北京大学
PEKING UNIVERSITY

Correlated fluctuations near the QCD critical endpoint

Lijia Jiang

School of Physics, Peking University

I. Introduction

II. Dynamical modeling near the critical point---A brief review

III. Correlated Fluctuations along the Freezeout surface

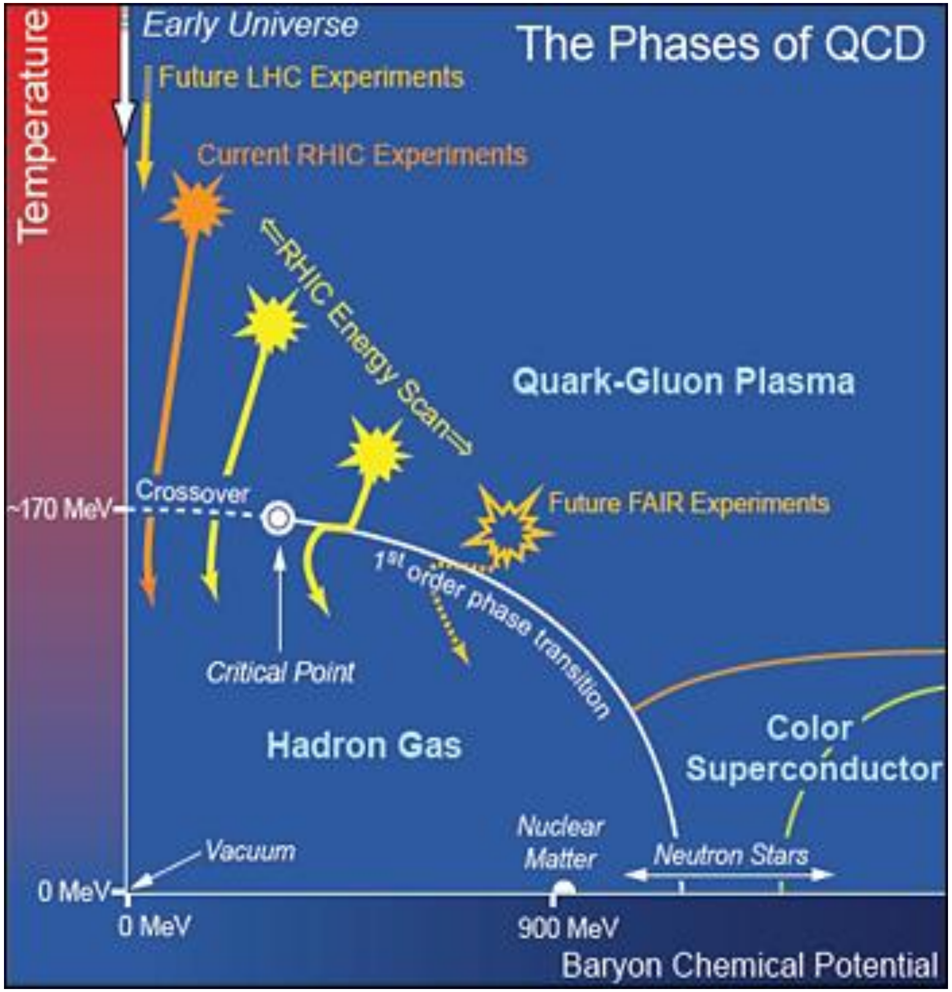
IV. Summary

QCD phase transition & CEP

Critical endpoint --- the landmark of the QCD phase diagram.

- **Lattice simulation :**
  - μ=0, finite T
  - **crossover**
- **Effective theories:**
  - (P)NJL, QM, FRG, DSE, RM)
  - finite T and μ
  - **first order**
  - **CEP** is predicted.

