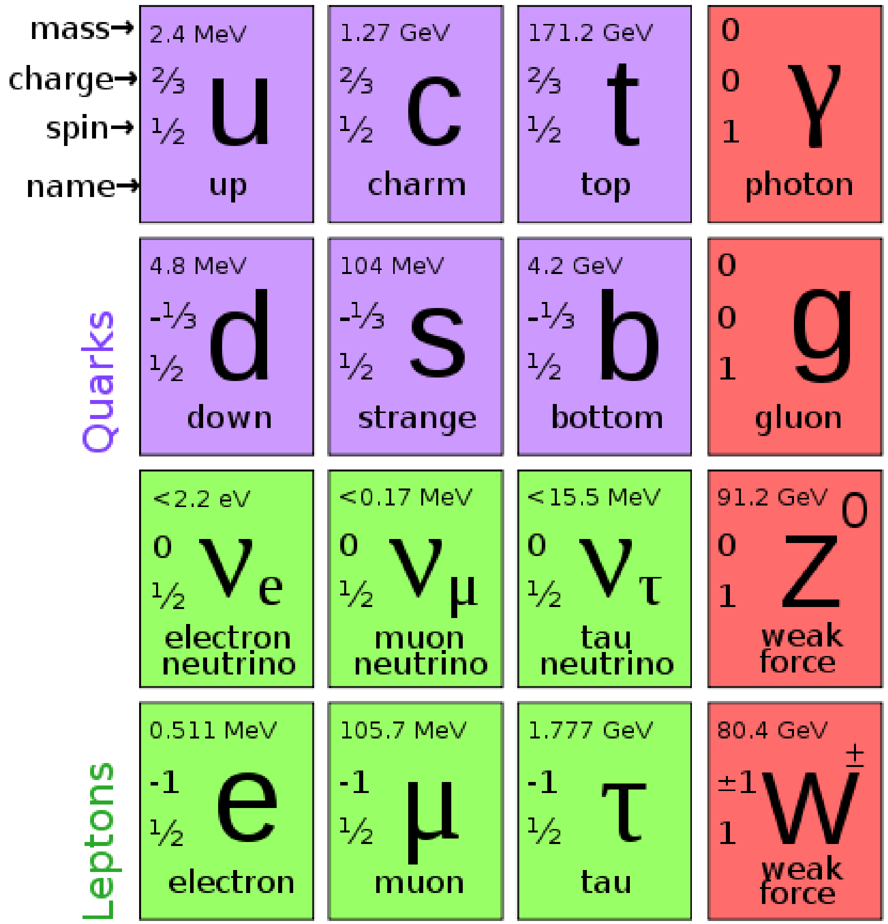
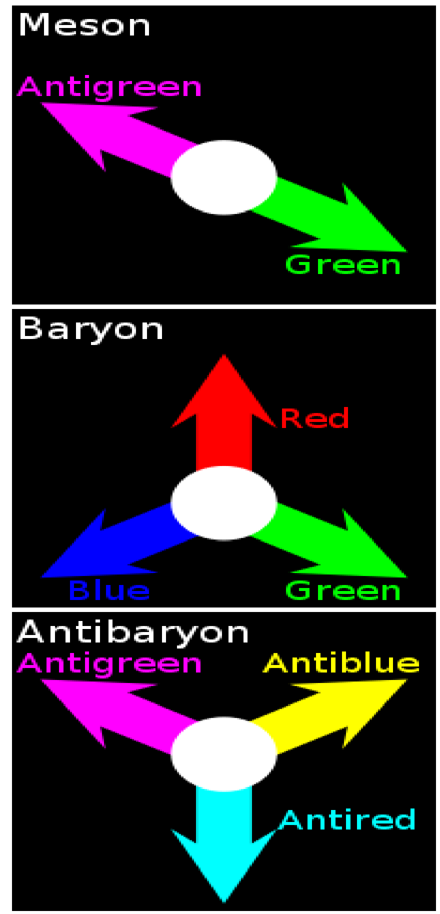
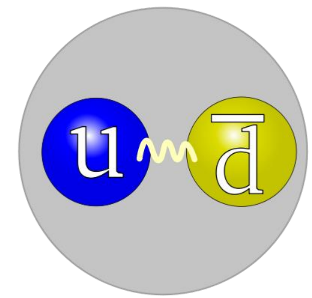
Quarks & gluons fundamental constituents of hadrons

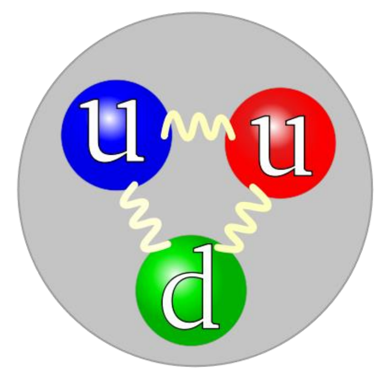




Mesons



Baryons



ATLAS & CMS, CERN/LHC ~126 GeV

Higgs Boson H^0

François Englert, Peter W. Higgs

The main missing block for the experimental validation of the SM is now in place

possible 4-quark state?
by

BESIII and Belle

$$e^+ e^- \longrightarrow \pi^+ \pi^- J/\psi$$

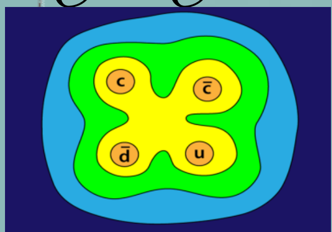
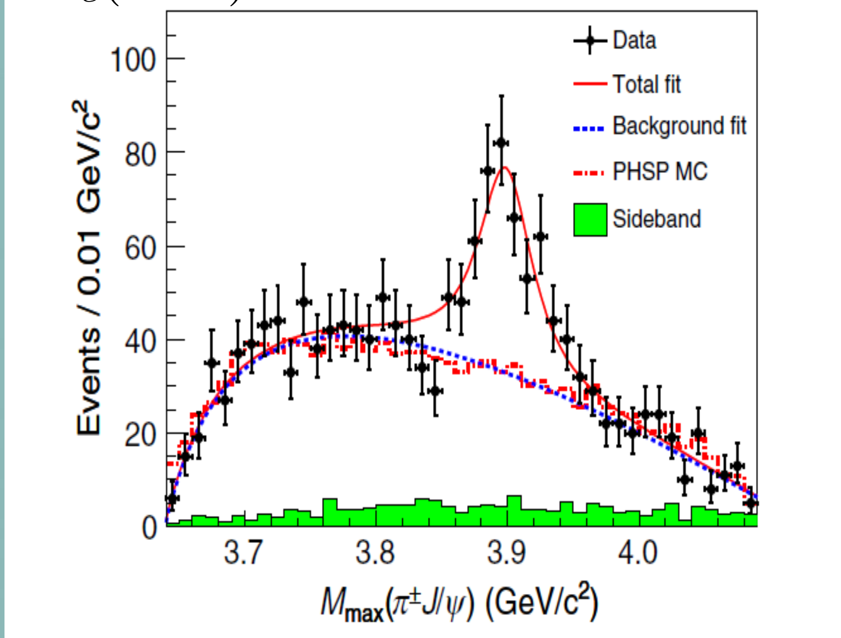


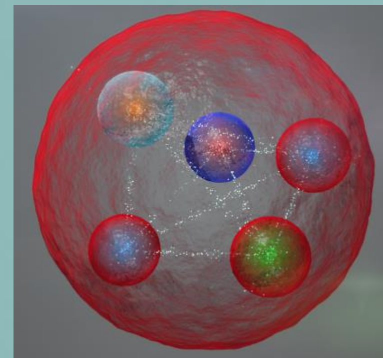
Image by BESIII coll.

$Z_c(3900)$



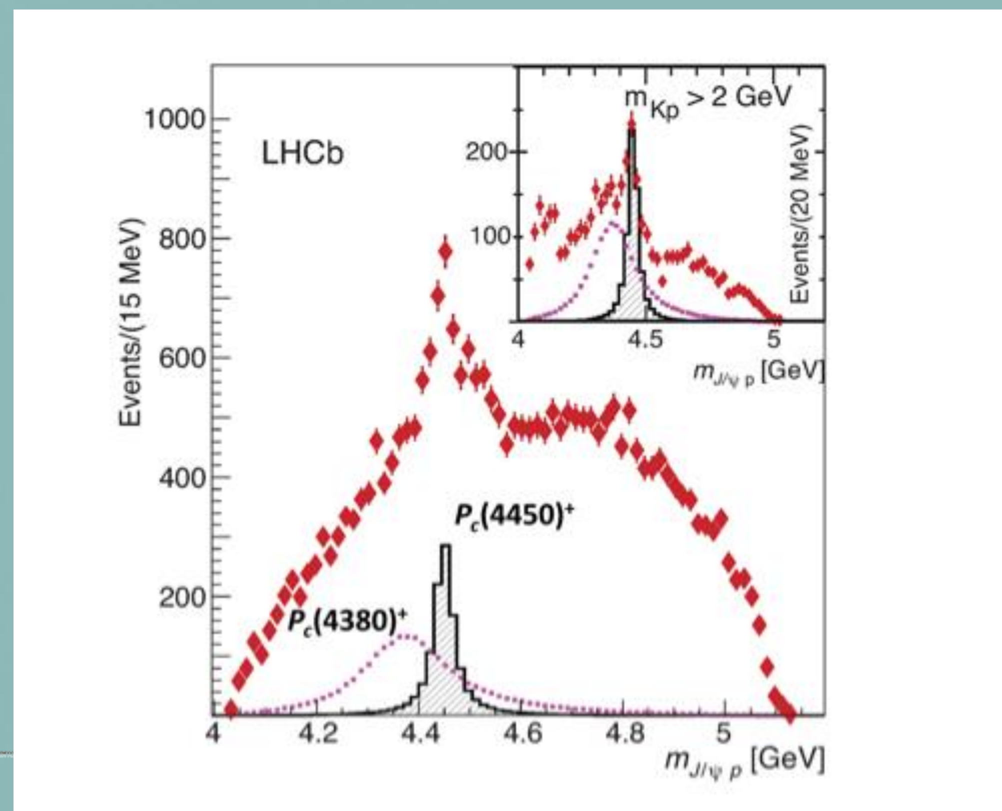
5-quark state just discovered

LHCb



$$\Lambda_b \to J/\psi\, p K$$

$$P_c = (c\bar{c}uud)$$



QCD $\quad SU_c(3)$ color $\quad : L(q_f^c, \bar{q}_f^c, A_\mu^c, m_f)$

Global symmetries

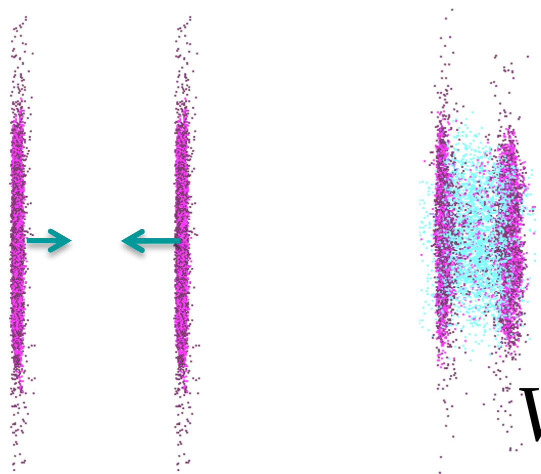
$U_B^1 \times U_S^1 \times U_Q^1$

$m_f \to \infty$

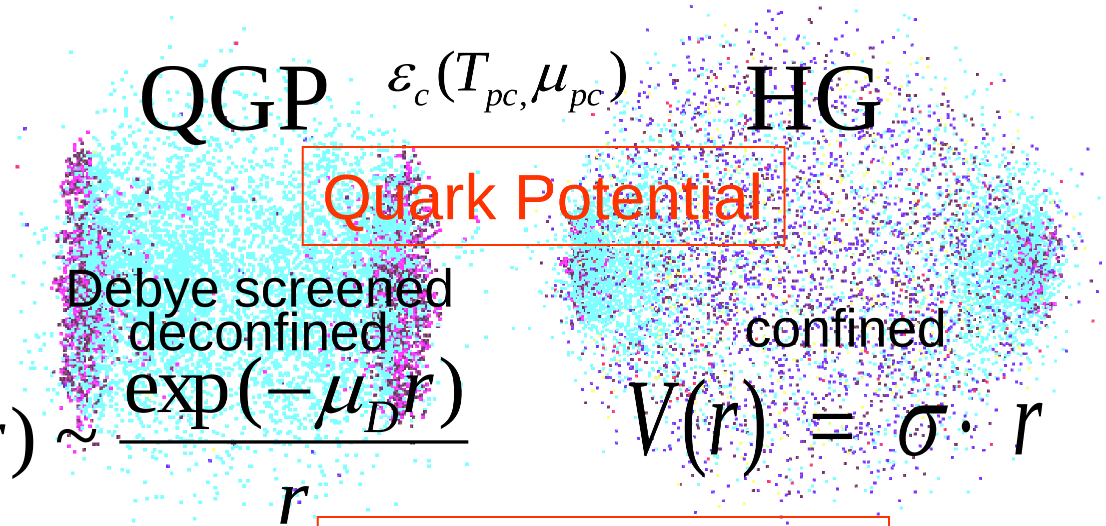
$m_{u,d} \to 0$

$Z(3)$ symmetry

Chiral $SU_L(2) \times SU_R(2)$ symmetry



A + A



QGP $\quad \varepsilon_c(T_{pc}, \mu_{pc})$ $\quad$ HG

Quark Potential

Debye screened
deconfined

$$V(r) \sim \frac{\exp(-\mu_D r)}{r}$$

confined

$$V(r) = \sigma \cdot r$$

Chiral Condensate

$< \bar{q}q > \approx 0$ $\qquad < \bar{q}q > \approx -225 MeV^3$

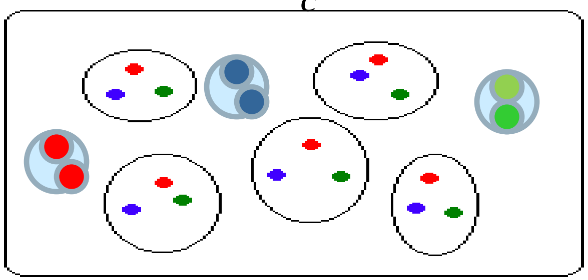
Chiral Symmetry: restored $\qquad$ broken

Critical Behaviour in Strongly Interacting Matter

Deconfinement and Chiral Symmetry restoration- expected within Quantum Chromodynamics (QCD)

cold hadrons gas

$$T << T_c$$



confined potential

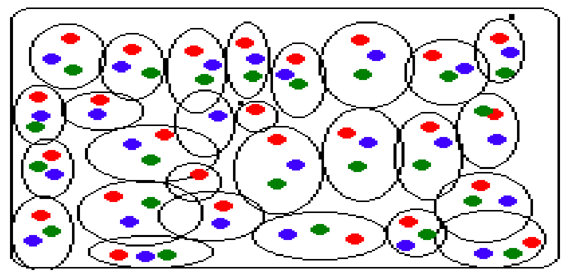
$$V(r) = \sigma \cdot r$$

chiral condensate

$$< \bar{\psi}\psi > \neq 0$$

critical region

$$T \approx T_c$$



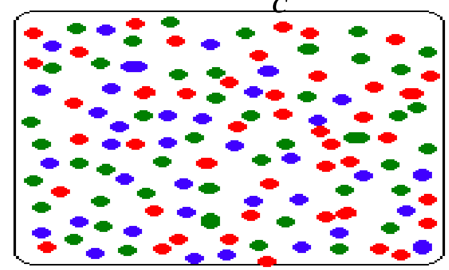
Collective dynamics

Transition appear for sufficiently large density

$$\varepsilon(T, \mu_B) \approx \varepsilon_N \approx \frac{m_N}{\frac{4}{3}\pi R_N^{\ 3}} \simeq 0.6 \frac{GeV}{fm^3}$$

QGP

$$T > T_c$$



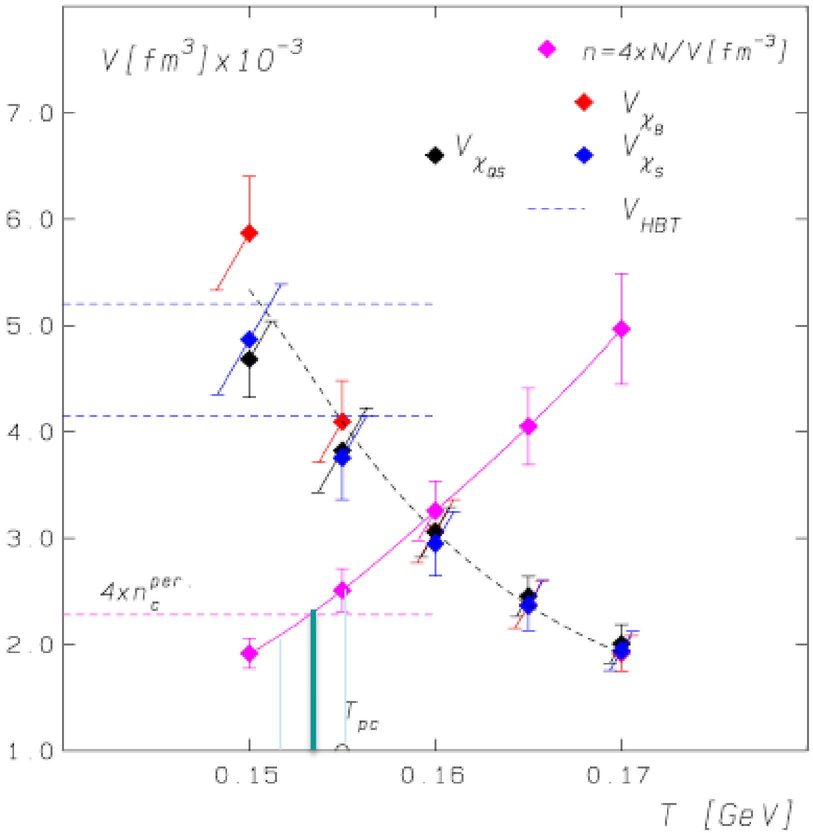
Debye screened potential

$$V(r) \sim \frac{\exp(-\mu_D r)}{r}$$

chiral condensate

$$< \bar{\psi}\psi > = 0$$

Particle density and percolation theory



- Density of particles at a given volume $n(T) = \frac{N_{total}^{exp}}{V(T)}$

- Total number of particles in HIC at LHC, ALICE

$$\langle N_t \rangle = 3\langle \pi \rangle + 4\langle p \rangle + 4\langle K \rangle + (2 + 4 \times 0.2175)\langle \Lambda_\Sigma \rangle + 4\langle \bar{\Xi} \rangle + 2\langle \bar{\Omega} \rangle,$$

$$\boxed{\langle N_t \rangle = 2486 \pm 146}$$

- Percolation theory: 3-dim system of objects of volume $V_0 = 4/3\pi R_0^3$

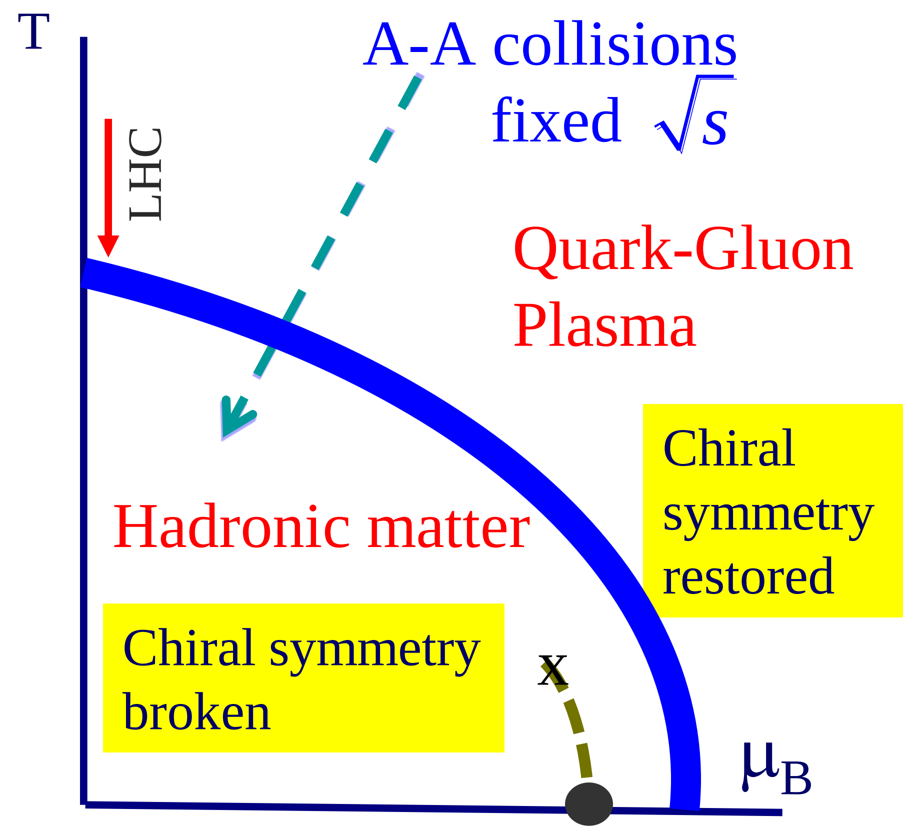
$$n_c = \frac{1.22}{V_0} \quad \text{take} \quad R_0 \approx 0.8\, fm \Rightarrow n_c \approx 0.57\ [fm^{-3}] \Rightarrow T_c^p \approx 154\ [MeV]$$

P. Castorina, H. Satz &K.R. Eur.Phys.J. C59 (2009)

QCD Phase diagram: from theory to experiment

Krzysztof Redlich University of Wroclaw

- QCD phase boundary in LGT and in effective models , its O(4) „scaling” & relation to freezeout in HIC
- Moments and probability distributions of conserved charges as probes of the criticality in QCD
- STAR data & expectations



1^{st} principle calculations:

$\mu, T << \Lambda_{QCD}$: χ – perturbation theory

$\mu, T >> \Lambda_{QCD}$: pQCD

$\mu_q < T$: LGT

Statistical Physics

Density Matrix
Partition Sum

$$\rho = \mathrm{e}^{-\frac{1}{T}(H-\mu_i N_i)}\,, \qquad Z = \hat{\mathrm{T}}\mathrm{r}\rho\,, \quad \hat{\mathrm{T}}\mathrm{r}(\ldots) = \sum_n \langle n|(\ldots)|n\rangle$$

Free energy &
Thermodynamics

$$F = -T\ln Z\,,$$
$$p = \frac{\partial(T\ln Z)}{\partial V}\,,$$
$$S = \frac{\partial(T\ln Z)}{\partial T}\,,$$

Net Charge

$$\bar{N}_i = \frac{\partial(T\ln Z)}{\partial \mu_i}\,,$$
$$E = -pV + TS + \mu_i \bar{N}_i$$

Densities

$$f = \frac{F}{V}, \quad p = -f, \quad s = \frac{S}{V}, \quad n_i = \frac{\bar{N}_i}{V}, \quad \epsilon = \frac{E}{V}$$

The partition function of QCD

$$Z(V,T,\mu;g,N_f,m_f) = \mathrm{Tr}(\mathrm{e}^{-(\mathrm{H}-\mu\mathrm{Q})/\mathrm{T}}) = \int \mathrm{DA\,D}\bar{\psi}\,\mathrm{D}\psi\;\mathrm{e}^{-\mathrm{S_g}[\mathrm{A}_\mu]}\mathrm{e}^{-\mathrm{S_f}[\bar{\psi},\psi.\mathrm{A}_\mu]}$$

$$\boxed{(F_g)^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^b_\nu}$$

Action

$$S_g[A_\mu] = \int_0^{1/T} d\tau \int_V d^3x\, \frac{1}{2}\mathrm{Tr}\, F_{\mu\nu}(x)F_{\mu\nu}(x),$$

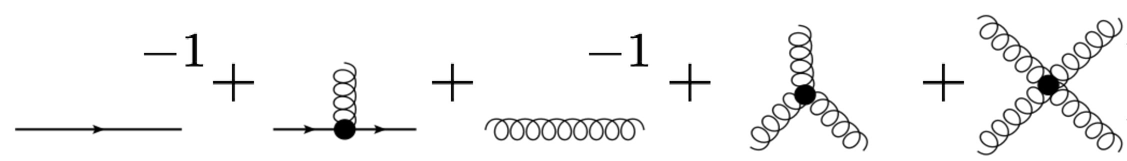
$$S_f[\bar{\psi},\psi,A_\mu] = \int_0^{1/T} d\tau \int_V d^3x \sum_{f=1}^{N_f} \bar{\psi}_f(x)\,(\gamma_\mu D_\mu + m_f - \mu_f\gamma_0)\,\psi_f(x)$$

$$D_\mu = \partial_\mu + igT^a A^a_\mu$$

$$A_\mu(\tau,\mathrm{x}) = A_\mu(\tau+\frac{1}{T},\mathrm{x}), \qquad \psi_f(\tau,\mathrm{x}) = -\psi_f(\tau+\frac{1}{T},\mathrm{x})$$

quark number $N^f_q = \bar{\psi}_f\gamma_0\psi_f$

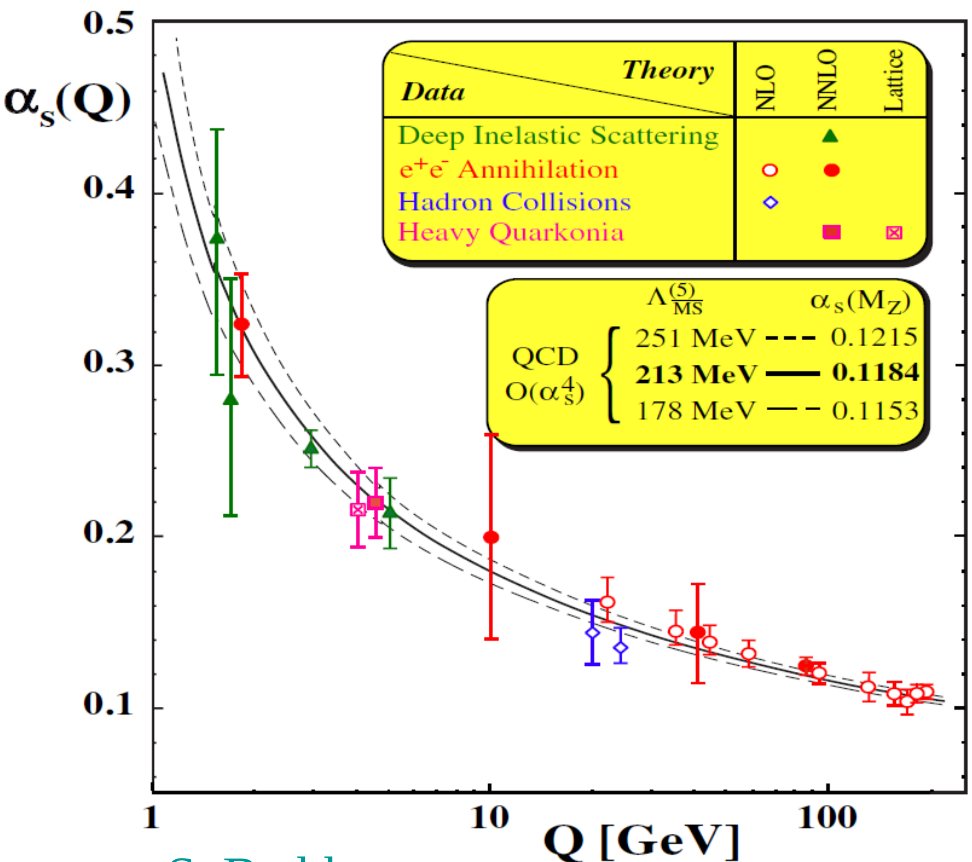
$$\mathcal{S}_{\mathrm{QCD}} = \int d^4x \;\Big($$



$\Big)$

Asymptotic freedom in QCD

QCD becomes perturbative at high energy



S. Bethke

The Nobel Prize in Physics 2004

David J. Gross, H. David Politzer, Frank Wilczek

The Nobel Prize in Physics 2004

David J. Gross

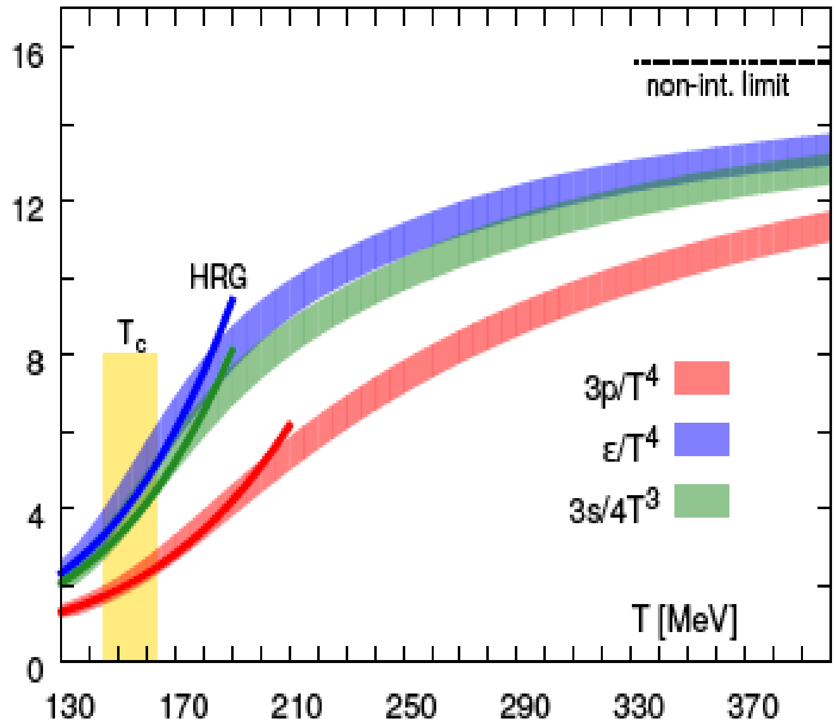
H. David Politzer

Frank Wilczek

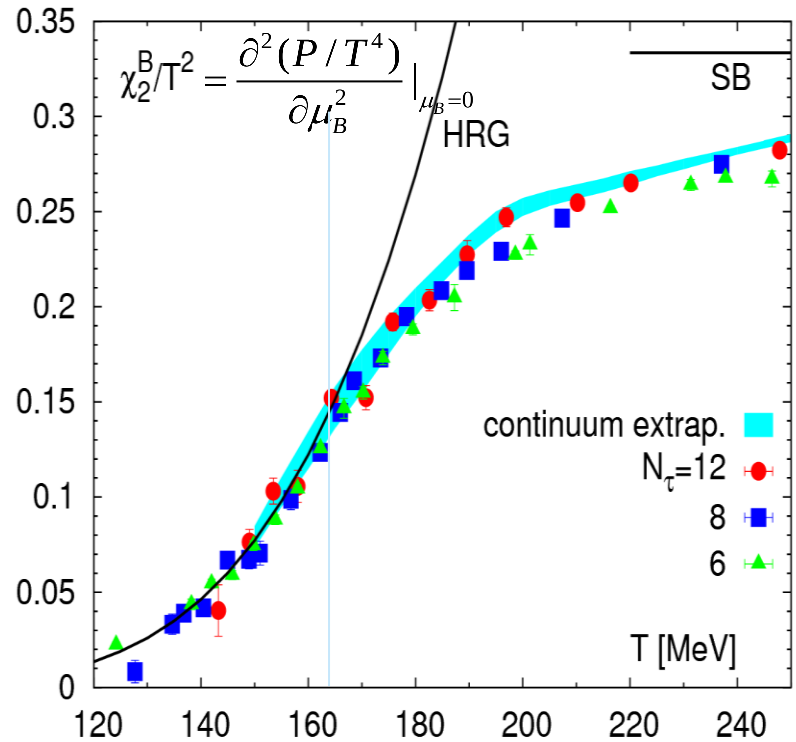
The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

Excellent description of the QCD Equation of States by Hadron Resonance Gas

A. Bazavov et al. HotQCD Coll. July 2014



F. Karsch et al. HotQCD Coll.



- “Uncorrelated” Hadron Gas provides an excellent description of the QCD equation of states in confined phase
- “Uncorrelated” Hadron Gas provides also an excellent description of net baryon number fluctuations

Chiral Transformations of QCD-Langrangian

$$L_{QCD}^{quark} = \bar{q}\gamma_\mu(i\partial_\mu - gA_\mu^a\lambda^a)q - m_q\bar{q}q$$

Decompose: $q = q_R + q_L$

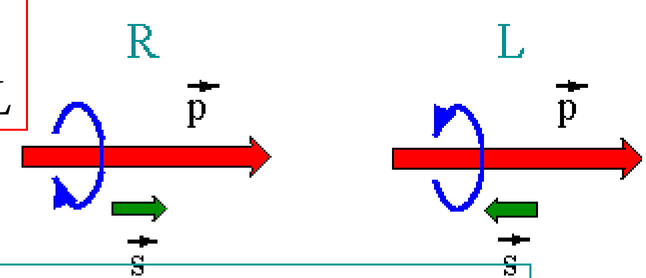
$$\bar{q}\gamma_\mu D_\mu q = \bar{q}_L\gamma_\mu D_\mu q_L + \bar{q}_R\gamma_\mu D_\mu q_R$$

$q_R = \frac{1}{2}(1+\gamma_5)q \quad q_L = \frac{1}{2}(1-\gamma_5)q$

Chiral transformations:

$\vec{s}\cdot\hat{p}\,|\,\vec{p},h> = h\,|\,\vec{p},h>$

$$q_R \to e^{-i\vec{\theta}_R\cdot\vec{\tau}/2}q_R \qquad q_L \to e^{-i\vec{\theta}_L\cdot\vec{\tau}/2}q_L$$



$SU_R(2) \times SU_L(2)$

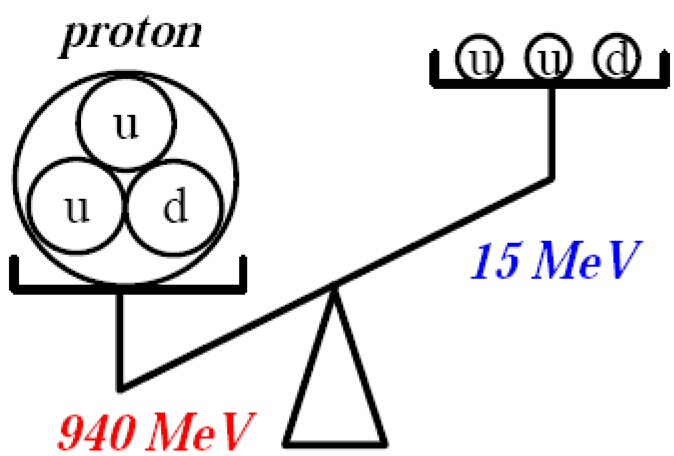
$$\bar{q}q = \bar{q}_L q_R + \bar{q}_R q_L$$

Breaks chiral symmetry:
invariant under
$SU_V(2) \quad (\theta_R = \theta_L)$

In QCD vacuum chiral symm. spontaneously broken

$$<\bar{q}q> \approx -(250MeV)^3$$

- origin of hadron masses?



