Isospin Matter

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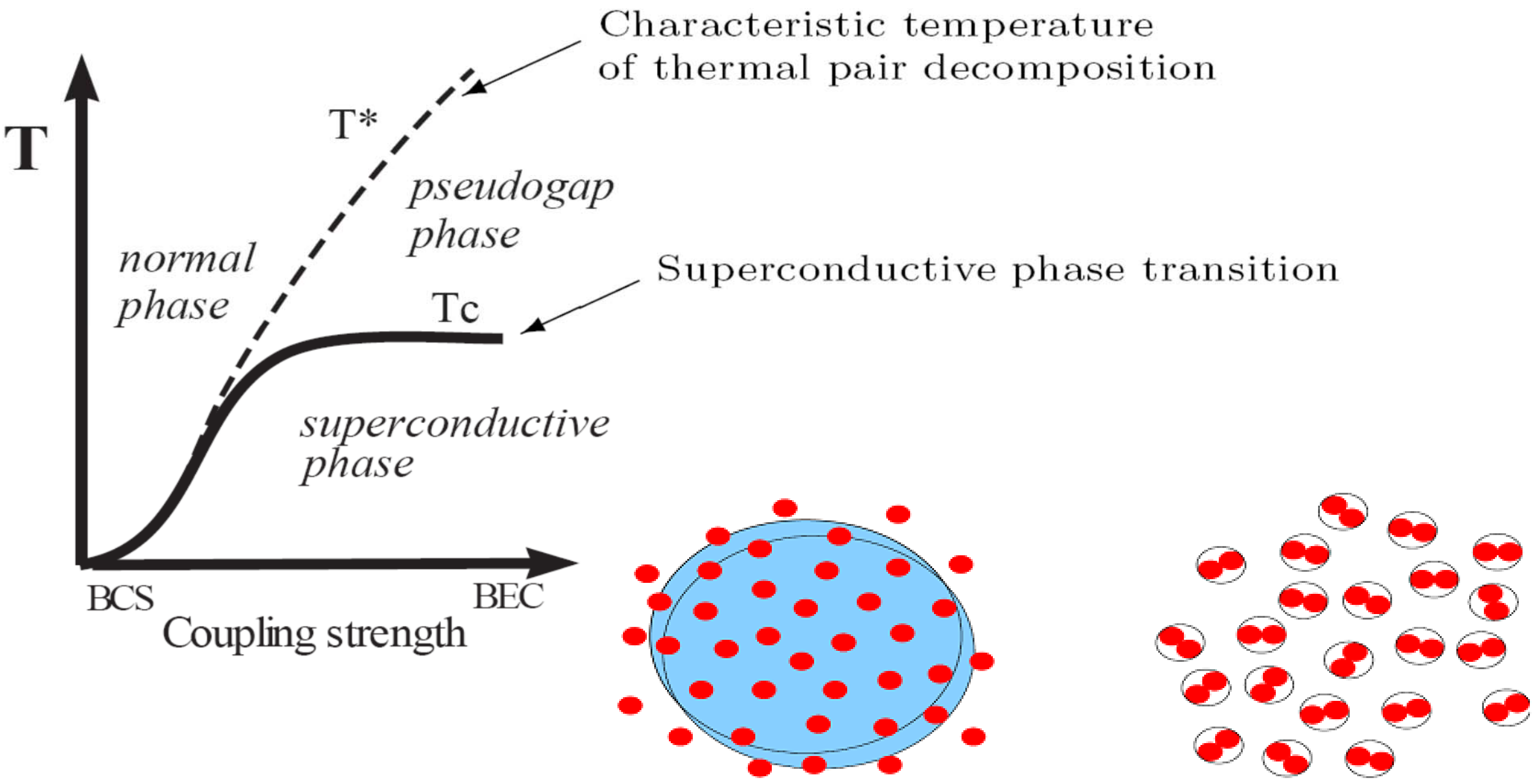
1) QCD Condensed Matter
2) Phase Diagram in Mean Field
3) BCS-BEC beyond the Mean Field
4) Fluctuations with Renormalization Group

reference: BCS-BEC Crossover in Relativistic Fermi Systems
by Lianyi He, Shijun Mao and Pengfei Zhuang
Int. J. Mod. Phys. A28(2013), 133054 (85 pages)

Pairing



$T^* = T_c$

$T^* >> T_c$

BEC-BCS crossover,
a way to understand the confinement-deconfinement phase transition?

Pairing in QCD

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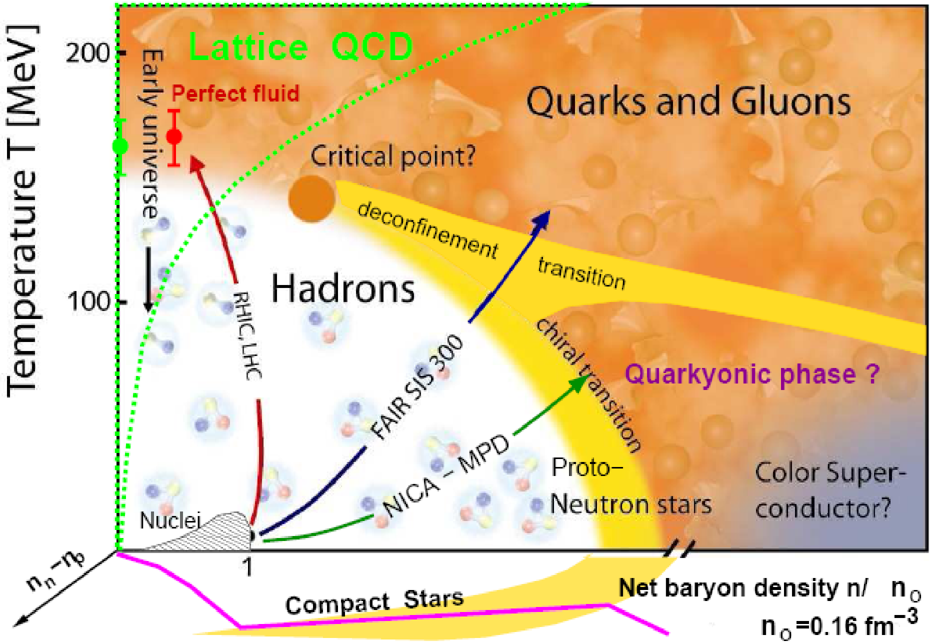
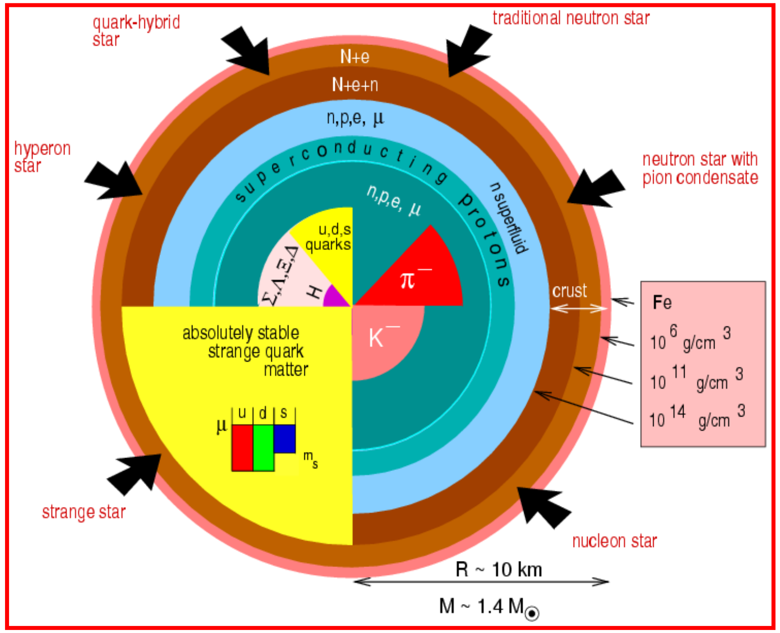




