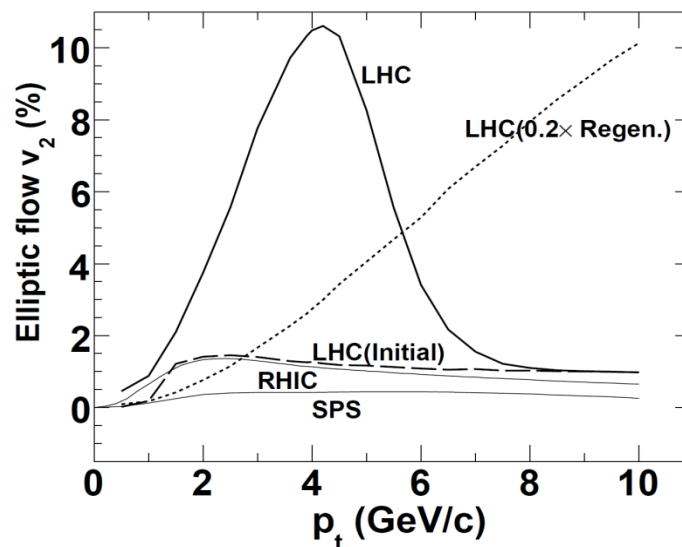
initial distribution with shadowing effect and Cronin effect

$$f_\Psi(\mathbf{x}, \mathbf{p}, \tau_0 | \mathbf{b}) = \frac{(2\pi)^3}{E_T \tau_0} \int dz_A dz_B \rho_A(\mathbf{x}_T, z_A) \rho_B(\mathbf{x}_T, z_B) \\ \times \mathcal{R}_g(x_1, \mu_F, \mathbf{x}_T) \mathcal{R}_g(x_2, \mu_F, \mathbf{x}_T - \mathbf{b}) \\ \times \bar{f}_\Psi^{pp}(\mathbf{x}, \mathbf{p}, z_A, z_B | \mathbf{b}), \quad (11)$$

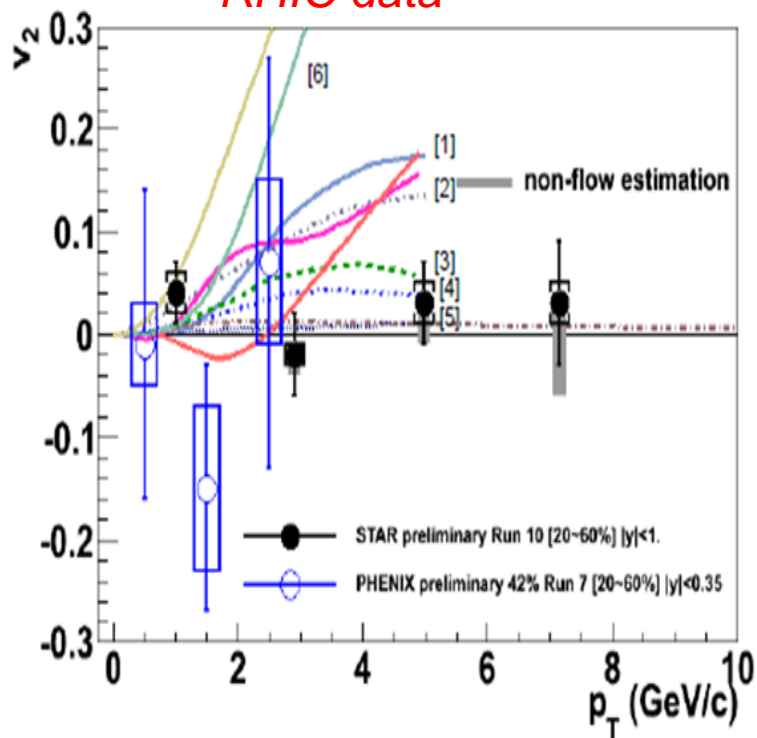
Elliptic Flow at RHIC and LHC

model

*5.5TeV central Pb+Pb,
Liu,Xu,Zhuang, NPA(2009)*



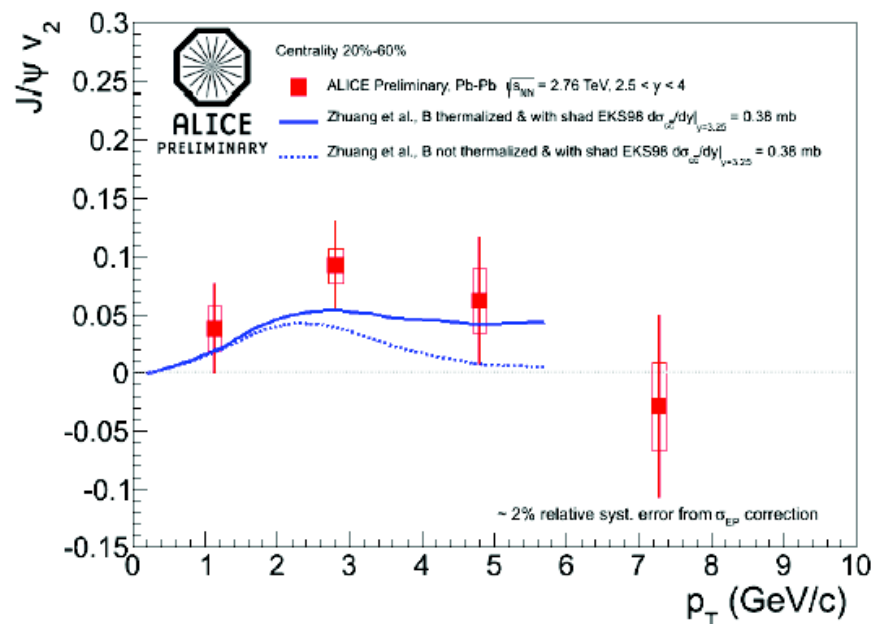
RHIC data



STAR, QM2011
PHENIX, QM2009

- [1] V. Greco, C.M. Ko, R. Rapp,
- [2] L. Ravagli, R. Rapp, PLB 65:
- [3] L. Yan, P. Zhuang, N. Xu, PR
- [4] X. Zhao, R. Rapp, 24th WWI
- [5] Y. Liu, N. Xu, P. Zhuang, Nuc
- [6] U. Heinz, C. Shen, private co