➢ **The location of CEP? The signals?**



Theoretical predictions on signals of CEP

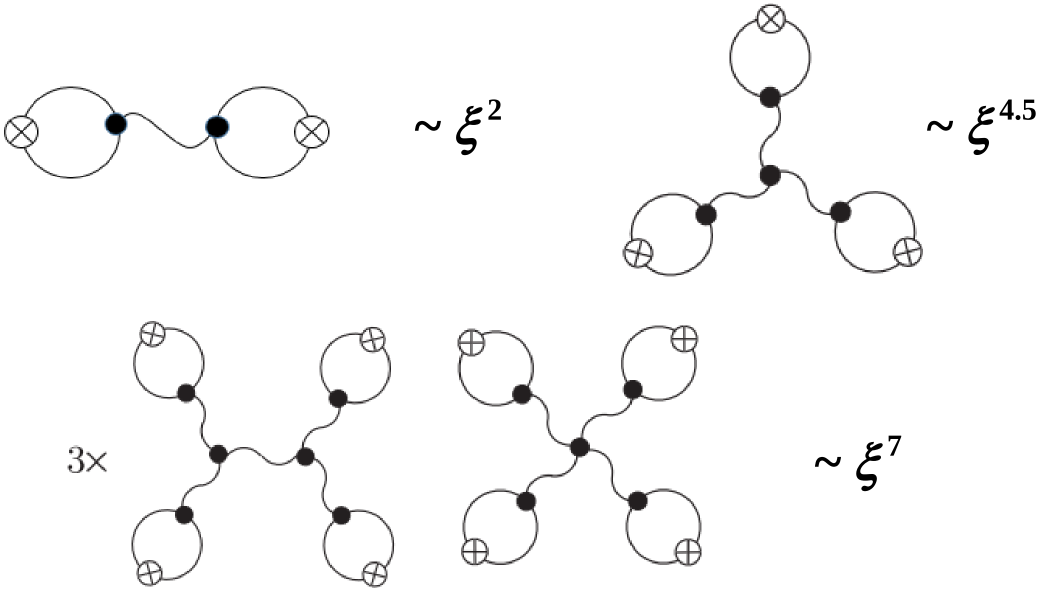
[M. Stephanov, PRL 102, 032301(2009)]

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \quad \Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots\right].$$

$$\langle \sigma_0^2 \rangle = \frac{T}{V}\xi^2 \qquad \langle \sigma_0^3 \rangle = \frac{2\lambda_3 T}{V}\xi^6; \qquad \langle \sigma_0^4 \rangle_c = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$

Critical Fluctuations of particles:

$$\left\langle (\delta N)^2 \right\rangle \sim \xi^2$$
$$\left\langle (\delta N)^3 \right\rangle \sim \xi^{4.5}$$
$$\left\langle (\delta N)^4 \right\rangle \sim \xi^7$$

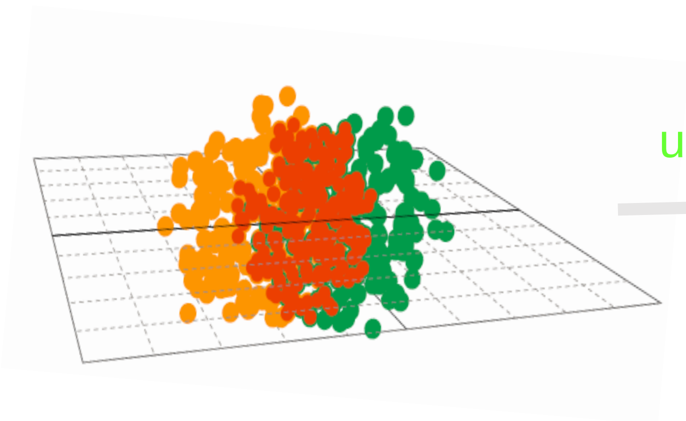


Static and infinite system at critical endpoint : $\xi \to \infty$

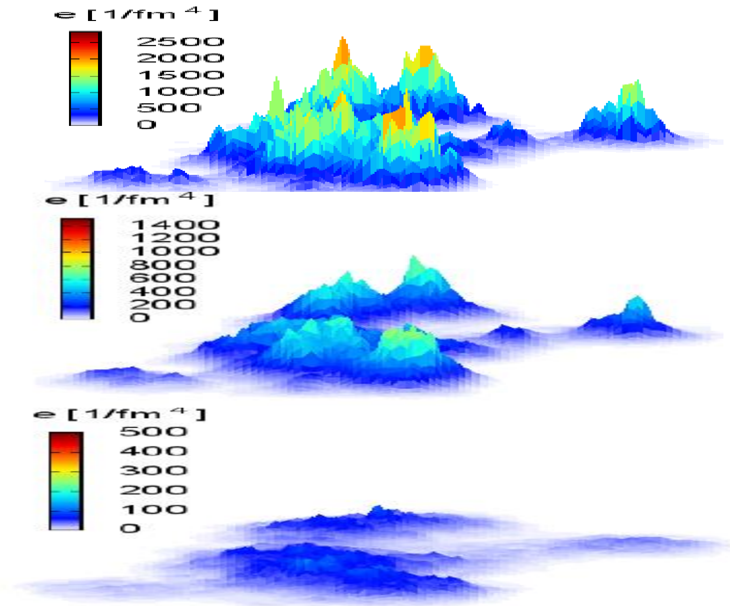
Fireball, finite size & finite evolution time: $\xi \sim O\ (3\ fm)$