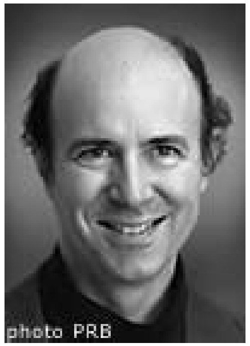
new phenomena in BCS-BEC crossover of QCD:

relativistic systems, anti-fermion contribution, rich inner structure (color, flavor), medium dependent mass, ……

色超导的特点

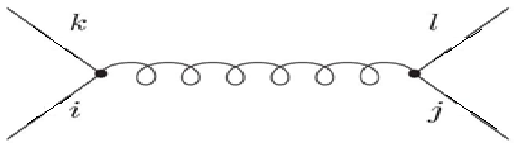
QED Condensed Matter: 由BCS理论，两个电子通过交换光子是排斥相互作用，吸引相互作用是通过交换集体激发模式－声子来实现的, 所以超导是*low temperature superconductivity*。

QCD Condensed Matter:



Frank Wilczek

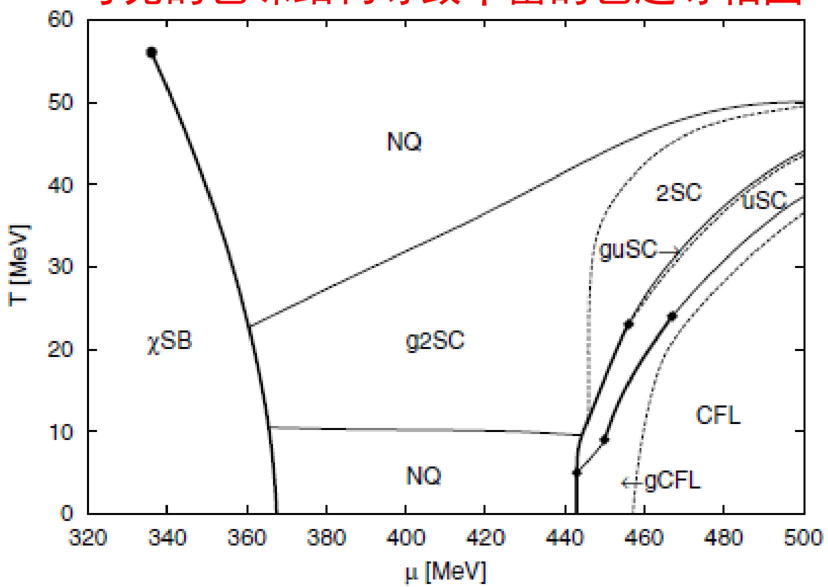
$$\mathscr{L}_{QCD} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m_q)\psi + g\bar{\psi}A_{\mu}^{a}T_{a}\gamma^{\mu}\psi - \frac{1}{4}F_{a}^{\mu\nu}F_{\mu\nu}^{a}$$



$$\sim (T_a)_{ki}(T_a)_{lj} = -\frac{N_c+1}{4N_c}\left(\delta_{jk}\delta_{il} - \delta_{ik}\delta_{jl}\right) + \frac{N_c-1}{4N_c}\left(\delta_{jk}\delta_{il} + \delta_{ik}\delta_{jl}\right)$$

第一项在交换初态或末态的两个夸克色指标时是反对称的，是吸引相互作用。表明在单胶子交换的层次就使得两个夸克可以配对，形成色超导，所以超导是*high temperature superconductivity* 。

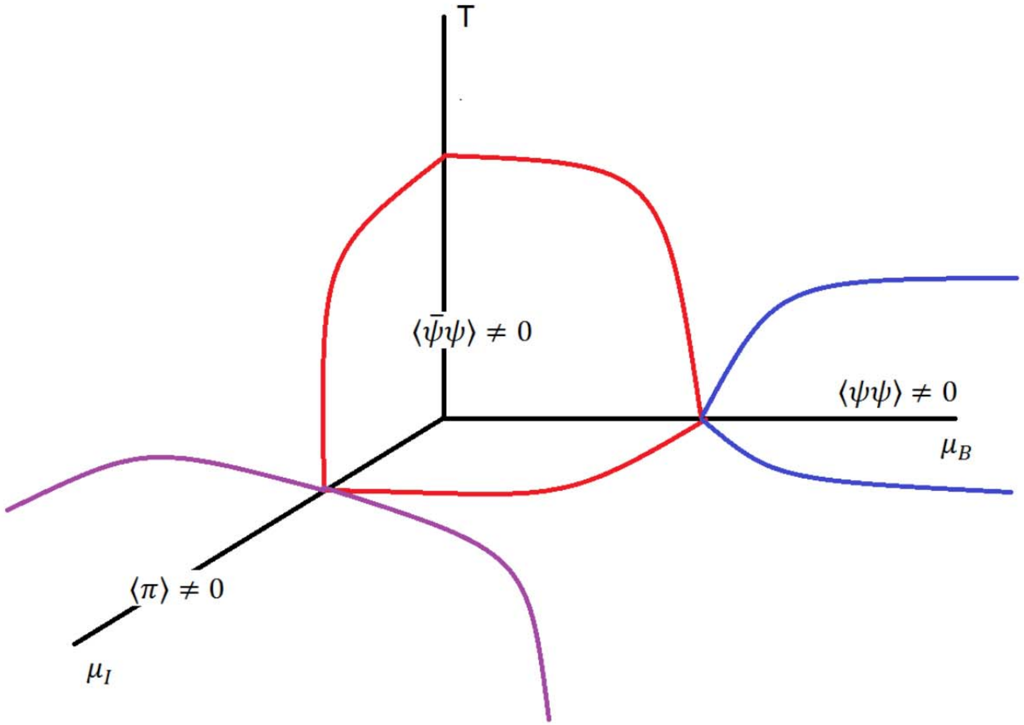
夸克的色味结构导致丰富的色超导相图



Ruster, Werth, Buballa, Shovkovy, Rischke，PRD72，034004(2005) *condensate* $\langle \bar{q} q \rangle \rightarrow$ *chiral symmetry breaking*

condensate $\langle q q \rangle \rightarrow$ *color symmetry breaking*

condensate $\langle \pi \rangle \rightarrow$ *isospin symmetry breaking*



EFT at Finite Density

- at finite T, vacuum excitation, reliable lattice QCD simulations

  $T_c = 155\ MeV$ H.Ding et al.

- at finite μ_B and μ_I, vacuum condensation, color superconductor and pion superfluid, not yet precise lattice result at finite baryon density, we have to consider effective models.

  lattice calculation at finite isospin density:

  J.B.Kogut and D.K.Sinclair, Phys. Rev. D66 (2002)034505.
  J.B.Kogut and D.K.Sinclair, Phys. Rev. D70 (2004)094501.