LHC data

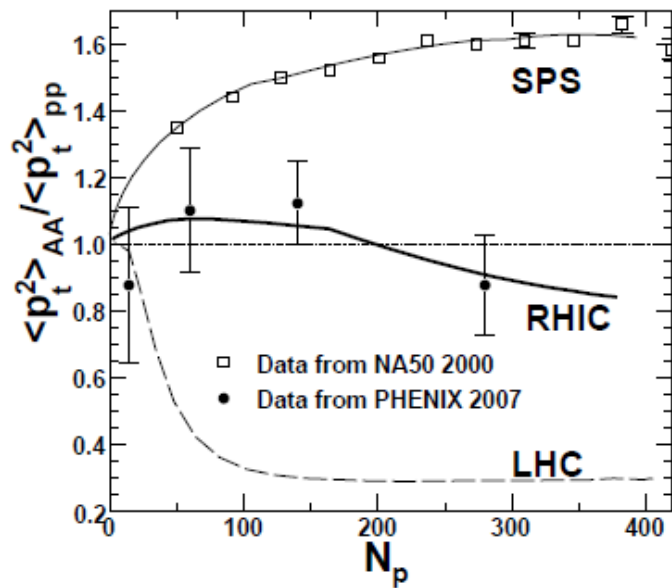


Averaged Transverse Momentum at SPS, RHIC and LHC

model

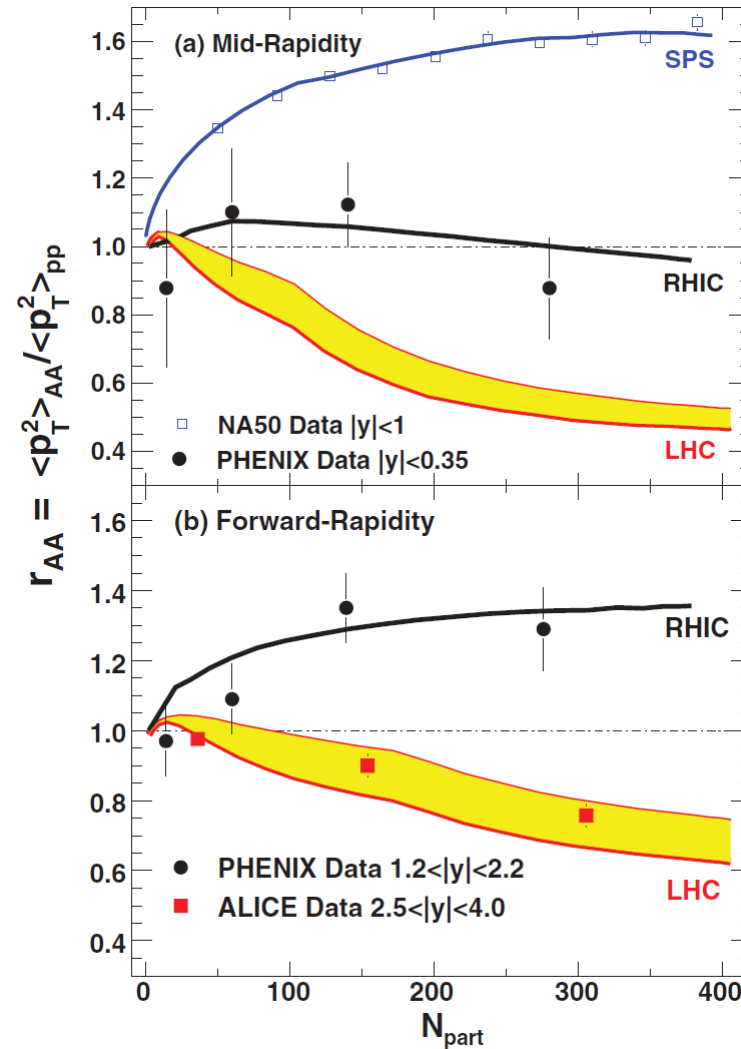
$$r_{AA} = \frac{\langle p_t^2 \rangle_{AA}}{\langle p_t^2 \rangle_{pp}}$$

$r_{AA} \begin{cases} > 1 & \text{SPS} \\ \approx 1 & \text{RHIC} \\ < 1 & \text{LHC} \end{cases}$ at midrapidity



5.5TeV central Pb+Pb, Zhou, Xu, Zhuang, NPA(2009)

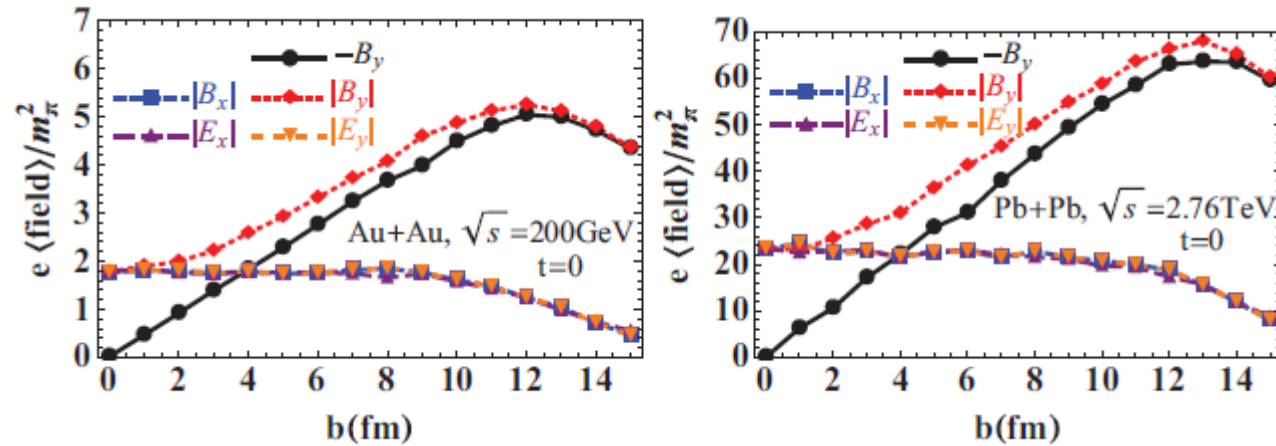
LHC data



Zhou, Xu, Xu, Zhuang, PRC(2014)

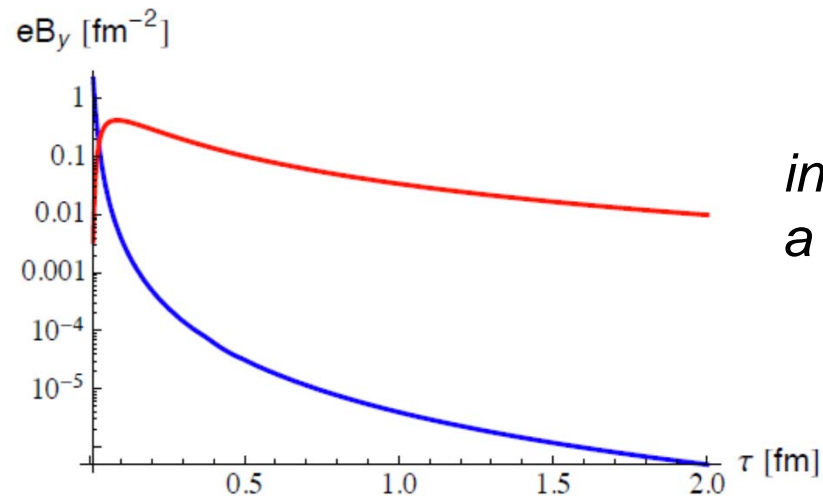
Strong Magnetic Field in A+A Collisions

Deng and Huang, 2012:



strongest magnetic field in nature but lasts ~ 0.1 fm/c

Gursoy, Kharzeev, and Rajagopal, 2014:



inducting medium may lead to a long life time !

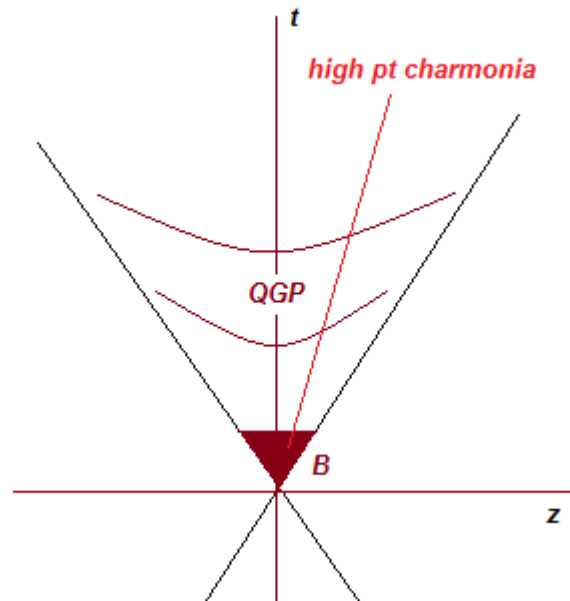
Charmonia as a Probe of the B Field

Guo, Shi, Xu, Xu, Zhuang, 2015

ideal probes of the B field:

- *created in the early stage,*
- *sensitive to the B field,*
- *not affected by the later hot medium.*

high p_t charmonium is such an ideal probe !



Idea:

during the evolution of a $c\bar{c}$ state $|c\bar{c}\rangle$,

$$i \frac{\partial}{\partial t} |c\bar{c}\rangle(\vec{x}) = \hat{H}(B(t)) |c\bar{c}\rangle(\vec{x})$$

the B field leads to an anisotropic production probability

$$| \langle \Psi | c\bar{c} \rangle(\vec{x}) |^2$$

→ anisotropic charmonium formation !