- current quark masses (cf. QCD scale ~ 200 MeV)

$$\underbrace{\begin{matrix} m_u & \lesssim & m_d & < & m_s \\ 4\,\mathrm{MeV} & & 7\,\mathrm{MeV} & & 100\,\mathrm{MeV} \end{matrix}}_{\text{light quarks}} \ll \underbrace{\begin{matrix} m_c & \ll & m_b & \ll & m_t \\ 1.2\,\mathrm{GeV} & & 4\,\mathrm{GeV} & & 180\,\mathrm{GeV} \end{matrix}}_{\text{heavy quarks}}$$

- chiral symmetry in u, d-quark sector ($m_{u,d}/\Lambda_{\mathrm{QCD}} \ll 1$)

$$\mathcal{L} = \mathcal{L}(q_L) + \mathcal{L}(q_R) + m_{u,d}\,(\bar{q}_L q_R + \bar{q}_R q_L) \quad \sim \quad SU_L(2) \otimes SU_R(2)$$

Is it manifest in hadron spectra? ... NO

Order parameter of chiral symmetry restoration

effective quark mass shift

Measures dynamically generated „constituent" quark mass: T=0 quarks „dress" with gluons

in hot medium dressing „melts"

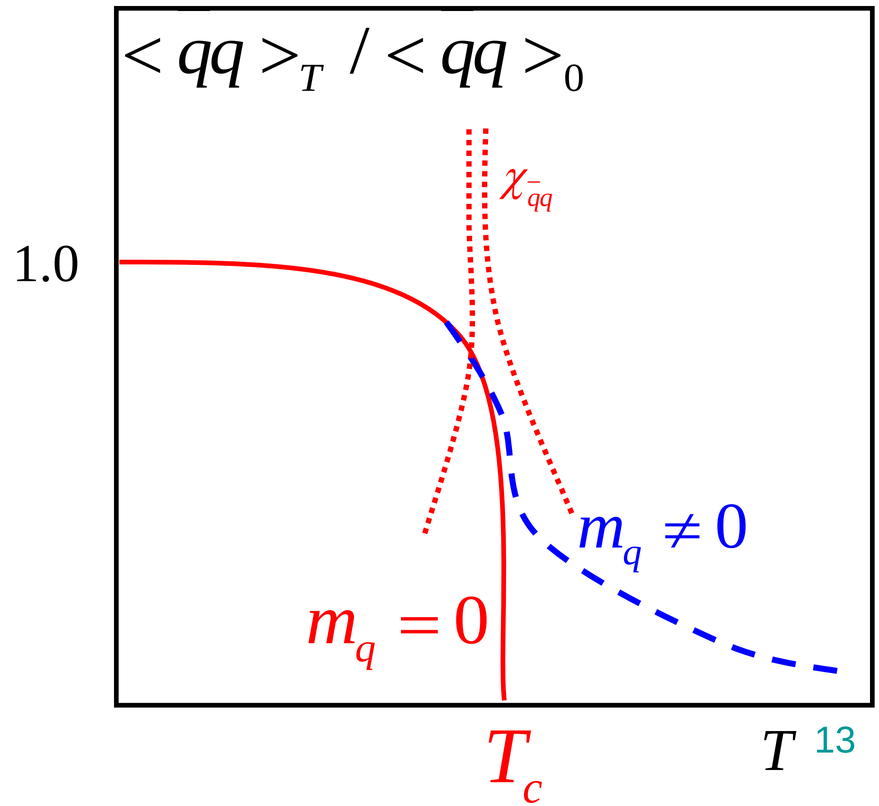
$$< \bar{q}q >= \begin{cases} 0 \Leftrightarrow \textit{chiral}\ \text{symmetry restored}\ T > T_c \\ \neq 0 \Leftrightarrow \textit{chiral}\ \text{symmetry broken}\ T < T_c \end{cases}$$

Consider chiral susceptibility:

$$\chi_{\bar{q}q} = \frac{\partial^2 P(T, \vec{\mu}, m_q)}{\partial {m_q}^2}$$

$$= < (\bar{q}q)^2 > - < \bar{q}q >^2$$

to determine the position of the chiral phase transition:



Z(N) transformation of QCD Lagrangian

$$SU(N)\ \text{gauge tr.}\ \Omega\ :\ D_\mu \to \Omega D_\mu \Omega^\dagger\,, \quad \psi \to \Omega\psi$$

$$\Omega \in SU(N) \quad \Rightarrow \quad \Omega^\dagger\Omega = \mathbf{1}\ \text{and}\ \det\Omega = 1$$

– a simple gauge tr.

$$\Omega_c = e^{i\phi}\mathbf{1} \qquad\qquad \det\Omega_c = 1$$

$$\phi = \frac{2\pi j}{N}\,, \quad j = 0, 1, \cdots, (N-1)\ :\ Z(N)\ \text{symmetry}$$

– $Z(N)$ at $T \neq 0$: imaginary time τ (0 - $\beta = 1/T$)

$$\text{gluon}\ :\ A_\mu(\beta, \vec{x}) = A_\mu(0, \vec{x}) \quad \text{periodic BC}$$
$$\text{quark}\ :\ \psi(\beta, \vec{x}) = -\psi(0, \vec{x}) \quad \text{anti-periodic BC}$$

Ω_c violates BC:

$$A_\mu^{\Omega_c}(\beta, \vec{x}) = A_\mu(0, \vec{x})\,, \quad \psi^{\Omega_c}(\beta, \vec{x}) \neq -\psi(0, \vec{x})$$

$\Rightarrow$ quark breaks $Z(N)$ symmetry

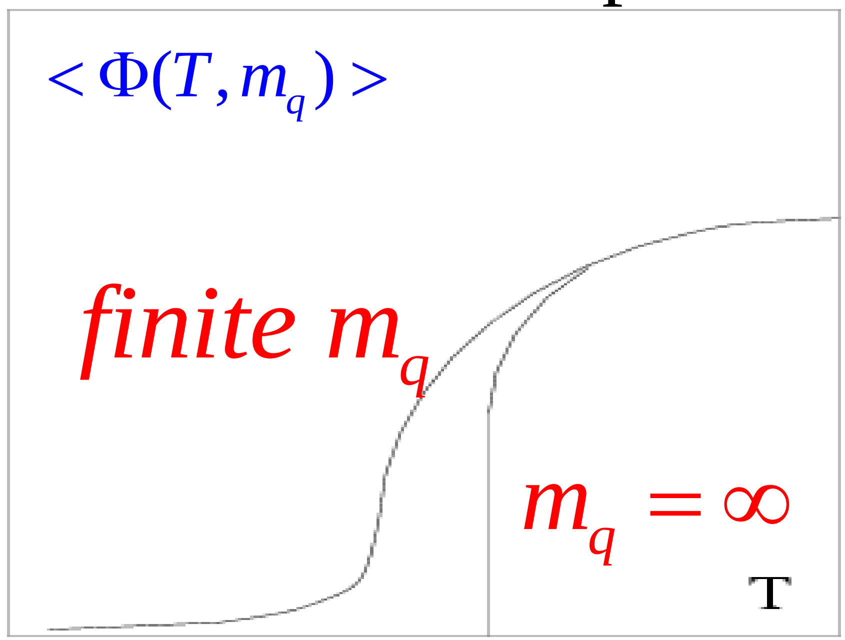
Polyakov loop and deconfinemnet

$$\Phi \doteq TrL(\vec{x}) = \frac{1}{N_c} Tr(P\exp[i\int_0^\beta d\tau A_4(\vec{x},\tau)])$$

Z(N)- transformation : $\boxed{L \Rightarrow c_N L}$ $\boxed{c_N = e^{2\pi\, i\, k/N} \in Z(N)}$

$$<\Phi> \approx e^{-F_q/T} = \begin{cases} 0 \Leftrightarrow \text{confined phase} \quad T < T_c \\ \neq 0 \Leftrightarrow \text{deconfined phase} \; T > T_c \end{cases}$$

L. McLerran & B. Svetitsky

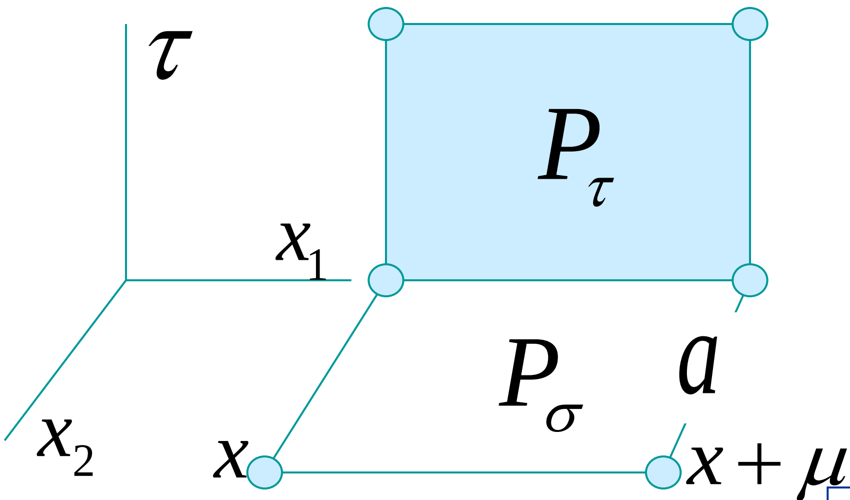


Consider fluctuations:

$$\chi_L \equiv <\Phi^2> - <\Phi>^2$$

to determine the position of the phase transition:

Lattice QCD



$$U_\mu(x) \in SU(N)$$

$$T^{-1} = N_\tau a$$

$$V = N_\sigma^3 a^3$$

$$U(x,0) = U(x,\beta)$$

$$\beta = \frac{2N}{g^2}$$

$$S_G = \sum (P_\sigma + P_\tau + h.c.)$$

$$Z(T,V,\mu) = \int dU \ \mathrm{e}^{-\beta \ \mathrm{S_g}[U]} \ \det[M]$$

$$M = M\ (U,\mu)$$

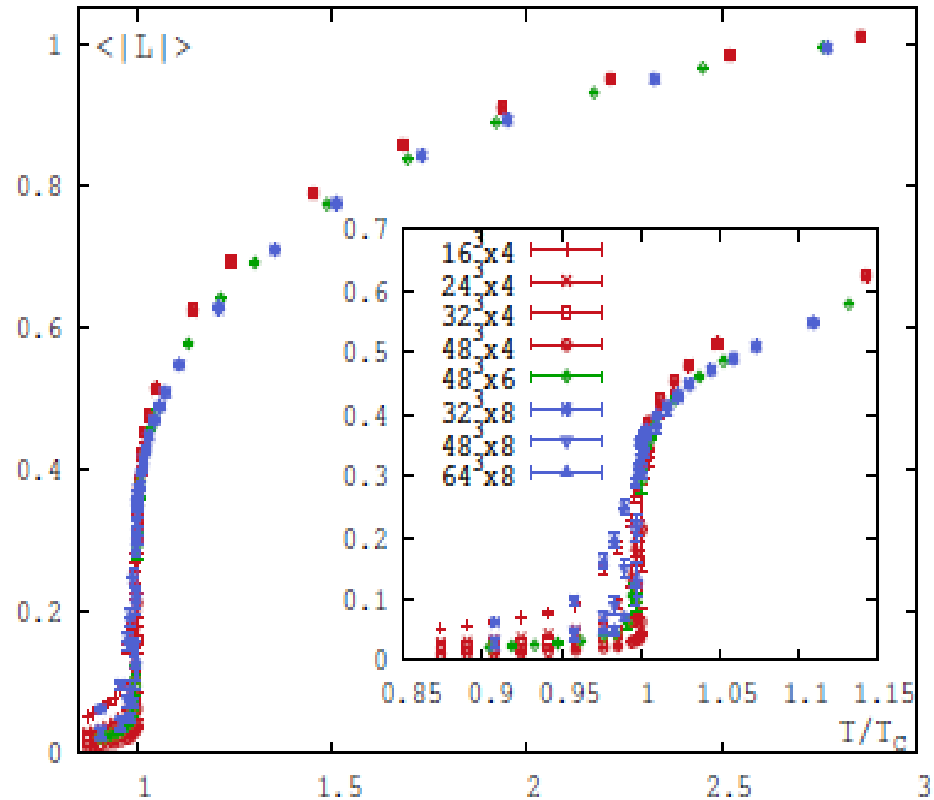
Polyakov loop on the lattice needs renormalization

- Introduce bare Polyakov loop

$$L_{\vec{x}}^{\text{bare}} = Tr \prod_{\tau=0}^{N_\tau - 1} U_{(\vec{x},\tau),\hat{\tau}}$$

$$L^{\text{bare}} = \frac{1}{N_\sigma^3} \sum_{\vec{x}} L_{\vec{x}}^{\text{bare}}$$

- Renormalized ultraviolet divergence $L^{\text{ren}} = \left(Z(g^2)\right)^{N_\tau} L^{\text{bare}}$
- Usually one takes $\langle |L^{\text{ren}}| \rangle$ as an order parameter



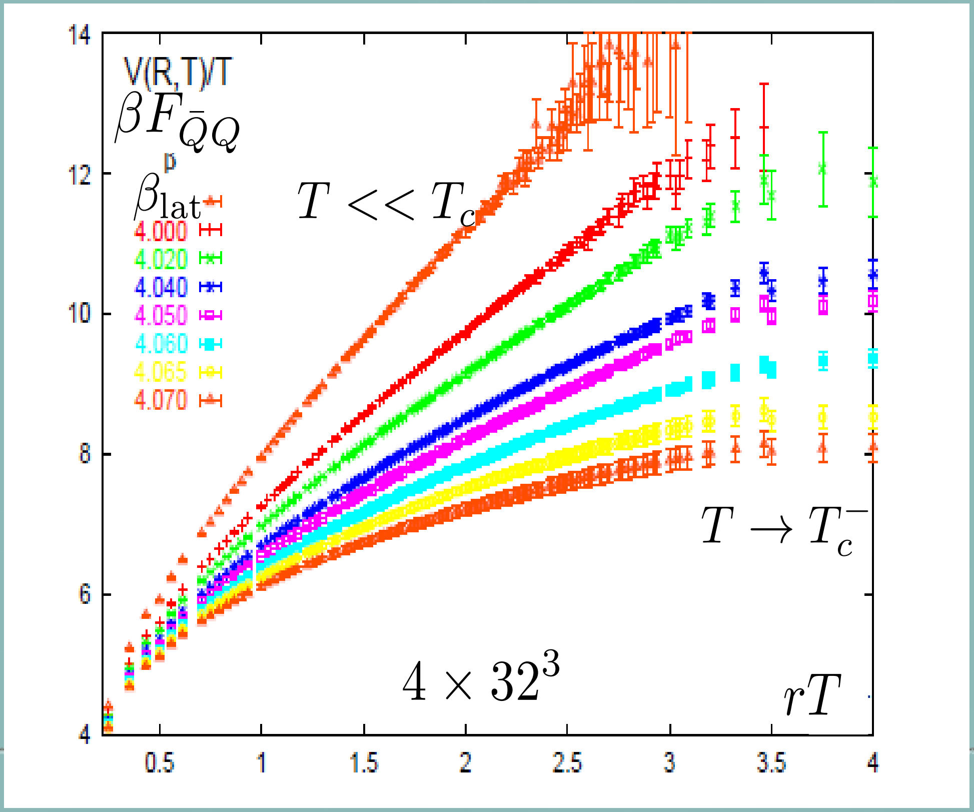
Heavy quark free energy

$$\langle L^{\mathrm{ren}}_{\vec{0}} L^{\dagger\mathrm{ren}}_{\vec{x}} \rangle = e^{-F^{\mathrm{ren}}_{q\bar{q}}(r=|\vec{x}|,T)/T} \stackrel{r\to\infty}{\to} |\langle L^{\mathrm{ren}} \rangle|^2 .$$

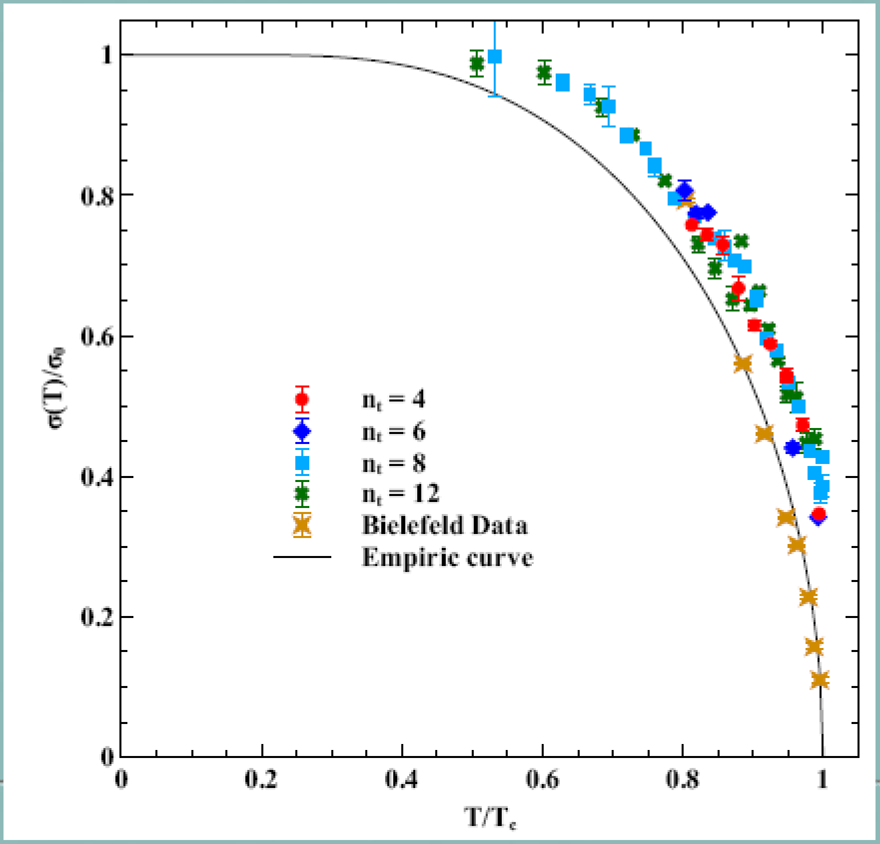
$$F_{QQ}(r) \approx \sigma(T)\cdot r + \frac{a}{r}$$

$$|\langle L \rangle|^2 = e^{-\beta F_{Q\bar{Q}}[r\to\infty,T]}$$

Kaczmarek *et.*



N. Cardoso* and P. Bicudo†



To probe deconfinement : consider fluctuations

- Fluctuations of modulus of the Polyakov loop

$$T^3\chi_A = \frac{N_\sigma^3}{N_\tau^3}\left(\langle|L^{\rm ren}|^2\rangle - \langle|L^{\rm ren}|\rangle^2\right)$$

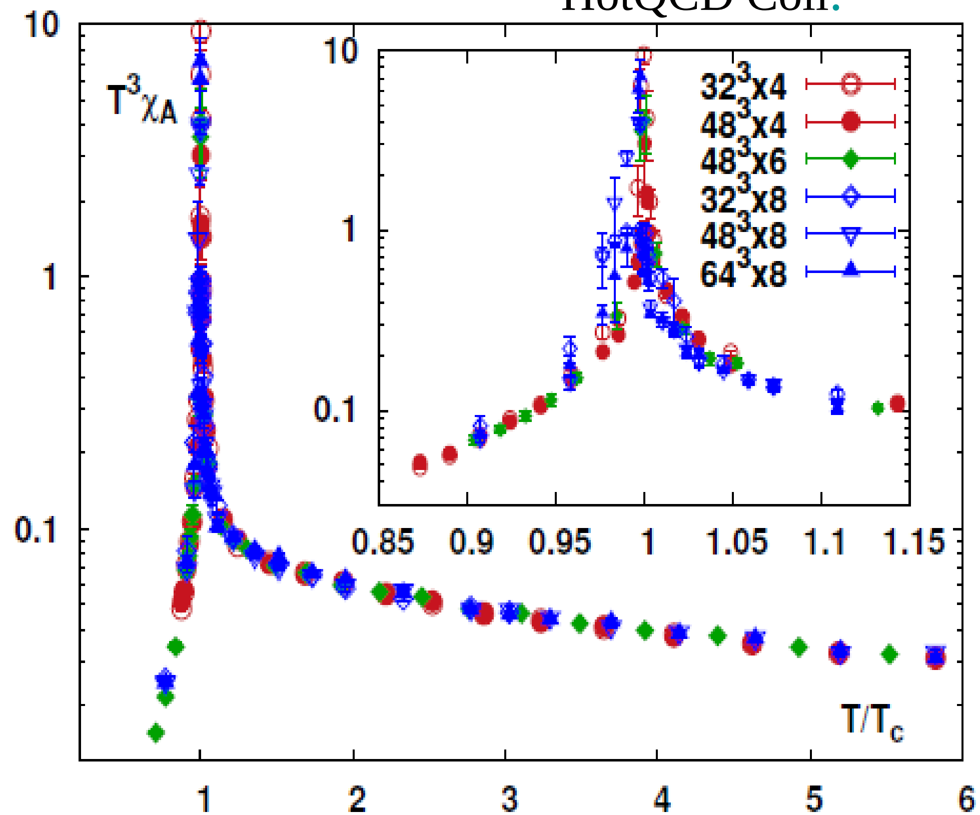
However, the Polyakov loop

$$L = L_R + iL_I$$

Thus, one can consider fluctuations of the real χ_R and the imaginary part χ_I of the Polyakov loop.

SU(3) pure gauge: LGT data

HotQCD Coll.



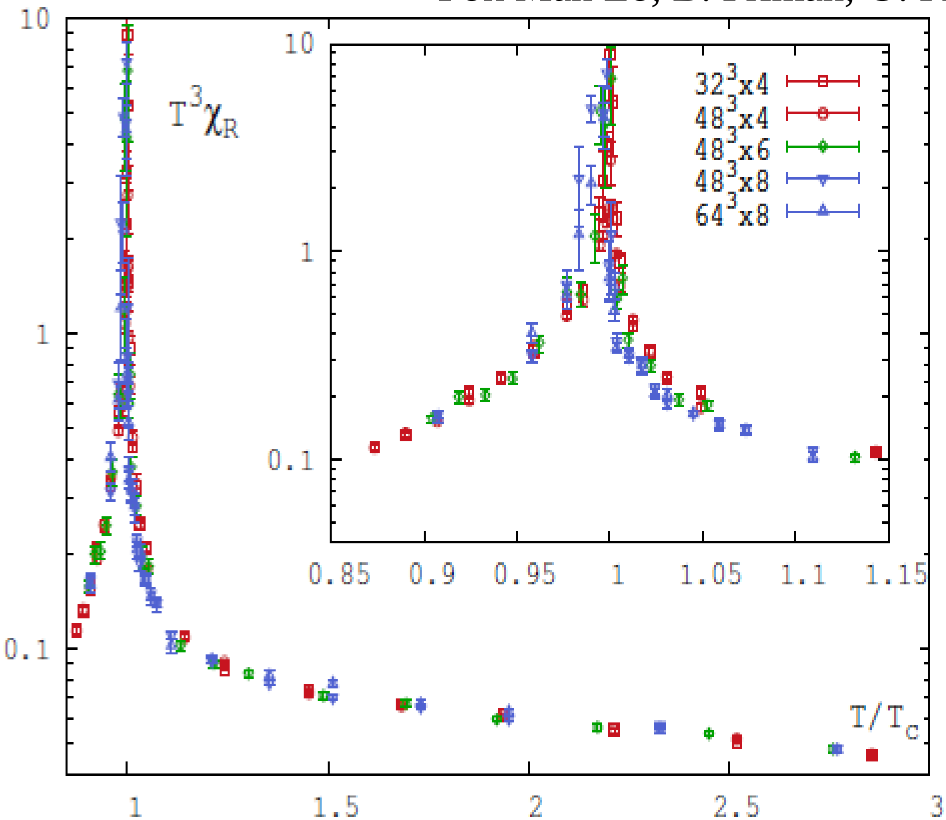
Fluctuations of the real and imaginary part of the renormalized Polyakov loop

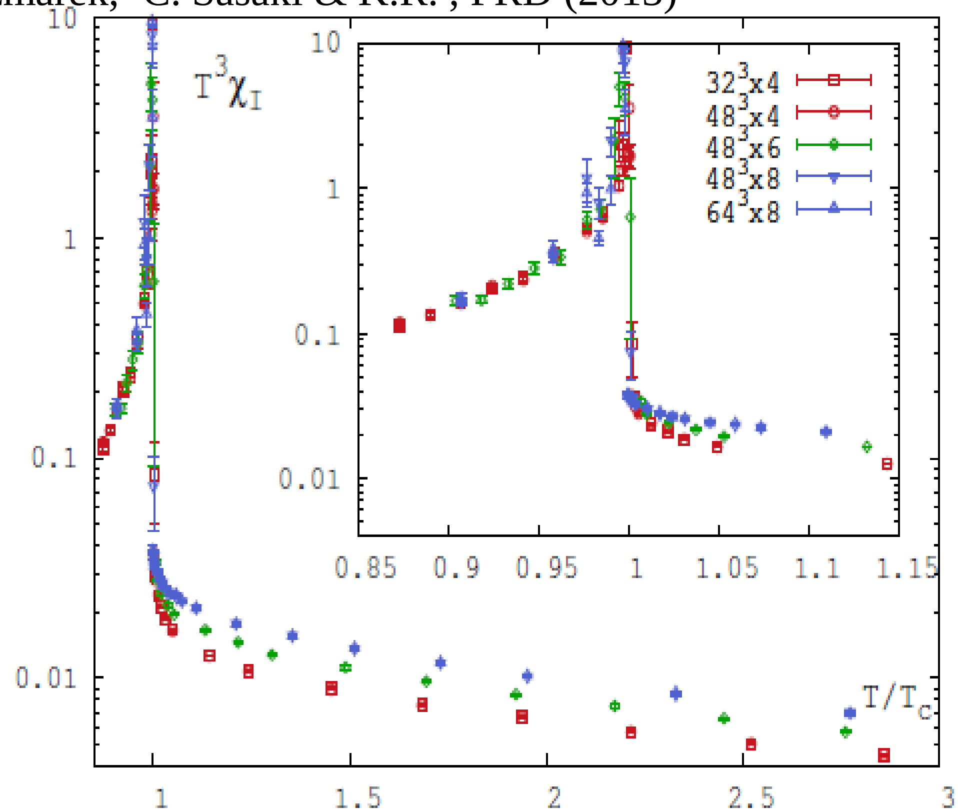
- Real part fluctuations
- Imaginary part fluctuations

$$T^3\chi_R = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_R^{\rm ren})^2\rangle - \langle L_R^{\rm ren}\rangle^2\right]$$

$$T^3\chi_I = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_I^{\rm ren})^2\rangle - \langle L_I^{\rm ren}\rangle^2\right]$$

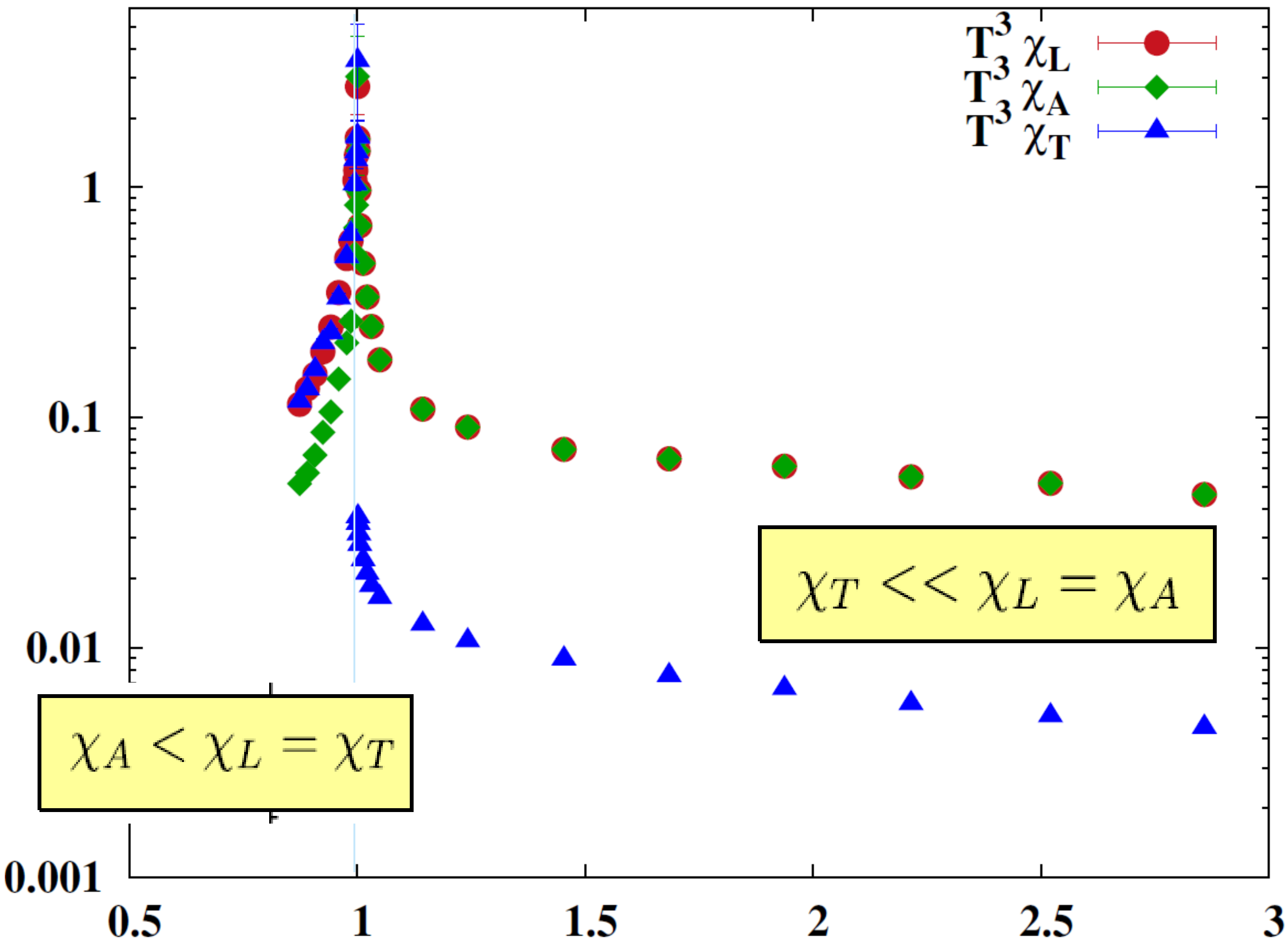
Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)





To probe deconfinement : consider fluctuations

$$T^3\chi_A = \frac{N_\sigma^3}{N_\tau^3}\left(\langle|L^{\rm ren}|^2\rangle - \langle|L^{\rm ren}|\rangle^2\right)$$



- the Polyakov loop

$$L = L_R + iL_I$$

- Consider fluctuations of

real $\chi_L = \chi_R$

modulus $\chi_A = \chi_{|L|}$

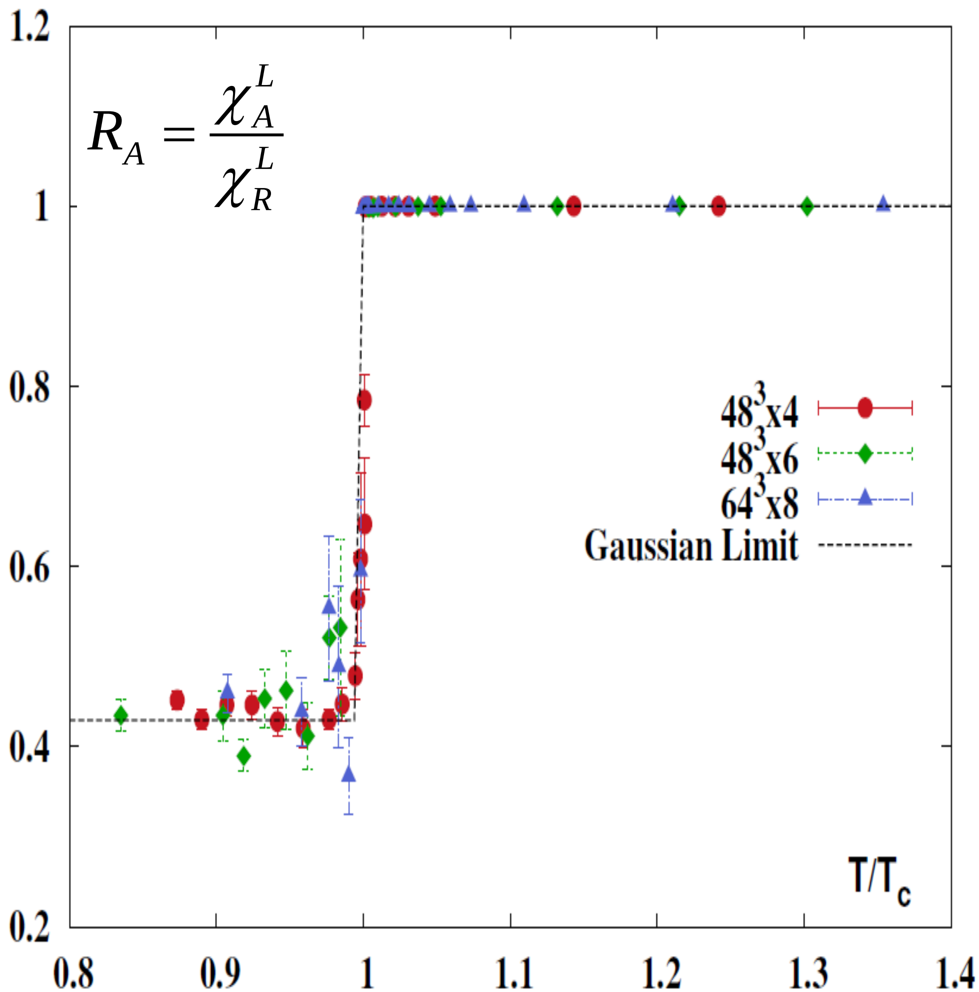
imaginary $\chi_T = \chi_I$

and take their ratios:

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)

Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the deconfined phase $R_A \approx 1$

Indeed, in the real sector of Z(3)

$$L_R \approx L_0 + \delta L_R \quad \text{with} \quad L_0 =< L_R >$$

$$L_I \approx L_0^I + \delta L_I \quad \text{with} \quad L_0^I = 0, \ \text{thus}$$

$$\chi_R^L = V < (\delta L_R)^2 >, \quad \chi_I^L = V < (\delta L_I)^2 >$$

Expand the modulus,

$$| L |= \sqrt{L_R{}^2 + L_I{}^2} \approx L_0(1 + \frac{\delta L_R}{L_0} + \frac{(\delta L_I)^2}{2L_0{}^2})$$

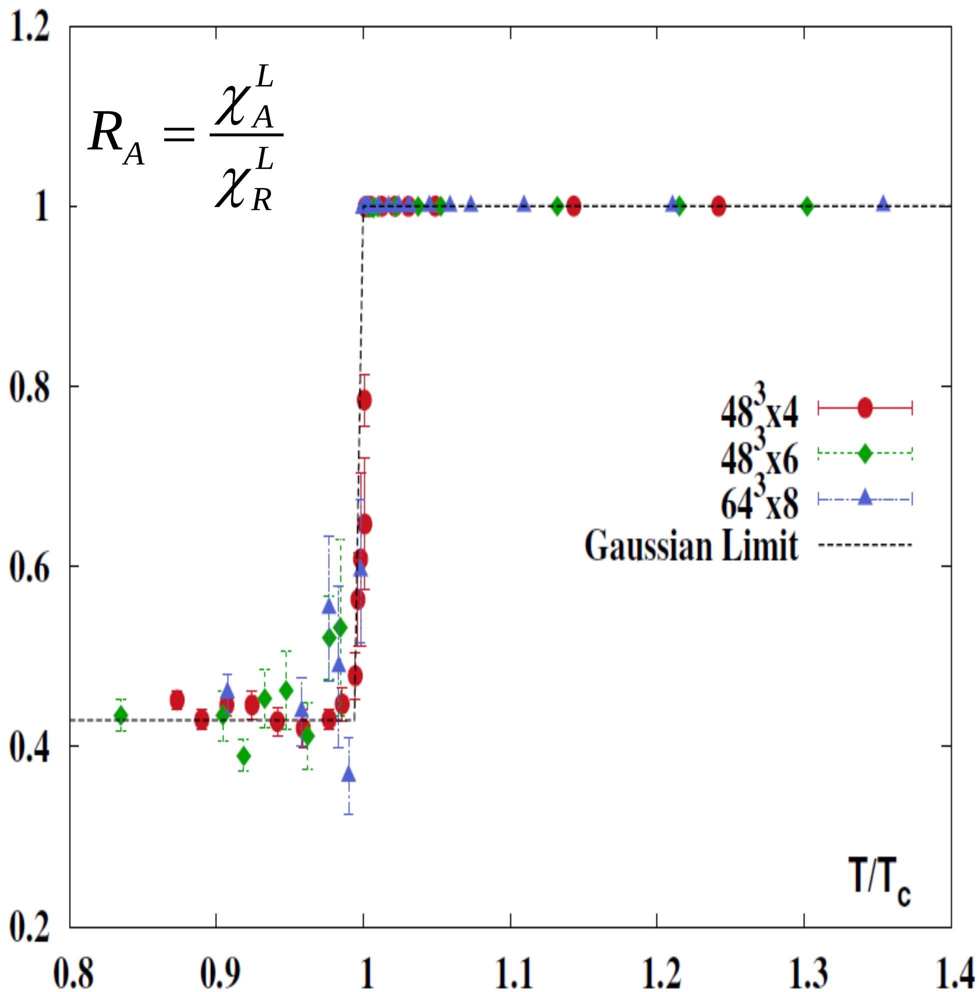
get in the leading order

$$\boxed{<| L |^2 > - <| L |>^2 \approx < (\delta L_R)^2 >}$$

thus $\chi_A \approx \chi_R$

Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the confined phase $R_A \approx 0.43$

Indeed, in the Z(3) symmetric phase, the probability distribution is Gaussian to the first approximation, with the partition function

$$Z = \int dL_R dL_I e^{VT^3[\alpha(T)(L_R^2+L_I^2)]}$$

Thus $\chi_R = \frac{1}{2\alpha T^3}$, $\chi_I = \frac{1}{2\alpha T^3}$ and

$\chi_A = \frac{1}{2\alpha T^3}(2-\frac{\pi}{2})$, consequently

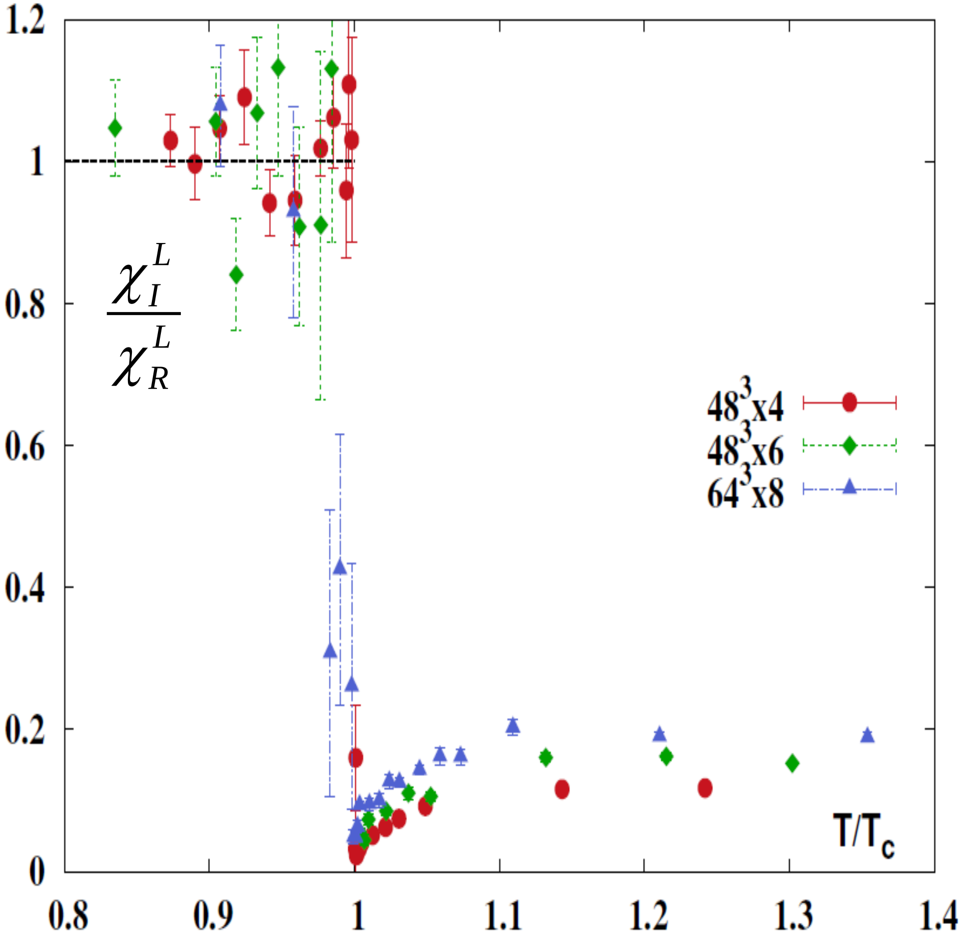
$$R_A^{SU(3)} = (2-\frac{\pi}{2}) = 0.429$$

In the SU(2) case $R_A^{SU(2)} = (2-\frac{2}{\pi}) = 0.363$

is in agreement with MC results

Ratio Imaginary/Real of Polyakov loop fluctuations

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the confined phase for any symmetry breaking operator its average vanishes, thus

$$\chi_{LL} = \langle L^2 \rangle - \langle L \rangle^2 = 0$$ and

$$\chi_{LL} = \chi_R - \chi_I$$ thus $\boxed{\chi_R = \chi_I}$

- In deconfined phase the ratio of $\chi_I / \chi_R \neq 0$ and its value is model dependent

Ratio Imaginary/Real of Polyakov loop fluctuations

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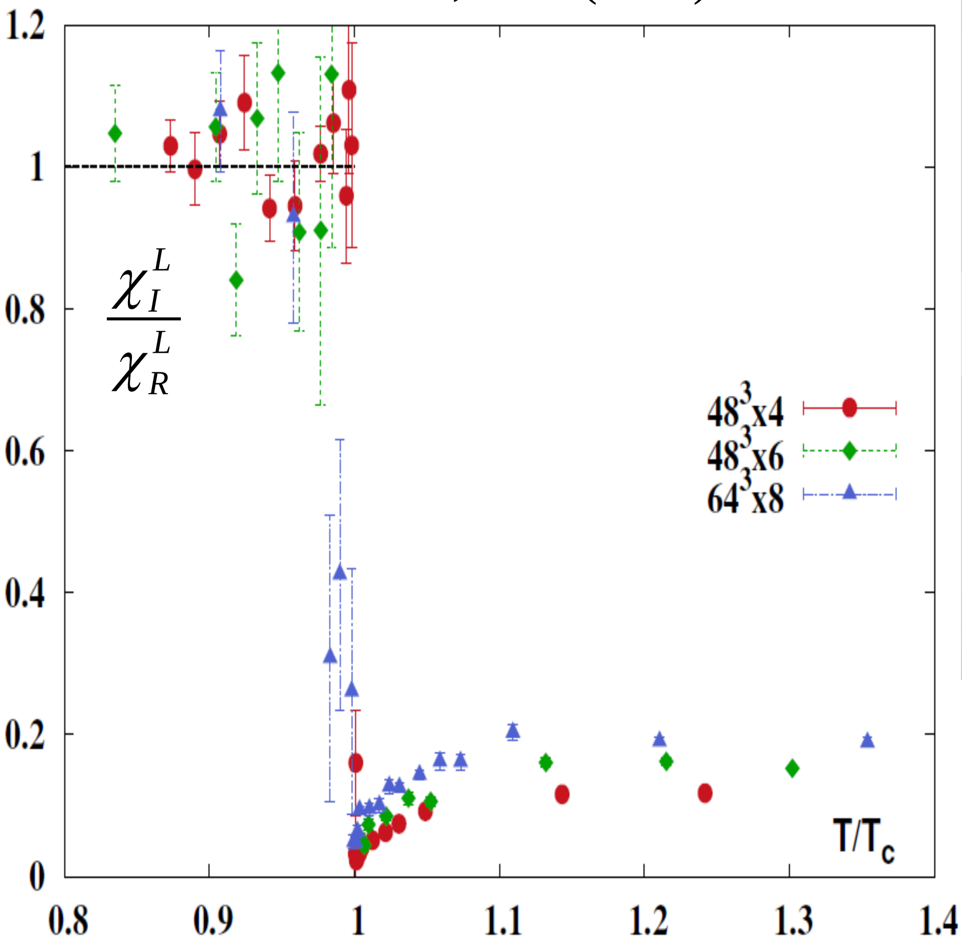
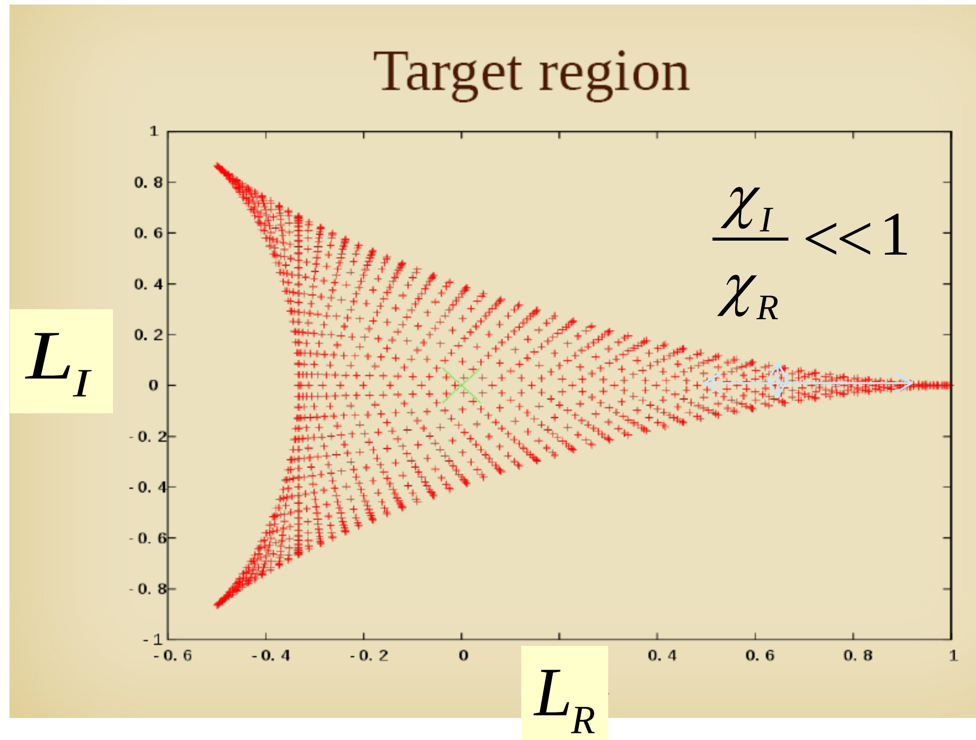