- Nambu-Jona-Lasinio (NJL) model inspired by the BCS theory is expected to well describe the chiral, color and pion condensates.

Y. Nambu

$$L_{NJL} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0\right)\psi + G\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\tau_i\gamma_5\psi\right)^2\right)$$

Y.Nambu and G.Jona-Lasinio, Phys. Rev. 122, 345(1961) and 124, 246(1961)
U.Vogl and W.Weise, Prog. Part. Nucl. Phys. 27, 195(1991)
S.P.Klevansky, Rev. Mod. Phys. 64, 649(1992)
M.K.Volkov, Phys. Part. Nucl. 24, 35(1993)
T.Hatsuda and T.Kunihiro, Phys. Rep. 247, 221(1994)
M.Buballa, Phys. Rep. 407, 205(2005)

Quarks in Mean Field

NJL at finite isospin density

$$L_{NJL} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma_0\right)\psi + G\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\tau_i\gamma_5\psi\right)^2\right)$$

quark chemical potentials

$$\mu = \begin{pmatrix} \mu_u & 0 \\ 0 & \mu_d \end{pmatrix} = \begin{pmatrix} \mu_B/3 + \mu_I/2 & 0 \\ 0 & \mu_B/3 - \mu_I/2 \end{pmatrix}$$

chiral and pion condensates with finite pair momentum

$$\sigma = \langle\bar{\psi}\psi\rangle = \sigma_u + \sigma_d, \quad \sigma_u = \langle\bar{u}u\rangle, \quad \sigma_d = \langle\bar{d}d\rangle$$

$$\pi_+ = \sqrt{2}\langle\bar{u}i\gamma_5 d\rangle = \frac{\pi}{\sqrt{2}}e^{2i\vec{q}\cdot\vec{x}}, \quad \pi_- = \sqrt{2}\langle\bar{d}i\gamma_5 u\rangle = \frac{\pi}{\sqrt{2}}e^{-2i\vec{q}\cdot\vec{x}}$$

quark propagator in MF

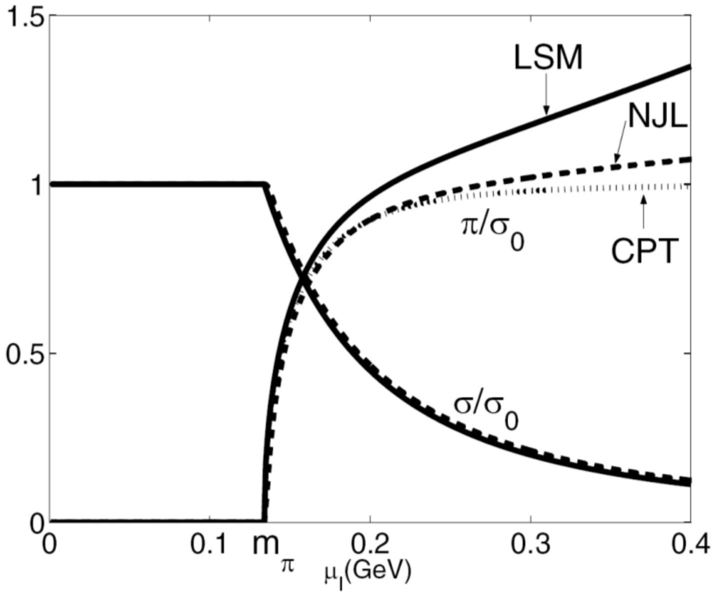
$$S^{-1}(p,\vec{q}) = \begin{pmatrix} \gamma^\mu p_\mu - \vec{\gamma}\cdot\vec{q} + \mu_u\gamma_0 - m & 2iG\pi\gamma_5 \\ 2iG\pi\gamma_5 & \gamma^\mu k_\mu + \vec{\gamma}\cdot\vec{q} + \mu_d\gamma_0 - m \end{pmatrix} \qquad m = m_0 - 2G\sigma$$

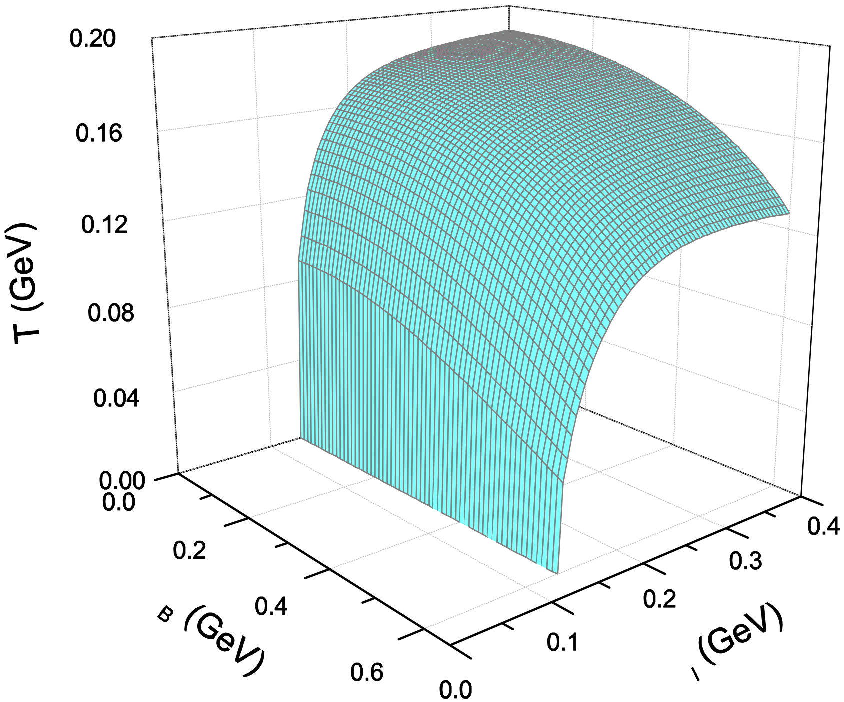
thermodynamic potential and gap equations:

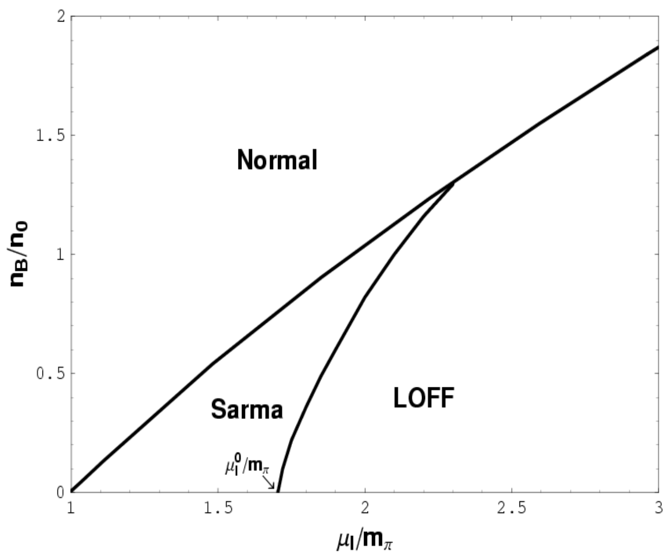
$$\Omega = G(\sigma^2 + \pi^2) - \frac{T}{V}\mathrm{Tr}\,\mathrm{Ln}\,S^{-1}$$

$$\frac{\partial\Omega}{\partial\sigma_u} = 0, \ \frac{\partial^2\Omega}{\partial\sigma_u^2} \geq 0, \qquad \frac{\partial\Omega}{\partial\sigma_d} = 0, \ \frac{\partial^2\Omega}{\partial\sigma_d^2} \geq 0, \qquad \frac{\partial\Omega}{\partial\pi} = 0, \ \frac{\partial^2\Omega}{\partial\pi^2} \geq 0, \qquad \frac{\partial\Omega}{\partial q} = 0, \ \frac{\partial^2\Omega}{\partial q^2} \geq 0$$