Time-dependent Schroedinger Equation

$$1) i \frac{\partial}{\partial t} \Phi(t) = \hat{H} \Phi(t), \quad \hat{H} = \frac{(\vec{p}_c - q_c \vec{A}_c)^2}{2m_c} + \frac{(\vec{p}_{\bar{c}} - q_{\bar{c}} \vec{A}_{\bar{c}})^2}{2m_c} - \frac{(q_c \vec{s}_c + q_{\bar{c}} \vec{s}_{\bar{c}}) \cdot \vec{B}}{m_c} + V_{c\bar{c}}(r)$$

$$V_{c\bar{c}}(r) = -\frac{\alpha}{r} + \sigma r + \beta e^{-\gamma r} \vec{s}_c \cdot \vec{s}_{\bar{c}}$$

parameters are determined in vacuum, [see Alford, Strickland, 2013](#).

$$2) \quad \vec{R} = \frac{\vec{r}_c + \vec{r}_{\bar{c}}}{2}, \quad \vec{r} = \vec{r}_c - \vec{r}_{\bar{c}}, \quad \vec{P} = \vec{p}_c + \vec{p}_{\bar{c}}, \quad \vec{p} = \frac{\vec{p}_c - \vec{p}_{\bar{c}}}{2},$$

kinetic momentum $\vec{P}_k = \vec{P} - q_c \vec{A}_c - q_{\bar{c}} \vec{A}_{\bar{c}}$, conserved momentum $\vec{P}_{ps} = \vec{P} + q_c \vec{A}_c + q_{\bar{c}} \vec{A}_{\bar{c}}$,

$$\hat{H} = \hat{H}_0 + \hat{H}_B, \quad \hat{H}_B = -\frac{(q_c \vec{s}_c + q_{\bar{c}} \vec{s}_{\bar{c}}) \cdot \vec{B}}{m_c} - \frac{q_c}{2m_c} (\vec{P}_{ps} \times \vec{B}) \cdot \vec{r} + \frac{q^2}{4m_c} (\vec{B} \times \vec{r})^2$$

3) expanding Φ in terms of the charmonium states:

$$\Phi(\vec{P}_{ps}, \vec{R}, \vec{r}, t) = \frac{1}{\sqrt{2\pi}} e^{i\left(\vec{P}_k \cdot \vec{R} - \frac{\vec{P}_{ps} t}{4m_c}\right)} \sum_{\psi} C_{\psi}(\vec{P}_{ps}, t) e^{-iE_{\psi} t} \psi(\vec{r}),$$

$$\hat{H}_0 \psi(\vec{r}) = E_{\psi} \psi(\vec{r})$$

$$\frac{d}{dt} C_{\psi}(\vec{P}_{ps}, t) = \sum_{\psi'} e^{i(E_{\psi} - E_{\psi'}) t} C_{\psi'}(\vec{P}_{ps}, t) \int d^3 \vec{r} \psi^*(\vec{r}) \hat{H}_B \psi'(\vec{r})$$

the probability for the $c\bar{c}$ to be in the charmonium state $|\psi\rangle$ is $|C_{\psi}(\vec{P}_{ps}, t)|^2$

Initial Condition

magnetic field in heavy ion collisions:

$$B = \begin{cases} B\vec{e}_y, & 0 < t < t_B \text{ and } \frac{x^2}{(R_A - b/2)^2} + \frac{y^2}{(b/2)^2} + \frac{\gamma^2 z^2}{(b/2)^2} < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$b = 8 \text{ fm}, \quad R_A = 6.6 \text{ fm}, \\ \text{LHC:} \quad 25m_\pi, \quad 0.2 \text{ fm}, \quad 1400$$

initial wave function determined by $p + p$ collisions:

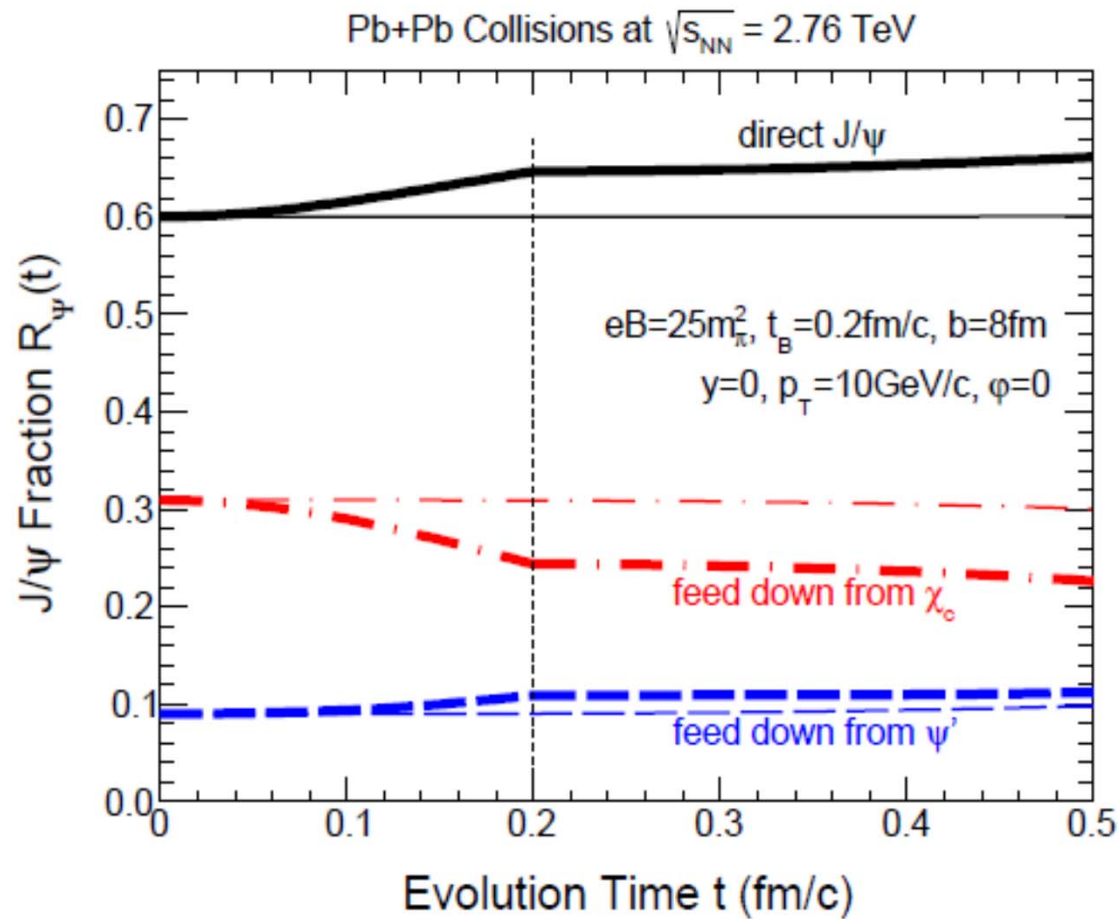
$$\Phi(\vec{r}, t = 0) = (2\pi\sigma^2)^{-\frac{3}{4}} e^{-\frac{(\vec{r}-\vec{r}_0)^2}{4\sigma^2}} = \sum_\psi C_\psi \psi(\vec{r}), \\ \vec{r}_0 = r_0(\sin\theta_0 \cos\varphi_0, \sin\theta_0 \sin\varphi_0, \cos\theta_0),$$

r_0 and σ are determined by the feedback from ψ' and χ_c in vacuum:

$$R(\chi_c) = 30\% \quad \text{and} \quad R(\psi') = 10\% \quad (\text{LHCb})$$

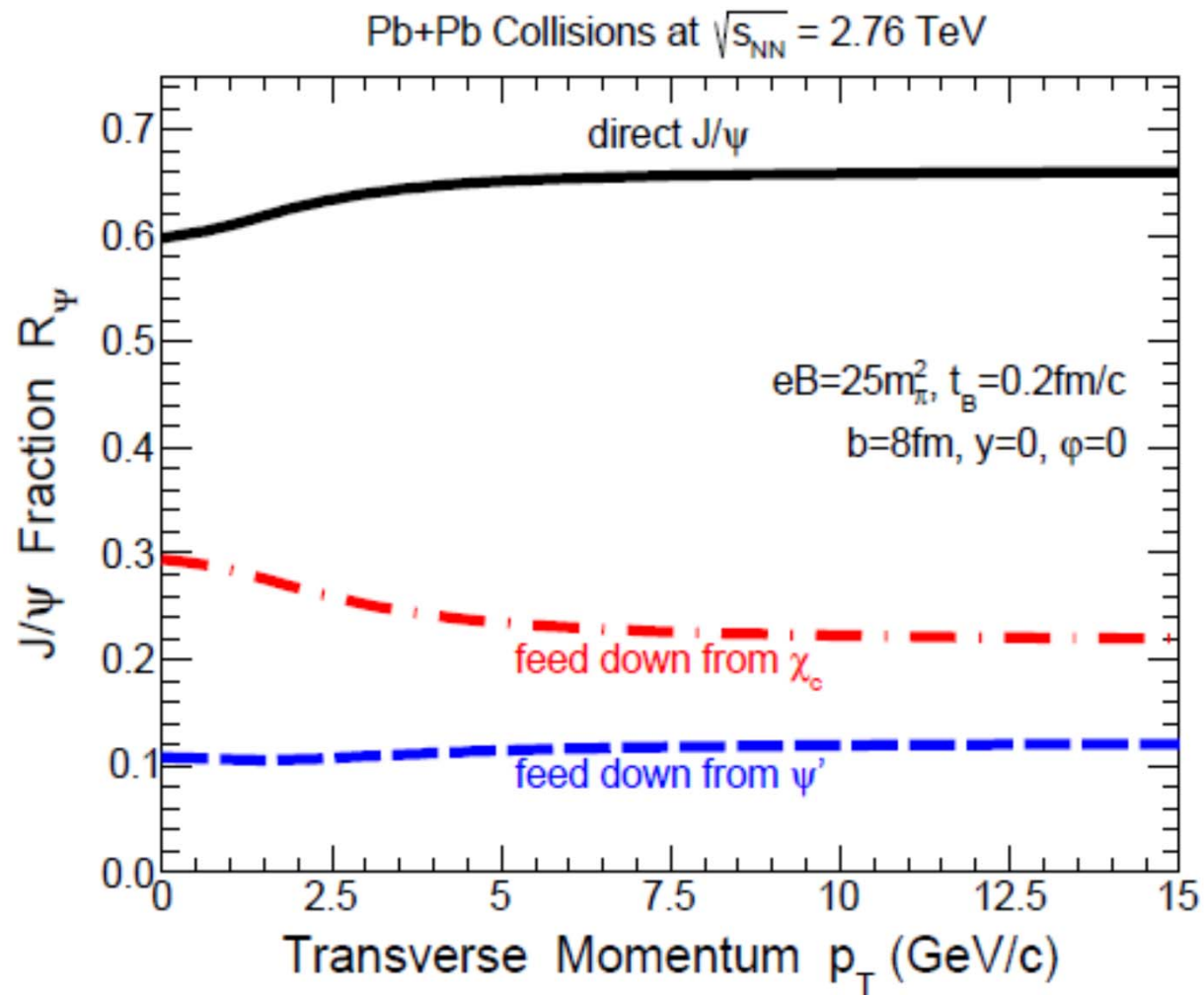
Time Evolution of the Feedback from ψ' and χ_c

$$R_{\Psi}(t) = \frac{|C_{\Psi}(t)|^2 \mathcal{B}(\Psi \rightarrow J/\psi)}{\sum_{\Psi} |C_{\Psi}(t)|^2 \mathcal{B}(\Psi \rightarrow J/\psi)}$$



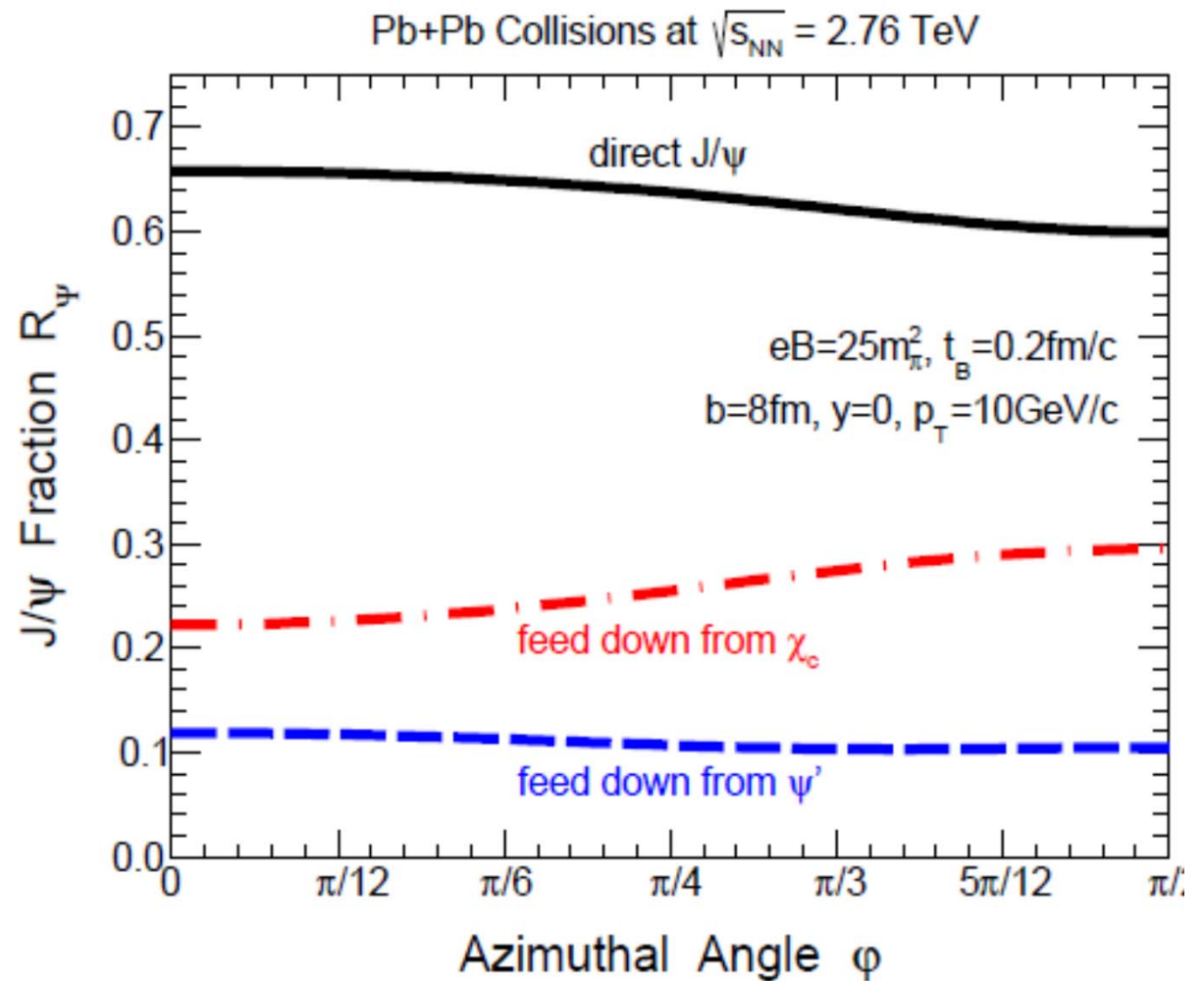
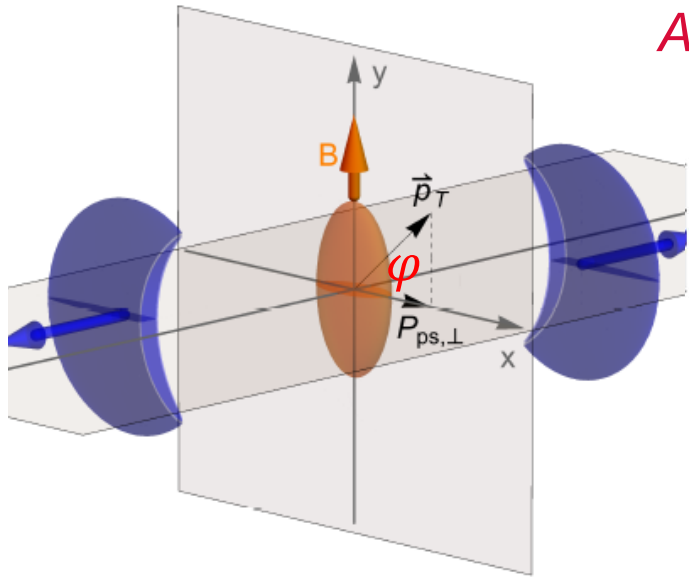
ψ' and J/ ψ enhancement by 10% and χ_c suppression by 23% .