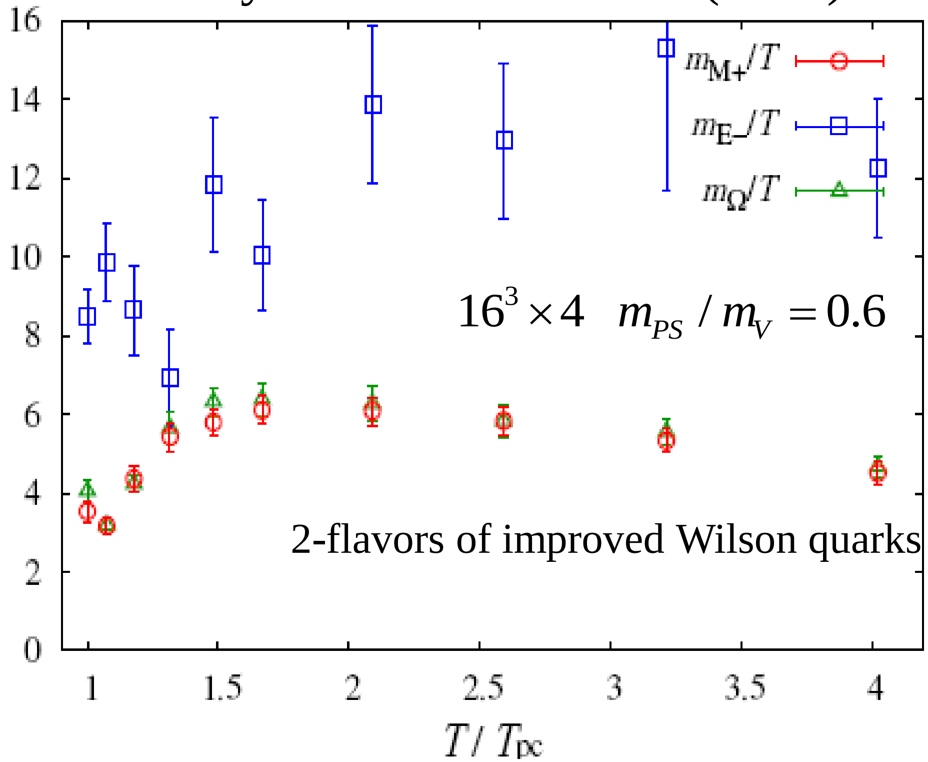
- In confined phase



Due to Z(3) symmetry breaking in confined phase, the fluctuations of transverse Polyakov loop strongly suppressed,

Ratio Imaginary/Real and gluon screening

WHOT QCD Coll:
Y. Maezawa[1], S. Aoki[2], S. Ejiri[3], T. Hatsuda[4], N. Ishii[4], K. Kanaya[2], N. Ukita[5] and T. Umeda[6]
Phys. Rev. D81 091501 (2010)



- In the confined phase

$$\chi_{R,(I)} = 4\pi \int dr\ r^2 C_{R,(I)}(r)$$

$$C_{R,(I)}(r) = < L_{R,(I)}(r) L_{R,(I)}(0) >_c$$

- WHOT QCD Coll. (Y. Maezawa et al.)

$$C_{R,(I)}(r)_{r\to\infty} \to \gamma_{R,(I)}(T) \frac{e^{-M_{R,(I)} r}}{rT}$$

and WHOT-coll. identified $M_{R(I)}$ as the magnetic and electric mass:

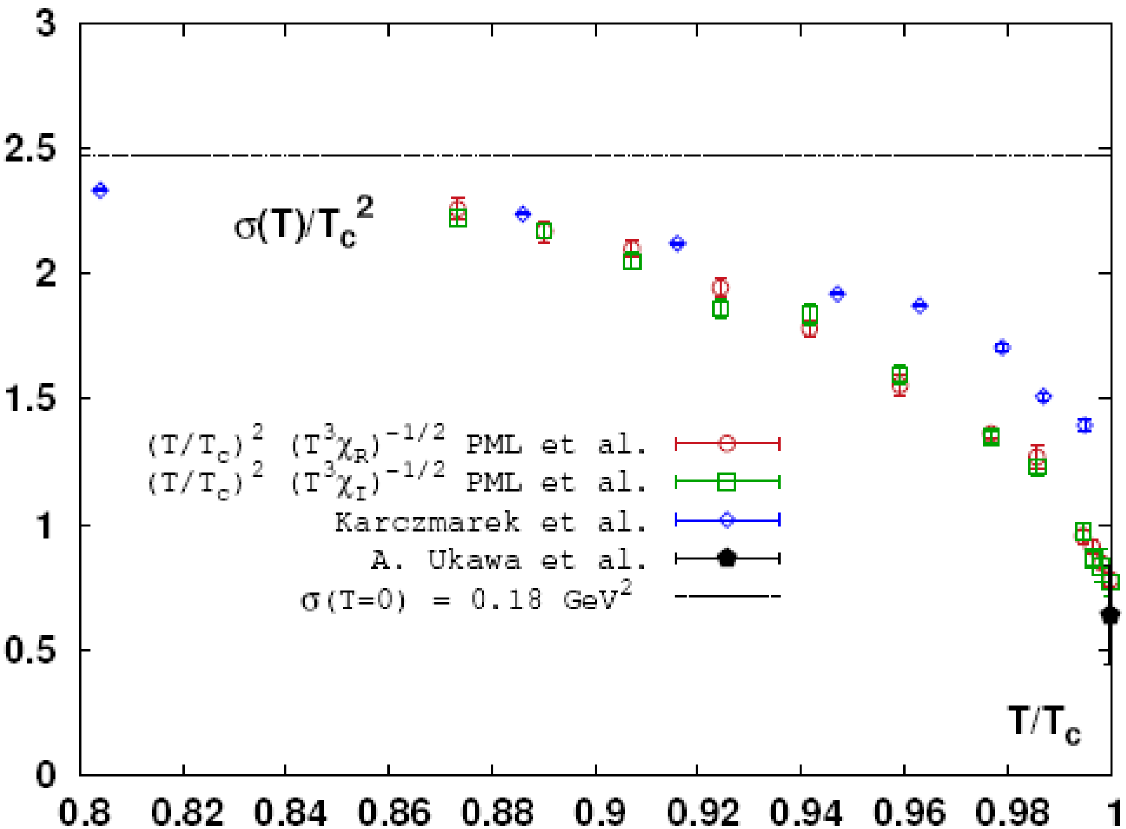
$$\chi_I \propto 1/m_E^2, \quad \chi_R \propto 1/m_M^2$$

Since

$$\boxed{m_E^2 >> m_M^2 \Rightarrow \chi_I << \chi_R}$$

String tension from the PL susceptibilities

Pok Man Lo, et al. (in preparation)



- $T < T_c \Rightarrow \chi_I = \chi_R$

$$\chi_{R,(I)} = 4\pi \int dr\ r^2 C_{R,(I)}(r)$$

- Common mass scale for $C_{R,(I)}(r)$

$$C_{R,(I)}(r) \approx \frac{e^{-M\ r}}{4\pi r T}$$

- In confined phase a natural choose for M

$$M = b / T$$

string tension

$$b(T)/T_c^2 \approx (T/T_c)^2 (T^3 \chi_{R,(I)})^{-1/2}$$

Ratios of the Polyakov loop fluctuations are excellent probes for deconfinement

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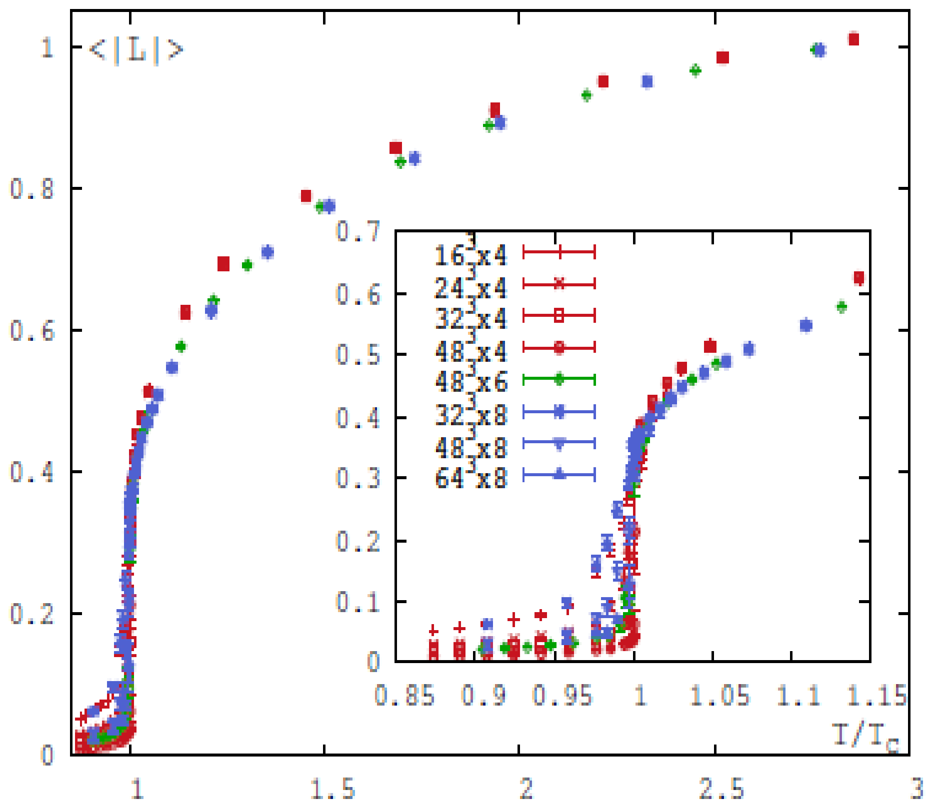
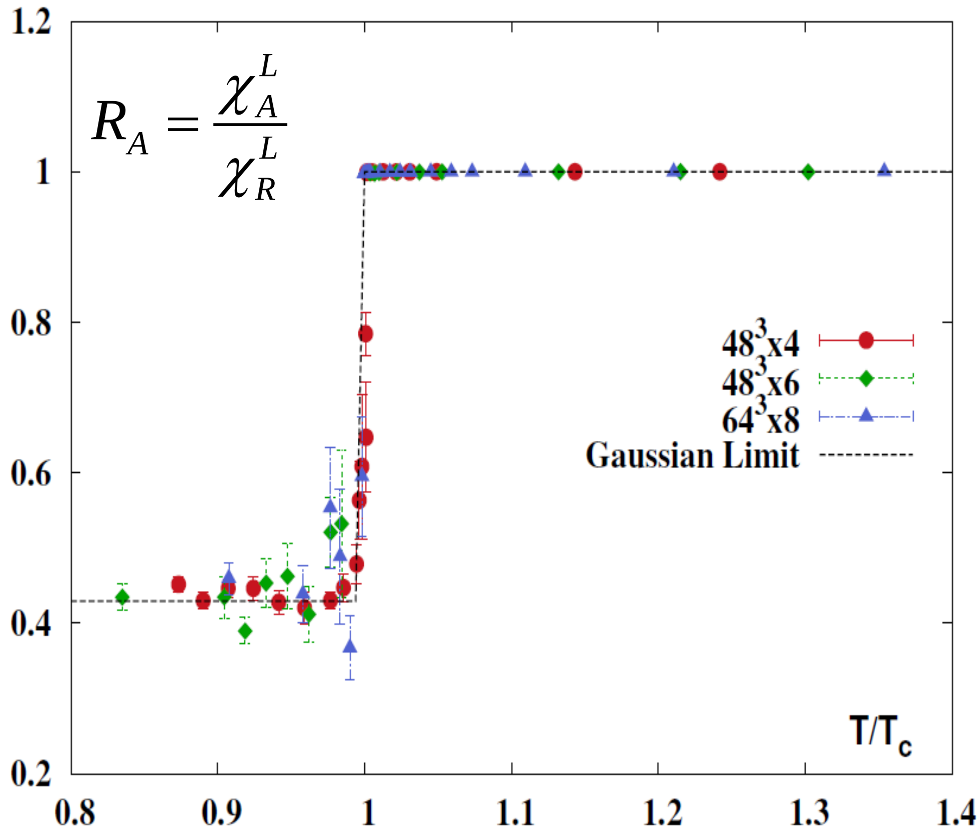




- How the above properties are modified when including quarks?

Modelling QCD phase diagram

- Preserve chiral $SU_L(N_f)\times SU_R(N_f)$ symmetry
  with $<\bar{\psi}\psi>$ condensate as an order parameter

- Preserve center $Z(N_c)$ symmetry with Polyakov loop

$$L=\frac{1}{N_c}Tr(P\exp[i\int_0^{\beta}d\tau A_4(\vec{x},\tau)])$$

as an order parameter

Polyakov loop dynamics ⟺ Confinement (R. Pisarski (2000))

Synthesis

Nambu & Jona-Lasinio ⟺ Spontaneous Chiral symmetry Breaking

K. Fukushima (2004)
C. Ratti & W. Weise (07)

PNJL Model

Effective chiral models and gluon potential

$$S = \int_0^{\beta=1/T} d\tau \int_V d^3x [i\bar{q}(\gamma_\mu \partial_\mu - A_\mu \delta_{\mu 4})q - V^{\mathrm{int}}(q,\bar{q}) + \mu_q q^+ q - U(L,L^*)]$$

$U(L,L^*) -$ the $Z(3)$ invariant Polyakov loop potential

(C. Sasaki et al; J. Pawlowski et al,.)

$V^{\mathrm{int}}(q,\bar{q}) -$ the SU(2)xSU(2) χ –invariant quark interactions described through:

- Nambu-Jona-Lasinio model ⇨ PNJL chiral model

  K. Fukushima; C. Ratti & W. Weise; B. Friman , C. Sasaki ., ….

- coupling with meson fields ⇨ PQM chiral model

  B.-J. Schaefer, J.M. Pawlowski & J. Wambach; B. Friman, V. Skokov, ...

- FRG thermodynamics of PQM model:

  B. Friman, V. Skokov, B. Stokic & K.R., …..

Effective QCD-like models

$$L_{PNJL} = \bar{q}(iD_\mu - m)q + G_S[(\bar{q}q)^2 + (\bar{q}i\vec{\tau}\gamma_5 q)^2] - G_V^{(S)}(\bar{q}\gamma_\mu q)^2$$

$$-G_V^{(V)}(\bar{q}\vec{\tau}\gamma_\mu q)^2 + \mu_q q^+ q + \mu_I q^+ \tau_3 q - U(\Phi[A], \bar{\Phi}[A], T)$$

K. Fukushima, C. Ratti & W. Weise, B. Friman & C. Sasaki , ., ...

B.-J. Schaefer, J.M. Pawlowski & J. Wambach; B. Friman et al.

$$L_{PQM} = \bar{q}(iD_\mu - g[\sigma + i\gamma_5 \vec{\tau}\vec{\pi}])q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2$$

$$-U(\Phi[A], \bar{\Phi}[A], T) - U(\sigma, \vec{\pi}^2)$$

$$D_\mu = \partial_\mu - i\delta_{\mu 0} A_0 \qquad \Phi = \frac{1}{N_c} Tr(P \exp[i\int d\tau A_4(\vec{x}, \tau)])$$

Polyakov loop

Extendet PNJL model and its mean field dynamics

$$L_{NJL} = \bar{q}(iD_\mu - m)q + G_S[(\bar{q}q)^2 + (\bar{q}i\vec{\tau}\gamma_5 q)^2] - G_V^{(S)}(\bar{q}\gamma_\mu q)^2$$

$$-G_V^{(V)}(\bar{q}\vec{\tau}\gamma_\mu q)^2 + \mu_q q^+ q + \mu_I q^+ \tau_3 q - U(\Phi[A], \bar{\Phi}[A], T)$$

$$D_\mu = \partial_\mu - i\delta_{\mu 0} A_\mu \qquad \Phi = \frac{1}{N_c} Tr(P\exp[i\int d\tau A_4(\vec{x},\tau)])$$

Polyakov loop

G_S, G_V^S, G_V^V : Strength of quarks interactions in scalar and vector sector

- Thermodynamic potential: mean-field approximation

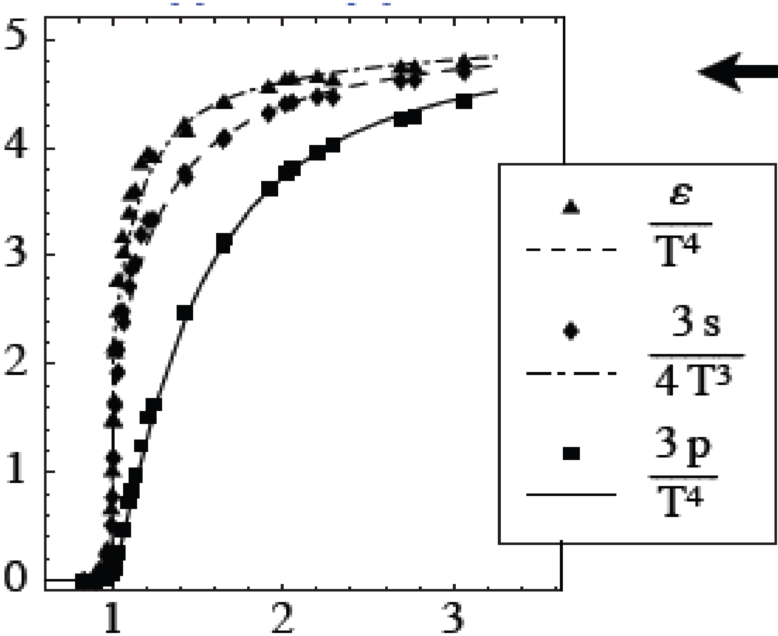
$$\Omega = \Omega(T, M_{(u,d)}, \tilde{\mu}_q, \tilde{\mu}_I, <\Phi>, <\bar{\Phi}>)$$

  - $M_{u,d} \sim <\bar{q}q>$: dynamical (u,d)-quark masses, shifted chemical potentials $\tilde{\mu}_i$ and thermal averages of Polyakov loops $<\Phi>$ obtained from the stationary conditions: $\partial\Omega(T,\vec{x})/\partial x_i = 0$

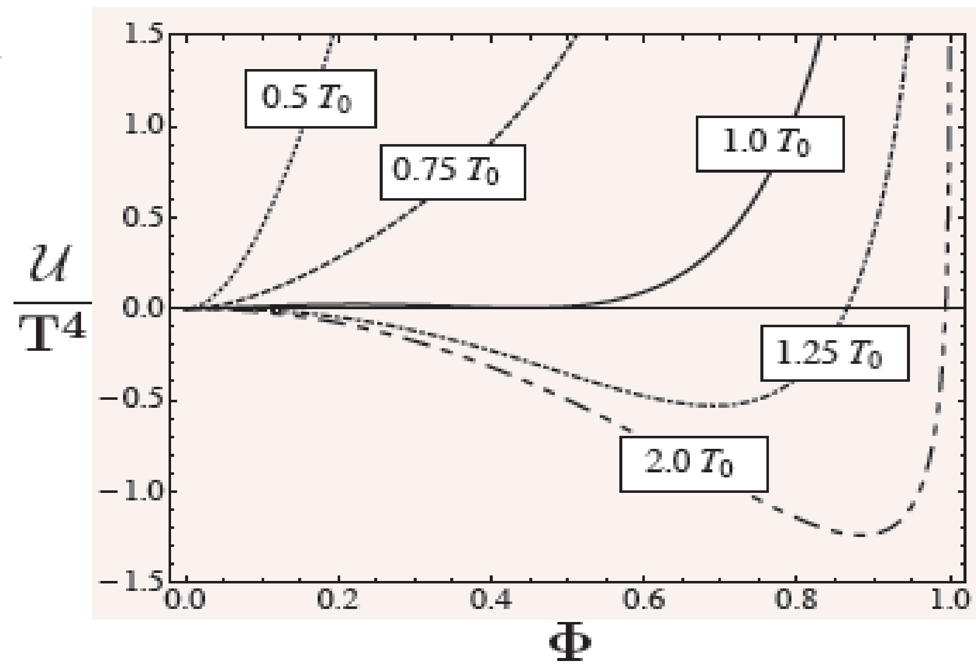
Polyakov loop parameters, fixed from the pure glue Lattice Thermodynamics

$$-P^G/T^4 = U(\Phi\ ,\Phi^*) = -b_2(T)\ \Phi^*\Phi - b_3(T)\ (\Phi^{*3}+\Phi^3) + b_4(T)\ (\Phi^*\Phi)^2$$

- $b_k(T)$ – fixed to reproduce pure SU(3) lattice results



lattice results:
O. Kaczmarek et al. PLB 543 (2002) 41



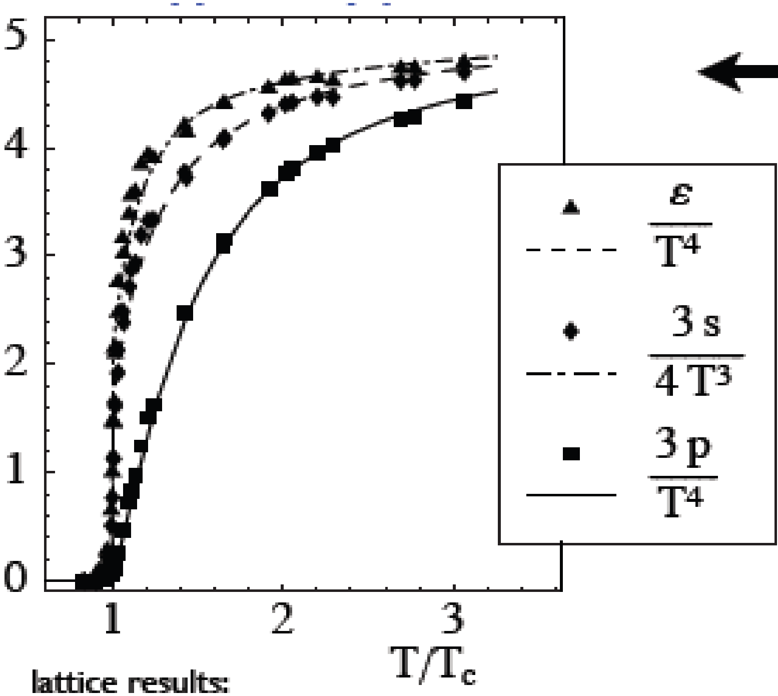
C. Ratti & W. Weise 07

- Polynomial potential results in $\chi_I > \chi_R$ thus is not applicable!

Polyakov loop parameters, fixed from a pure glue Lattice Thermodynamics

$$U(L\ ,L^{*}) = -b_2(T)\ \ L^{*}L - b_3(T)\ \ \ln[M(L^{*},L)]$$

- $b_k(T)$ – fixed to reproduce pure SU(3) lattice results



lattice results:
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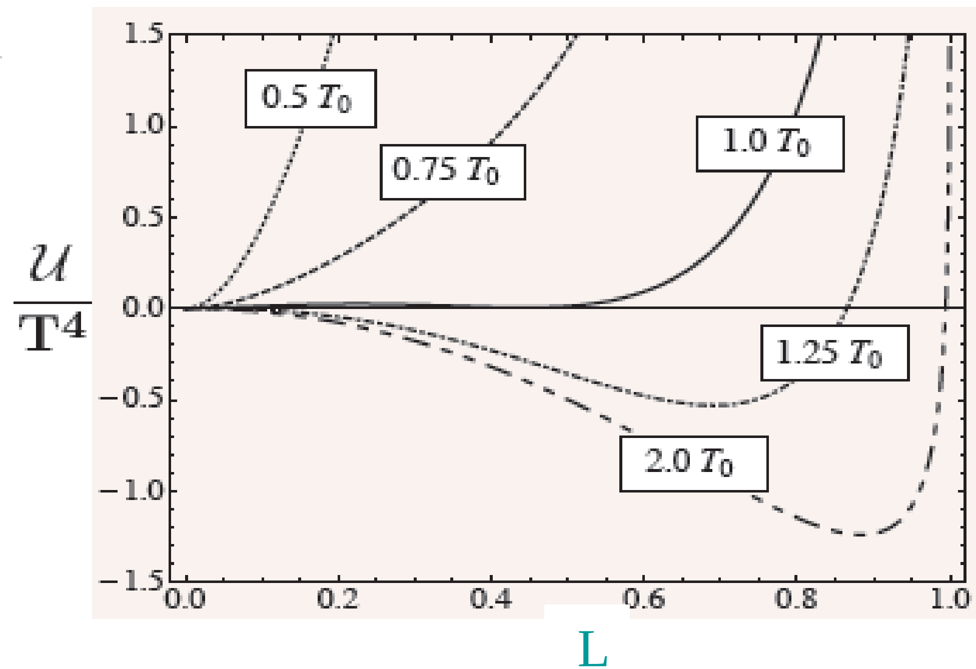


K. Fukushima, C. Ratti & W. Weise

L

Effective Polyakov loop Potential from Y-M Lagrangian

Chihiro Sasaki & K.R.

Deriving partition function from YM Lagrangian

$$Z = \int \mathcal{D}A_\mu \mathcal{D}C \mathcal{D}\bar{C} \exp\left[i \int d^4x \mathcal{L}\right] , \quad \mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}$$

1. employ background field method. (Gross, Pisarski & Yaffe)

$$A_\mu = \bar{A}_\mu + g\breve{A}_\mu$$

2. collect terms quadratic in quantum fields.

$$\mathcal{L}^{(2)} = -\frac{1}{2}\breve{A}^a_\alpha \left[\delta_{ab} g^{\alpha\beta}\partial^2 - f_{abc}\left(\partial^\beta \bar{A}^{\alpha,c} + 2g^{\alpha\beta}\bar{A}^c_\mu \partial^\mu\right) + f_{ac\bar{c}} f_{cb\bar{d}} g^{\alpha\beta} \bar{A}^{\bar{c}}_\mu \bar{A}^{\mu,\bar{d}} + 2f_{abc}\bar{A}^{\alpha\beta,c}\right] \breve{A}_\beta{}^b$$

3. consider a constant uniform background $\bar{A}_0$.

$$\bar{A}^a_\mu = \bar{A}^a_0 \delta_{\mu 0} , \quad \bar{A}_0 = \bar{A}^3_0 T^3 + \bar{A}^8_0 T^8$$

4. calculate propagator inverse and diagonalize it.

.

5. from Minkowski to Euclidean space: carry out Matsubara summation.

$$\sum_n \ln\det\left(D^{-1}\right) = \ln\det\left(1 - \hat{L}_A e^{-|\vec{p}|/T}\right)$$

$$\hat{L}_A = \text{diag}\left(1\,,1\,,e^{i(\phi_1-\phi_2)}\,,e^{-i(\phi_1-\phi_2)}\,,e^{i(2\phi_1+\phi_2)}\,,e^{-i(2\phi_1+\phi_2)}\,,e^{i(\phi_1+2\phi_2)}\,,e^{-i(\phi_1+2\phi_2)}\right)$$

- thermodynamic potential (gluon part)

$$\boxed{\Omega_g = 2T\int\frac{d^3p}{(2\pi)^3}\text{tr}\ln\left(1 - \hat{L}_A\, e^{-|\vec{p}|/T}\right)}$$

traced Polyakov loops $\Phi = \text{tr}\hat{L}_F/N_c\,,\ \bar{\Phi} = \text{tr}\hat{L}_F^{\dagger}/N_c$ (gauge invariant)

full thermodynamics potential: $\Omega = \Omega_g + \Omega_{\text{Haar}}$

$$\begin{aligned}
\Omega_g &= 2T\int\frac{d^3p}{(2\pi)^3}\ln\left(1 + \sum_{n=1}^{7} C_n\, e^{-n|\vec{p}|/T} + e^{-8|\vec{p}|/T}\right)\,,\\
\Omega_{\text{Haar}} &= -a_0 T\ln\left[1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2\right]\,,\\
C_1 &= C_7 = 1 - N_c^2\bar{\Phi}\Phi\,,\quad C_2 = C_6 = 1 - 3N_c^2\bar{\Phi}\Phi + N_c^3\left(\bar{\Phi}^3 + \Phi^3\right)\,,\\
C_3 &= C_5 = -2 + 3N_c^2\bar{\Phi}\Phi - N_c^4\left(\bar{\Phi}\Phi\right)^2\,,\\
C_4 &= 2\left[-1 + N_c^2\bar{\Phi}\Phi - N_c^3\left(\bar{\Phi}^3 + \Phi^3\right) + N_c^4\left(\bar{\Phi}\Phi\right)^2\right]
\end{aligned}$$

$\Rightarrow$ energy distributions solely determined by group characters of SU(3)

Thermodynamics

- high temperature limit: $\Phi \to 1 \Rightarrow$ non-int. gluon gas

$$\Omega_g(\Phi = \bar{\Phi} = 1) = 16T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - e^{-|\vec{p}|/T}\right)$$

- any finite temperature in confined phase: $\Phi = 0$ thus $\Omega_{\text{Haar}} = 0$

$$\Omega_g(\Phi = \bar{\Phi} = 0) \sim 2T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-|\vec{p}|/T}\right)$$

wrong sign! $\Rightarrow$ unphysical EoS $s, \epsilon < 0$

Gluons are NOT correct dynamical variables below T_c!

cf. PNJL/PQM: quarks are suppressed but exist at any T.

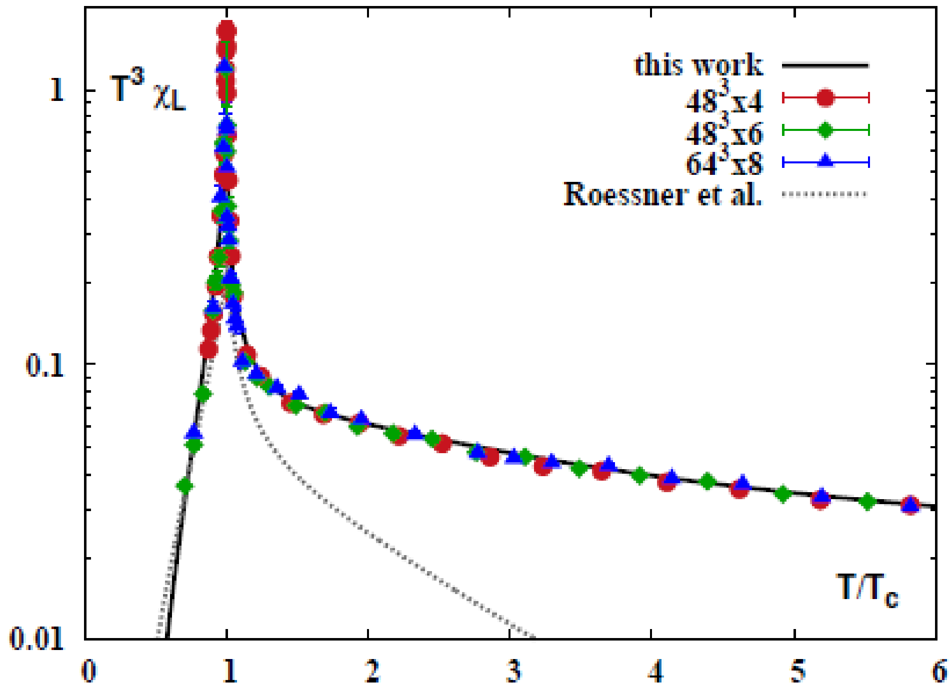
- **higher representations of Polyakov loop**
  - non-vanishing in confined phase *within mean field approx.*
  - do not condense when energy distributions are expressed in fund. rep.

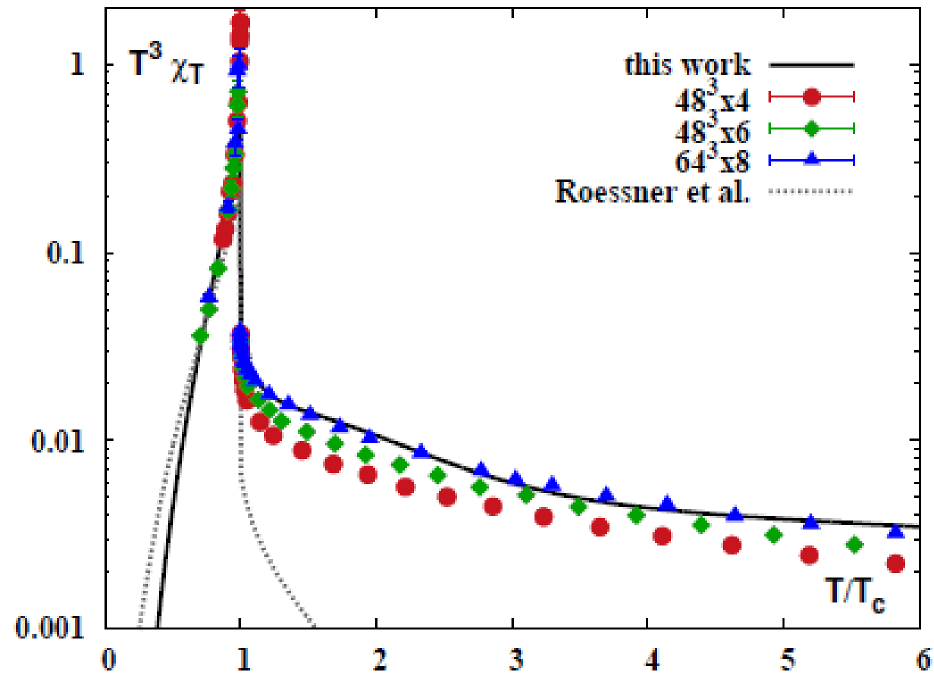
$\Rightarrow$ the correct physics restored!

- The minimal potential needed to incorporate Polyakov loop fluctuations

$$\frac{U(L,\bar{L})}{T^4} = -\frac{1}{2}a(T)\bar{L}L + b(T)\ln M_H(L,\bar{L})$$
$$+\frac{1}{2}c(T)(L^3+\bar{L}^3) + d(T)(\bar{L}L)^2,$$

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.