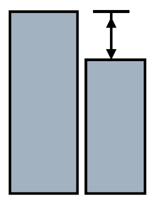
Phase Diagram of Pion Superfluid







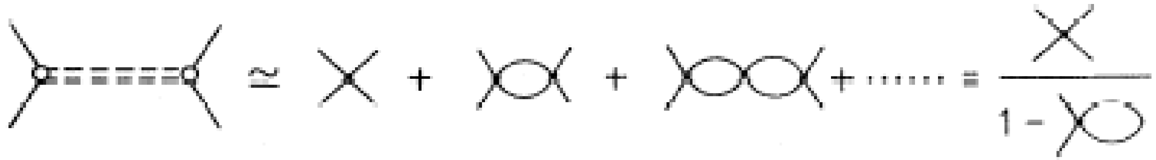
1) μ_I controls the Fermi surface of pion superfluid, but μ_B governs the Fermi surface mismatch



2) homogeneous ($\vec{q}=0$) and inhomogeneous ($\vec{q}\neq 0$) pion superfluid

Quantum Fluctuations (Mesons) in RPA

meson propagator at RPA



considering all possible channels in the bubble summation

meson polarization functions

$$\Pi_{mn}(k)=i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left(\Gamma_m^* S(p+k)\Gamma_n S(p)\right) \qquad \Gamma_m=\begin{cases}1, & m=\sigma\\ i\tau_+\gamma_5, & m=\pi_+\\ i\tau_-\gamma_5, & m=\pi_-\\ i\tau_3\gamma_5, & m=\pi_0\end{cases}$$

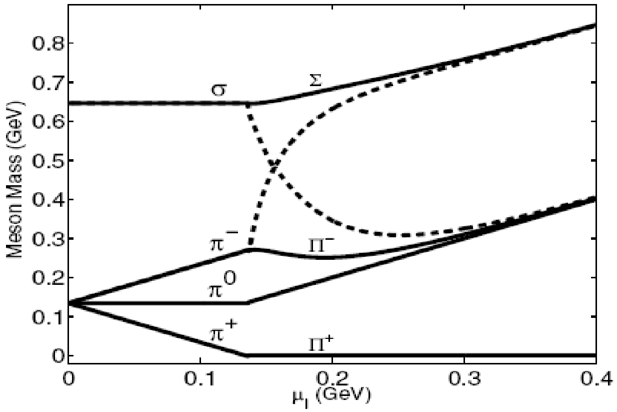
pole of the propagator determines meson masses M_m

$$\det\begin{pmatrix}1-2G\Pi_{\sigma\sigma}(k) & -2G\Pi_{\sigma\pi_+}(k) & -2G\Pi_{\sigma\pi_-}(k) & -2G\Pi_{\sigma\pi_0}(k)\\ -2G\Pi_{\pi_+\sigma}(k) & 1-2G\Pi_{\pi_+\pi_+}(k) & -2G\Pi_{\pi_+\pi_-}(k) & -2G\Pi_{\pi_+\pi_0}(k)\\ -2G\Pi_{\pi_-\sigma}(k) & -2G\Pi_{\pi_-\pi_+}(k) & 1-2G\Pi_{\pi_-\pi_-}(k) & -2G\Pi_{\pi_-\pi_0}(k)\\ -2G\Pi_{\pi_0\sigma}(k) & -2G\Pi_{\pi_0\pi_+}(k) & -2G\Pi_{\pi_0\pi_-}(k) & 1-2G\Pi_{\pi_0\pi_0}(k)\end{pmatrix}_{k_0=M_m,\vec{k}=0}=0$$

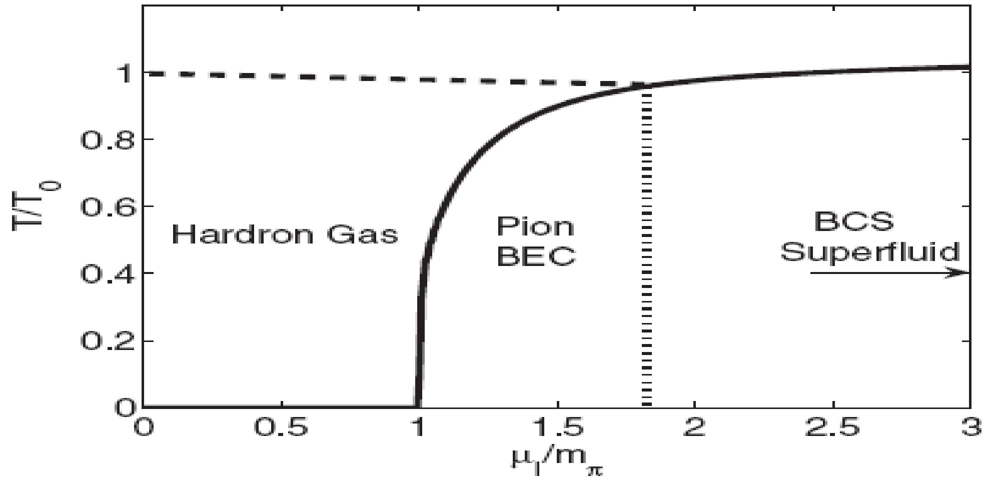
σ,π_+,π_- *are no longer the eigen modes of the system in pion superfluid phase, the new eigen modes* $\bar{\sigma},\bar{\pi}_+,\bar{\pi}_-$ *are linear combinations of* σ,π_+,π_-