P_t dependence



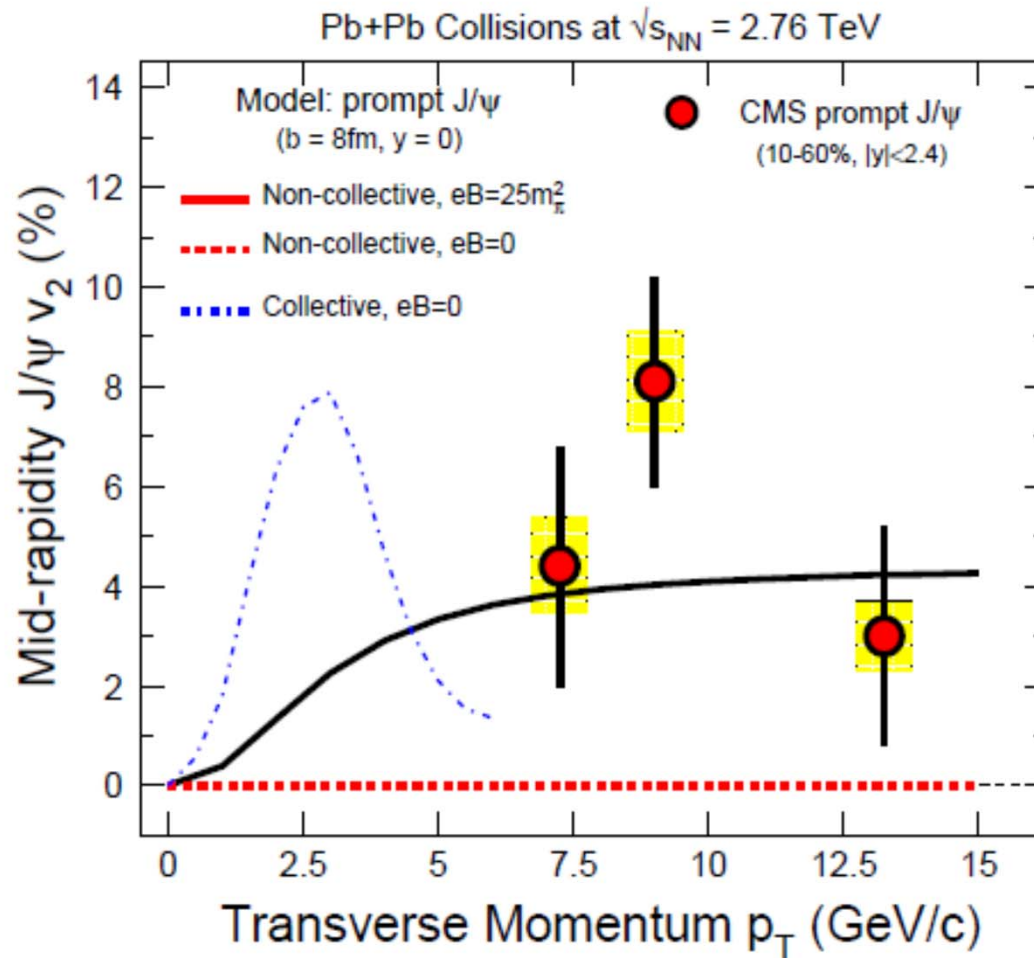
strong enhancement or suppression at high P_t , due to the Lorentz force.

Anisotropic Formation



- 1) *anisotropic charmonium formation in the B field.*
- 2) *strong enhancement or suppression at $\varphi = 0$, but almost no change at $\varphi = \pi/2$.*

Non-collective v_2 at High P_t



Non-collective v_2 at high P_t , created by the B field

Ω_{ccc} He, Liu, Zhuang, PLB(2015)

Ω_{ccc} is the ground bound state of 3 charm quarks,
it is not yet found experimentally.

1) Ω_{ccc} production in a p+p collision needs at least 3 pairs of $c\bar{c}$, the production cross section is small even at LHC energy.

see for instance J.D.Bjorken 1986 and Y.Chen, 2011.

2) However, *coalescence among uncorrelated charm quarks in A+A collisions* leads to a large production cross section,

$$N(\Omega_{ccc}) \sim N_c^3$$

it may become most probable to discover Ω_{ccc} in A+A !

Ω_{ccc} , a Clean Signal of QGP

Current signals of QGP:

*jet quenching,
J/ψ suppression,
strangeness enhancement,
electromagnetic probes,*

However, they are produced in both p+p and A+A, the signal is only the quantitative difference between p+p and A+A.

If Ω_{ccc} is observed, it is a clean signal of QGP, since it can not be produced in p+p at the same energy.

Ω_{ccc} , Computable Coalescence Probability

The coalescence probability for light hadrons is usually assumed to be a Gaussing distribution, and the width is taken as the particle's radius.

However, the calculation for heavy quarks becomes theoretically solid. The Wigner function (the coalescence probability) for Ω_{ccc} can be calculated via Schroedinger equation.

3-body Schroedinger Equation

$$\hat{H}\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = E_T\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$

$$\hat{H} = \sum_{i=1}^3 \frac{\hat{\mathbf{P}}_i^2}{2m_c} + V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sum_{i<j} V_{cc}(\mathbf{r}_i, \mathbf{r}_j). \quad V_{cc} = V_{c\bar{c}}/2. \quad V_{c\bar{c}}(\mathbf{r}_i, \mathbf{r}_j) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma|\mathbf{r}_{ij}|,$$

- 1) α, σ, m_c can be determined by fitting the charmonium spectra,
- 2) coalescence happens at T_c , assuming $V(T_c) = V(0)$

1) $\vec{r}_1, \vec{r}_2, \vec{r}_3 \rightarrow \vec{R}$ (baryon coordinate), \vec{r}_x, \vec{r}_y (relative coordinates)

$$\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) \rightarrow \Phi(\vec{R})\psi(\vec{r}_x, \vec{r}_y)$$

2) $\vec{r}_x, \vec{r}_y \rightarrow r, \theta_x, \phi_x, \theta_y, \phi_y, \alpha = \arctg \frac{r_y}{r_x}$ (hyperspherical coordinates)

Problem:

since $V(|\vec{r}_i - \vec{r}_j|)$ depends on the 5 angles, one can not separate the relative motion into a radial part and an angular part.

Hyperspherical Symmetry

E.Nielsen et al., Phys. Rep. 347, 373(2001), I.narodetskii et al., JETP Lett. 90, 232(2009)

Taking the averaged potential

$$v(r) = \frac{8}{\pi} \int_0^{\pi/2} \sum_{i < j} V_{cc}(\sqrt{2}r \sin \alpha) \sin^2 \alpha \cos^2 \alpha d\alpha$$

to replace $V(|\vec{r}_i - \vec{r}_j|)$, one separates the relative motion into a radius equation

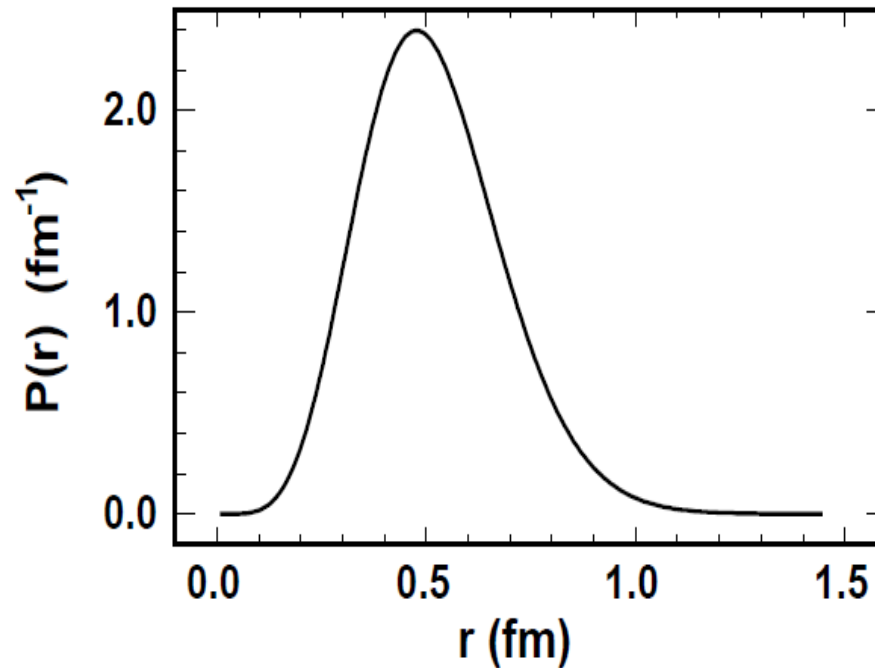
$$\left[\frac{1}{2m_c} \left(-\frac{d^2}{dr^2} - \frac{5}{r} \frac{d}{dr} \right) + v(r) \right] \varphi(r) = E\varphi(r)$$

and an angular equation.