Thermodynamics of PQM model under MF approximation in the large quark mass limit

- Thermodynamic potential has pure gluon and quark-antiquark contribution

$$\Omega = \Omega_g + \Omega_q + \Omega_{\bar{q}} + \Omega_{Haar}$$

- Fermion contribution to thermodynamic potential

$$\boxed{\Omega_q \approx \int d^3p \ln[1 + 3L\, e^{-(E_q+\mu)/T} + 3L^* e^{-2(E_q-\mu)/T} + e^{-3(E_q+\mu)/T}]}$$

- Consider a limit of large quark mass

$$\Omega_q + \Omega_{\bar{q}} \approx N_f \int d^3p (L\, e^{-(E_q+\mu)/T} + L^* e^{-2(E_q-\mu)/T}]$$

$$\Omega_q + \Omega_{\bar{q}} \approx h(N_f, M_q, T) \cdot L_R$$

- Where the Polyakov loops obtained from gap equation

$$\frac{\partial \Omega}{\partial L} = 0 \qquad \frac{\partial \Omega}{\partial \bar{L}} = 0$$

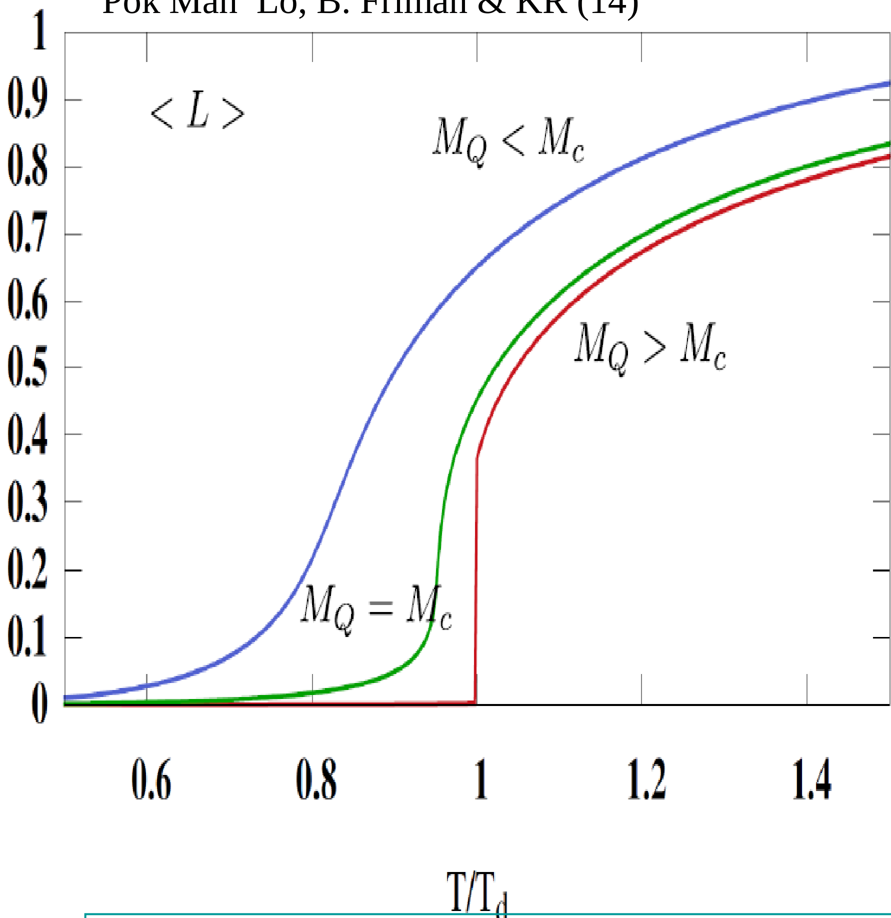
- Modelling the partition function form QCD in background field approach

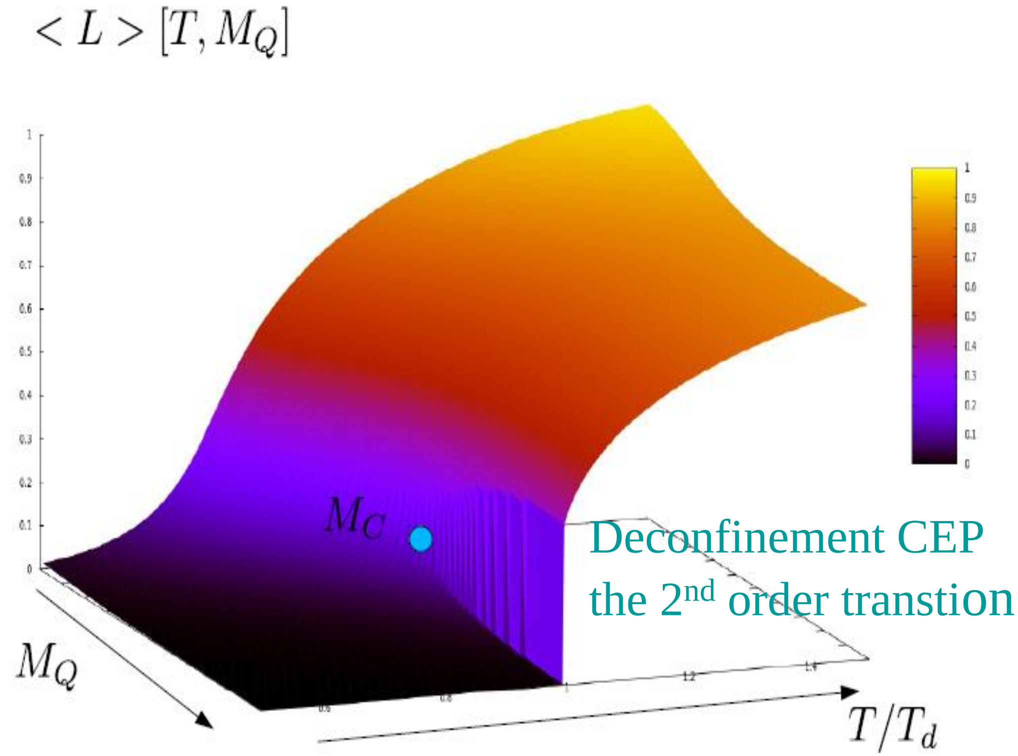
$$Z = \int dL dL^{+} e^{-\beta V U(L,L^{+}) + \ln\det[Q_f]}$$

$$\ln\det[Q_f] \approx -h_{eff} L_R$$

$$h_{eff} \approx N_f (M_q / T)^2 K_2(M_q / T)$$

Pok Man Lo, B. Friman & KR (14)

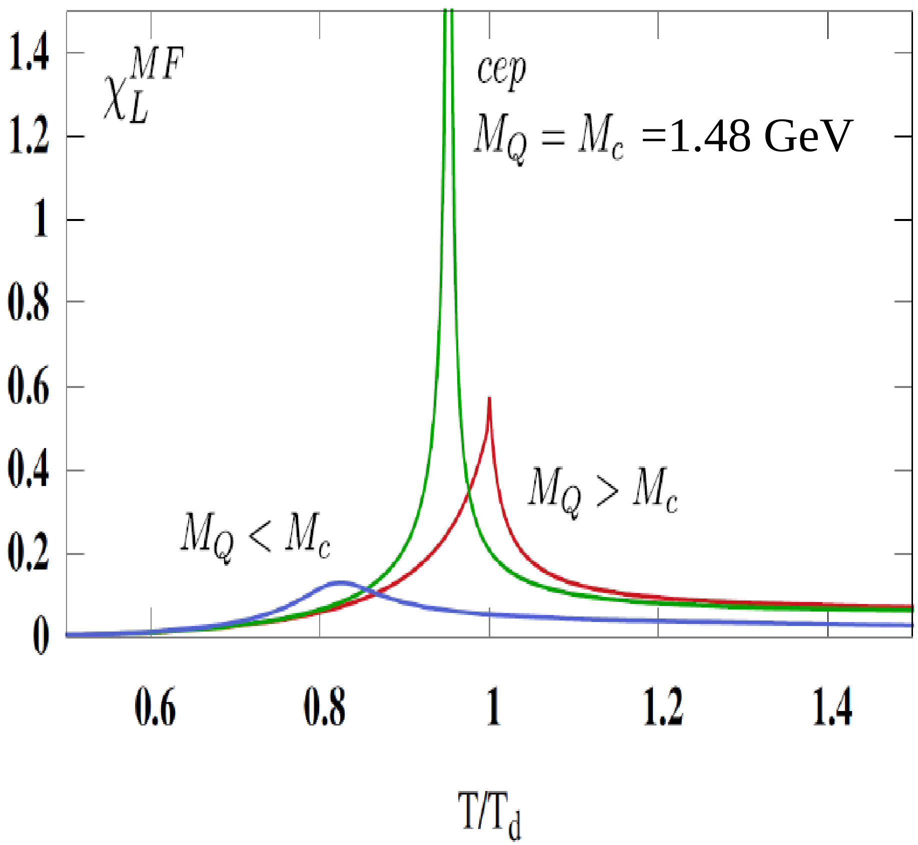


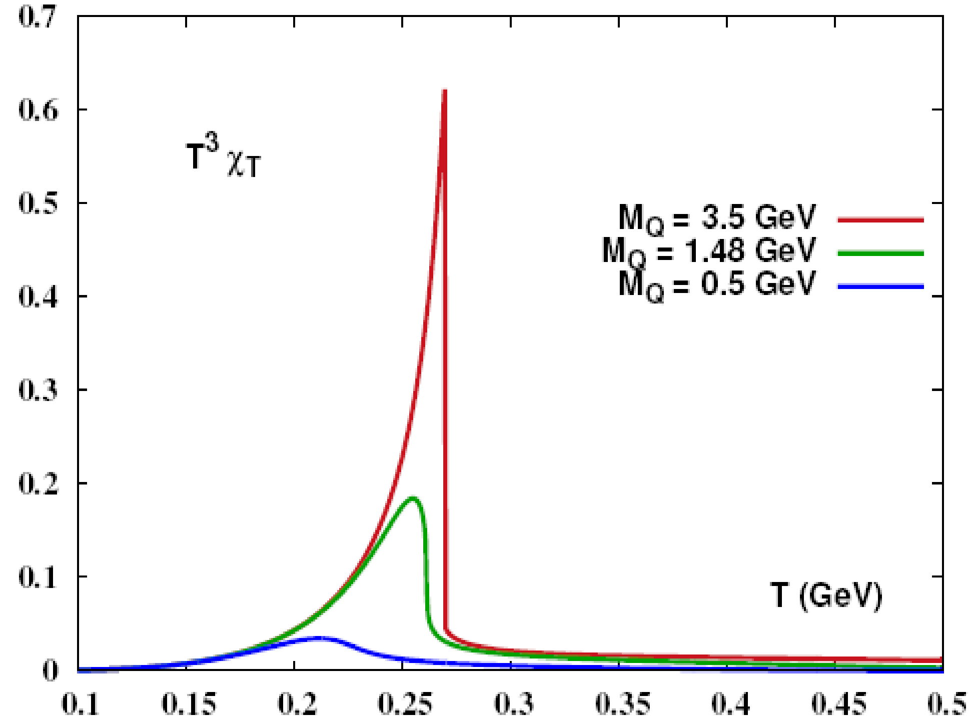


Deconfinement CEP appears in effective QCD at $M_Q \approx 1.5$ GeV

Susceptibility at the deconfinement critical endpoint

Pok Man Lo, B. Friman & KR (14)





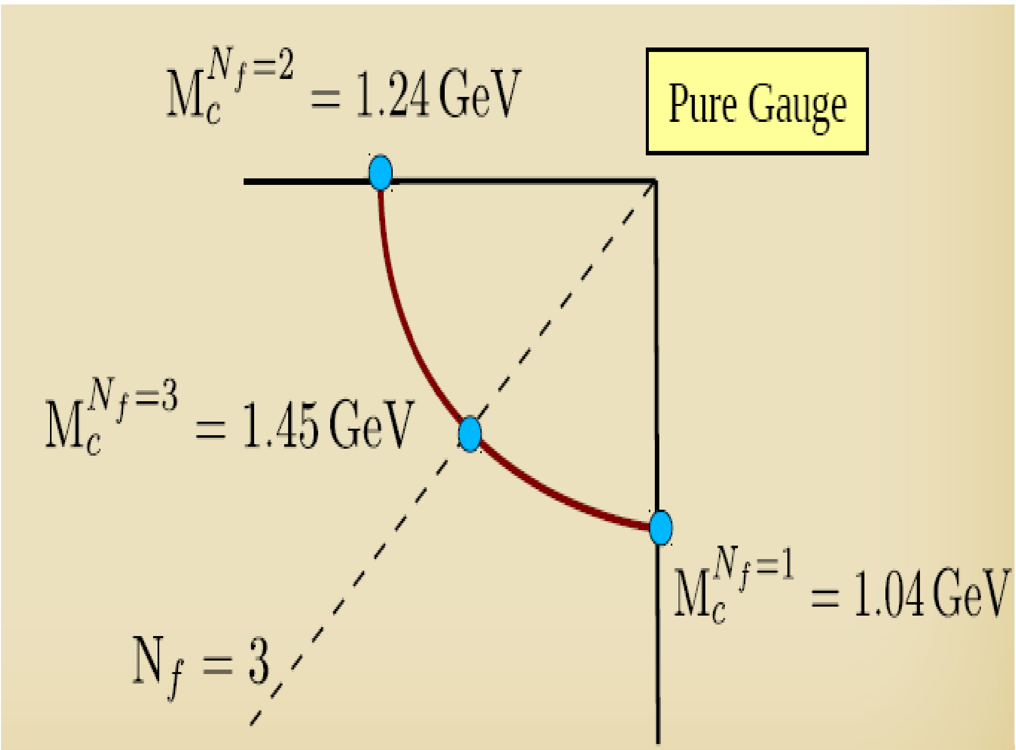
- Divergent longitudinal susceptibility at the critical point

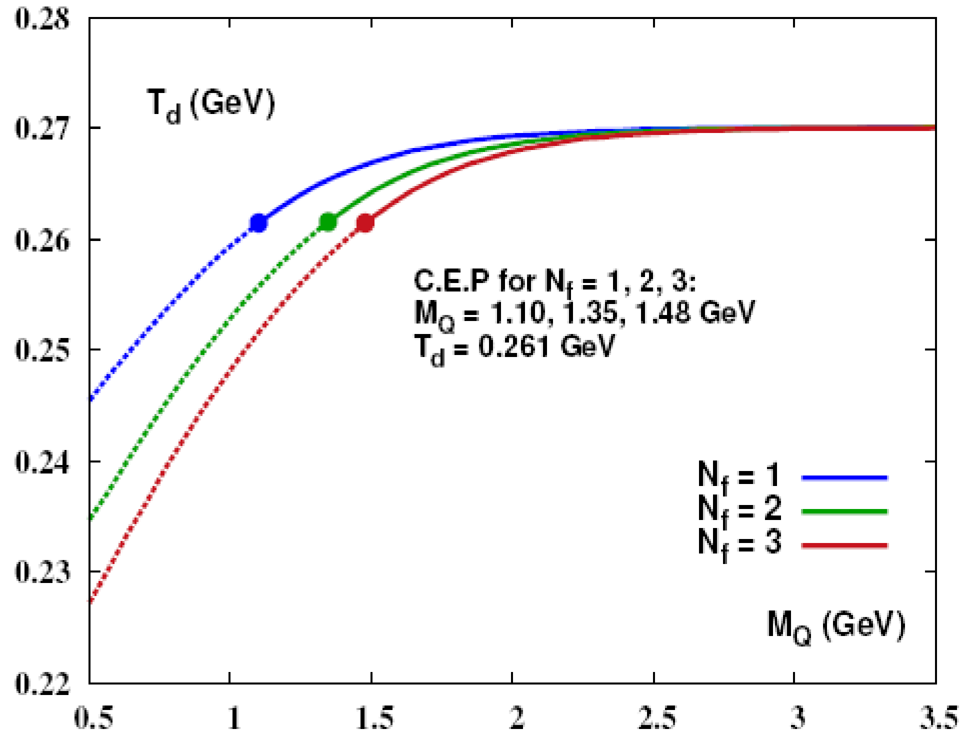
See also LGT results for the posstion of CEP

H. Saito, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno, and T. Umeda, Phys. Rev. D **84** (2011) 054502

Critical masses and temperature values

Pok Man Lo, et al.





- Different values then in the matrix model by

$$M_c^{N_f=3} \approx 2.5\,\mathrm{GeV}$$

$$T_c^{\mathrm{de}} \approx 0.27\,\mathrm{GeV}$$

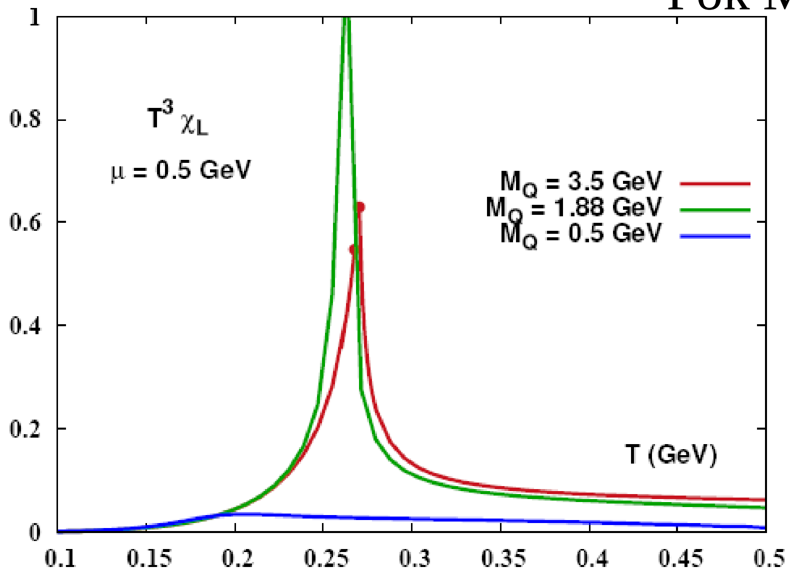
K. Kashiwa, R. Pisarski and V. Skokov, Phys. Rev. D85 (2012)

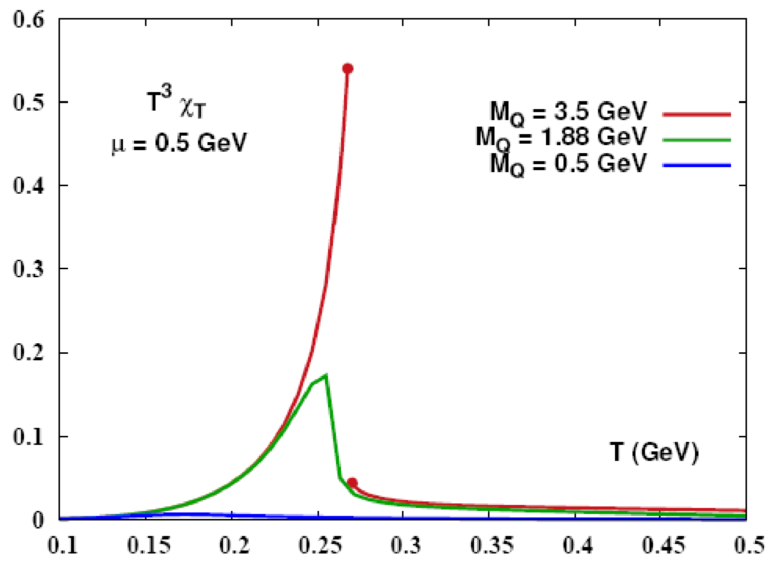
LGT C. Alexandrou et al. (99) $M_c^{N_f=3} \approx 1.4 GeV$

Phase diagram for large quark mass and finite density

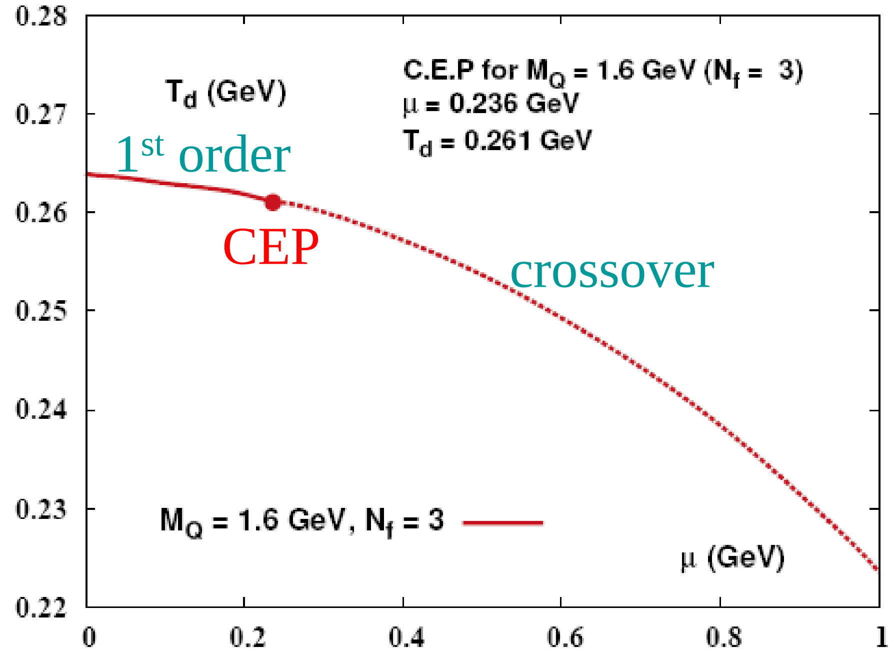
- Susceptibilities:

Pok Man Lo, et al.





- Phase diagram



- Critical temperature at CEP is μ independent of , however $\mu_c(m)$ is strongly changing with the quark mass

$$\Omega_q + \Omega_{\bar{q}} \approx h(N_f, M_q, T) \cdot L_L \cosh(\mu)$$

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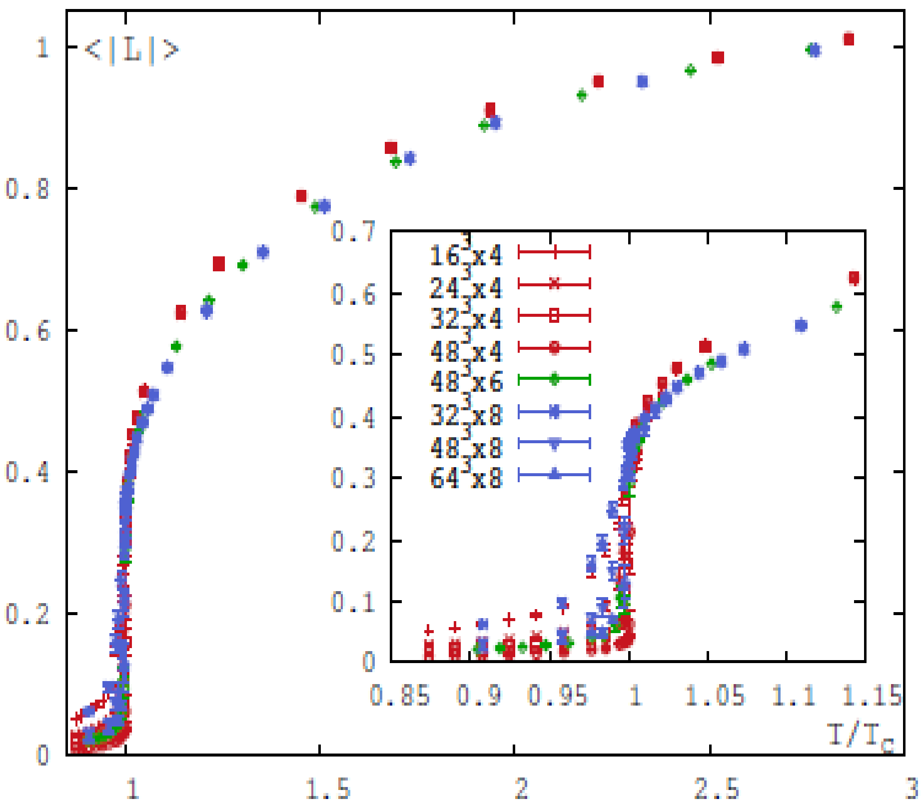
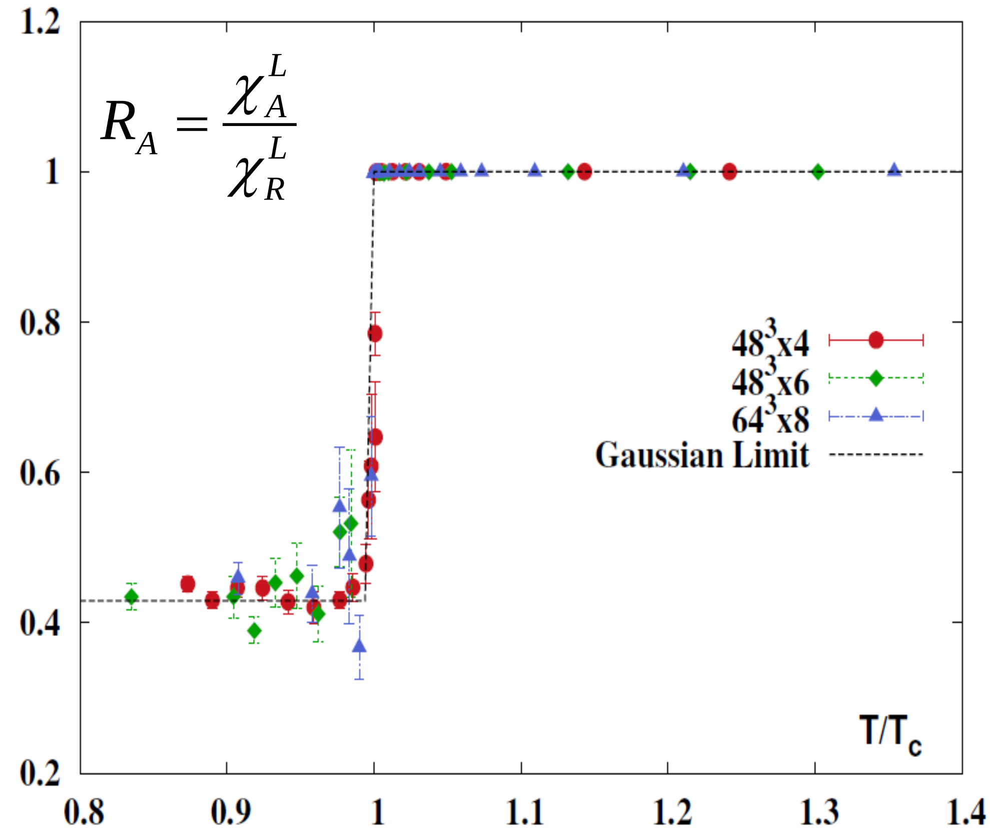
C. Sasaki & K.R. , PRD (2013)

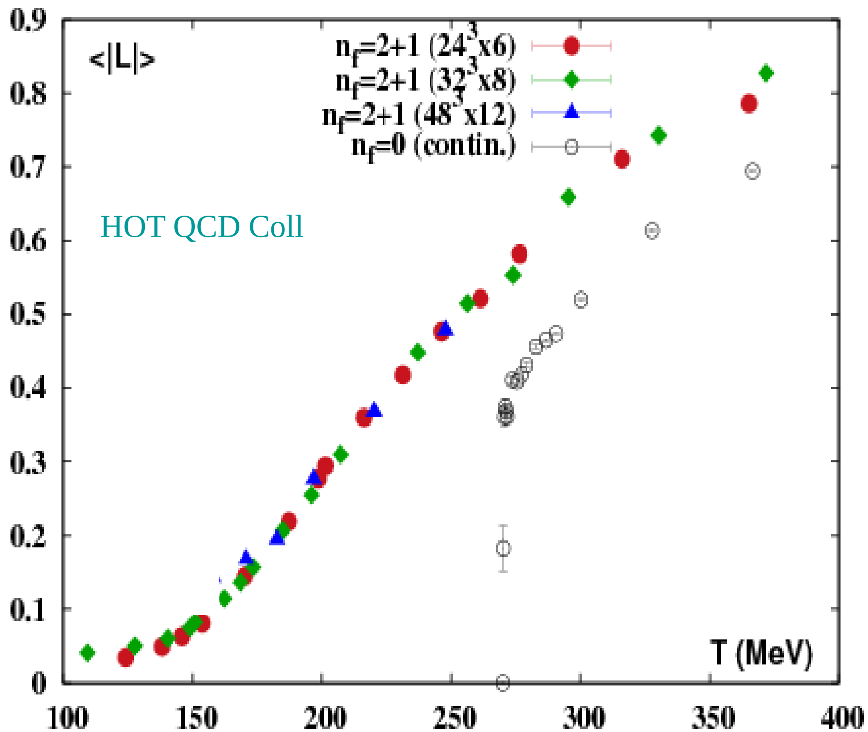


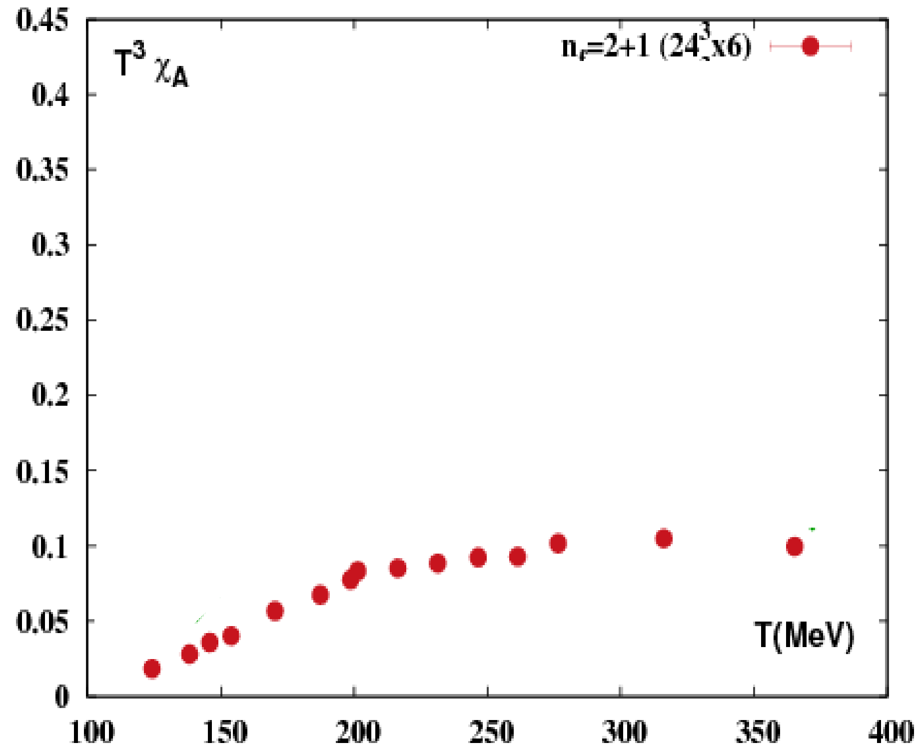


- How the above properties are modified when including quarks?

Polyakov loop and fluctuations in QCD

- Smooth behavior for the Polyakov loop and fluctuations
  ⇨ difficult to determine where is “deconfinement”



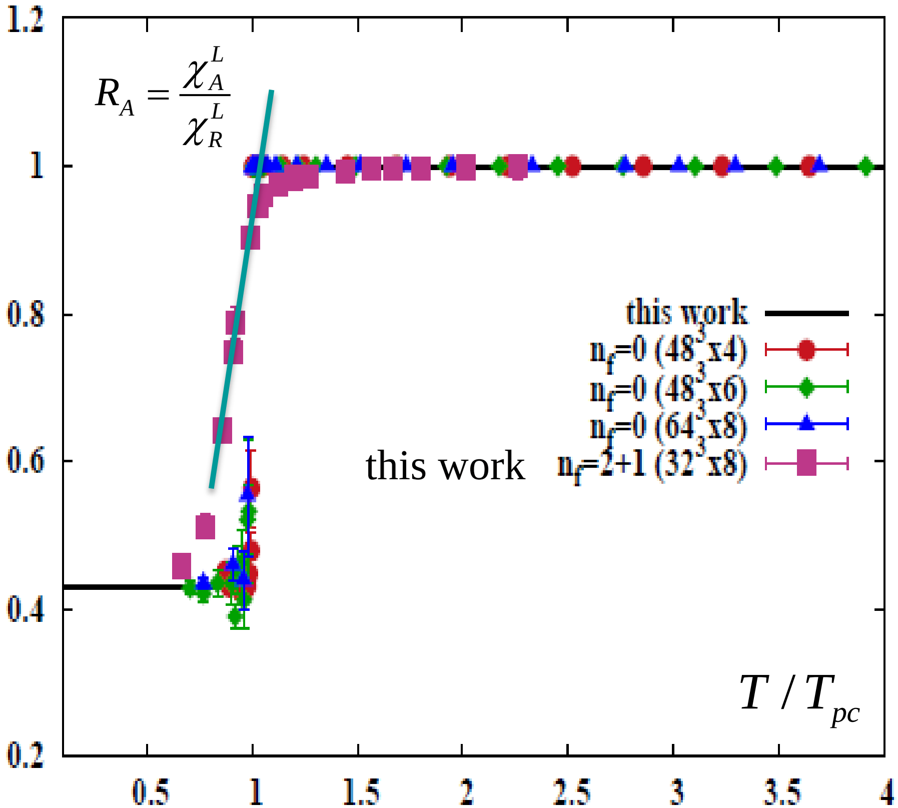


The inflection point at $T_{dec} \approx 0.22 GeV$

The influence of fermions on the Polyakov loop susceptibility ratio

- Z(3) symmetry broken, however ratios still showing deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.



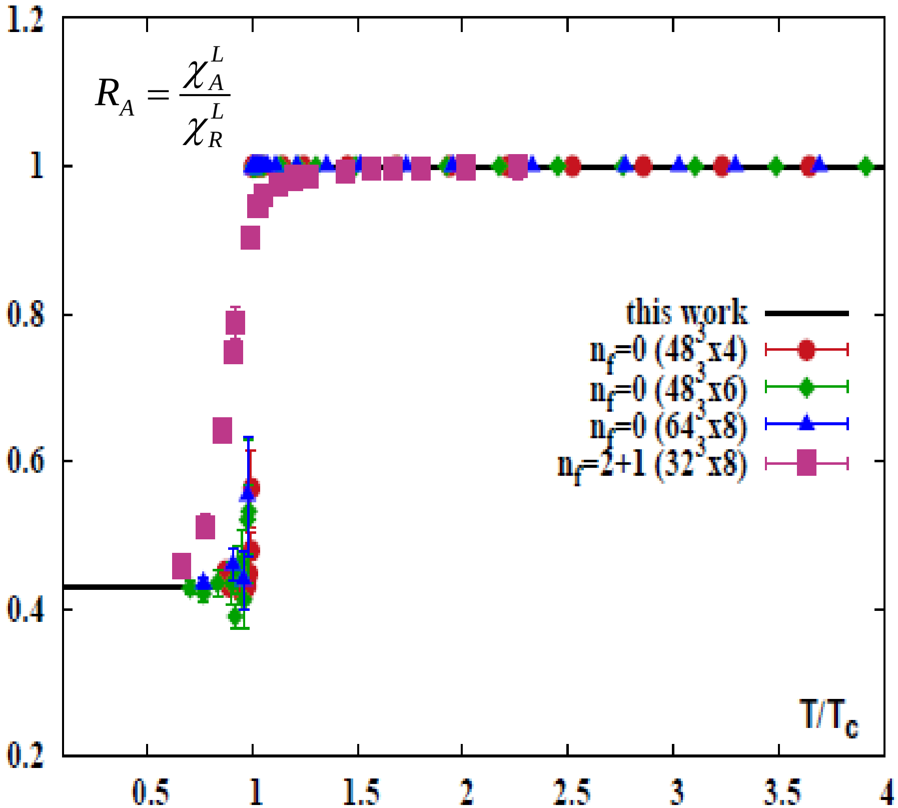
- Change of the slope in the narrow temperature range signals color deconfinement

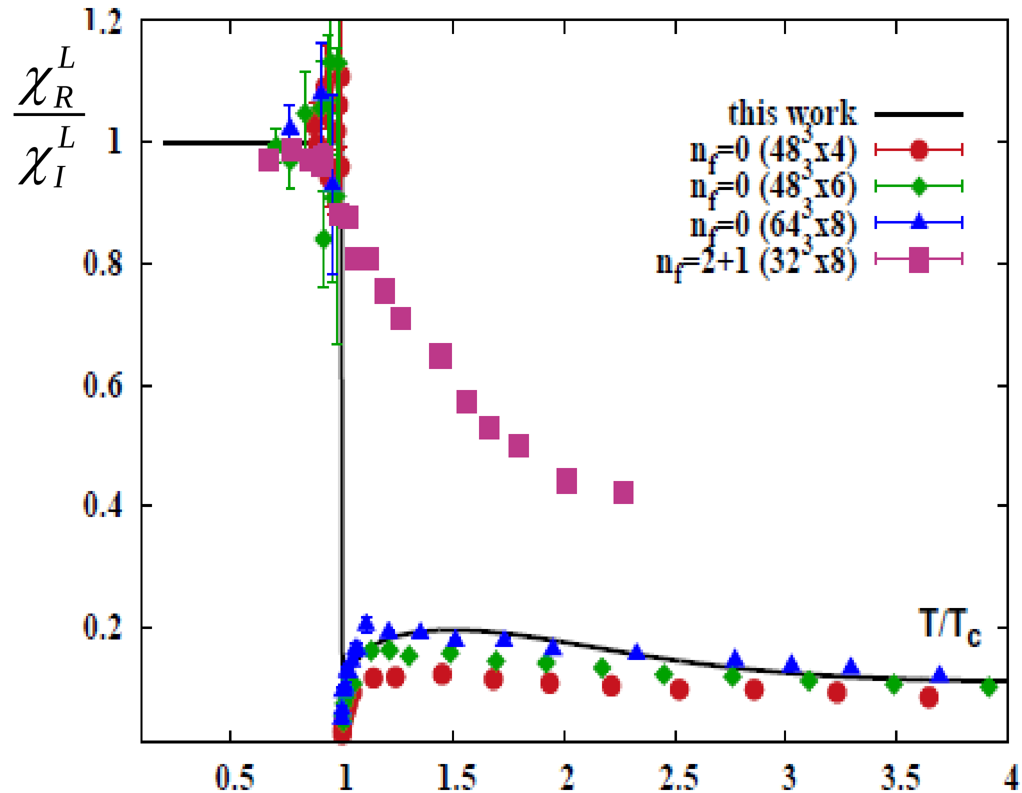
- Dynamical quarks imply smoothening of the susceptibilities ratio, between the limiting values as in the SU(3) pure gauge theory

The influence of fermions on ratios of the Polyakov loop susceptibilities

- Z(3) symmetry broken, however ratios still showing the transition
- Change of the slopes at fixed T

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.





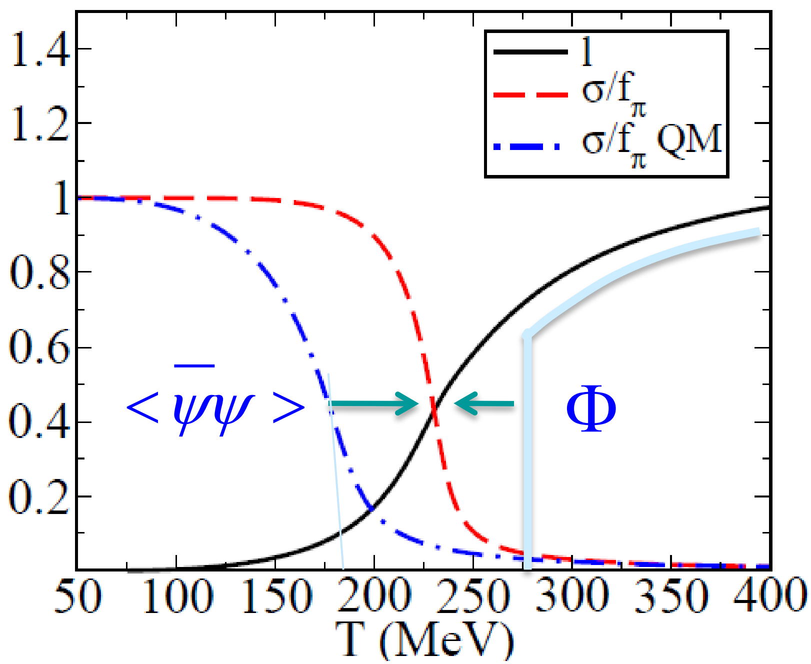
Thermodynamics of PQM model under MF approximation

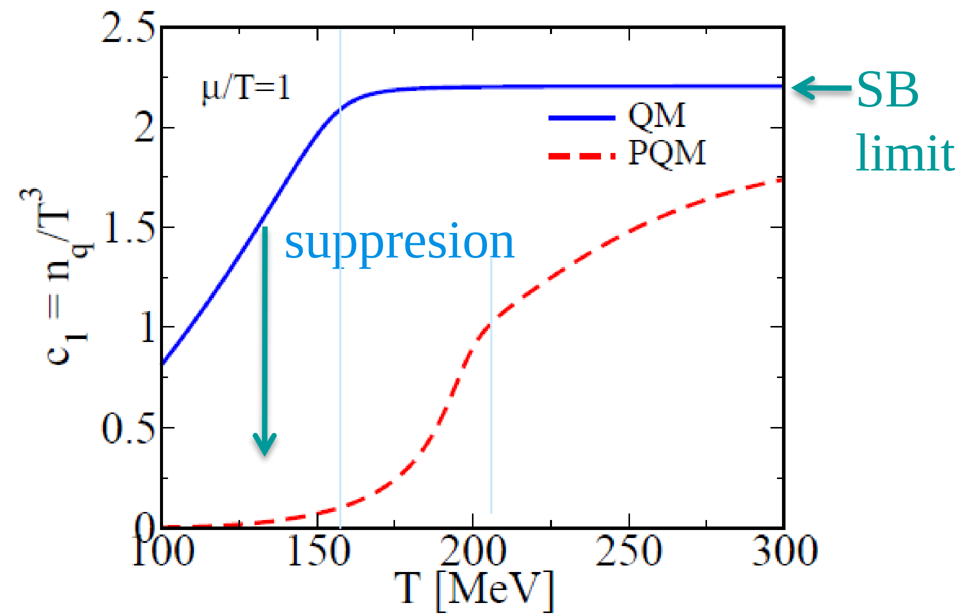
- Fermion contribution to thermodynamic potential

$$\Omega_{q\bar{q}} \approx \int d^3p(\ln[1+3\Phi\, e^{-(E_q+\mu)/T} + 3\Phi^* e^{-2(E_q-\mu)/T} + e^{-3(E_q+\mu)/T}] +(q \leftrightarrow \bar{q})$$

Entanglement of deconfinement and chiral symmetry

Suppresion of thermodynamics due to „statistical confinement”

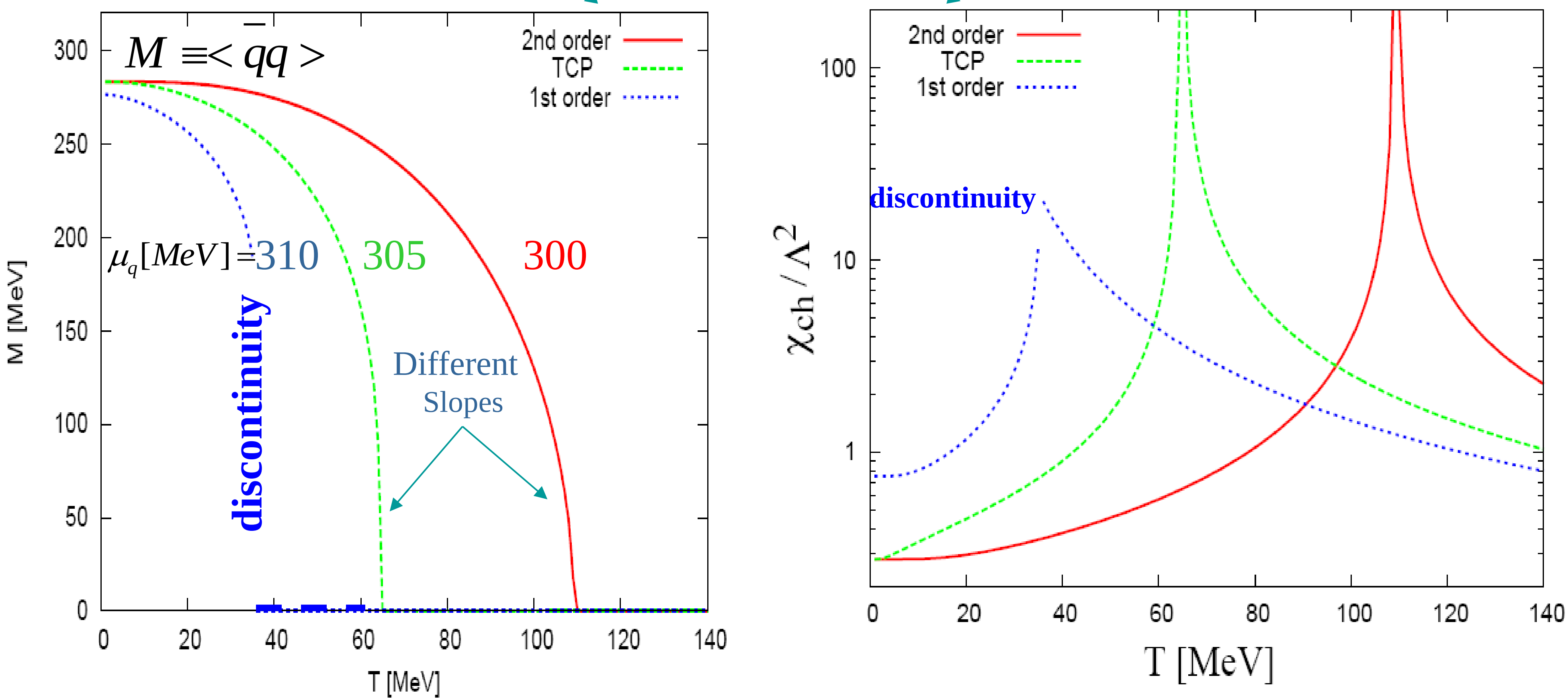




Chiral Symmetry Restoration – Order Parameter

Fixed μ_q

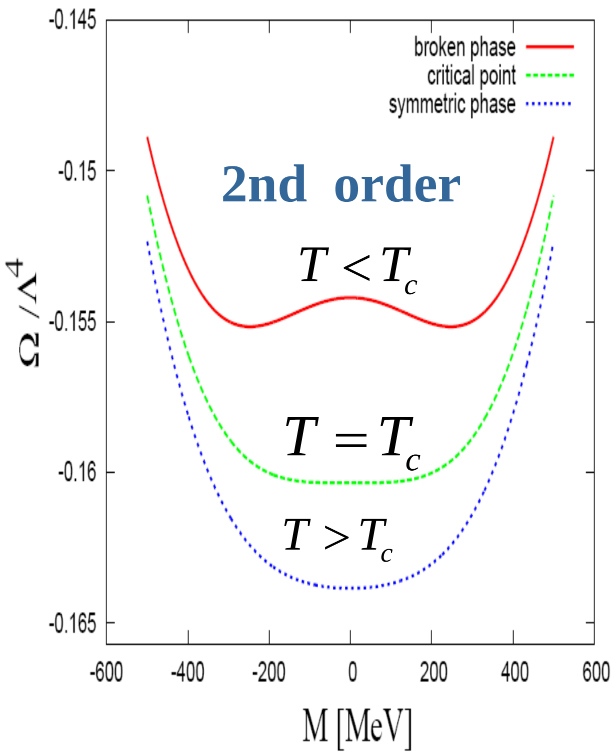
B. Friman, C. Sasaki et al.