BCS-BEC Crossover of Pion Superfluid

meson mass, Goldstone mode

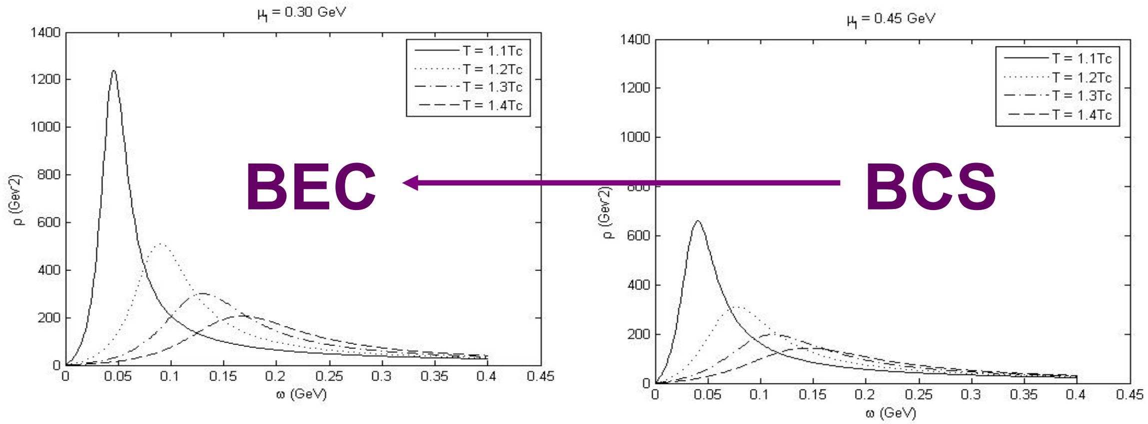


phase diagram

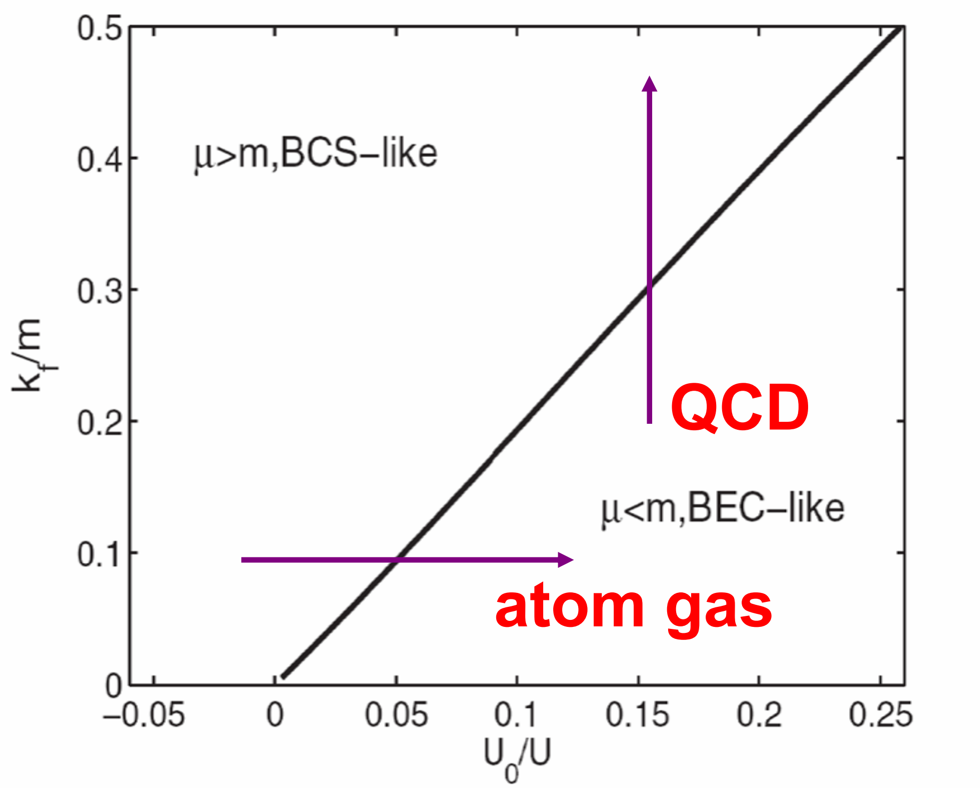


meson spectra function

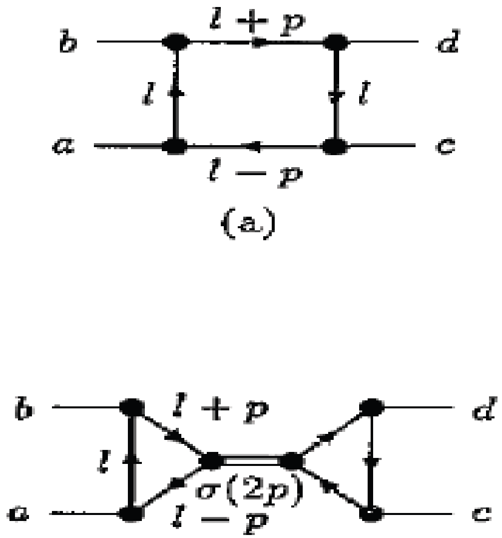
$$\rho(\omega,\vec{k}) = -2\,\mathrm{Im}\,D(\omega,\vec{k})$$

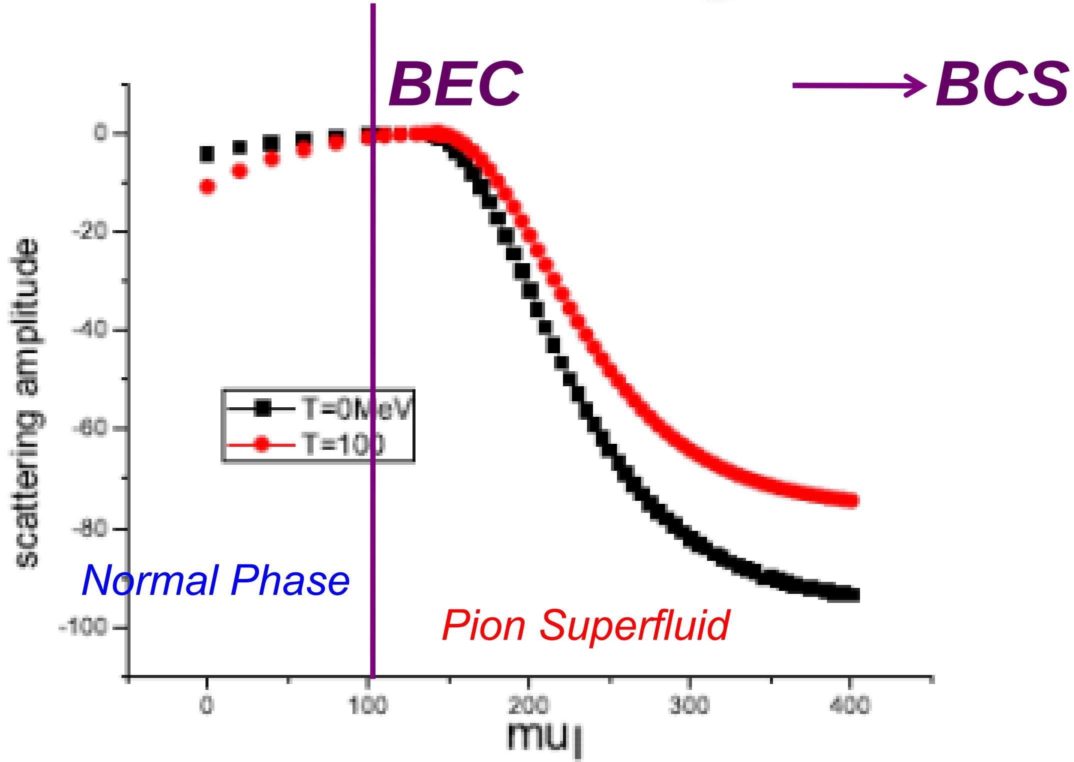


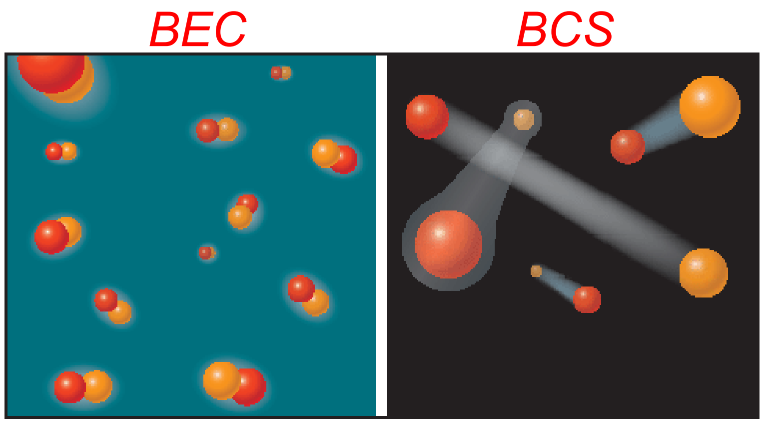
In non-relativistic case, the BCS-BEC crossover is induced only by changing the coupling.
In relativistic case, however, the BCS-BEC crossover can be induced by changing either the coupling or the density.