Wave Function

$$m_{\Omega} = 4.75 \text{ GeV} \text{ (4.8 GeV from LQCD)} \quad \epsilon_{\Omega} = 900 \text{ MeV}$$

$$P(r) = |\varphi(r)|^2 r^5$$

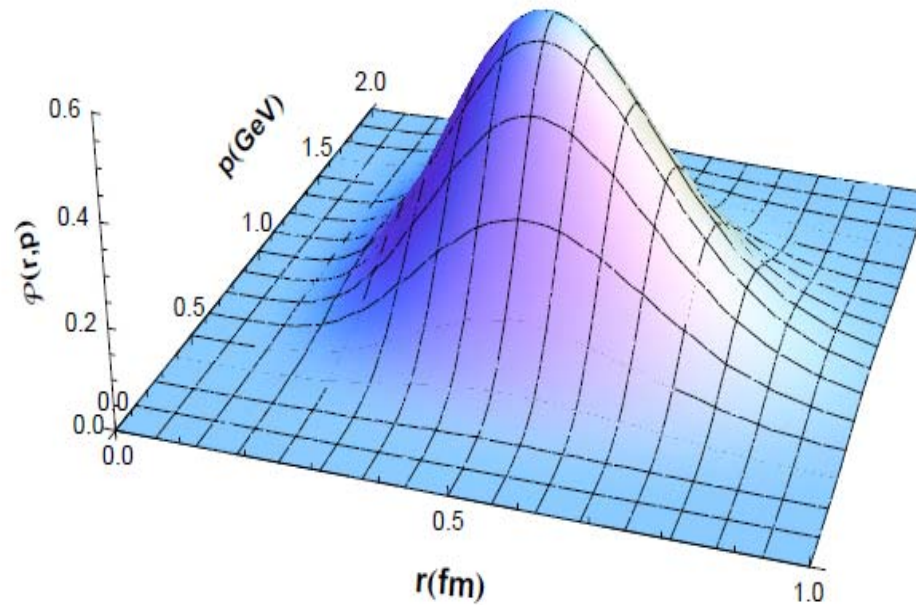


$$\langle r_{\Omega} \rangle = 0.5 \text{ fm} \simeq \langle r_{J/\psi} \rangle$$

Wigner Function

$$W(\mathbf{r}, \mathbf{p}) = \int d^6\mathbf{y} e^{-i\mathbf{p}\cdot\mathbf{y}} \psi\left(\mathbf{r} + \frac{\mathbf{y}}{2}\right) \psi^*\left(\mathbf{r} - \frac{\mathbf{y}}{2}\right)$$

$$\mathcal{P}(r, p) = \frac{1}{24\pi} r^5 p^5 \int_0^\pi W(r, p, \theta) \sin^4 \theta d\theta$$



Coalescence

$$\frac{dN}{d^3\mathbf{P}} = C \int \frac{d^3\mathbf{R}}{(2\pi)^3} \int \frac{d^3\mathbf{r}_x d^3\mathbf{r}_y d^3\mathbf{p}_x d^3\mathbf{p}_y}{(2\pi)^6} f(r_1, p_1) f(r_2, p_2) f(r_3, p_3) W(\mathbf{r}_x, \mathbf{r}_y, \mathbf{p}_x, \mathbf{p}_y)$$

1) Coalescence happens on the hadronization hypersurface Σ determined by

$$T(R_\mu) = T_c$$

$$\frac{dN}{d^2\mathbf{P}_T d\eta} = C \int_{\Sigma} \frac{P^\mu d\sigma_\mu(R)}{(2\pi)^3} \int \frac{d^4r_x d^4r_y d^4p_x d^4p_y}{(2\pi)^6} f(r_1, p_1) f(r_2, p_2) f(r_3, p_3) V(\mathbf{r}_x, \mathbf{r}_y, \mathbf{p}_x, \mathbf{p}_y)$$

$$d\sigma_0 = \left(R_T \frac{\partial\tau}{\partial\eta} \sinh\eta + R_T \tau \cosh\eta \right) dR_T d\phi d\eta,$$

$$d\sigma_1 = \left(\tau \frac{\partial\tau}{\partial\phi} \sin\phi - R_T \tau \frac{\partial\tau}{\partial R_T} \cos\phi \right) dR_T d\phi d\eta,$$

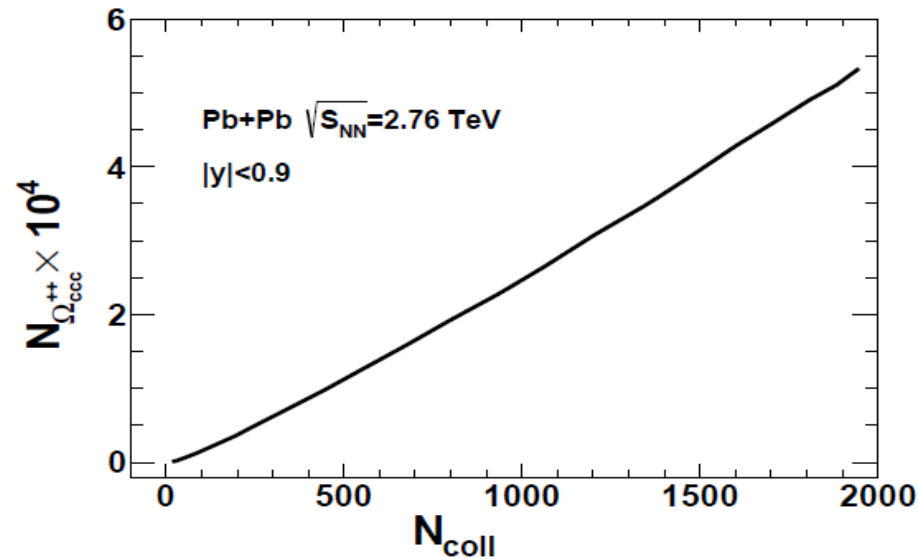
$$d\sigma_2 = - \left(\tau \frac{\partial\tau}{\partial\phi} \cos\phi + R_T \tau \frac{\partial\tau}{\partial R_T} \sin\phi \right) dR_T d\phi d\eta$$

$$d\sigma_3 = - \left(R_T \frac{\partial\tau}{\partial\eta} \cosh\eta + R_T \tau \sinh\eta \right) dR_T d\phi d\eta.$$

2) Charm quark (thermal) distribution $f(\vec{r}, \vec{p}) = \frac{1}{e^{p^\mu u_\mu/T} + 1}$

3) Local T and u_μ are from $\partial_\mu T^{\mu\nu} = 0$ + QCD EoS

Numerical Results at LHC



Effective cross section per binary collision:

$$\sigma_{\Omega} \equiv \frac{N_{\Omega}}{N_{coll} \Delta\eta} \sigma_{pp} \quad \sigma_{pp} = 62 \text{ mb} \rightarrow \sigma_{\Omega} = 9 \text{ nb}$$

1) In comparison with $p+p$ (Bjorken 1986, Chen 2011):

$$\sigma_{\Omega} = 0.06-0.13 \text{ nb at } 7 \text{ TeV}$$

$$0.1-0.2 \text{ nb at } 14 \text{ TeV}$$

2) In comparison with J/ψ and Υ in $A+A$:

$$\sigma_{J/\psi} = 1900 \text{ nb} \quad \text{and} \quad \sigma_{\Upsilon} = 3.4 \text{ nb}$$

Conclusions

1) Quarkonium TMD, especially v_2 and $r_{AA} = \frac{\langle p_t^2 \rangle_{AA}}{\langle p_t^2 \rangle_{pp}}$, can distinguish hot mediums between SPS, RHIC and LHC: from p_t broadening to p_t suppression.

2) It is most probable to discover Ω_{ccc} in heavy ion collisions at LHC, and the discovery is a clean signature of the quark-gluon plasma formation.

3) The anisotropic charmonium production at high p_t is a signal of the initially created magnetic field.

We need more precise study on quarkonia at RHIC and LHC!