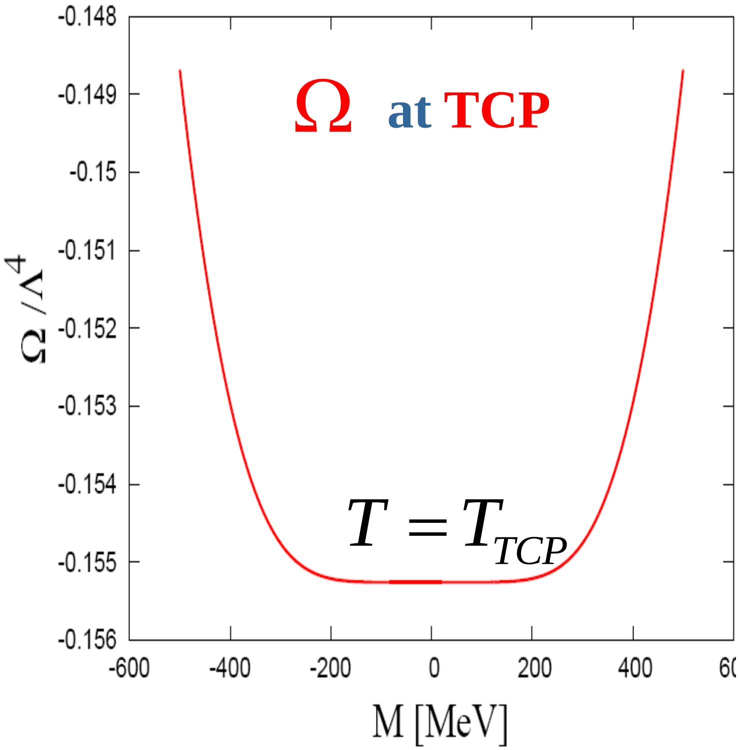


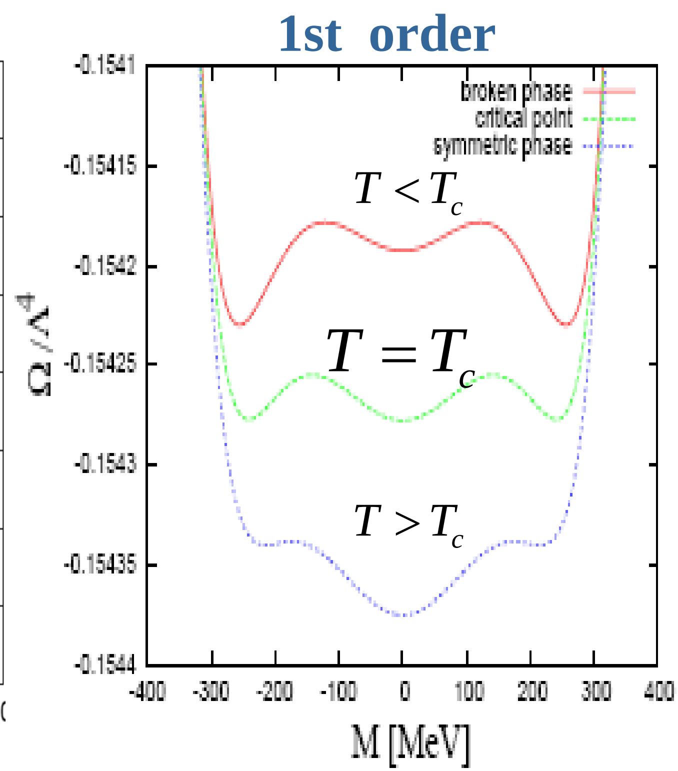
Divergence of the chiral susceptibility at the 2nd order transition and at the TCP

Discontinuity of the chiral susceptibility: at the 1st order transition

Effective Thermodynamic Potentials







Flattening of the potential at TCP: **indeed expanding thermodynamic Ω near $M = 0$**

$$\Omega(T, \vec{\mu}, M) \approx a_2(T, \vec{\mu}) M^2 + a_4(T, \vec{\mu}) M^4 + a_6(T, \vec{\mu}) M^6$$

Landau – Ginzburg potential

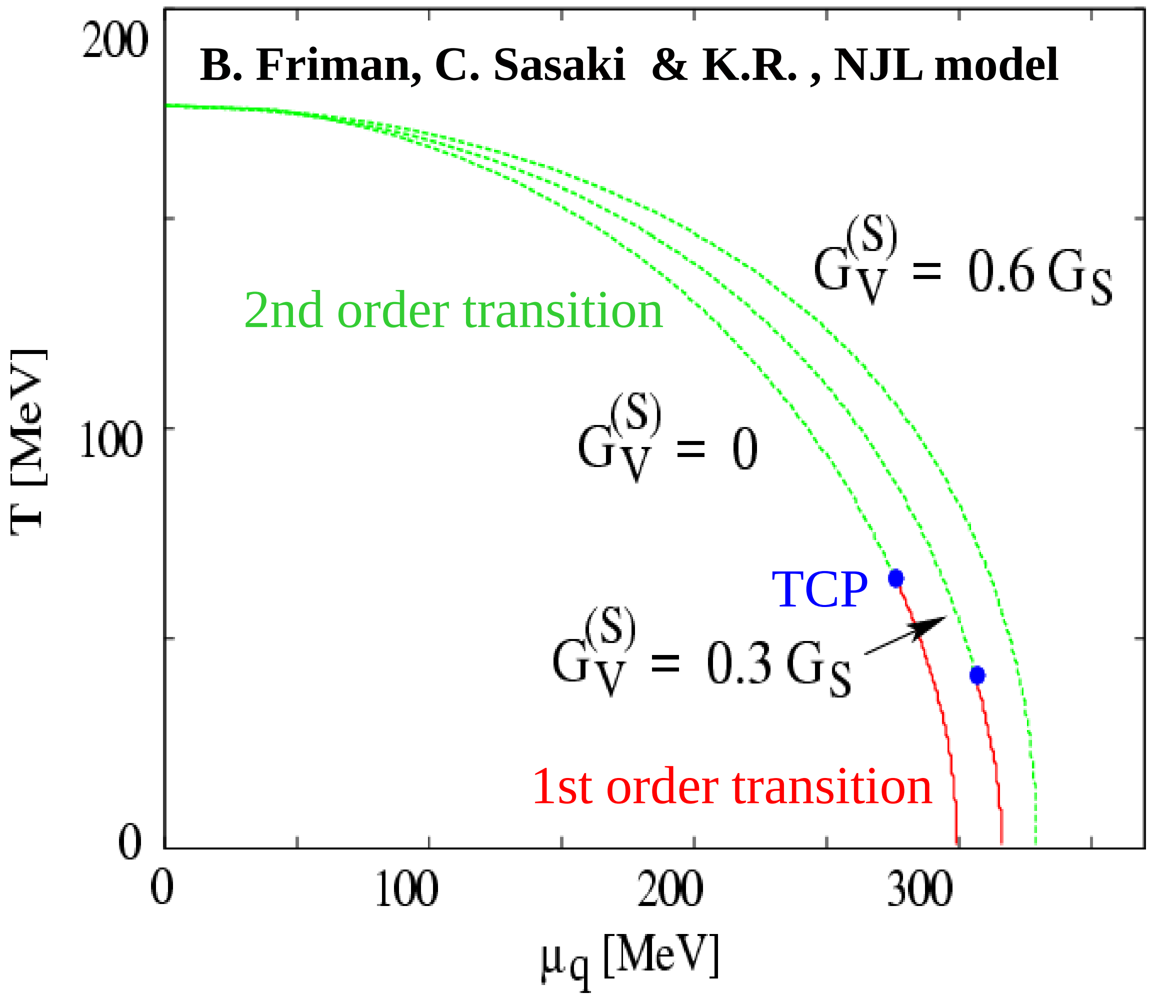
finds:

$a_2(T_c, \vec{\mu}_c) = 0 \qquad a_4(T_c, \vec{\mu}_c) \neq 0$ **2nd order**

$a_2(T_c, \vec{\mu}_c) = 0 \qquad a_4(T_c, \vec{\mu}_c) = 0$ **TCP**

$a_6 > 0$

Generic Phase diagram for effective chiral Lagrangians



$G_V^{(S)} \rightarrow$ quantifies repulsive interactions between quarks

- Generic structure of the phase diagram as expected in QCD and in different chiral models see eg.:J. Berges & Rajagopal; M. Alford et al; C.Ratti & W. Weise; B. J. Schaefer & J. Wambach; M. Buballa &D.Blaschke; B. Friman, C. Sasaki at al., M.Stephanov et al.,……
- Quantitative properties of the phase diagram and the position of TCP are strongly model dependent
- Large G_V^S no TCP at finite T
- $m_q \neq 0$ acts as an external magnetic field and destroys the 2nd order transition to the cross-over and moves TCP to CEP

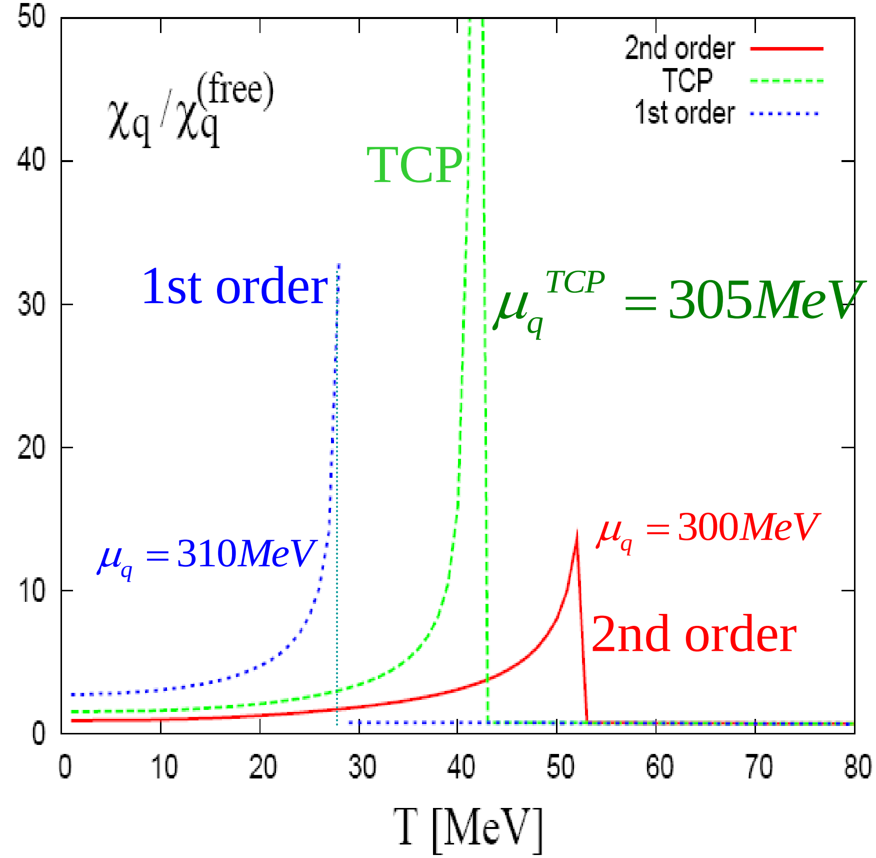
Susceptibilities of conserved charges

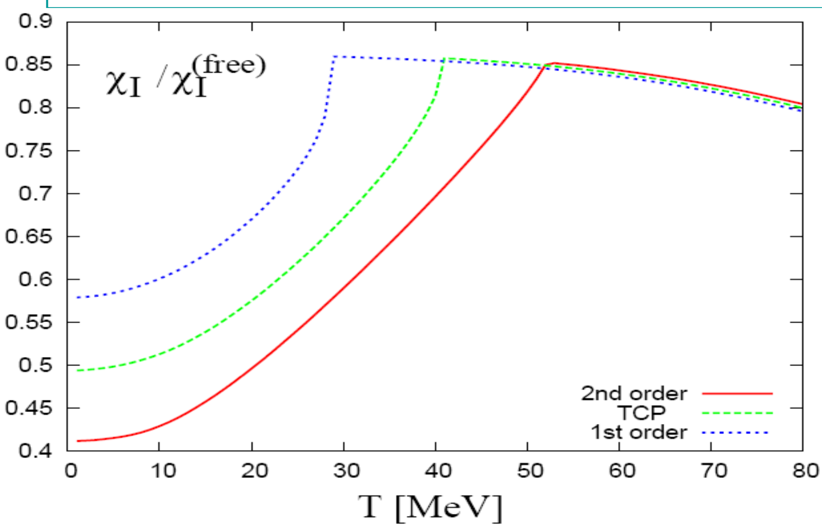
- Net quark-number χ_q, isovector χ_I and electric charge χ_Q fluctuations $\chi_A = \langle A^2 \rangle - \langle A \rangle^2$

$$\chi_q = \frac{\partial^2 P}{\partial \mu_q^{\,2}} \qquad \chi_I = \frac{\partial^2 P}{\partial \mu_I^{\,2}}$$

$$\boxed{\chi_Q = \frac{1}{36}\chi_q + \frac{1}{4}\chi_I + \frac{1}{6}\frac{\partial^2 P}{\partial \mu_q \partial \mu_I}}$$

C. Sasaki, B. Friman & K.R.





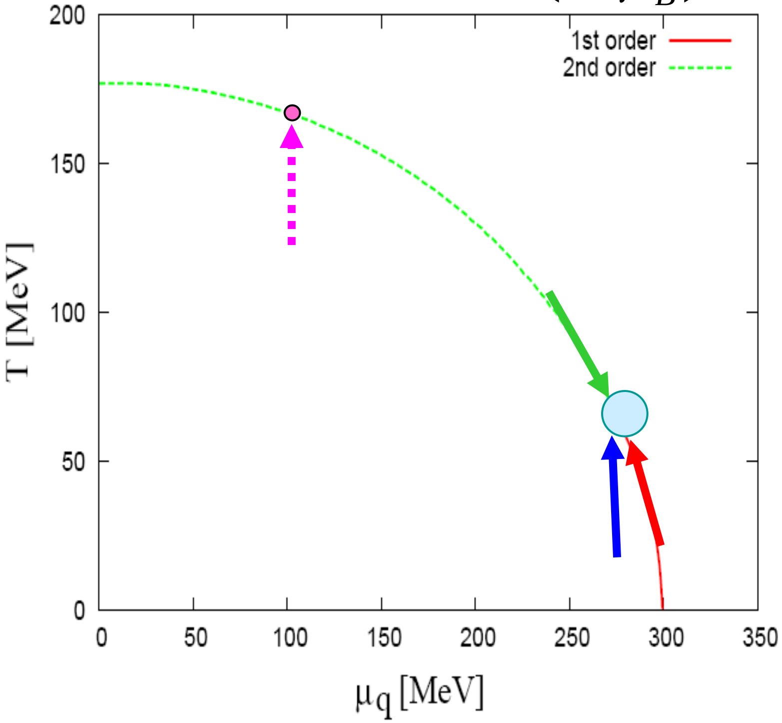
No mixing of isospin density with the sigma field due to isospin conservation

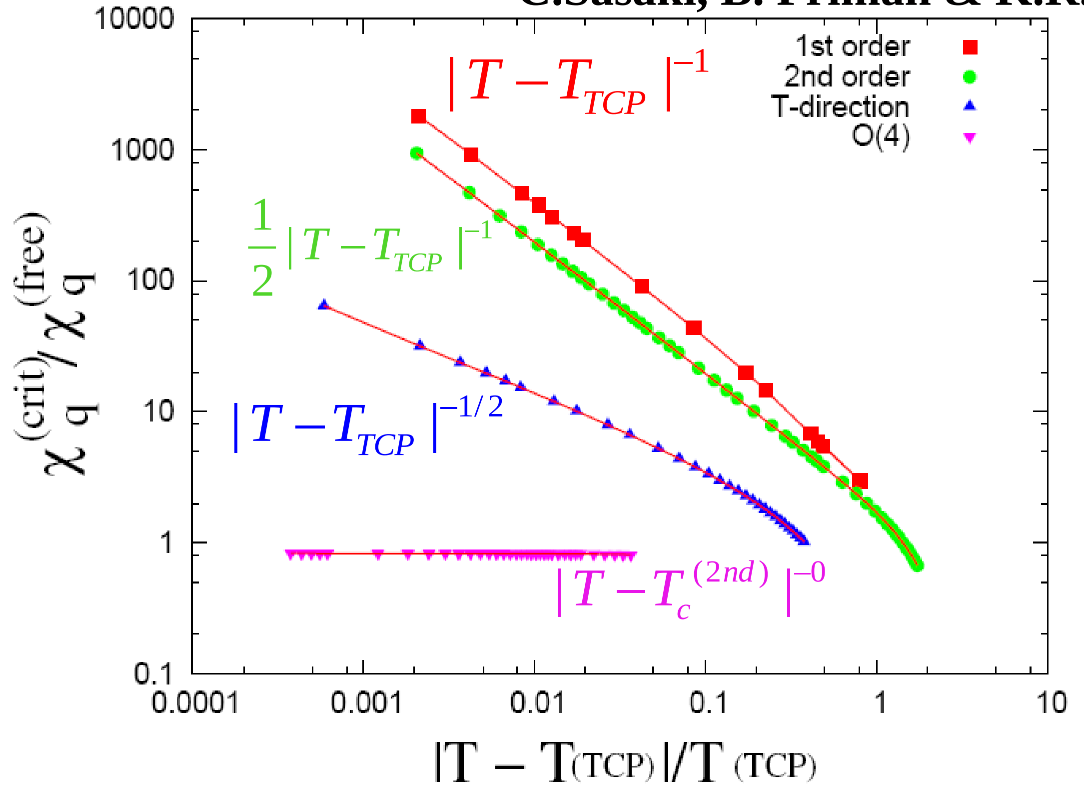
Hatta & Stephanov

Scaling properties: $\chi_q = a + b\,|T - T_{T_c}|^{-\theta}$

The strength of the singularity at TCP depends on direction in (T,μ_B) plane

C.Sasaki, B. Friman & K.R.





$\chi_q \propto |T - T_{TCP}|^{-1}$ **along 1st order line**

$\chi_q \propto |T - T_{TCP}|^{-1/2}$ **any direction not parallel**

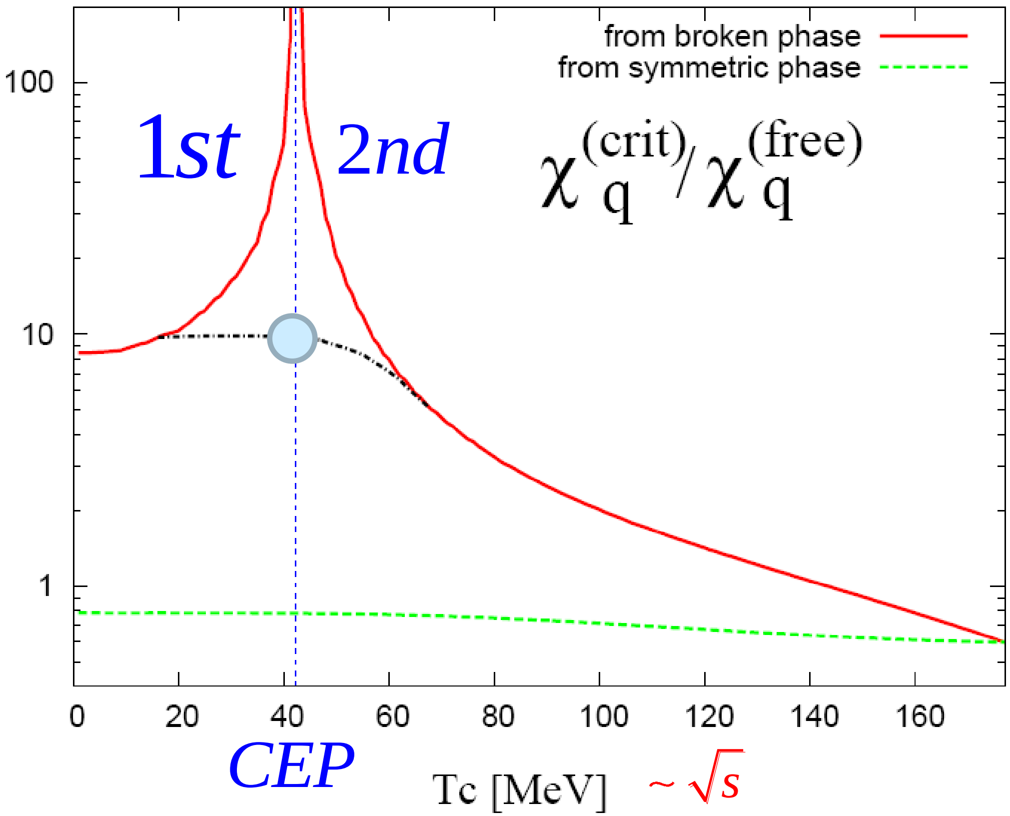
$\chi_q \propto \frac{1}{2}|T - T_{TCP}|^{-1}$ **along 2nd order line**

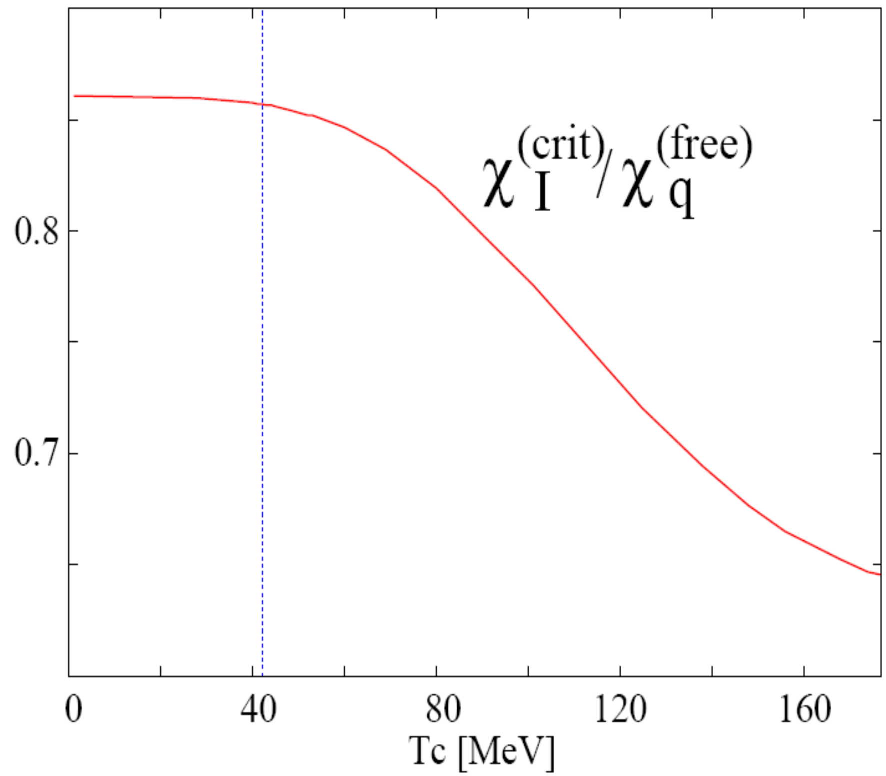
See also Y. Hatta, T. Ikeda

Quark and isovector fluctuations along critical line

To find CEP surch for a non-monotonic behavior of the net quark number susceptibility as a function of $T_c = T_c(\mu_c)$ or in heavy ion, A-A collisions as a function of $\sqrt{s}$

C. Sasaki, B. Friman & K.R.



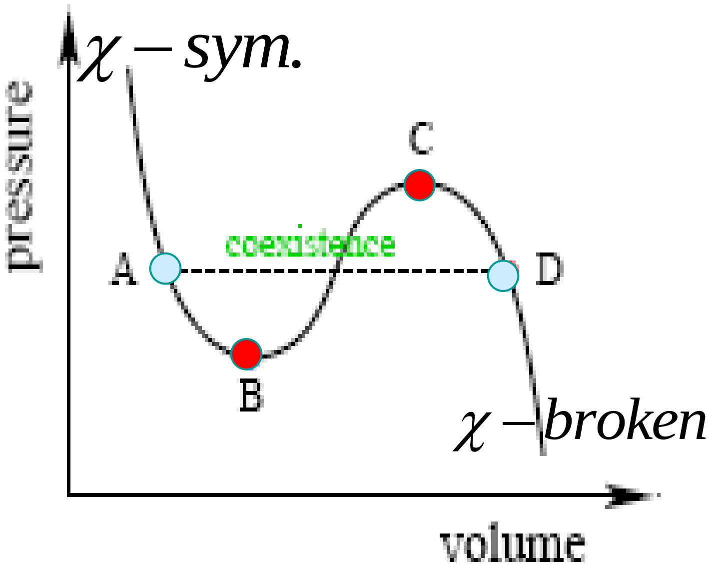


$\chi_q(T_c, \mu_c(T_c))$ **sensitive probes of CEP**

Non-singular behavior at CEP of $\chi_I(T_c, \mu_c(T_c))$

The nature of the 1st order chiral phase transition

instability of a system:



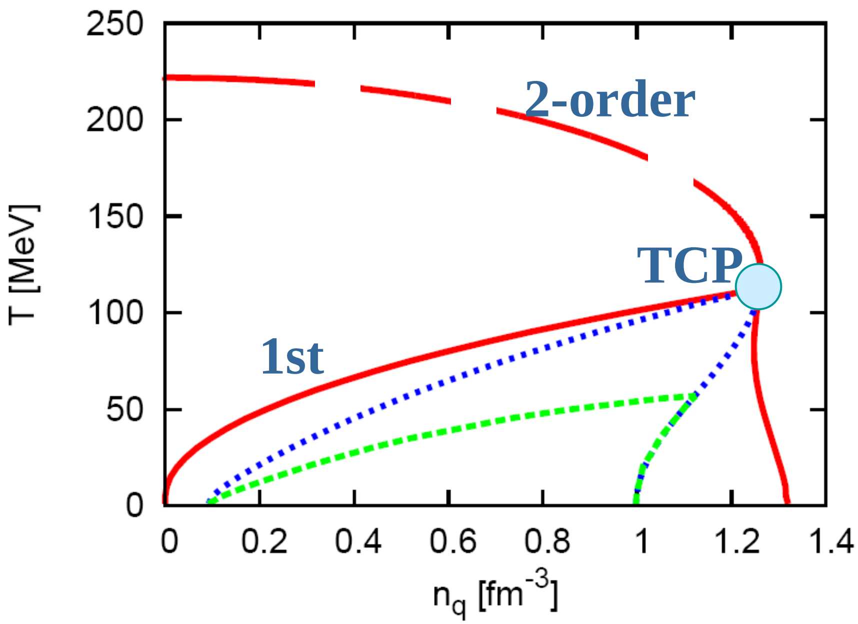
$\partial P / \partial V < 0$	:	stable
$\partial P / \partial V > 0$	:	unstable
$\partial P / \partial V = 0$	:	spinodal

A-B: supercooling (symmetric phase)
B-C: non-equilibrium state
C-D: superheating (broken phase)

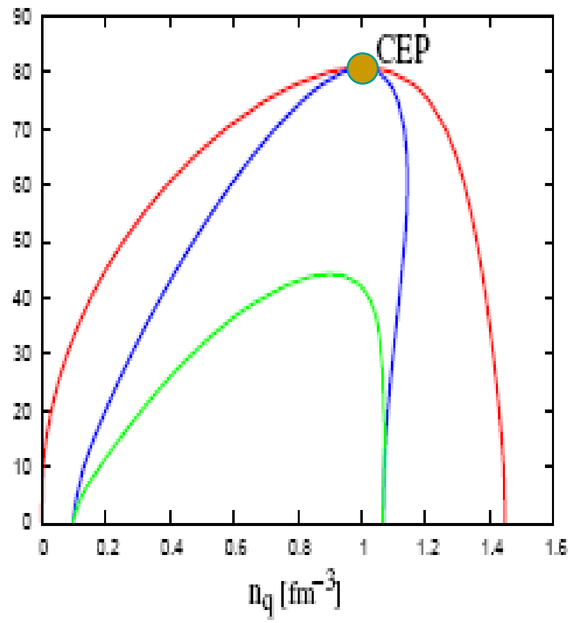
Phase diagram and spinodals

B. Friman, C. Sasaki & K.R.

$m_q = 0$



$m_q \neq 0$



critical end point (CEP) :
$T = 81$ MeV, $\mu = 330$ MeV

spinodal lines :

$$\left(\frac{\partial P}{\partial V}\right)_T = 0 \quad : \text{isothermal}$$

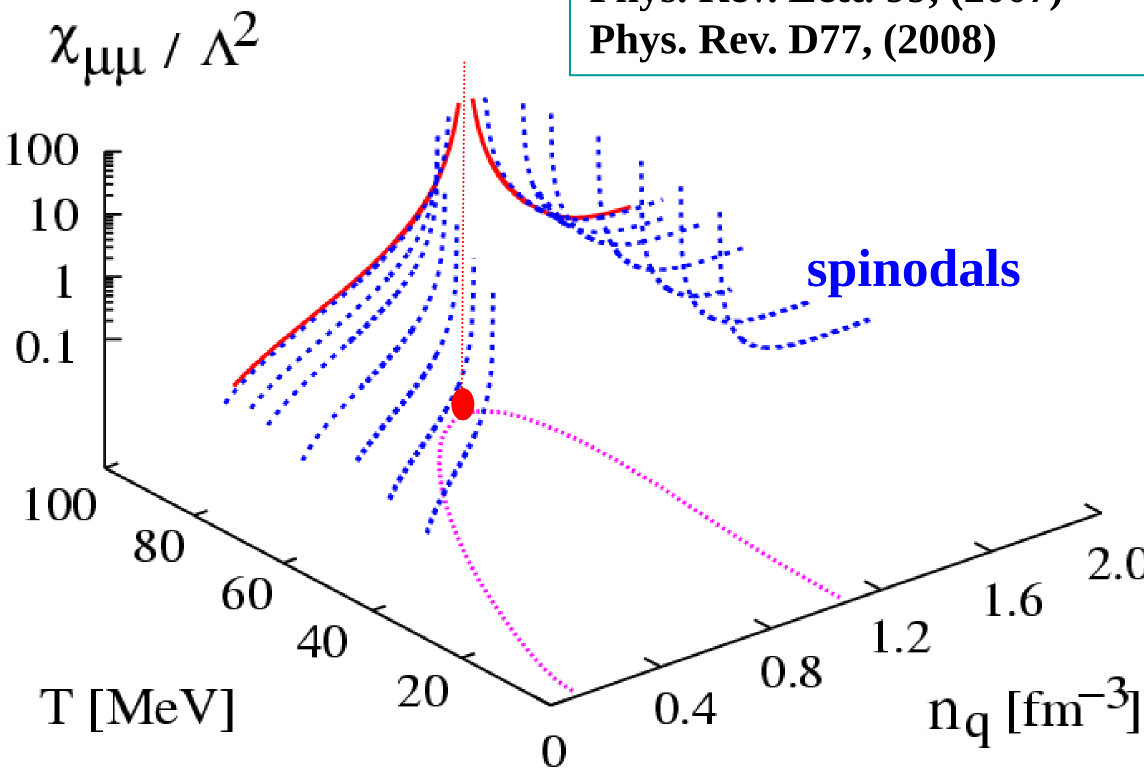
$$\left(\frac{\partial P}{\partial V}\right)_S = 0 \quad : \text{isentropic}$$

$$\left(\frac{\partial P}{\partial V}\right)_T = \left(\frac{\partial P}{\partial V}\right)_S + \frac{T}{C_V}\left[\left(\frac{\partial P}{\partial T}\right)_V\right]^2$$

mixed phase - Maxwell construction

Net-quark fluctuations on spinodals

B. Friman, C. Sasaki & K.R.
Phys. Rev. Lett. 99, (2007)
Phys. Rev. D77, (2008)



at any spinodal points:

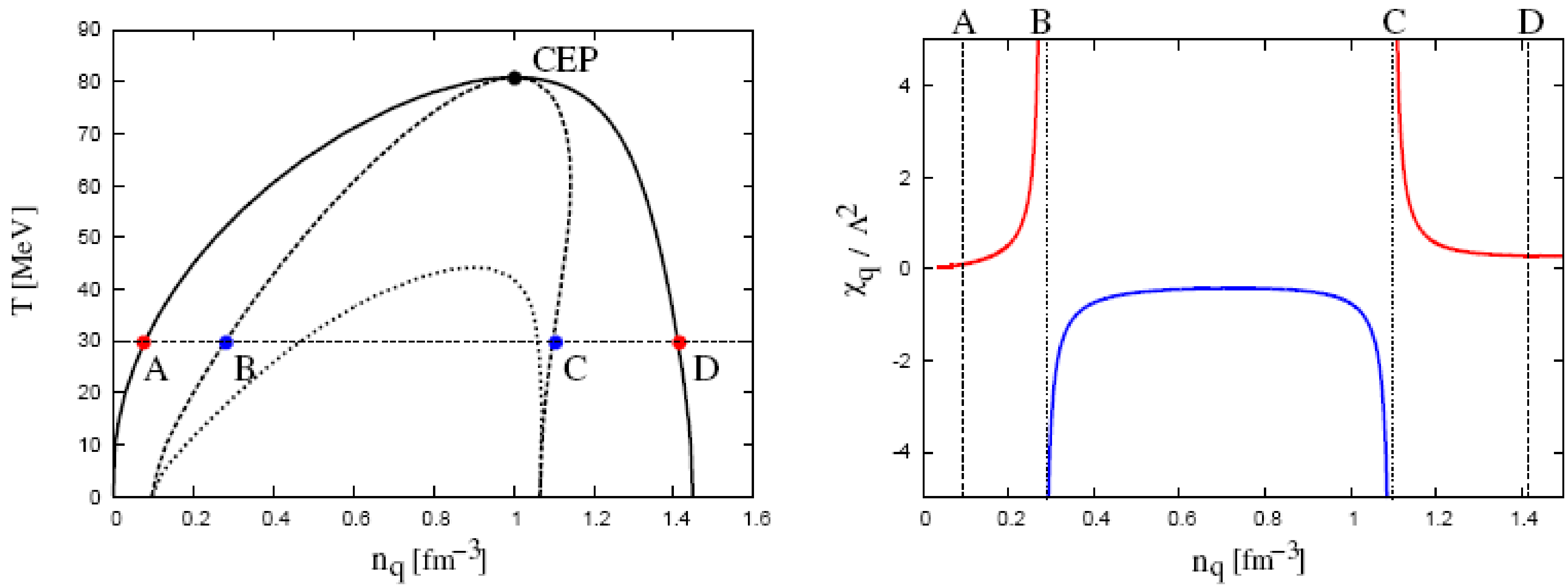
$$\frac{\partial P}{\partial V}\Big|_T = -\frac{n_q^{\,2}}{V}\frac{1}{\chi_q}$$

Singularity at CEP are the remnant of that along the spinodals

$$\chi_q \sim \left|\frac{\mu-\mu_c}{\mu_c}\right|^{-\gamma} \quad \text{with} \quad \gamma_{m_q=0} = \begin{cases} 1/2 \ \ (0.53) & TCP \\ 1/2 & 1st \end{cases}, \gamma_{m_q\neq 0} = \begin{cases} 2/3 \ \ (0.78) & CEP \\ 1/2 & 1st \end{cases}$$