π – π scattering and BCS-BEC Crossover







BCS: overlapped molecules, large π – π cross section
BEC: identified molecules, ideal Boson gas limit

Yukawa Potential

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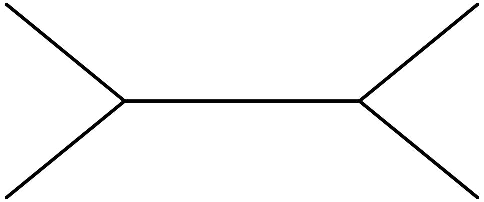
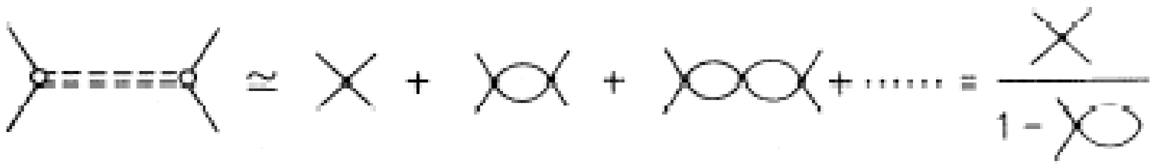
nucleon potential (1934-1935)



$$V(r) \sim \frac{e^{-Mr}}{r}$$

quark potential



$$V(r) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} TrD(0,\vec{k}^2)$$

Applications in Nuclear Matter:

- *Friedel oscillations in nuclear matter (Alonso, et al., 1989,1994)*
- *Complex poles and oscillatory potential (Sivak et al., 2001)*
- *N-N potential including rho meson exchange (Mornas et al., 2001)*
- *Effect of meson width on Yukawa potential (Flambaum, Shuryak, 2007)*

Vacuum, Matter and Pole Contributions

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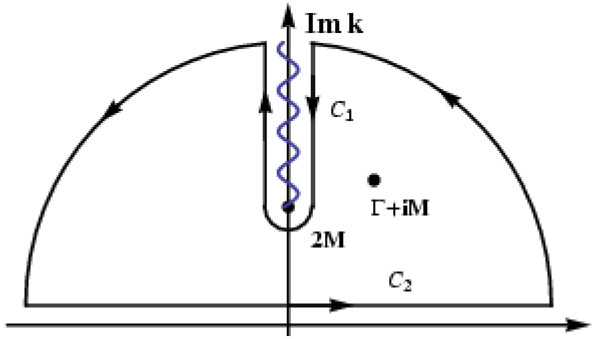


$$V_m(r) = V_m^1(r) + V_m^2(r) + V_m^3(r).$$

vacuum part:

$$V_m^1(r) = \frac{G}{2\pi^2 r}\int_{2M}^{\infty} dk k e^{-kr}\,\mathrm{Im}\left[\frac{1}{1-2G\Pi_m(0,(ik+\epsilon)^2)} - \frac{1}{1-2G\Pi_m(0,(ik-\epsilon)^2)}\right]$$

matter part:

$$V_m^2(r) = -\frac{G}{2\pi^2 r}\mathrm{Im}\int_{-\infty}^{\infty} dk k e^{ikr}\left[\frac{1}{1-2G\Pi_m(0,k^2)} - \frac{1}{1-2G\Pi_m(0,(k+i\epsilon)^2)}\right]$$

pole part:

$$V_m^3(r) = \frac{1}{2\pi}\frac{e^{-M_m r}}{r}\frac{a_m\cos(\Gamma_m r) + b_m\sin(\Gamma_m r)}{a_m^2 + b_m^2}$$

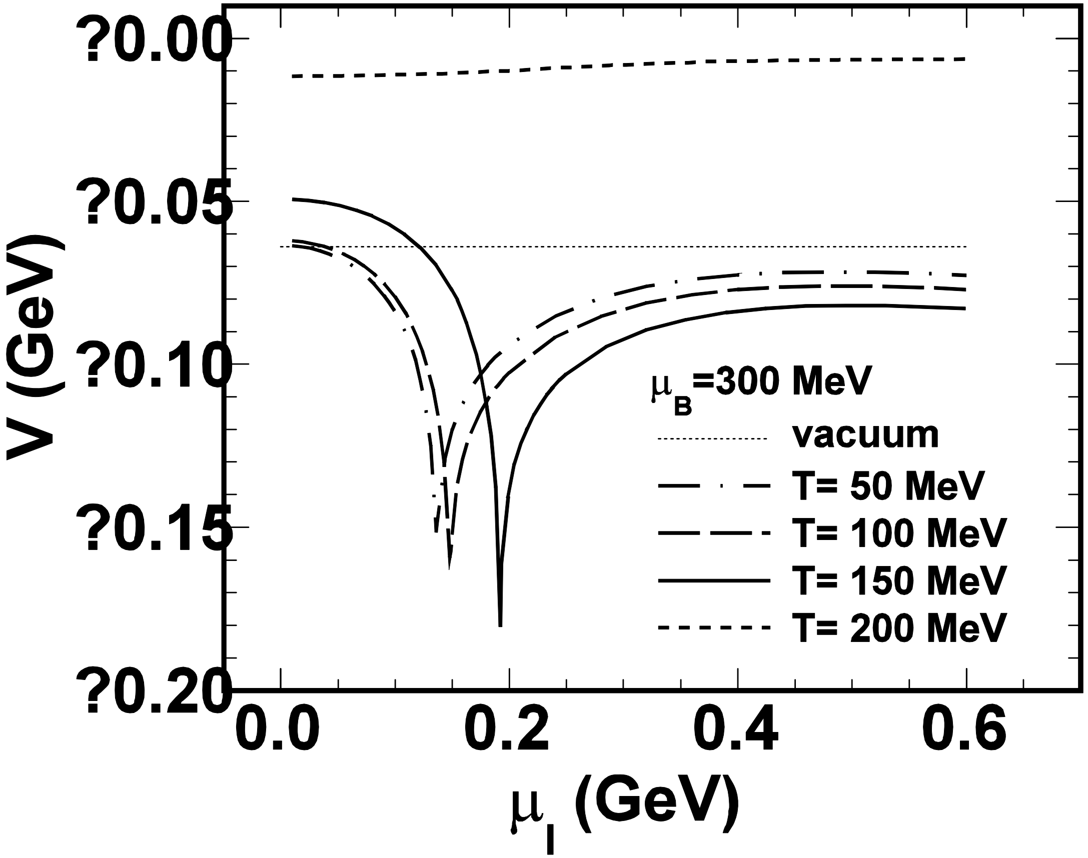
screening mass and width:

$$1 - 2G\Pi_m(0,(iM_m+\Gamma_m)^2) = 0, \qquad \left.\frac{\partial\Pi_m(0,k^2)}{\partial k^2}\right|_{k=\Gamma_m+iM_m} = a_m + ib_m.$$

limit of the Yukawa part:

$$V_m^3(r) = \frac{1}{2\pi}\frac{e^{-M_m r}}{a_m r}.$$

Quark Potential in Pion Superfluid

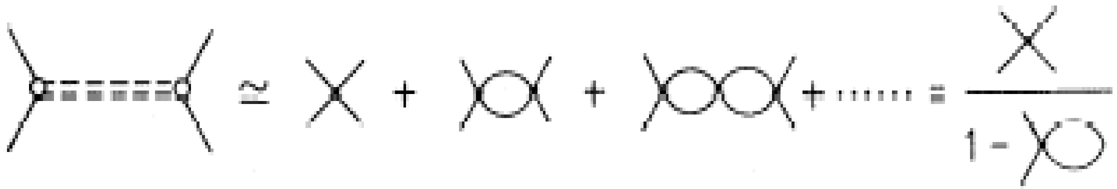


1) the maximum potential is located at the phase boundary .

2) the potential in pion superfluid is non-zero at extremely high isospin density.

Quantum Fluctuations above Mean Field

- loop summation
  - ♣ mean field (classical) approximation
  - ♣ Gaussing fluctuations
  - ♣ loop summation (hard thermal loop resummation, hard dense loop resummation, RPA, DSE, ……



- lattice QCD
- renormalization group
  - ♣ model independent critical phenomena
  - ♣ symmetry based non-perturbative treatment

momentum scale: k

action including quantum fluctuations in the region $[k, \infty]$:

Γ_k , $\int_0^\infty dp \rightarrow \int_k^\infty dp \sim \int_0^\infty R_k(p)dp$

regulator R_k to suppress the low momentum fluctuations

flow equation (J.Berges, H.Gies, D.F.Litim, N.Tetradis, C.Wetterich, …):

$$\partial_k \Gamma_k = \frac{1}{2} Tr\left[\left(\Gamma_k^2 + R_k\right)^{-1} \partial_k R_k\right]$$

$$\Gamma_k{}^2 = \frac{\delta^2 \Gamma}{\delta \varphi^2}, \quad R_k = (k^2 - p^2)\theta(k^2 - p^2)$$

$\lim_{k\rightarrow\infty} \Gamma_k$ (classical action) $\xrightarrow{flow\ equation}$ $\lim_{k\rightarrow 0} \Gamma_k$ (full action)

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$$\mathcal{L} = Tr\left(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi\right) - \lambda_1 \left(Tr \Phi^\dagger \Phi\right)^2 - \lambda_2 Tr(\Phi^\dagger \Phi)^2 + \underline{c\left(\det \Phi + \det \Phi^\dagger\right)} + \underline{Tr(H(\Phi + \Phi^\dagger))}$$

explicit $U_A(1)$ breaking

explicit chiral breaking

$$\Phi = T^a \Phi^a = T^a(\sigma^a + i\pi^a), \quad a = 0, \dots, 8$$

Gell-Mann matrices: T_a

scalar fields σ_a: $\sigma, f_0, a_0^0,\ a_0^\pm, \bar{\kappa}^0, \kappa^0, \kappa^\pm$

pseudoscalar fields π_a: $\eta, \eta', \pi^0, \pi^\pm, \bar{K}^0, K^0, K^\pm$

mass and coupling constants: $m^2, c, \lambda_1, \lambda_2$

Mean Field (classical) Approximation

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- *field shifts:* $\phi \to \langle\phi\rangle + \phi, \quad \mathcal{L}(\phi) \to \mathcal{L}(\phi, \langle\phi\rangle)$
- *classical potential:*

$$U(\bar\sigma) = \frac{m^2}{2}\bar\sigma_a^2 - G_{abc}\bar\sigma_a\bar\sigma_b\bar\sigma_c + \frac{1}{3}F_{abcd}\bar\sigma_a\bar\sigma_b\bar\sigma_c\bar\sigma_d - h_a\bar\sigma_a$$

meson masses generated by the condensates:

$$(M_S^2)_{ab} = m^2\delta_{ab} - 6G_{abc}\bar\sigma_c + 4F_{abcd}\bar\sigma_c\bar\sigma_d,$$
$$(M_P^2)_{ab} = m^2\delta_{ab} + 6G_{abc}\bar\sigma_c + 4H_{abcd}\bar\sigma_c\bar\sigma_d$$

$$G_{abc} = \frac{c}{6}\left[d_{abc} + \frac{9}{2}d_{000}\delta_{a0}\delta_{b0}\delta_{c0} - \frac{3}{2}\left(\delta_{a0}d_{0bc} + \delta_{b0}d_{a0c} + \delta_{c0}d_{ab0}\right)\right]$$
$$F_{abcd} = \frac{\lambda_1}{4}\left(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}\right) + \frac{\lambda_2}{8}\left(d_{abe}d_{ecd} + d_{ade}d_{ebc} + d_{ace}d_{ebd}\right)$$
$$H_{abcd} = \frac{\lambda_1}{4}\delta_{ab}\delta_{cd} + \frac{\lambda_2}{8}\left(d_{abe}d_{ecd} + f_{ade}d_{ebc} + f_{ace}d_{ebd}\right)$$

gap equations to determine the condensates: $\partial U/\partial\langle\phi\rangle = 0$

parameters are fixed by meson masses and PCAC in vacuum.

- ***however, Goldstone mechanism is broken at low T in mean field !***

quantum averaged action:

$$\Gamma_k[\langle\phi\rangle] = \int d^4x \left[\mathrm{Tr}\left(Z_k \partial_\mu \langle\phi\rangle^\dagger \partial^\mu \langle\phi\rangle\right) + U_k(\langle\phi\rangle) + \cdots\right]$$

neglecting

1) *space-time dependence of the condensates ($\langle\phi\rangle = const$),*
2) *wave function renormalization ($Z_k = 1$),*
3) *high order contribution*

$$\Gamma_k = \int d^4x U_k(\langle\phi\rangle) = V U_k(\langle\phi\rangle)$$

$$\partial_k U_k = \frac{k^4}{6\pi^2} T \sum_n \mathrm{Tr}\left(D_{Sk} + D_{Pk}\right)$$

meson propagators $\left(\omega_n + k^2 + M^2\right)^{-1}$

expanding U_k around the physics condensates:

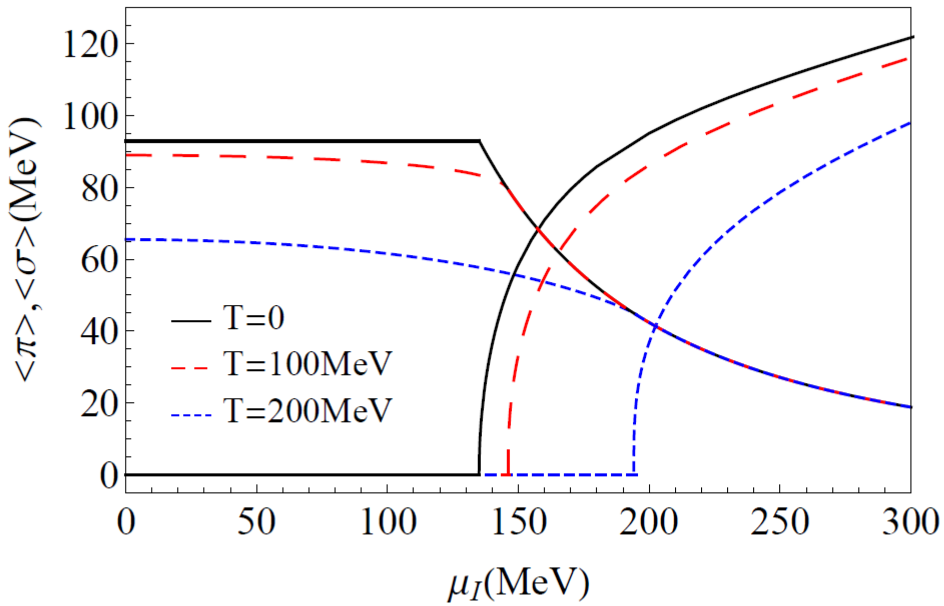
$$\langle\phi\rangle = \langle\phi\rangle_k + \delta\langle\phi\rangle_k$$

→ *4 flow equations for the 3 renormalization constants* $m_k, \lambda_{1k}, \lambda_{2k}$

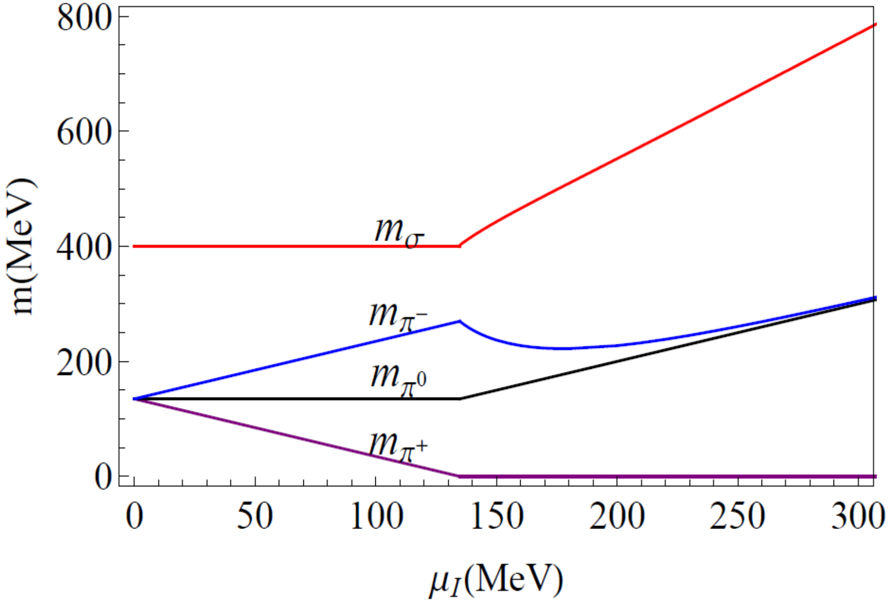
solution of the flow equations → evolution of the system

Pion Superfluid

phase diagram



meson masses



there is always a Goldstone mode in the pion superfluid phase.

critical exponent β

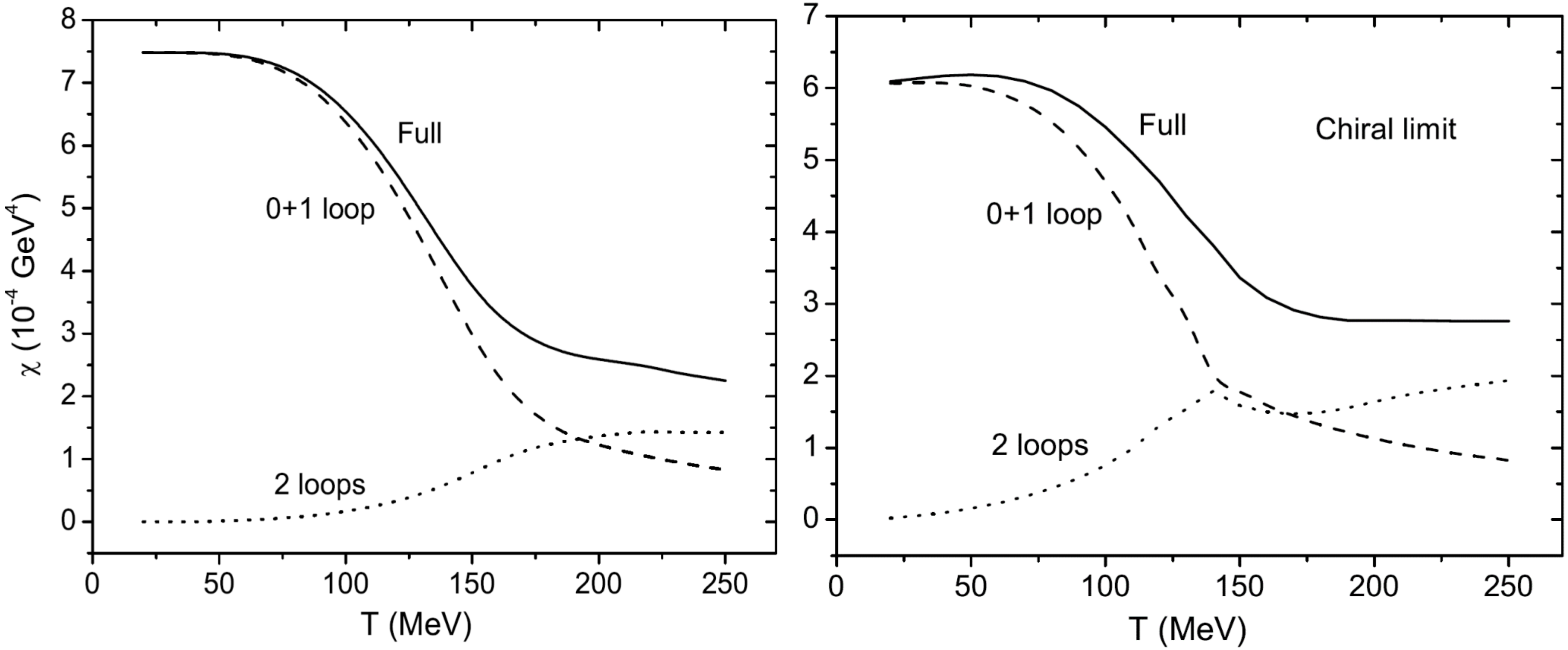
$T(MeV)$	0	50	100	150	200
$\mu_I^c(MeV)$	135	137.98	146.25	164.39	195.08
β	0.5	0.397	0.370	0.346	0.342

β with FRG is very different from 0.5 in mean field.

$U_A(1)$ *Restoration*

- *topological susceptibility*

$$\chi = \int d^4x \langle 0|T(Q(x)Q(0))|0\rangle$$



$U_A(1)$ symmetry is only partially restored even in chiral limit.

Conclusions

■ *there exists the pion superfluid at* $\mu_I > m_\pi$.

■ *there exists a BEC-BCS crossover in the strongly coupled pion superfluid.*

■ *the critical exponents with FRG are very different from the mean field results.*

■ $U_A(1)$ *symmetry is only partially restored even in chiral limit.*

■ *Possible applications in compact stars and intermediate energy nuclear collisions.*