I thank my colleagues and students !



Baoyi Chen



Xingyu Guo



Hang He



Yunpeng Liu



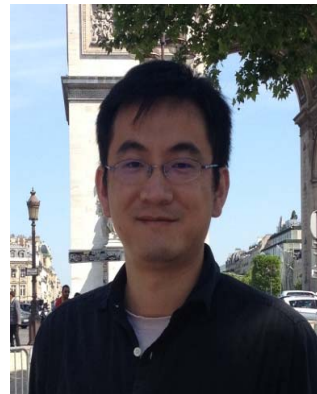
Shuzhe Shi



Nu Xu



Zhe Xu



Li Yan



Kai Zhou

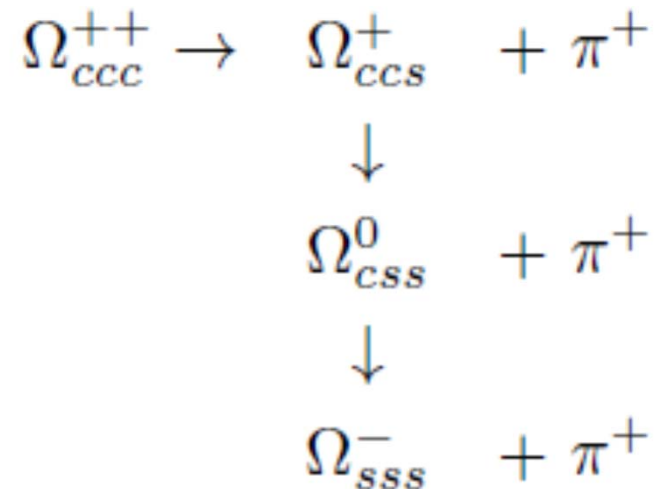


Xianglei Zhu

decay modes

Decay through weak interaction, for instance

nonleptonic cascade decay mode (Chen 2011):

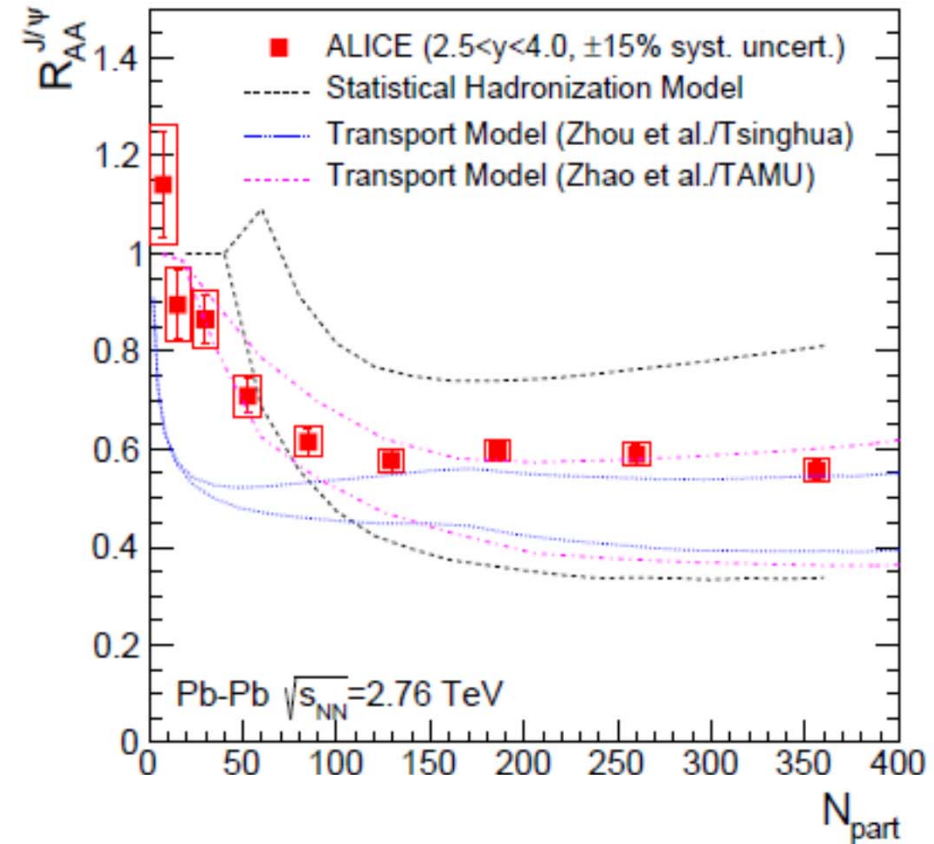
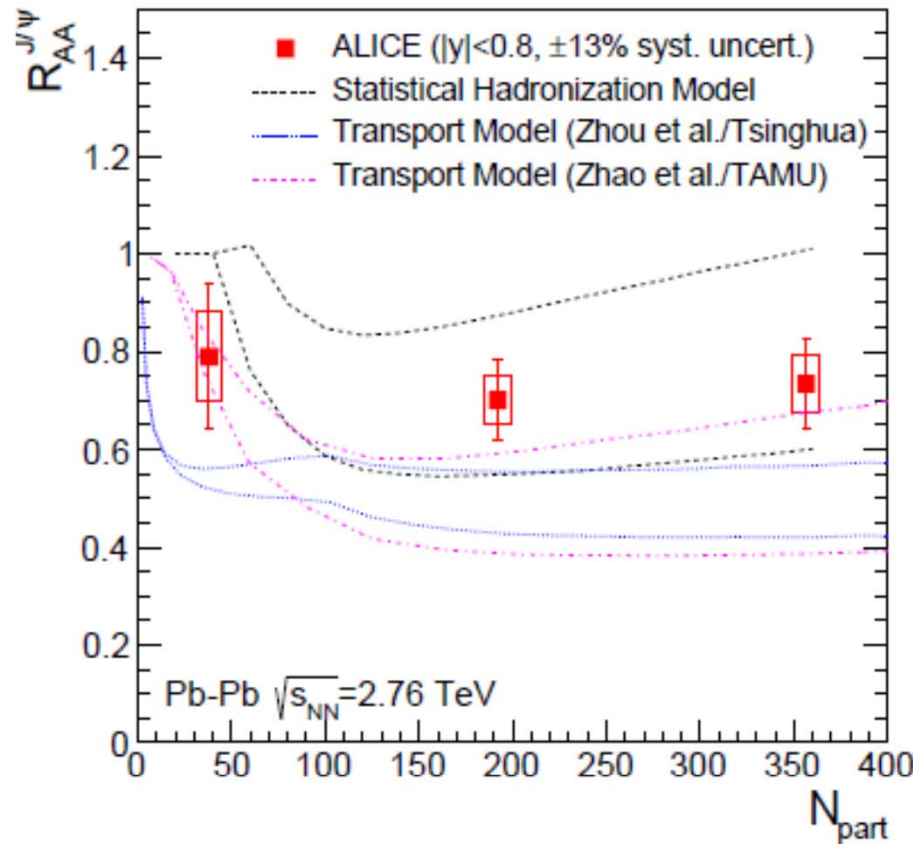


semileptonic decay mode (Bjorken, 1986):



midrapidity

forward rapidity



Both model categories reproduce the data ... $d\sigma_{c\bar{c}}/dy$ values rather different:

midrapidity: Stat. Hadr.: 0.3-0.4 mb

Transport: 0.5-0.75 mb (TAMU), 0.65-0.8 mb (Tsinghua)