Quark number susceptibility

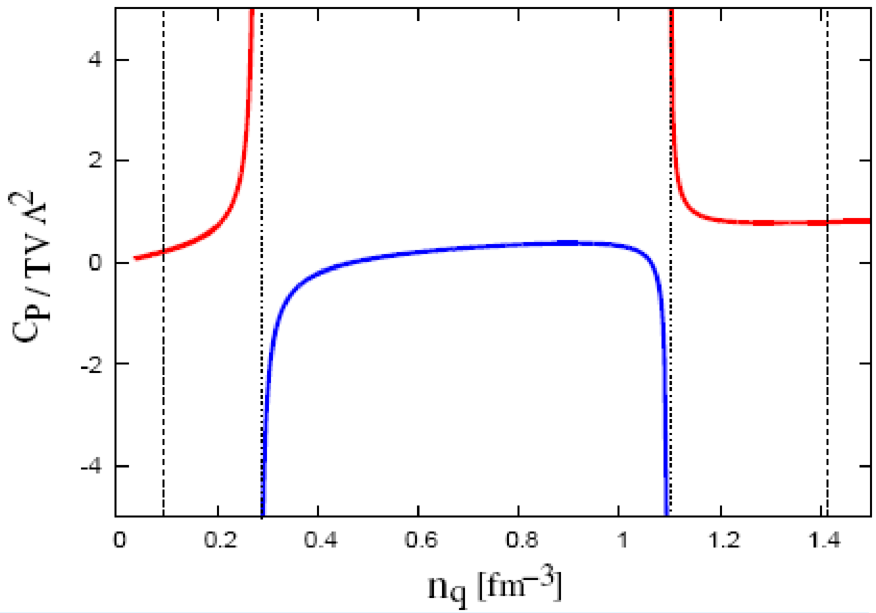
- deviation from equilibrium, large fluctuations induced by instabilities



- at 1st order transition point (A, D) : χ_q is finite
- at isothermal spinodal point (B, C) : χ_q diverges and changes its sign
  $\frac{\partial P}{\partial V} < 0$ for stable/meta-stable state $\Rightarrow$ $\frac{\partial P}{\partial V} > 0$ for unstable state
- in unstable region (B-C) : χ_q is finite and negative

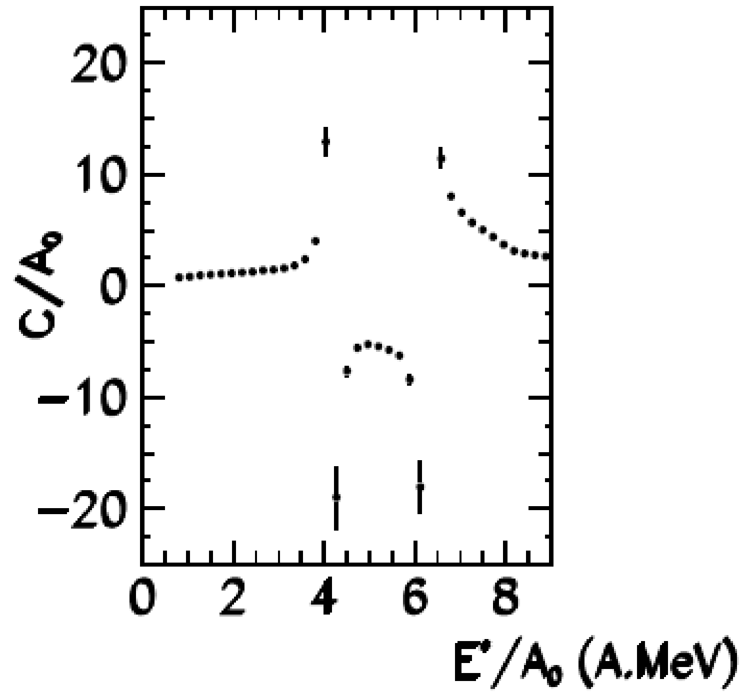
Experimental Evidence for 1st order transition

Specific heat for constant pressure:



$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = TV\left[\chi_{TT} - \frac{2s}{n_q}\chi_{\mu T} + \frac{s^2}{{n_q}^2}\chi_q\right]$$

Low energy nuclear collisions

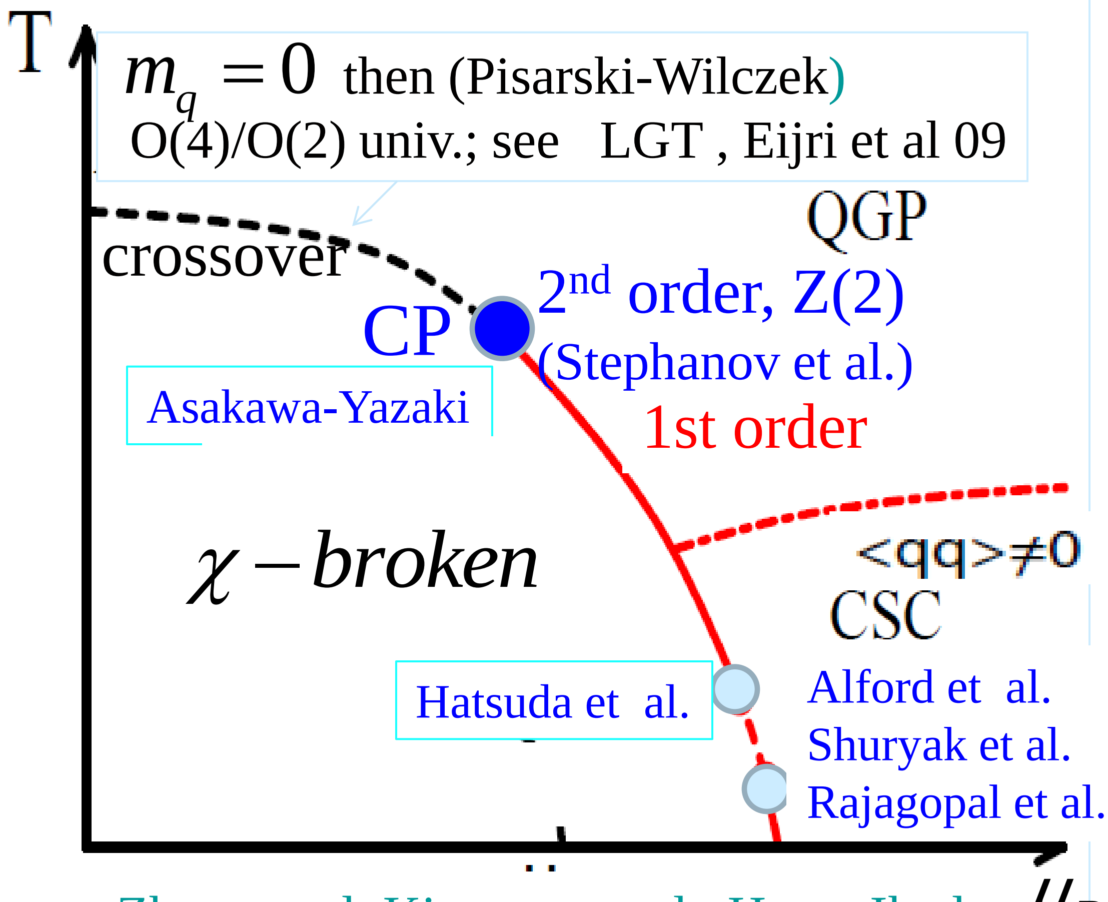


M. D'Agostino *et al.*, Phys. Lett. B 473, 219 (2000)

negative heat capacity : anomalously large fluctuations

$\Rightarrow$ an evidence of the 1st order liquid-gas phase transition

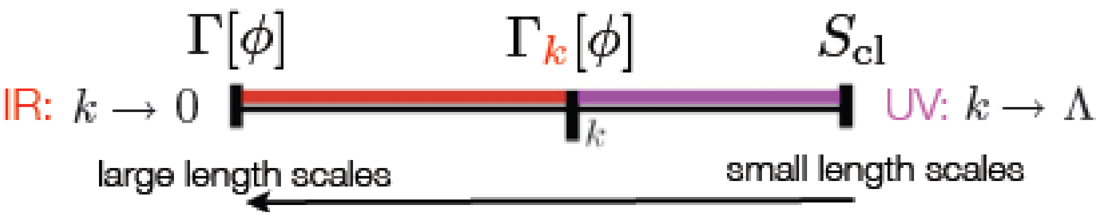
Generic Phase diagram from effective chiral Lagrangians



Zhang et al, Kitazawa et al., Hatta, Ikeda; Fukushima et al., Ratti et al., Sasaki et al., Blaschke et al., Hell et al., Roessner et al., ..

- The existence and position of CP and transition is model and parameter dependent !!
- Introducing di-quarks and their interactions with quark condensate results in CSC phase and dependently on the strength of interactions to new CP's

Including quantum fluctuations: FRG approach

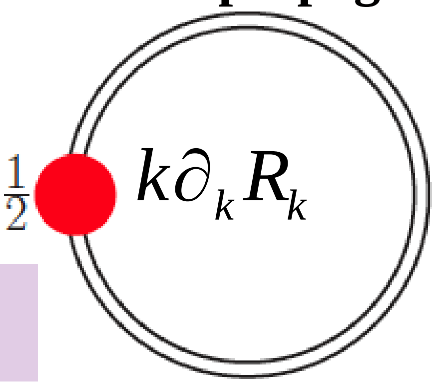


k-dependent full propagator

start at classical action and include quantum fluctuations successively by lowering **k**

FRG flow equation (C. Wetterich 93)

$$k\,\partial_k \Gamma_k[\phi] = \frac{1}{2}$$



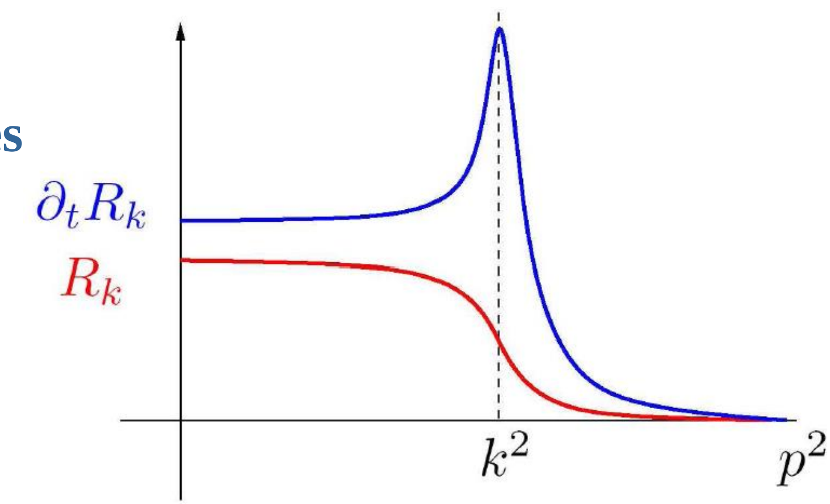
FRG review: J. Berges, N. Tetradis & C. Wetterich, Phys.Rept.363:223-386, '02

FRG formulation of PQM model **B. Stokic, V. Skokov, B. Friman, K.R.** , '10

$$k\partial_k\Gamma_k \equiv \partial_t\Gamma_k = \frac{1}{2}\mathrm{Tr}\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

$$\Gamma_k^{(2)} = \frac{\delta^2\Gamma_k}{\delta\phi\delta\phi}$$

Regulator function suppresses particle propagation with momentum Lower than **k**



$$\Omega(T,V) = \lim_{k\to 0}(\Omega_k = (T/V)\Gamma_k)$$

Quark-meson model w/ FRG approach

$$\mathscr{L}_{\mathrm{QM}} = \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi})]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 - U(\sigma,\vec{\pi})$$

Effective potential is obtained by solving *the exact flow equation* (Wetterich eq.) with the approximations resulting in the O(4) critical exponents

B.J. Schaefer & J. Wambach,; B. Stokic, B. Friman & K.R.

$$\partial_k \Omega_k(\sigma) = \frac{Vk^4}{12\pi^2}\left[\sum_{i=\pi,\sigma} \frac{d_i}{E_{i,k}}[1+2n_B(E_{i,k})] - \frac{2\nu_q}{E_{q,k}}[1-n_F(E^+_{q,k})-n_F(E^-_{q,k})]\right]$$

$$E_{\pi,k} = \sqrt{k^2+\Omega'_k}$$

$$E_{\sigma,k} = \sqrt{k^2+\Omega'_k+2\rho\Omega''_k}$$

$$E_{q,k} = \sqrt{k^2+2g^2\rho}$$

$$\Omega'_k \equiv \frac{\partial \Omega_k}{\partial(\sigma^2/2)}$$

Full propagators with $k < q < \Lambda$

$\Gamma_\Lambda = S_{\text{classical}}$

Integrating from $k=\Lambda$ to $k=0$ gives a full quantum effective potential

Put $\Omega_{k=0}(\sigma_{\min})$ into the integral formula for P(N)

O(4) scaling and critical behavior

- Near T_c critical properties obtained from the singular part of the free energy density

$$F = F_{reg} + F_S$$

h: external field and

$$t = \frac{T - T_c}{T_c} + \kappa \left(\frac{\mu}{T_c} \right)^2$$

with $F_S(t, h) = b^{-d} F(b^{1/\nu} t, b^{\beta\delta/\nu} h)$

- Phase transition encoded in the "equation of state"

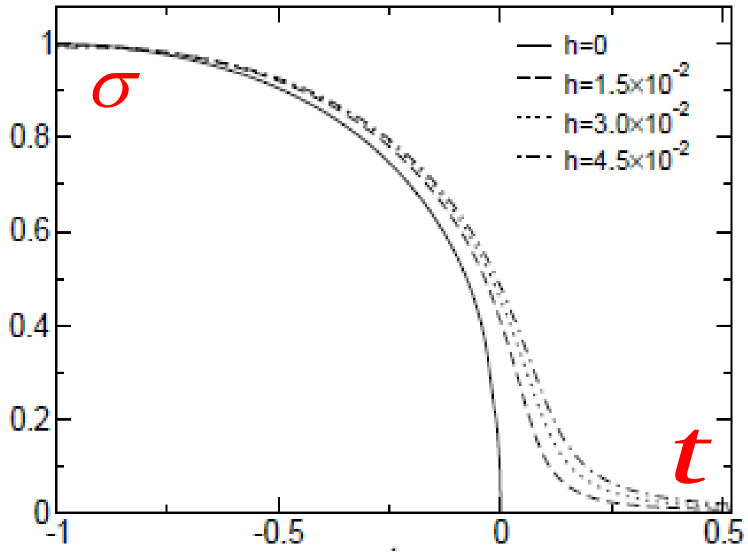
$$< \sigma > = -\frac{\partial F_s}{\partial h} \Rightarrow \quad < \sigma > = h^{1/\delta} F_h(z) \ , \quad z = t h^{-1/\beta\delta}$$

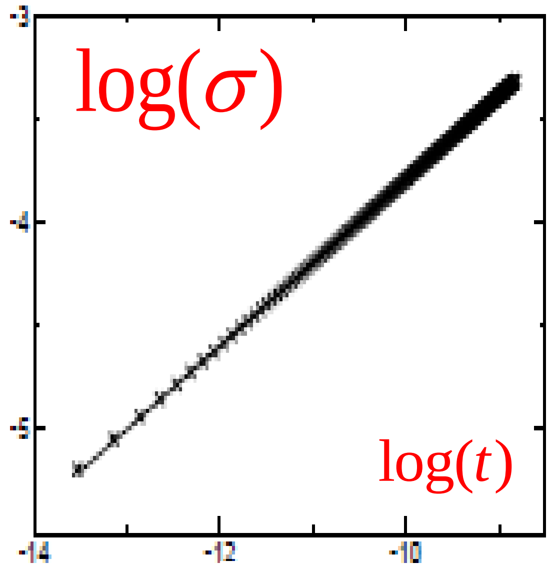
$$< \sigma > = | t |^{\beta} \ F_s^{'}(h \,| t |^{-\beta\delta})$$

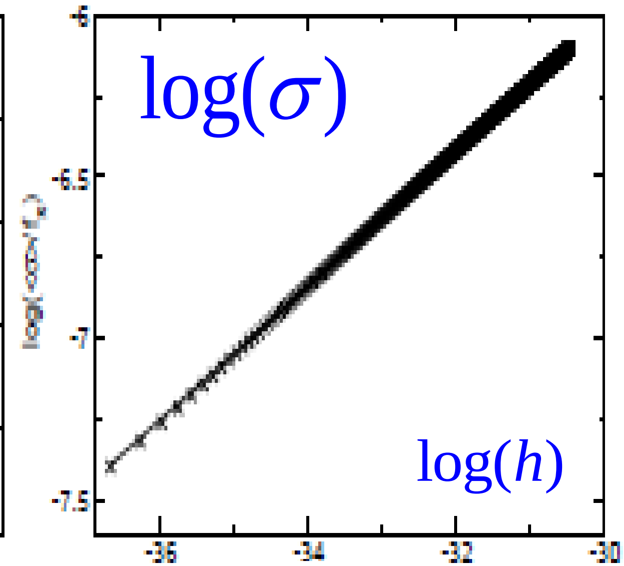
- Resulting in the well known scaling behavior of $< \sigma >$

$$< \sigma > = \begin{cases} B(-t)^{\beta}, h = 0, \ t < 0 & \text{coexistence line} \\ B h^{1/\delta} \ , \ t = 0, \ h > 0 & \text{pseudo-critical point} \end{cases}$$

FRG-Scaling of an order parameter in QM model







- The order parameter shows scaling. From the one slope one gets

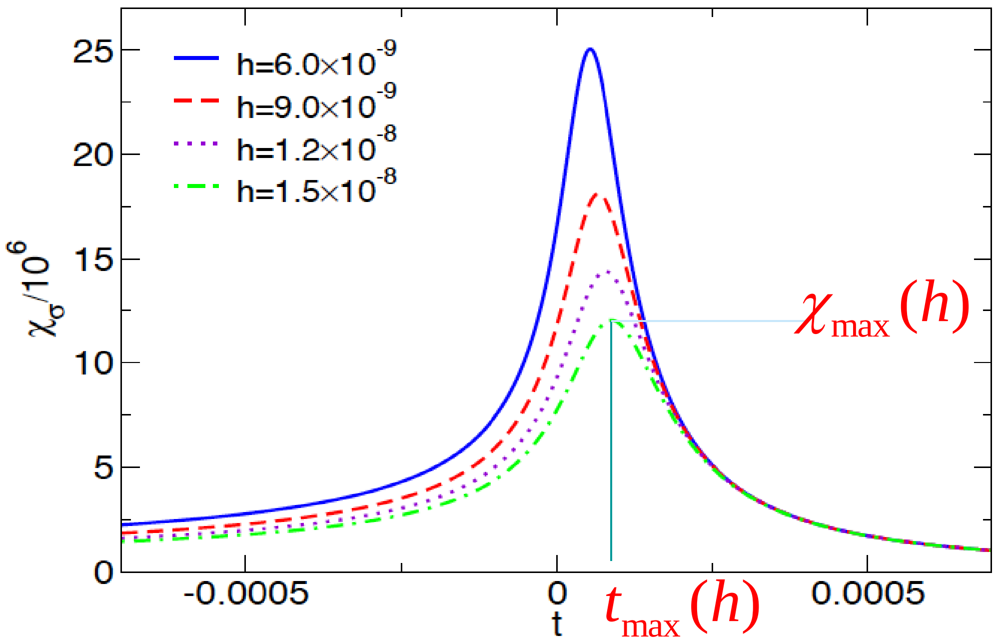
	β	δ
MF	0.5	3
FRG	0.401(1)	4.818(29)
LGT	0.3836(46)	4.851(22)

J. Engels et al., K. Kanaya et al.

- However we have neglected field-dependent wave function renormal. Consequently $\eta = 0$ and $\delta = 5$. The 3% difference can be attributed to truncation of the Taylor expansion at 3th order when solving FRG flow equation:

see D. Litim analysis for O(4) field Lagrangian

Fluctuations & susceptibilities



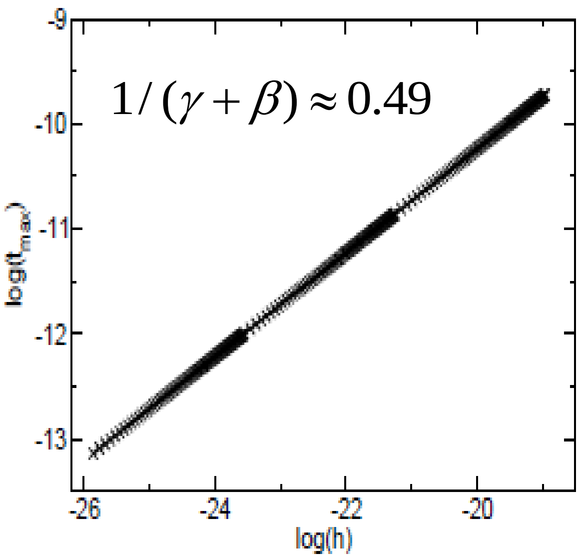
- Two type of susceptibility related with order parameter

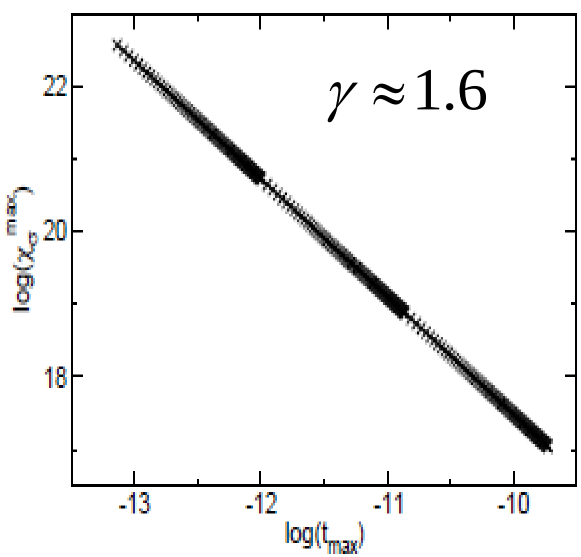
1. longitudinal

$$\chi_l = \chi_\sigma = \partial\sigma / \partial h$$

2. transverse

$$\chi_t = \chi_\pi = \sigma / h$$





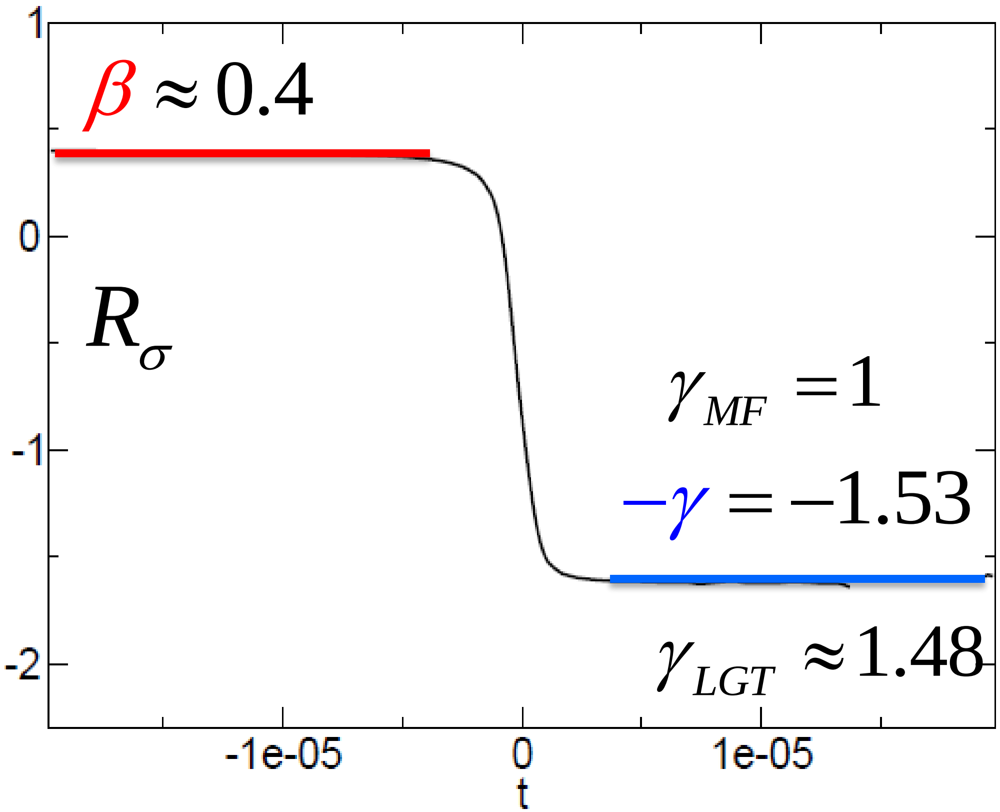
- Scaling properties at t=0 and $h \to 0$

$$\chi_\sigma \bullet \delta = \chi_\pi = B\ h^{1/\delta - 1}$$

$$t_{\max} \approx h^{1/(\gamma+\delta)}$$

$$\chi_\sigma(t_{\max}) \approx t_{\max}^{-\gamma}$$

Effective critical exponents



- Approaching T_c from the side of the symmetric phase, t >0, with small but finite h : from Widom-Griffiths form of the equation of state

$$\sigma = B_c h^{1/\delta} f(x)^{-1/\delta}, \quad x \doteq t\sigma^{-1/\beta}$$

- For $t > 0$ and $h \to 0 \Rightarrow \sigma \to 0$

$$\Rightarrow \quad x \to \infty \;\Rightarrow f(x) \approx x^{\gamma}$$

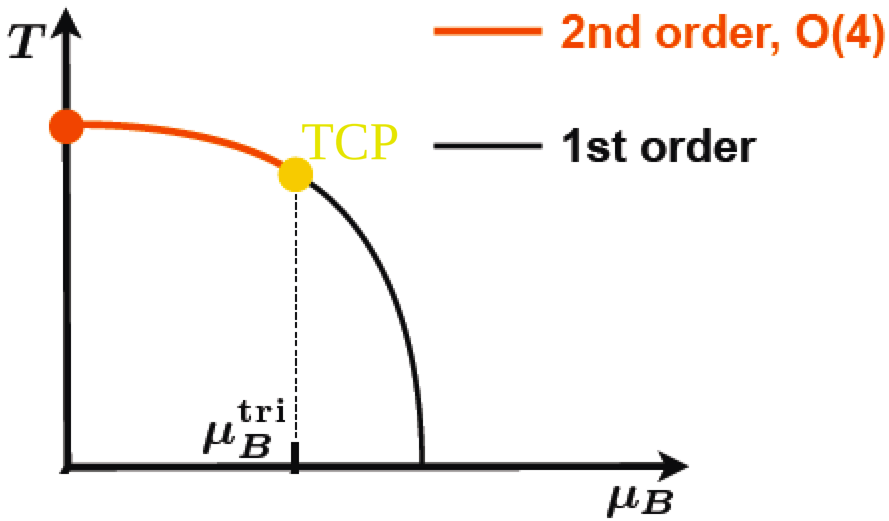
$$\Rightarrow \quad \sigma \sim t^{-\gamma} h$$, thus

$$<\sigma> = \begin{cases} B(-t)^{\beta}, h \to 0, & t < 0 \\ B_c t^{-\gamma} h \;\;, h \to 0, & t > 0 \end{cases}$$

Define: $R_\sigma := \dfrac{d\log(\sigma)}{d\log(t)} = \begin{cases} \beta & t < 0 \\ -\gamma & t > 0 \end{cases}$

QCD phase diagram and the O(4) criticality

Pisarki & Wilczek conjecture

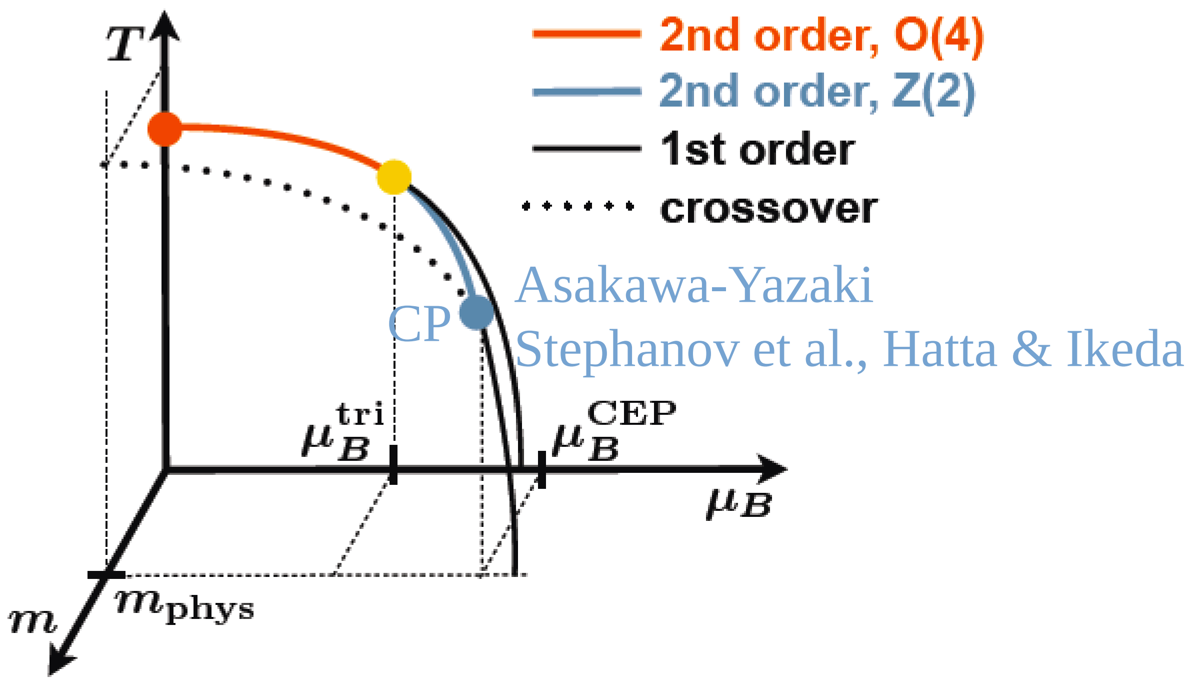


Expected phase diagram in the chiral limit, for massless u and d quarks:

TCP: Rajagopal, Shuryak, Stephanov
Y. Hatta & Y. Ikeda

- In QCD the quark masses are finite: the diagram has to be modified

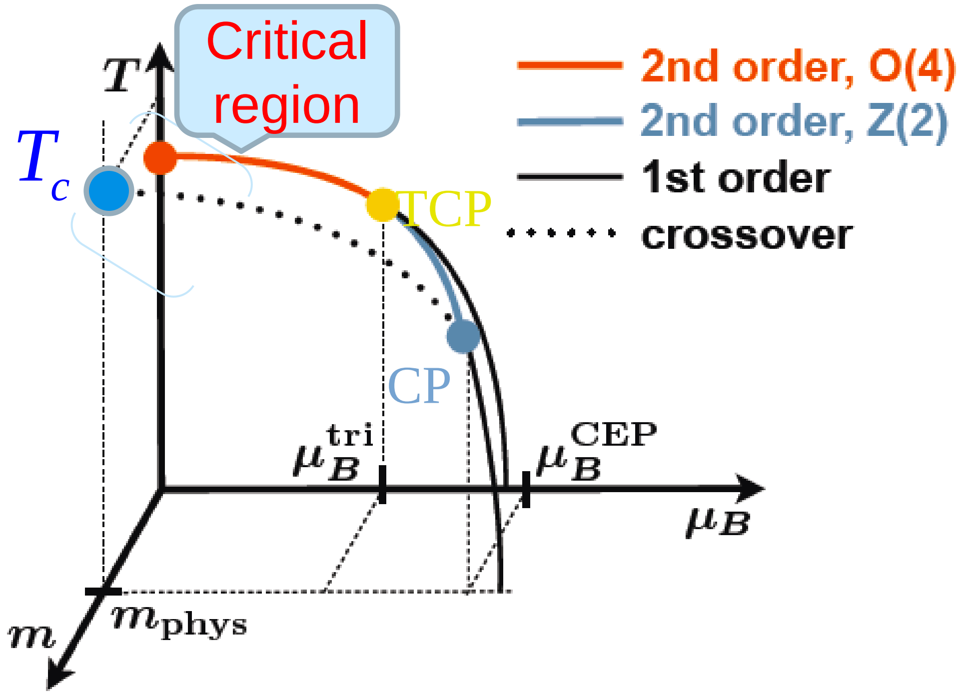
The phase diagram at finite quark masses



At the CP:
Divergence of Fluctuations, Correlation Length and Specific Heat

- The u,d quark masses are small
- Is there a remnant of the O(4) criticality at the QCD crossover line?

Deconfinement and chiral symmetry restoration in QCD



- The QCD chiral transition is crossover Y.Aoki, et al Nature (2006)
  and appears in the O(4) critical region

  O. Kaczmarek et.al. Phys.Rev. D83, 014504 (2011)

- Chiral transition temperature

  $$T_c = 155(1)(8) \text{ MeV}$$

  T. Bhattacharya et.al.
  Phys. Rev. Lett. 113, 082001 (2014)

- Deconfinement of quarks sets in at the chiral crossover

  A.Bazavov, Phys.Rev. D85 (2012) 054503

- The shift of T_c with chemical potential

$$\boxed{T_c(\mu_B) = T_c(0)[1 - 0.0066 \cdot (\mu_B / T_c)^2]}$$

Ch. Schmidt Phys.Rev. D83 (2011) 014504

See also:
Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, *et al.*
JHEP, 0906 (2009)

O(4) scaling and magnetic equation of state

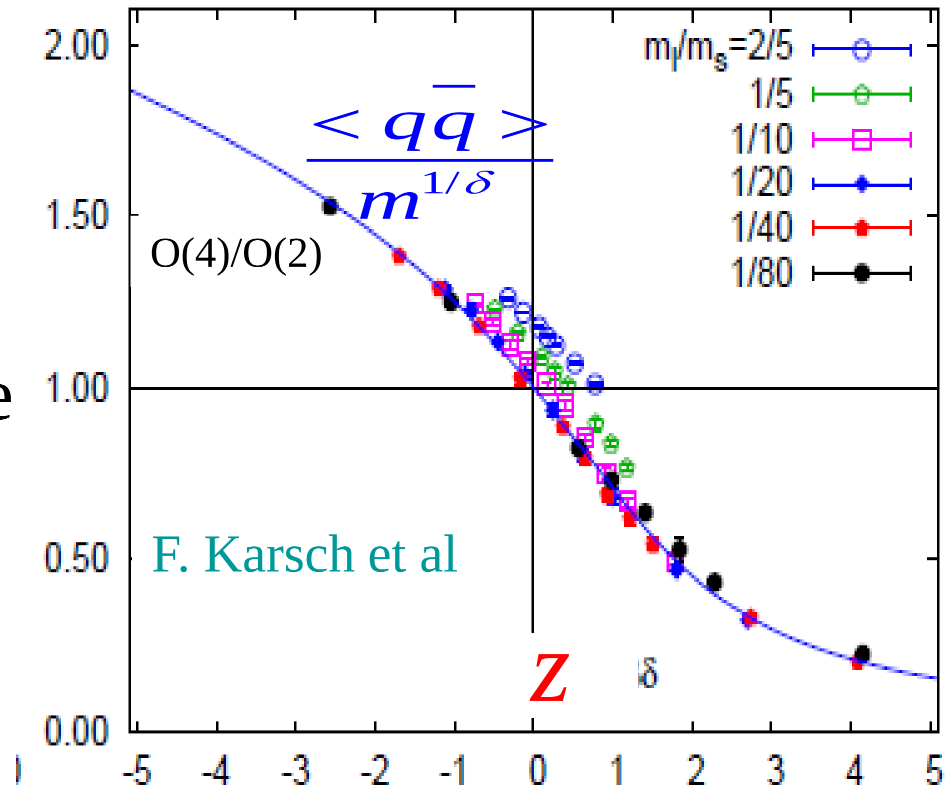
$$P = P_R(T, \mu_q, \mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t, b^{\beta\delta/\nu} h)$$

- Phase transition encoded in the magnetic equation of state

$$< q\bar{q} > = -\frac{\partial P}{\partial m} \Rightarrow \text{pseudo-critical line}$$

$$\boxed{\frac{< q\bar{q} >}{m^{1/\delta}} = f_s(z) \,, \quad z = t m^{-1/\beta\delta}}$$

QCD chiral crossover transition in the critical region of the O(4) 2nd order



universal scaling function common for all models belonging to the O(4) universality class: known from spin models J. Engels & F. Karsch (2012)

The endpoint of QCD in in LGT

Fodor & Katz 04

- Multiparameter reweighting:

renormalized physical operator

$$Z(\mu,\beta) = \int DU \exp(-S_g(\beta,U)) \det M(\mu,U)$$

$$\equiv \int DU \exp(-S_g(\beta,U)) \det M(\mu=0,U) \times \frac{\det M(\mu,U)}{\det M(\mu=0,U)}$$

Lee-Yang zeroes:

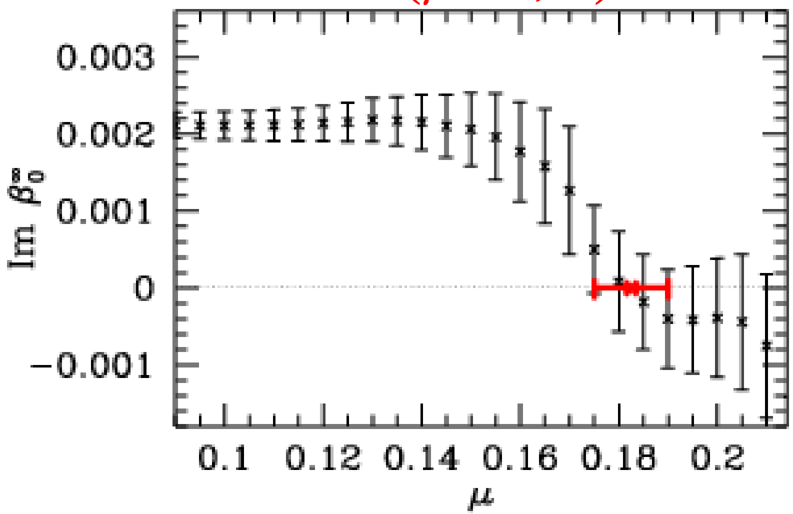
Finite volume V:

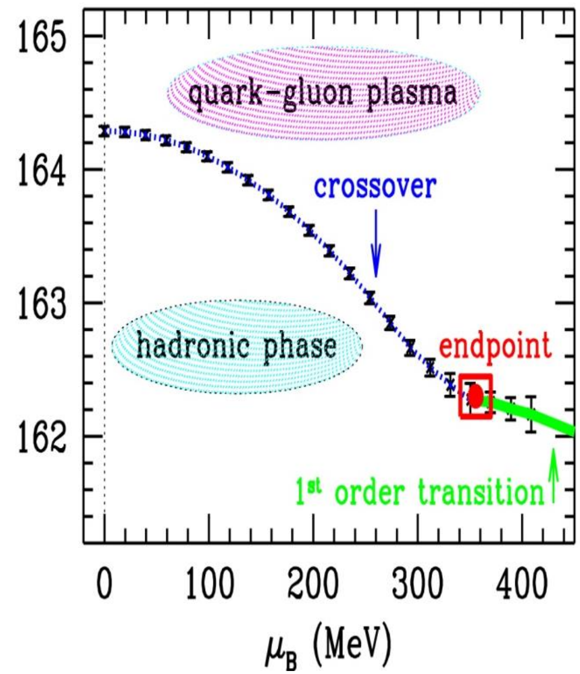
$$Z\left(\beta^*, V\right) = 0$$

If $V \to \infty$ and:

$\operatorname{Im} \beta^*(V \to \infty) = 0$ phase transition

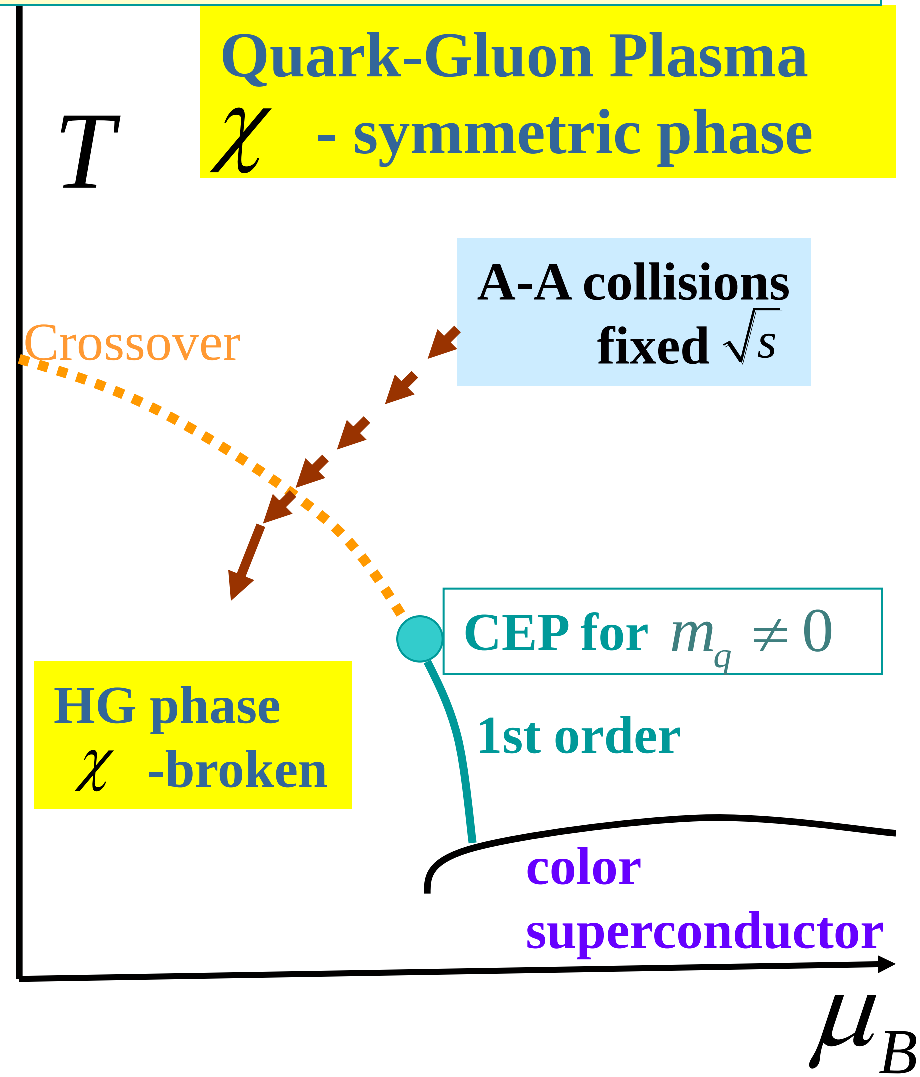
$\operatorname{Im} \beta^*(V \to \infty) \neq 0$ crossover transition





QCD Phase diagram: from theory to experiment

- QCD phase boundary in LGT & relation to freezeout in HIC
- Moments and probability distributions of conserved charges as probes of the criticality in QCD
- STAR data and potential discovery of CEP



Susceptibilities of net charge and order parameters

– The generalized susceptibilities probing fluctuations of net -charge number in a system and its critical properties

pressure: $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}, m_{u,d,s})$$

generalized susceptibilities $$\chi^{(i+j+k)} = \frac{\partial^{(i+j+k)} p / T^{\wedge 4}}{\partial T^i \partial \mu_x^j \partial m^i}:$$

Order parameter

$$< O_h >= \frac{1}{V} \frac{\partial \ln Z}{\partial h}$$

particle number density

$$\frac{n_q}{T^3} = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial \mu_q / T}$$

quark number susceptibility

$$\chi_q^{(2)} = \frac{\partial\, n_q / T^3}{\partial \mu_q / T}$$

4th order cumulant

$$\chi_q^{(4)} = \frac{1}{VT^3} \frac{\partial^4 \ln Z}{\partial (\mu_q / T)^4}$$

$$\chi_q^1 = \frac{1}{VT^3} < N >, \qquad \chi_q^2 = \frac{1}{VT^3} (< N^2 > - < N >^2) \qquad \chi_q^4 = \frac{1}{VT^3} (< (\delta N)^4 > -3 < (\delta N)^3 >)$$

expressed by $\boxed{N = N_q - N_{\bar{q}}}$ **and central moment** $\boxed{\delta N = N - < N >}$

Probing chiral criticality with charge fluctuations

- Due to expected O(4) scaling in QCD the free energy:

$$\boxed{P = P_R(T,\mu_q,\mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta\delta/\nu} h)}$$

- Generalized susceptibilities of net baryon number

$$c_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = c_R^{(n)} + c_S^{(n)} \quad \text{with} \quad \begin{array}{l} c_s^{(n)}\big|_{\mu=0} = d\; h^{(2-\alpha-n/2)/\beta\delta}\; f_\pm^{(n)}(z) \\ c_s^{(n)}\big|_{\mu\neq 0} = d\; h^{(2-\alpha-n)/\beta\delta}\; f_\pm^{(n)}(z) \end{array}$$

- At $\mu = 0$ only $c_B^{(n)}$ with $n \geq 6$ receive contribution from $c_S^{(n)}$
- At $\mu \neq 0$ only $c_B^{(n)}$ with $n \geq 3$ receive contribution from $c_S^{(n)}$

- $\boxed{c_B^{n=2} = \chi_B / T^2}$ Generalized susceptibilities of the net baryon number non critical with respect to O(4)

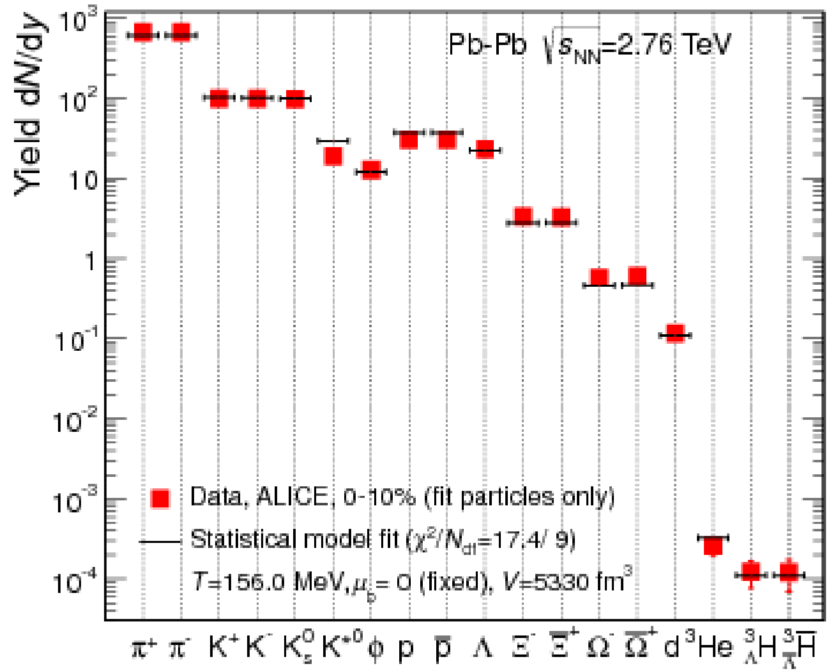
Thermal origin of particle yields with respect to HRG

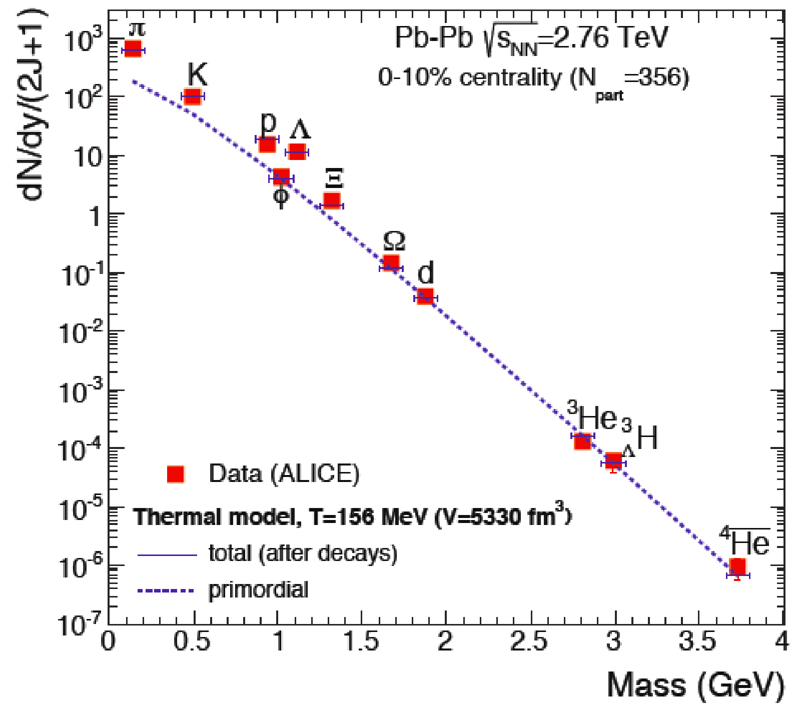
Rolf Hagedorn => the Hadron Resonace Gas (HRG):

"uncorrelated" gas of hadrons and resonances

$$< N_i > = V\,[n_i^{th}(T,\vec{\mu}) + \sum_K \Gamma_{K\to i}\, n_i^{th-Res.}(T,\vec{\mu})]$$

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel,

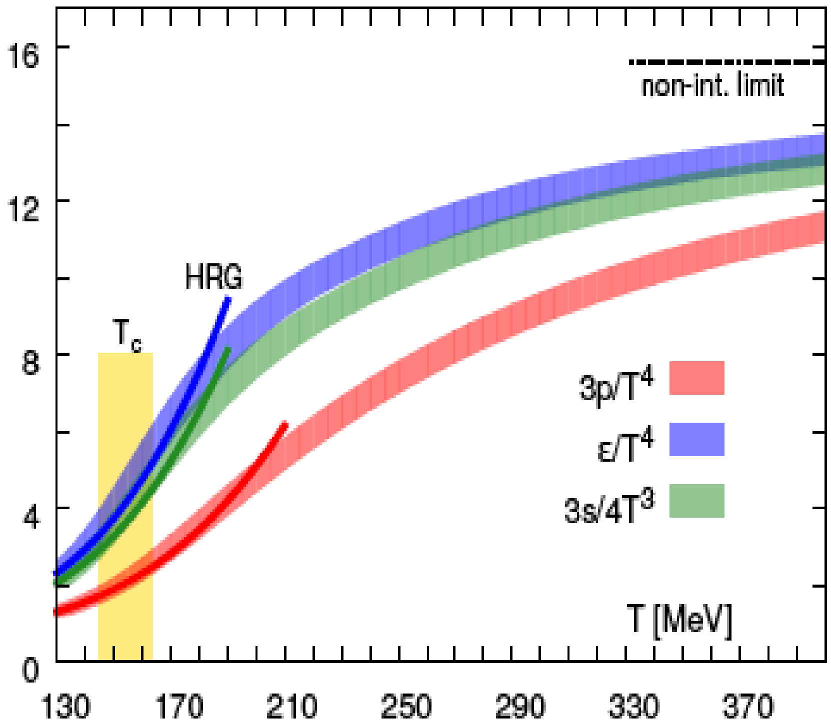




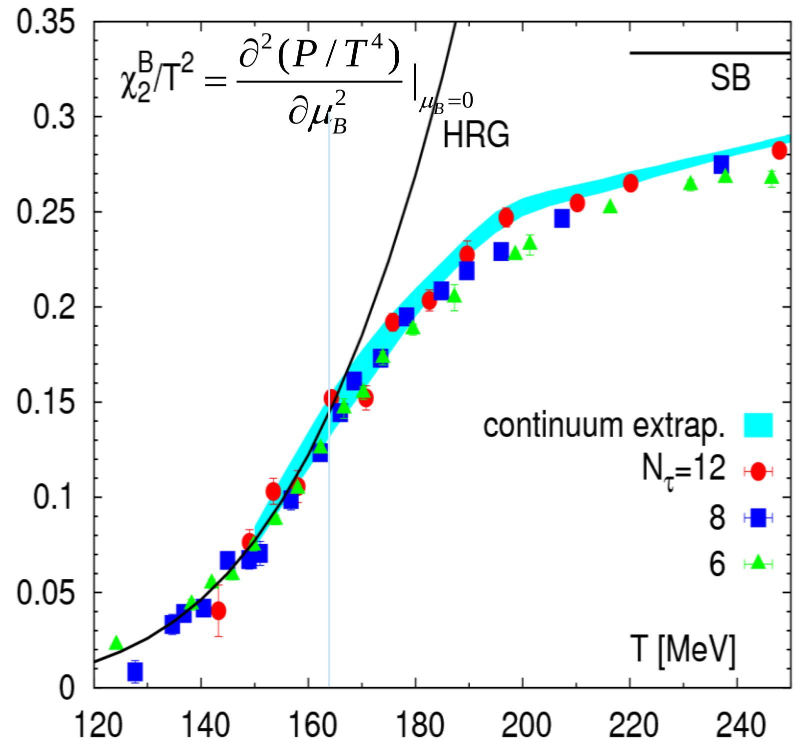
- Measured yields are reproduced with HRG at $T = 156$ MeV

Excellent description of the QCD Equation of States by Hadron Resonance Gas

A. Bazavov et al. HotQCD Coll. July 2014



F. Karsch et al. HotQCD Coll.



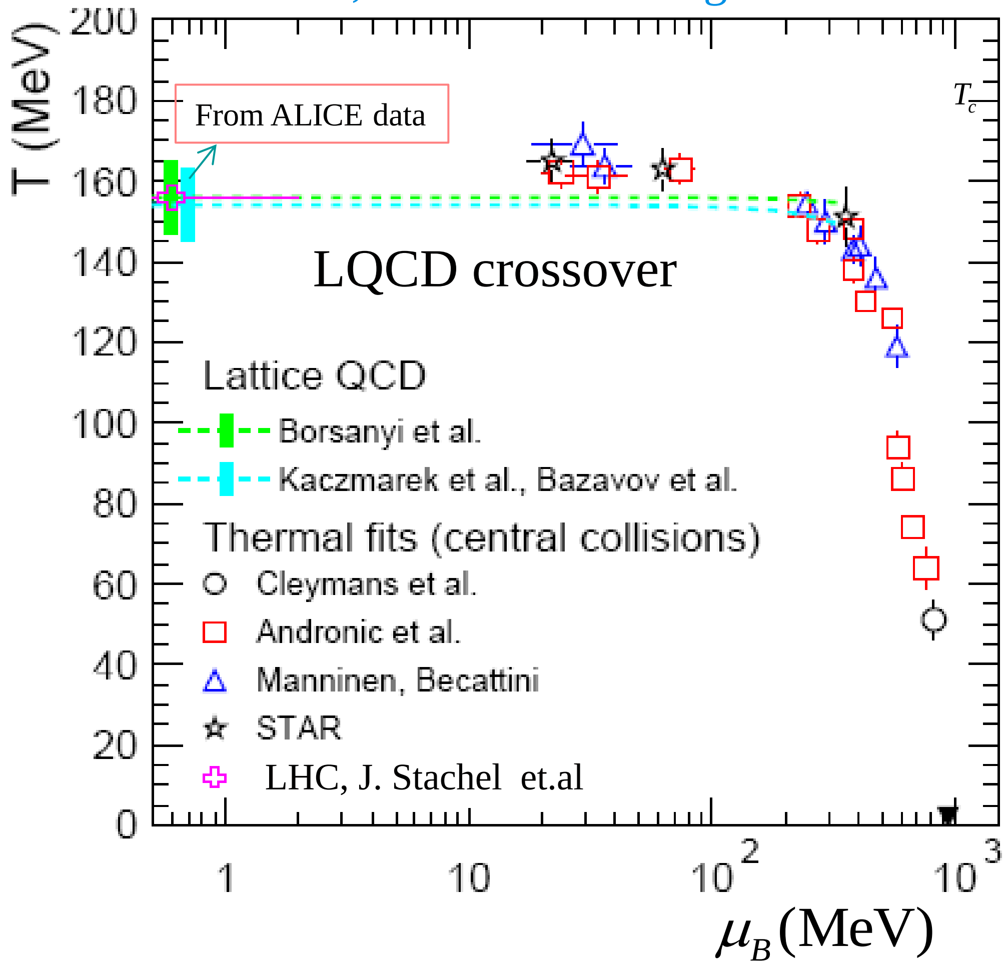
- "Uncorrelated" Hadron Gas provides an excellent description of the QCD equation of states in confined phase

- "Uncorrelated" Hadron Gas provides also an excellent description of net baryon number fluctuations

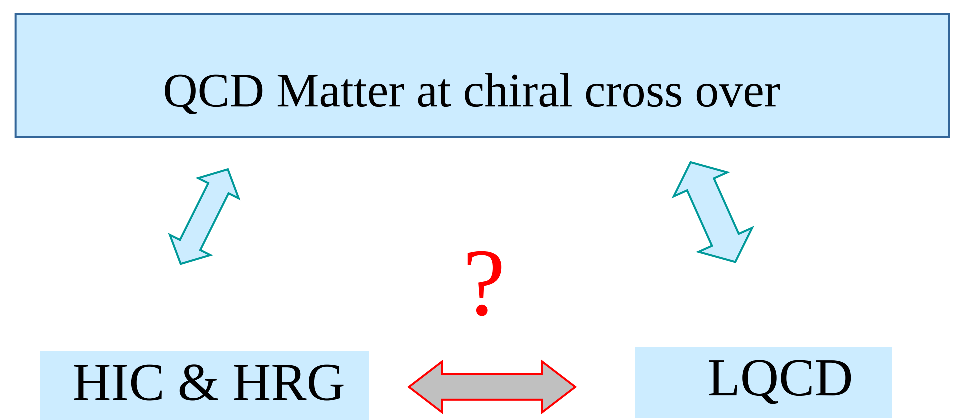
Chemical Freeze out and QCD Phase Boundary

Chemical freeze out defines a lower bound for the QCD phase boundary

A. Andronic, P. Braun-Munzinger & J. Stachel



- The QCD phase boundary coincides with chemical freeze out conditions obtained from HIC data analyzed with the HRG model



- The HRG should describe the QCD thermodynamics in the hadronic phase

Properties of fluctuatiosusns in HRG

F. Karsch & K.R.

Calculate generalized susceptibilities: $\chi_q^{(n)} = \frac{\partial^n [p(T,\vec{\mu})/T^4]}{\partial(\mu_q/T)^n}$
from Hadron Resonance Gas (HRG) partition function:

$$\frac{P}{T^4} = \frac{1}{\pi^2} \sum_i d_i (m_i/T)^2 K_2(m_i/T) \cosh[(B_i\mu_B + S_i\mu_S + Q_i\mu_Q)/T]$$

then,

$$\frac{\chi_B^{(4)}}{\chi_B^{(2)}} = 1, \qquad \frac{\chi_B^{(3)}}{\chi_B^{(1)}} = 1 , \qquad \frac{\chi^{(2)}}{\chi^{(1)}} \approx \coth(\mu_B / T) \quad \text{and} \quad \frac{\chi^{(3)}}{\chi^{(2)}} \approx \tanh(\mu_B / T)$$

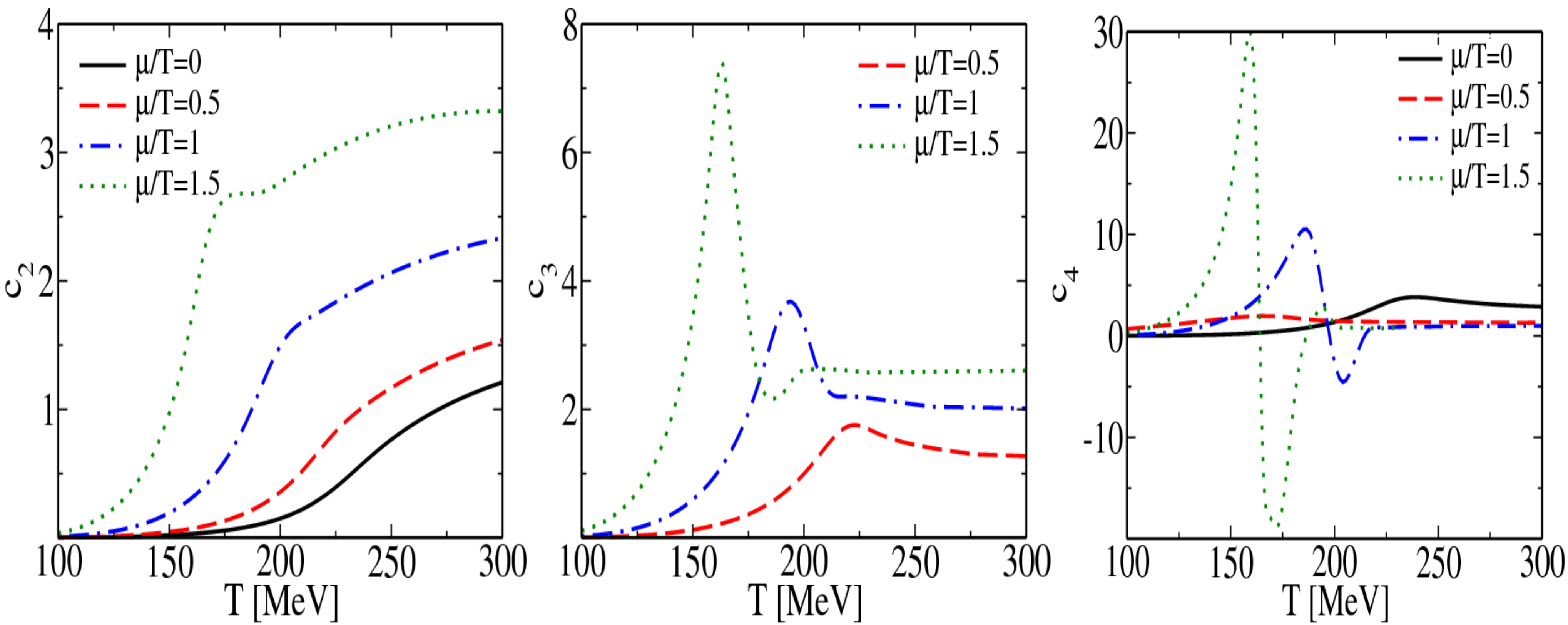
resulting in:

$$\frac{\sigma_q^2}{M_q} = \frac{\chi_q^{(2)}}{\chi_q^{(1)}}, \qquad S_q\sigma_q = \frac{\chi_q^{(3)}}{\chi_q^{(2)}}, \qquad \kappa_q\sigma_q^2 = \frac{\chi_q^{(4)}}{\chi_q^{(2)}}$$

Compare this HRG model predictions with STAR data at RHIC:

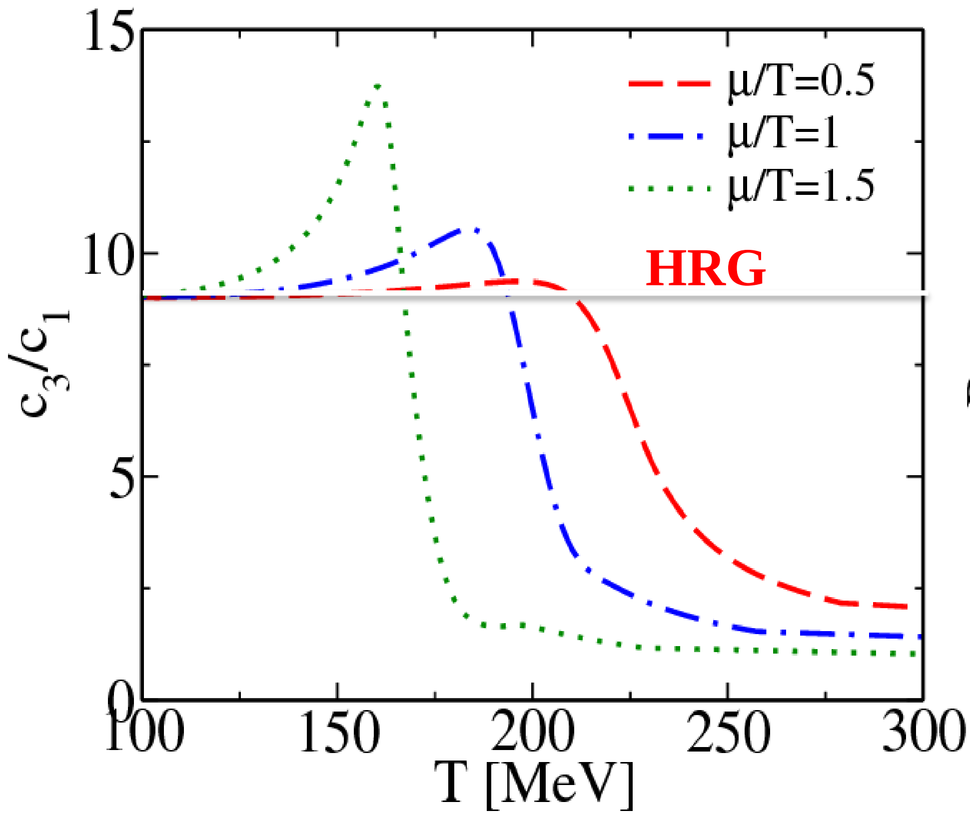
Quark number fluctuations at finite density

- Strong increase of fluctuations with baryon-chemical potential

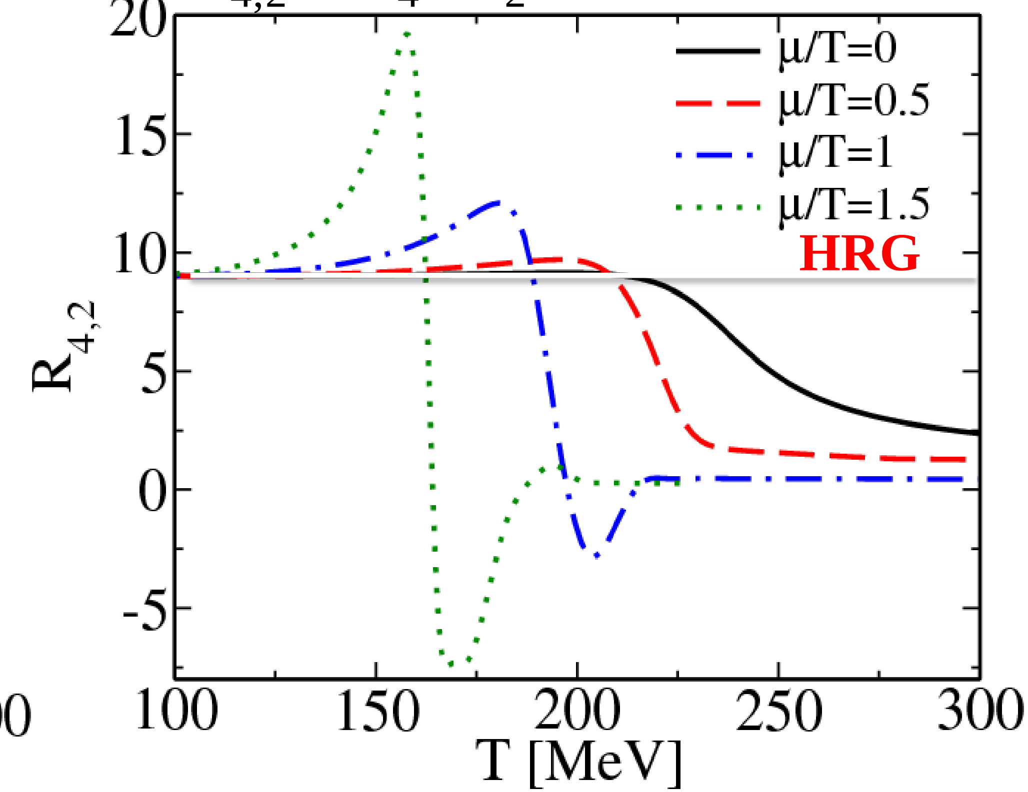


- In the chiral limit the c_3 and c_4 daverge at the O(4) critical line at finite chemical potential

Ratio of cumulants at finite density



$R_{4,2} = c_4 / c_2$

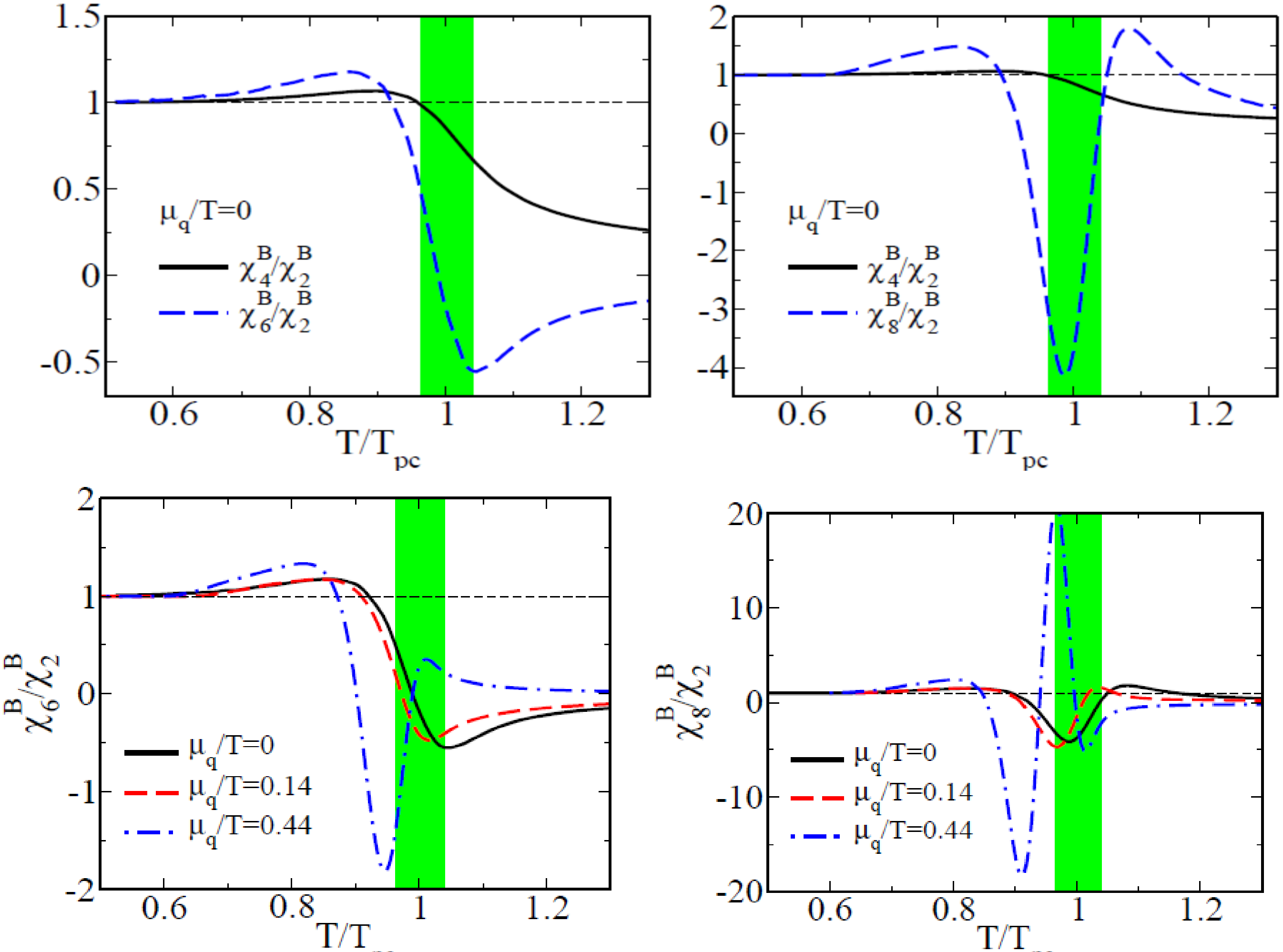


Deviations of the ratios of odd and even order cumulants from their asymptotic, low T-value, $c_4/c_2 = c_3/c_1 = 9$ are increasing with μ/T and the cumulant order

Properties essential in HIC to discriminate the phase change by measuring baryon number fluctuations !

Higher moments of baryon number fluctuations

B. Friman, K. Morita, V. Skokov & K.R.



- If freeze-out in heavy ion collisions occurs from a thermalized system close to the chiral crossover temperature, this will lead to a negative sixth and eighth order moments of net baryon number fluctuations.

These properties are universal and should be observed in HIC experiments at LHC and RHIC

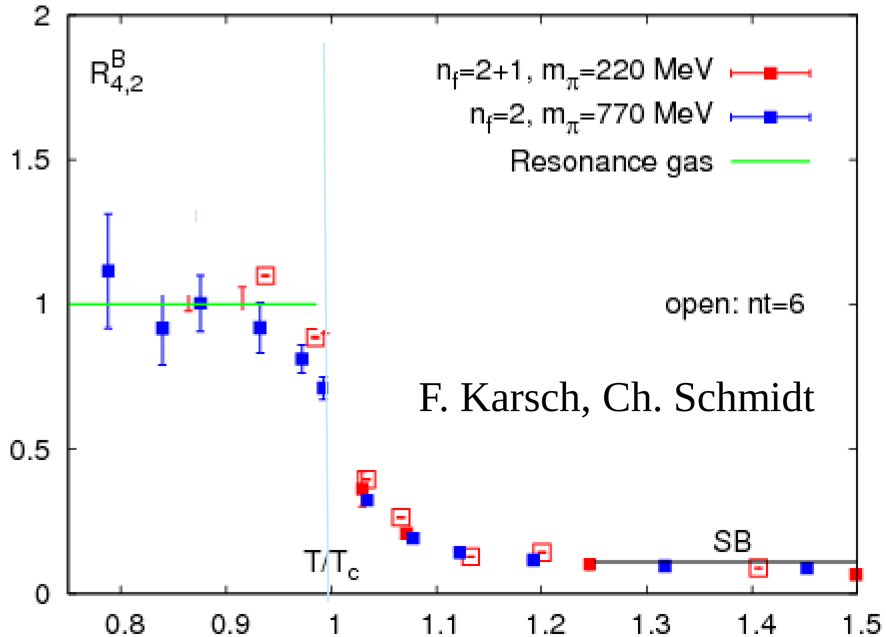
Figures: results of the PNJL model obtained within the Functional Renormalisation Group method

Kurtosis as an excellent probe of deconfinement

S. Ejiri, F. Karsch & K.R.

$$R_{4,2}{}^{B} = \frac{1}{9}\frac{c_4}{c_2}$$

Kurtosis



The $R_{4,2}{}^{B}$ measures the quark content of particles carrying baryon number

- HRG factorization of pressure:

$$P^B(T,\mu_q) = F(T)\cosh(3\mu_q/T)$$

consequently: $c_4/c_2 = 9$ in HRG

- In QGP, $SB = 6/\pi^2$
- Kurtosis=Ratio of cumulants

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

excellent probe of deconfinement

Kurtosis of net quark number density in PQM model

V. Skokov, B. Friman &K.R.

- For $T < T_c$
  the assymptotic value
  due to „confinement” properties

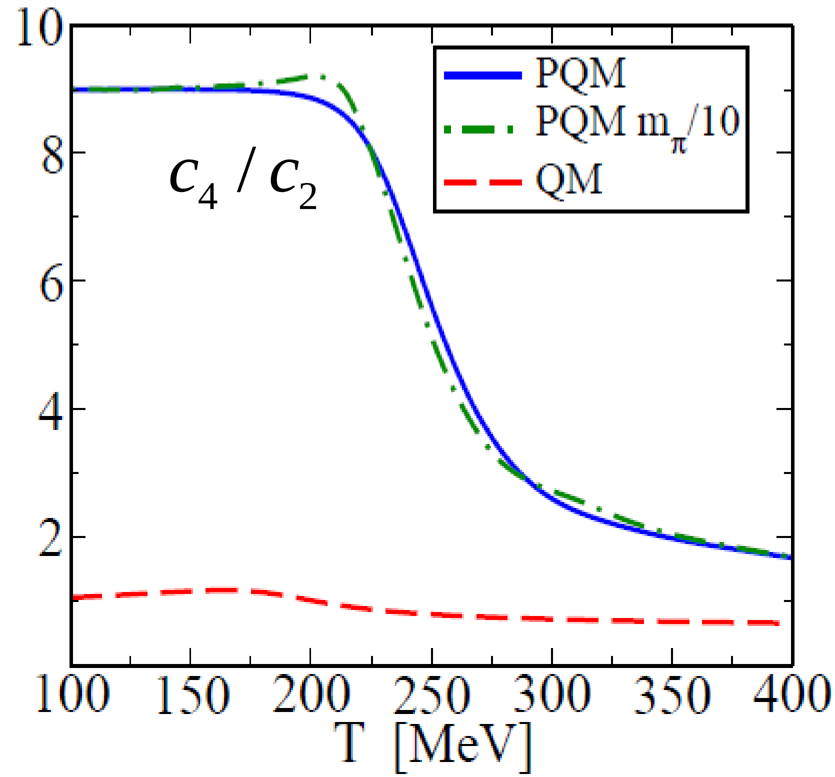
$$\frac{P_{q\bar{q}}(T)}{T^4} \approx \frac{2N_f}{27\pi^2}\left(\frac{3m_q}{T}\right)^2 K_2\left(\frac{3m_q}{T}\right)\cosh\frac{3\mu_q}{T}$$

$$\Rightarrow \quad c_4 / c_2 = 9$$

- For $T >> T_c$

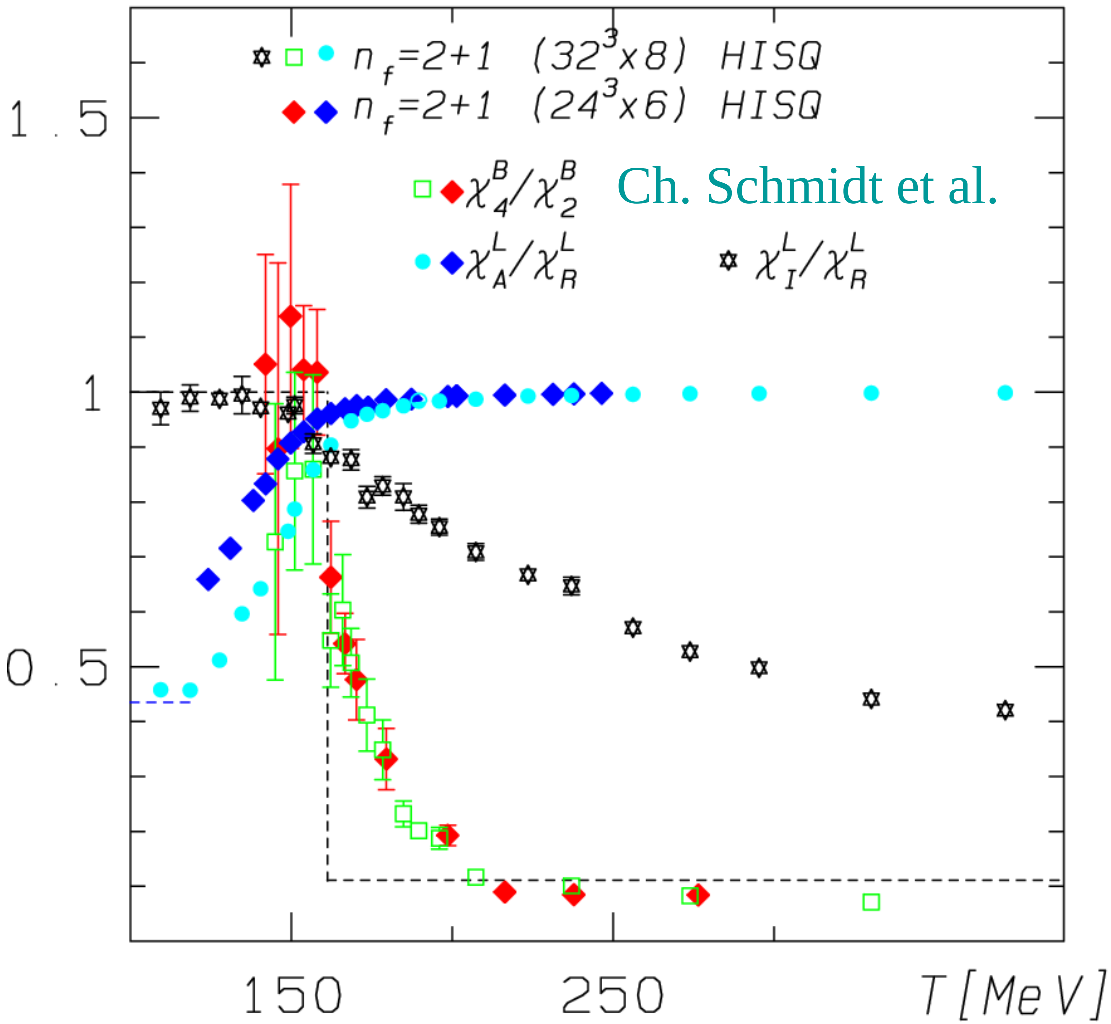
$$\frac{P_{q\bar{q}}(T)}{T^4} = N_f N_c\left[\frac{1}{2\pi^2}\left(\frac{\mu}{T}\right)^4 + \frac{1}{6}\left(\frac{\mu}{T}\right)^2 + \frac{7\pi^2}{180}\right]$$

$$\Rightarrow c_4 / c_2 = 6/\pi^2$$



- Smooth change with a very weak dependence on the pion mass

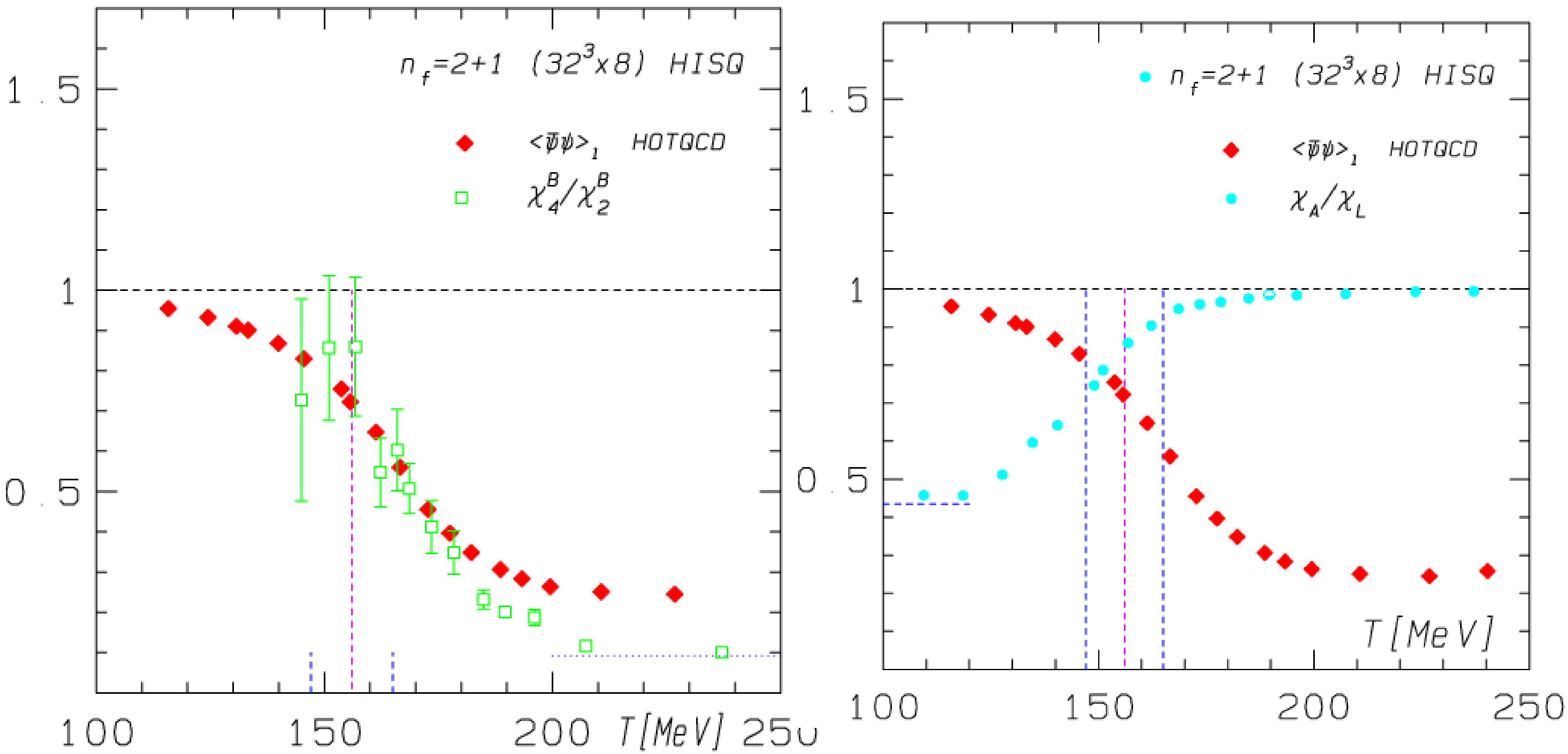
Polyakov loop susceptibility ratios still away from the continuum limit:



- The renormalization of the Polyakov loop susceptibilities is still not well described:

Still strong dependence on N_τ in the presence of quarks.