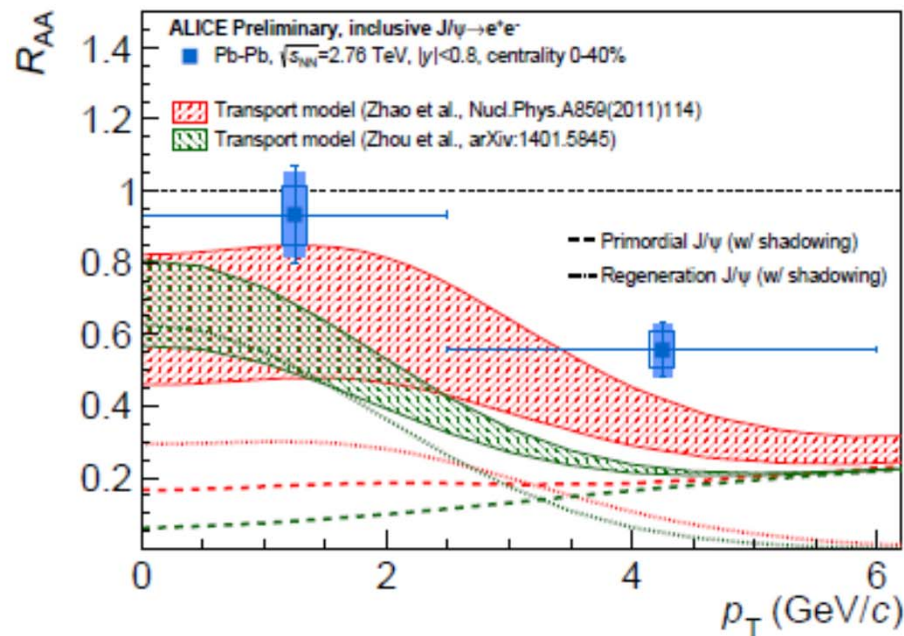
J/ψ vs. p_T - data and models

from Andronic, QM2014

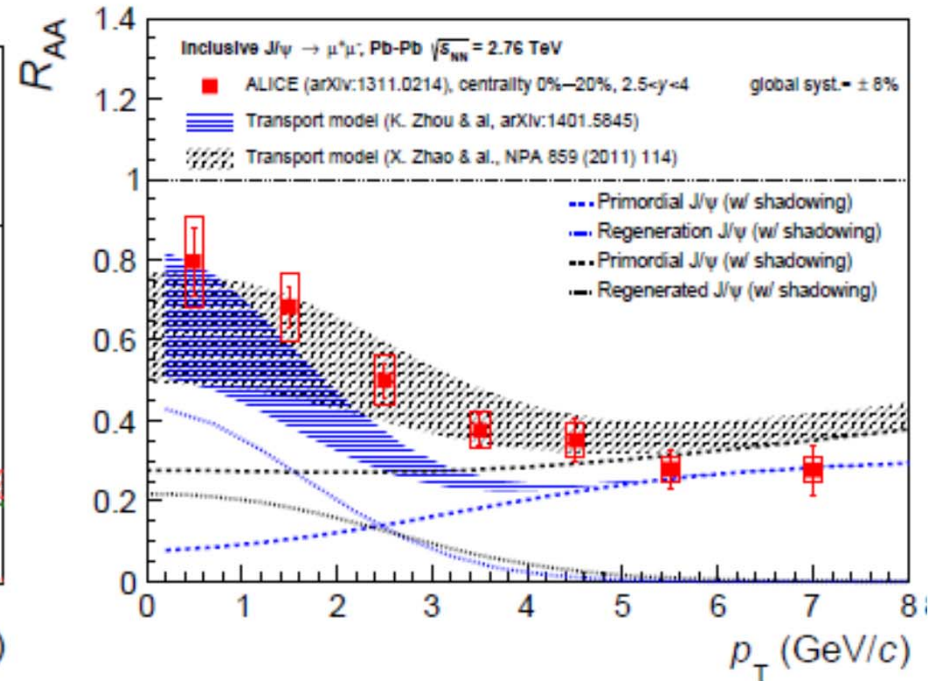
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A.Andronic@GSI.de

midrapidity

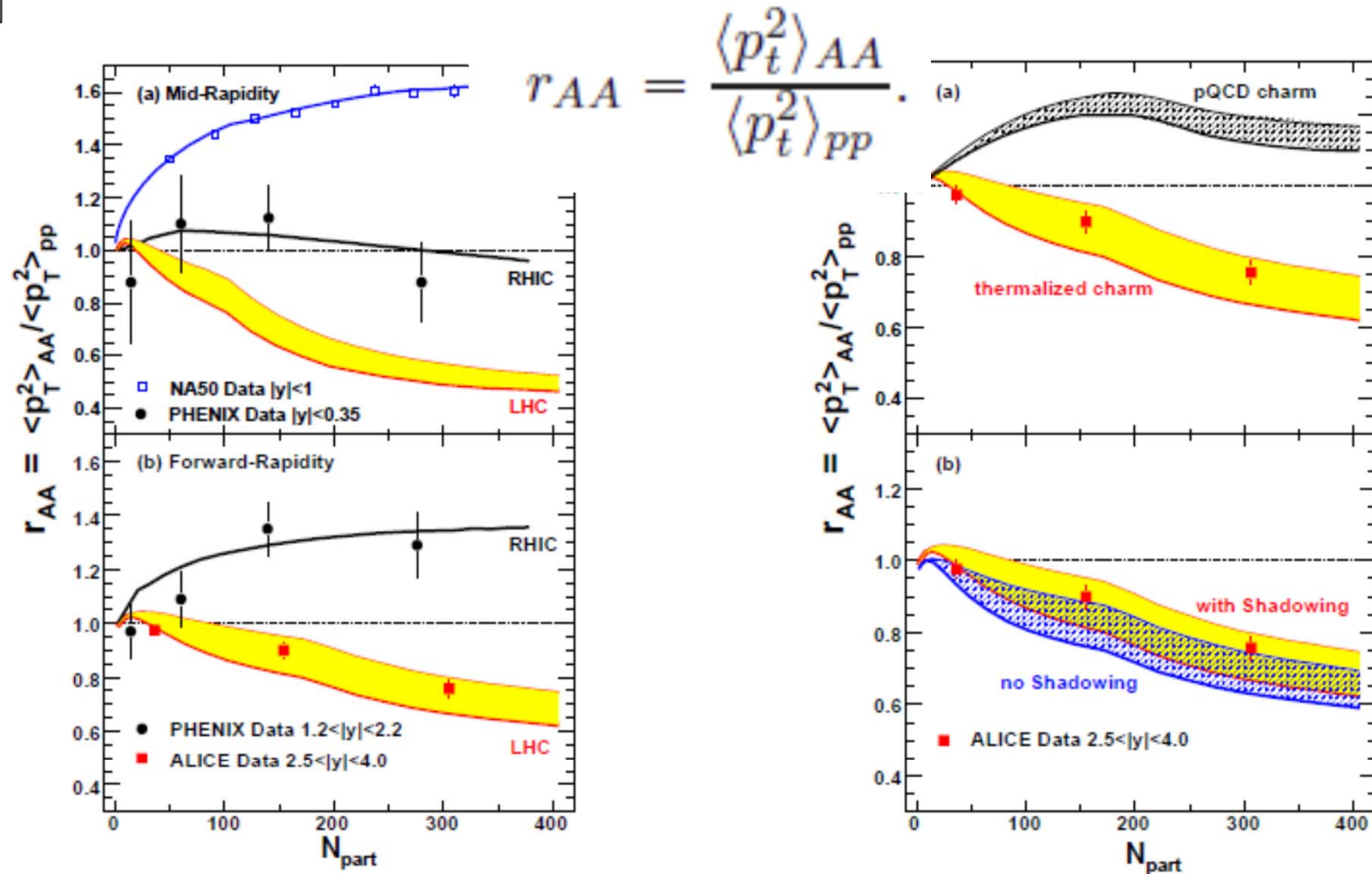


forward rapidity



(re)generation models describe the LHC data well ...with a healthy fraction of J/ψ newly produced

ALICE, arXiv:1311.0214 (& prelim., Book, HF 4)



Zhou et al., arXiv:1401.5845 the transport model reproduce the data very well
 ...requires thermalized charm quarks ("sanity check")

Heavy Quark Distribution

assuming thermalized charm quark distribution:

$$f_c(x, q) \sim \frac{1}{e^{q^\mu u_\mu/T} + 1}$$

$u_\mu(x)$ and $T(x)$ determined by hydrodynamics

equation for charm quark density

$$\partial_\mu (f_c u^\mu) = R_{gain} - R_{loss}$$

$$f_c(\tau_0) = \frac{T_A(\vec{x})T_B(\vec{x}-\vec{b})\cosh(\eta)}{\tau_0} \frac{d\sigma_{pp}^{c\bar{c}}}{d\eta}$$

$$R_{gain} = R_{gg \rightarrow c\bar{c}} + R_{q\bar{q} \rightarrow c\bar{c}}$$

(Nason, Dawson, Ellis, 1988)

R_{loss} : determined by detailed balance with R_{gain}

Zhou, Chen, Zhuang, 2015