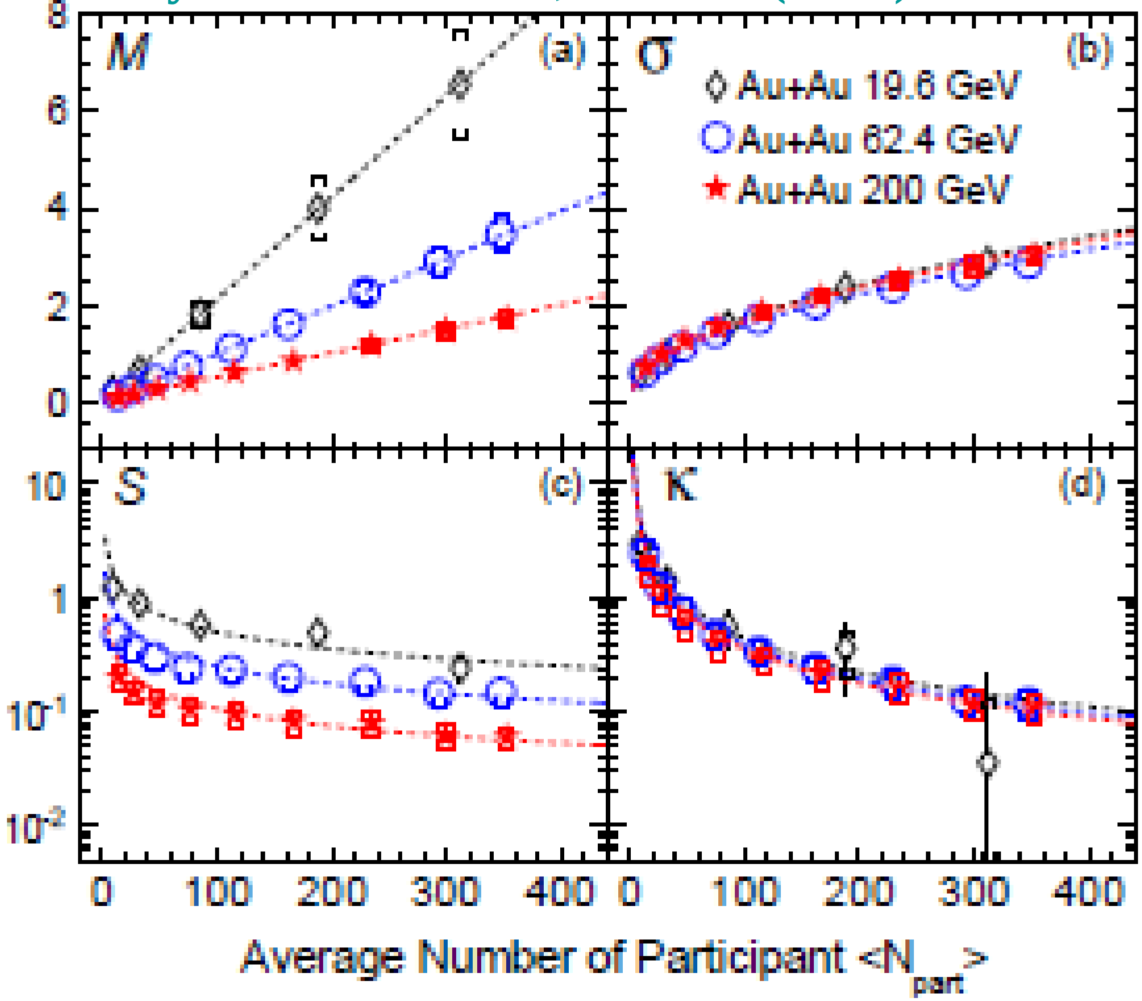
Interplay between deconfinement and chiral transition at finite temperature in LQCD



- Challenging and pioneering STAR data on net proton number, electric chage and strangeness fluctuations

STAR DATA ON MOMENTS of $B = p - \bar{p}$ FLUCTUATIONS

Phys. Rev. Lett. 105, 022302 (2010)



$$\delta N_B = N_B - M_B$$

- Mean

$$M_B = < N_p > - < N_{\bar{p}} >$$

- Variance

$$\sigma_B{}^2 = < (\delta N_B)^2 >$$

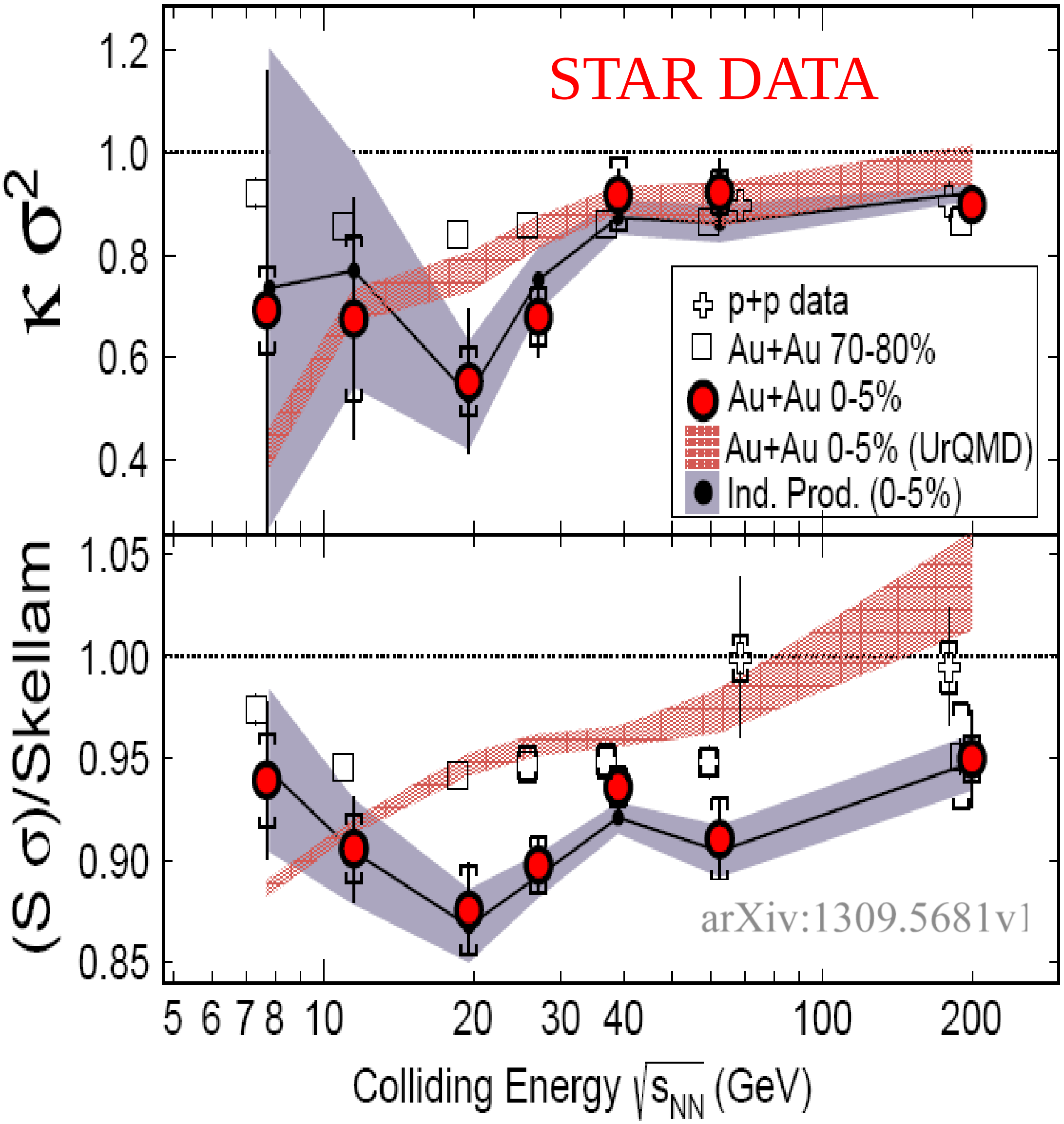
- Skewness

$$S_B = < (\delta N_B)^3 > / \sigma_B{}^3$$

- Kurtosis

$$\kappa_B = < (\delta N_B)^4 > / \sigma_B{}^4 - 3$$

STAR data on the first four moments of net baryon number



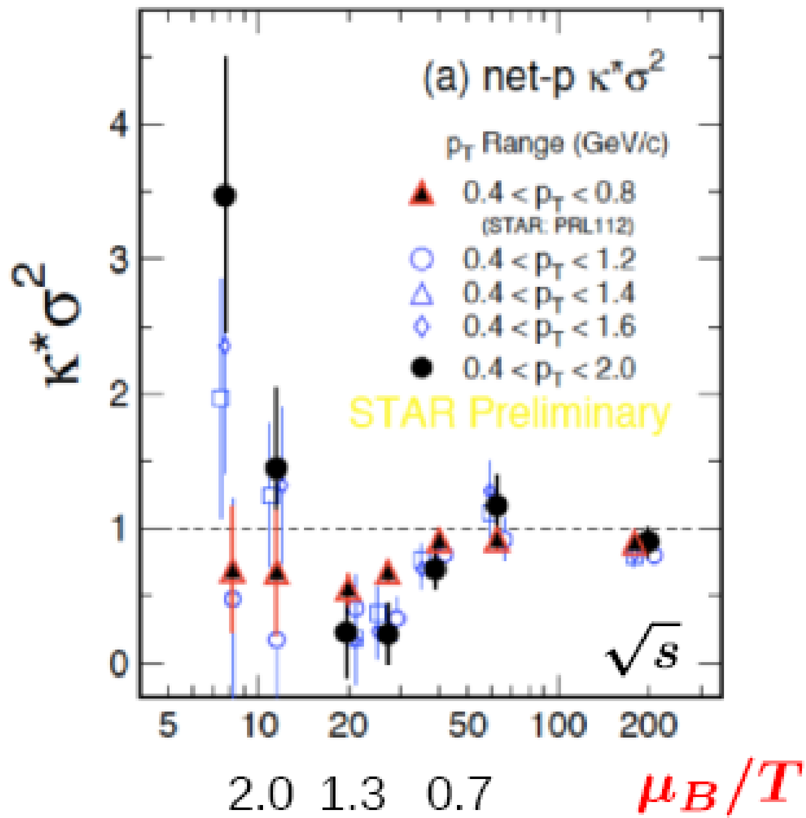
Deviations from the HRG

$$S\,\sigma = \frac{\chi_B^{(3)}}{\chi_B^{(2)}}\quad , \quad \kappa\,\sigma^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

$$S\,\sigma\,|_{HRG} = \frac{N_p - N_{\bar{p}}}{N_p + N_{\bar{p}}}\quad , \kappa\sigma\,|_{HRG} = 1$$

Data qualitatively consistent with the change of these ratios due to the contribution of the O(4) singular part to the free energy

Challenging and pioneering STAR data



- Can we understand this non-monotonic structure as an indication of criticality at chemical freezeout near QCD phase boundary ?
- Is such structure due to remnant of O(4) or Z(2) CP or bought?
- Is systematics of other conserved charges consistent with critical behavior

Moments obtained from probability distributions

- Moments obtained from probability distribution

$$< N^k >= \sum_N N^k P(N)$$

- Probability quantified by all cumulants

$$P(N) = \frac{1}{2\pi} \int_0^{2\pi} dy \exp[iyN - \chi(iy)]$$

Cumulants generating function: $\chi(y) = \beta V[p(T, y+\mu) - p(T,\mu)] = \sum_k \chi_k y^k$

- In statistical physics

$$P(N) = \frac{Z_C(N)}{Z_{GC}} e^{\frac{\mu N}{T}}$$

$$e^{i\phi S} H e^{i\phi S} = H \leftrightarrow \ [S,H]=0$$

conservation on the average

exact conservation

$$Z^{GC}(T,\mu_S,V) = Tr\ [e^{-\beta(H-\mu_S S)}]$$

$$Z_S^C(T,V) = Tr_S[e^{-\beta H}]$$

$$Z^{GC} = \sum_{S=-\infty}^{S=+\infty} e^{S\mu_S/T} Z_S^C$$

$$Z_S(T,V) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi e^{-iS\varphi} Z^{GC}(T,\frac{\mu_S}{T} \to i\varphi)$$

$$P(S) = \left(\frac{\bar{S}_1}{\bar{S}_{\bar{1}}}\right)^{\frac{S}{2}} \exp\left[\sum_{n=1}^{3}(\bar{S}_n + \bar{S}_{\bar{n}})\right] \sum_{i=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\left(\frac{\bar{S}_3}{\bar{S}_{\bar{3}}}\right)^{k/2} I_k(2\sqrt{\bar{S}_3\bar{S}_{\bar{3}}}) \left(\frac{\bar{S}_2}{\bar{S}_{\bar{2}}}\right)^{i/2} I_i(2\sqrt{\bar{S}_2\bar{S}_{\bar{2}}}) \left(\frac{\bar{S}_1}{\bar{S}_{\bar{1}}}\right)^{-i-3k/2} I_{2i+3k-S}(2\sqrt{\bar{S}_1\bar{S}_{\bar{1}}})$$

- Probability quantified by $S_n, \overline{S_n}$:mean numbers of charged 1,2 and 3 particles & their antiparticles

Probability distribution of the net baryon number

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

- For the net baryon number P(N) is described as Skellam distribution

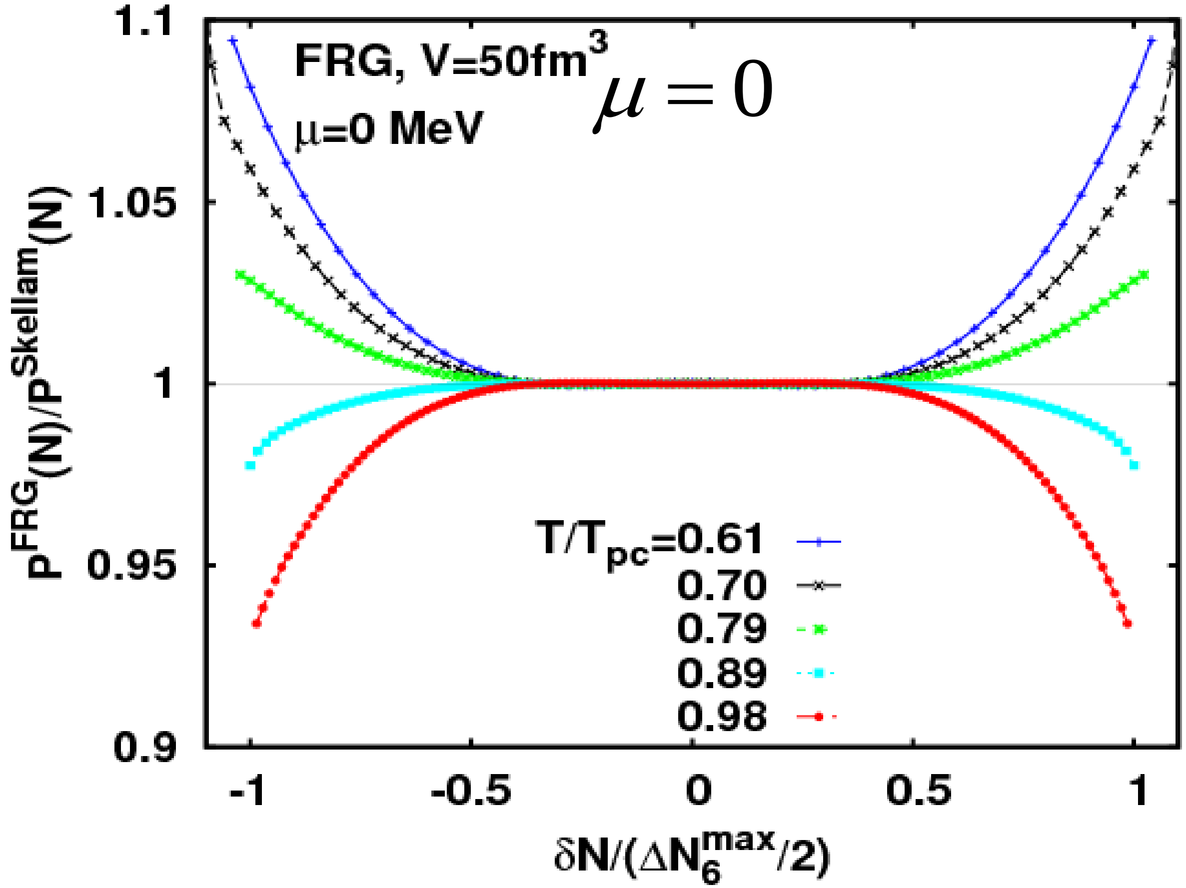
$$P(N) = \left(\frac{B}{\bar{B}} \right)^{N/2} I_N (2\sqrt{B\bar{B}}) \exp[-(B+\bar{B})]$$

- P(N) for net baryon number N entirely given by measured mean number of baryons B and antibaryons $\bar{B}$
- In Skellam distribution all cummulants expressed by the net mean $M = B - \bar{B}$ and variance $\sigma^2 = B + \bar{B}$

The influence of O(4) criticality on P(N) for $\mu = 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance at different T/T_{pc}

K. Morita, B. Friman &K.R. (PQM model within renormalization group FRG)



- Ratios less than unity near the chiral crossover, indicating the contribution of the O(4) criticality to the thermodynamic pressure

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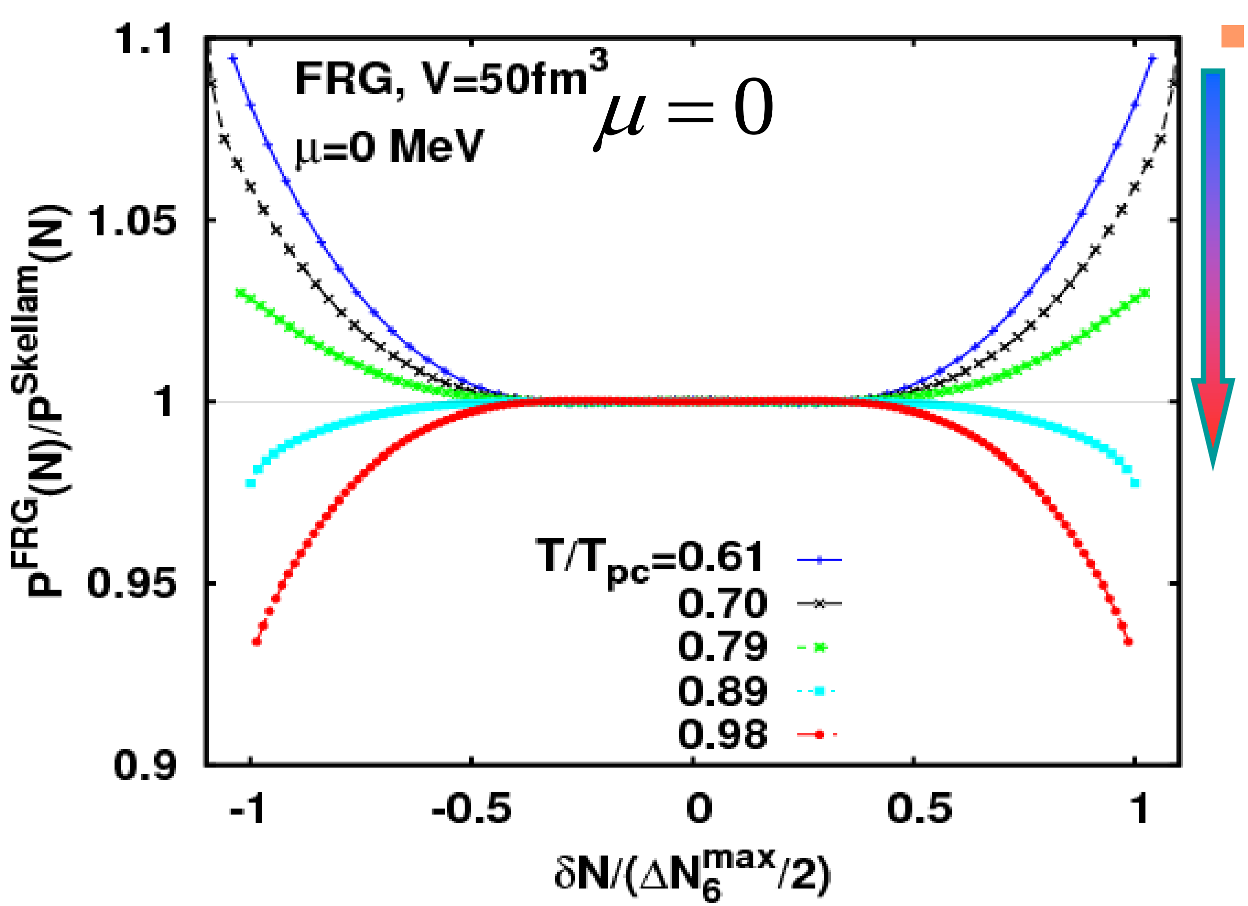

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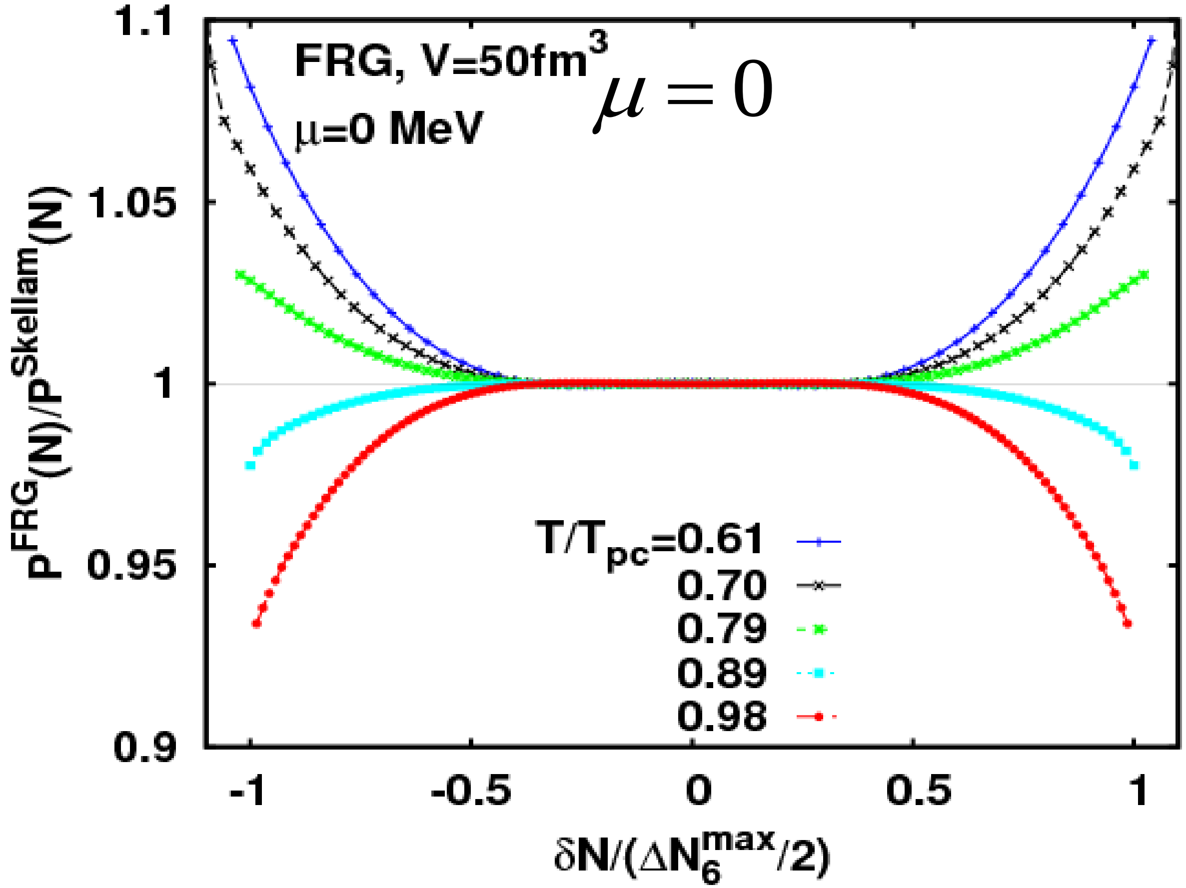
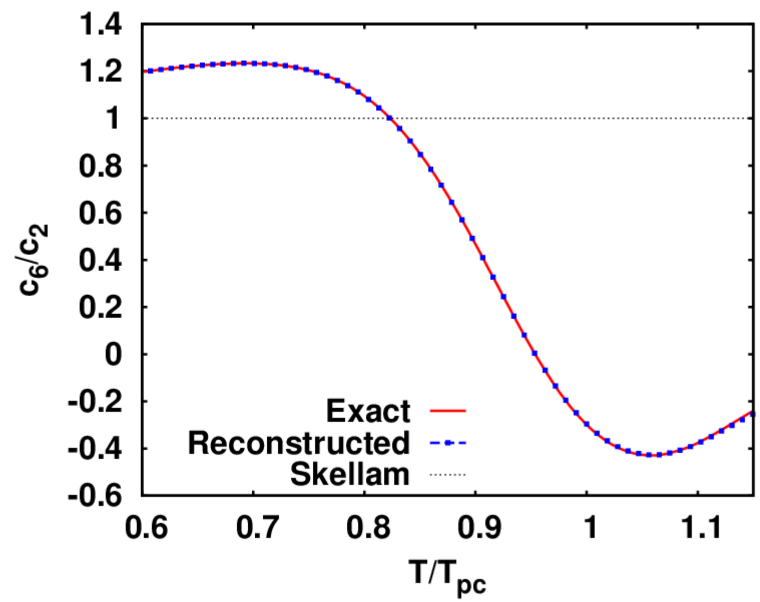


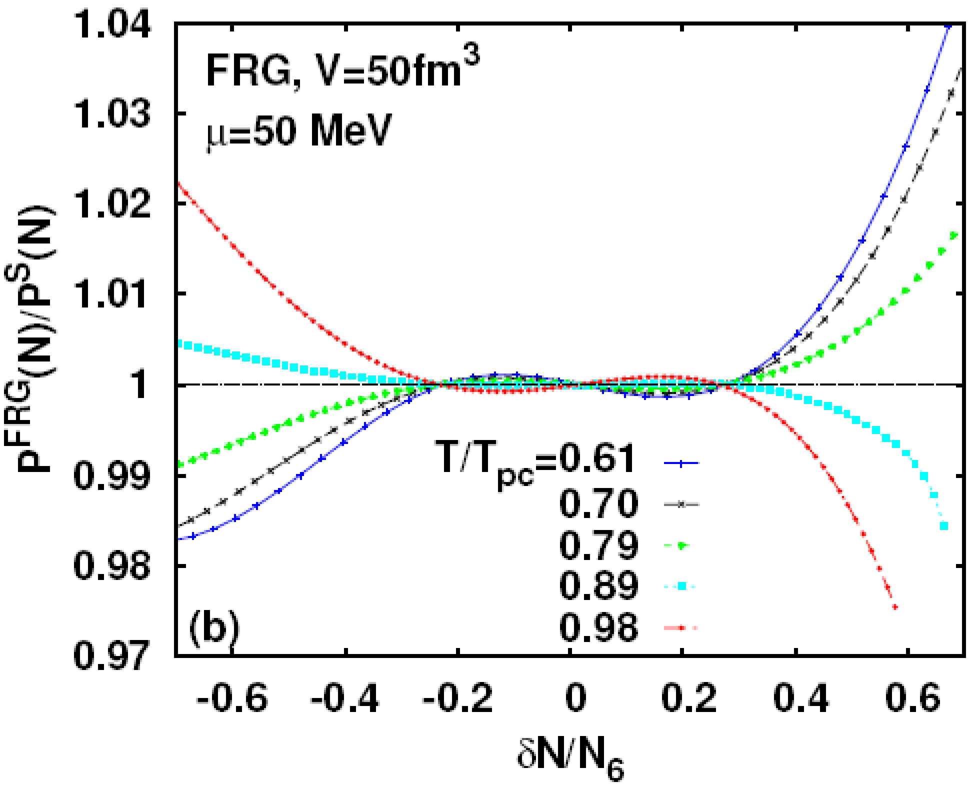
Ratio < 1 at larger |N| if $c_6/c_2 < 1$



The influence of O(4) criticality on P(N) at $\mu \neq 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance near $T_{pc}(\mu)$

K. Morita, B. Friman et al.

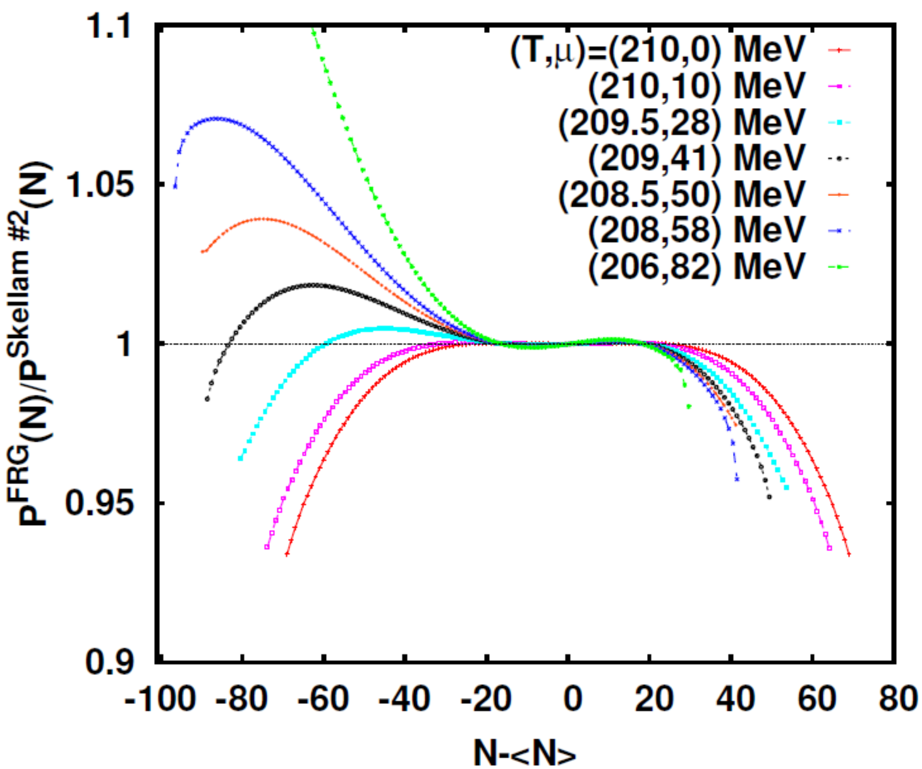


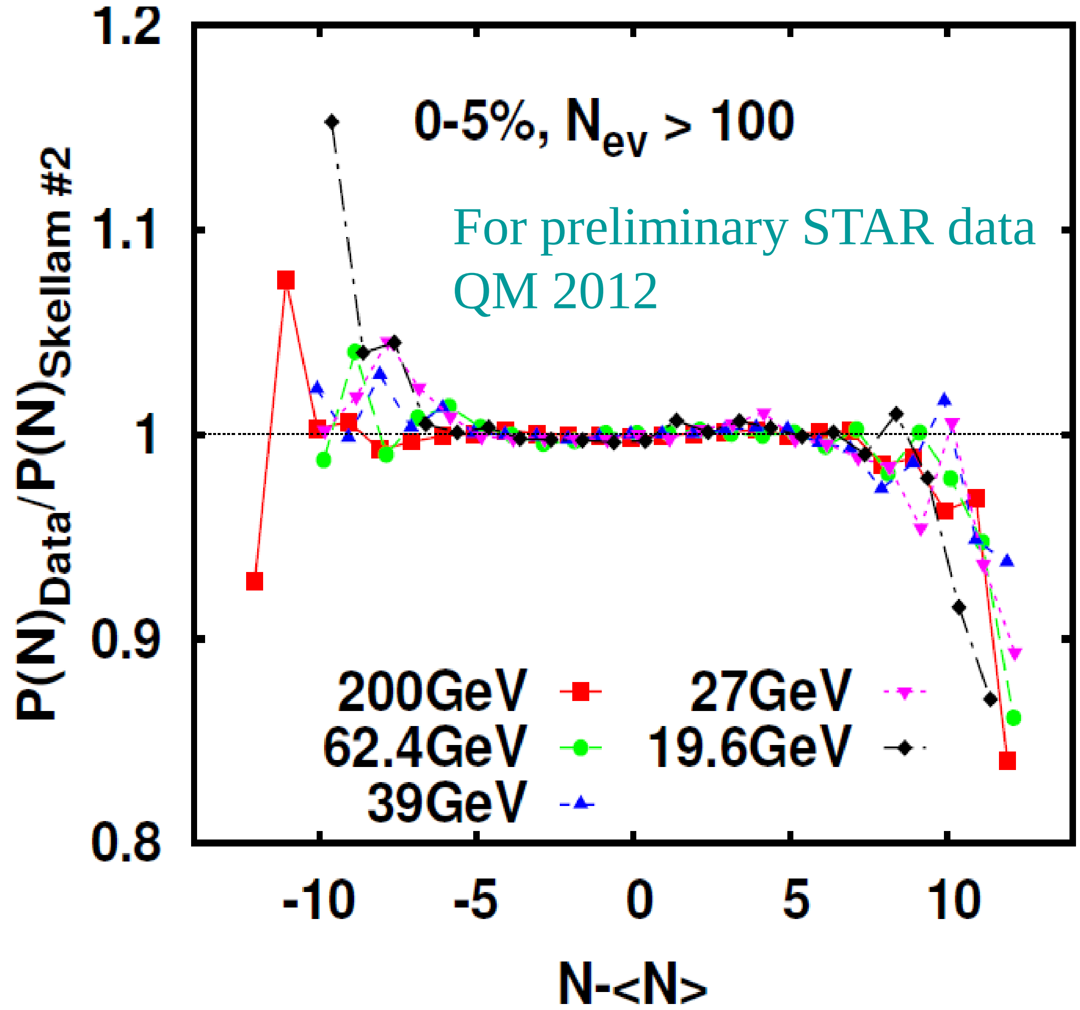
- Asymmetric P(N) $N > < N >$
- Near $T_{pc}(\mu)$ the ratios less than unity for

The influence of O(4) criticality on P(N) for $\mu \neq 0$

- In central collisions the probability behaves as being influenced by the chiral transition

K. Morita, B. Friman & K.R.





Summary

- Effective chiral Lagrangians provide a powerful tool to study the critical consequences of the chiral symmetry restoration in QCD, however

  to quantify the QCD phase diagram and the existence of the CEP/TCP requires the first principle LGT calculations
- A non-monotonic change of the net-quark susceptibility in HIC with the collision energy probes the existence of CEP

  However in non-equilibrium: due to spinodal instabilities the charge fluctuations are as well diverging

  - Large fluctuations signals 1st order transition
- Particle yields in HIC are of thermal origin
- HIC provide a lower bound for a phase boundary in QCD
- To observe remnants of deconfinement and O(4) chiral crossover and to discover CP =>measure higher order fluctuations of conserved charges!!