Fluctuations & Correlations at RHIC and the LHC

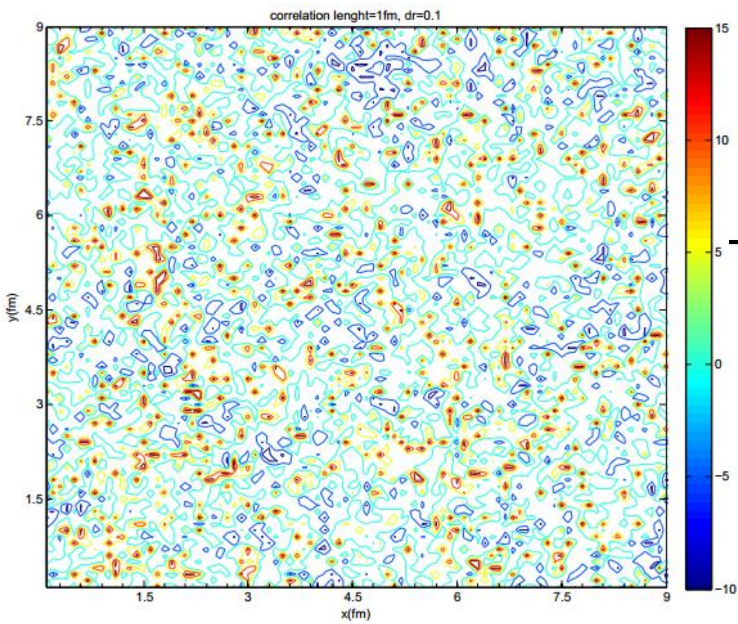
- **Initial fluctuations**



uncorrelated



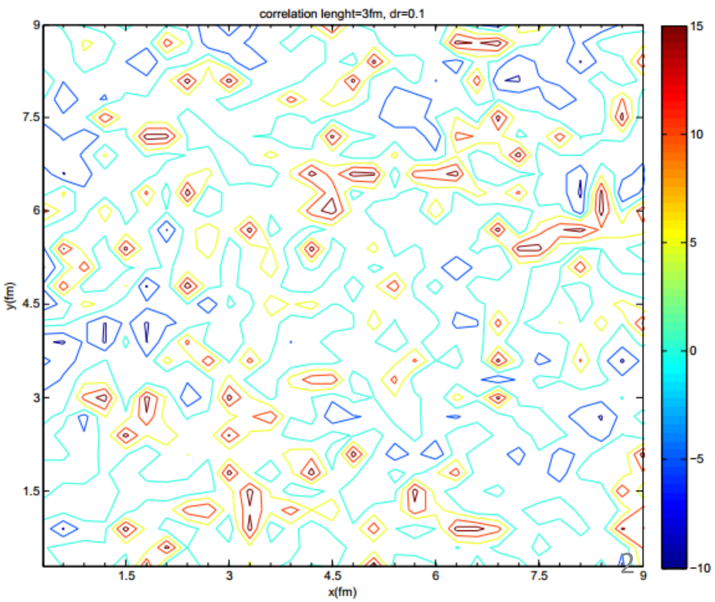
- **Critical fluctuations**



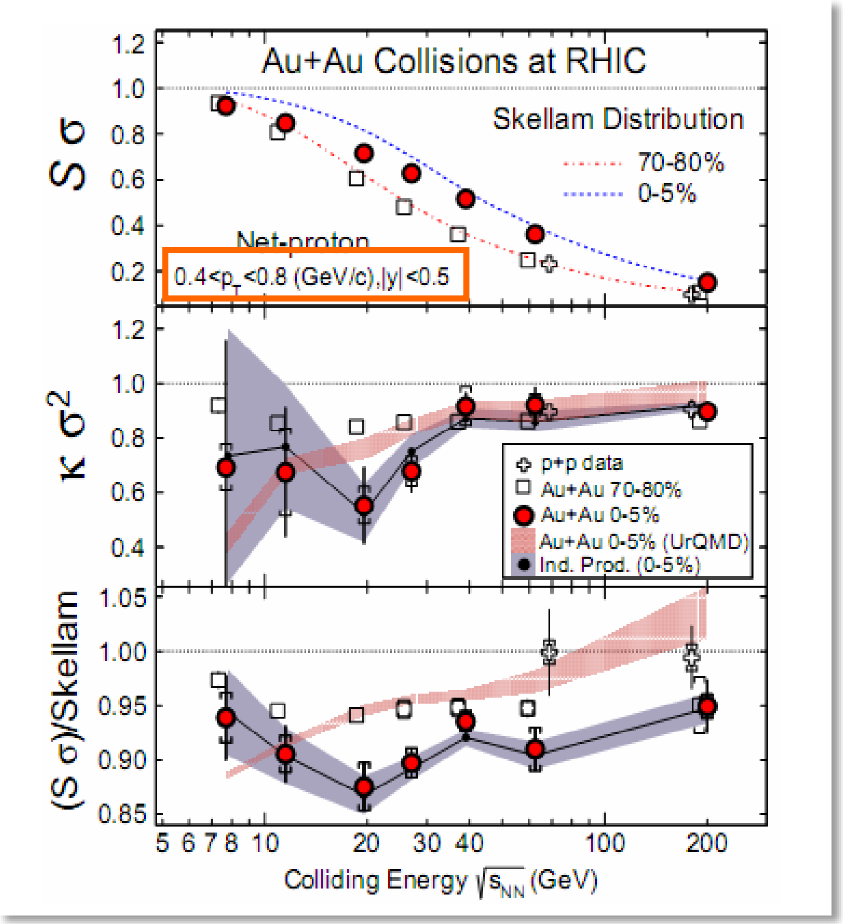
correlated

increase ξ

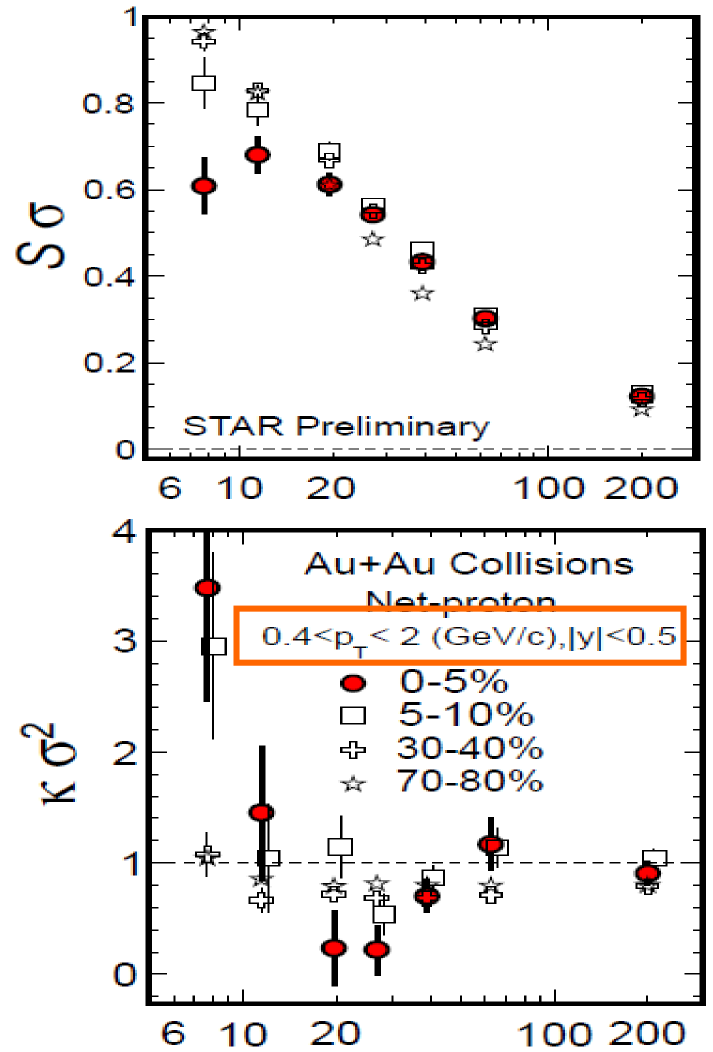
$$\langle \sigma_1 \sigma_2 \rangle \sim \frac{e^{-r/\xi}}{r}$$



STAR BES: Cumulant ratios



[STAR Collaboration, PRL, 112, 032302 (2013)]



[Xiaofeng Luo(for the STAR Collaboration), PoS(CPOD2014)019]

$$S\,\sigma = \frac{C_3}{C_2} \sim \chi_B^{(3)}/\chi_B^{(2)}$$

$$\kappa\sigma^2 = \frac{C_4}{C_2} \sim \chi_B^{(4)}/\chi_B^{(2)}$$

Static --> dynamical??

It is important to address the effects from **dynamical** evolutions!

Dynamical Modeling near the QCD critical point

-- a brief review

Chiral Hydrodynamics

[K. Paech, H. Stocker and A. Dumitru, PRC 68, 044907 (2003)]

$$\mathcal{L} = \bar{q}\left[i\gamma - m - g\left(\sigma + i\gamma_5\tau\pi\right)\right]q + \frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi\partial^\mu\pi\right] - U\left(\sigma,\pi\right)$$

Chiral field:

$$\partial_\mu\partial^\mu\sigma + \frac{\delta U}{\delta\sigma} = -g\langle\bar{q}q\rangle = -g\rho_s, \quad \text{with} \quad \rho_s = g\sigma d_q\int\frac{d^3p}{(2\pi)^3}\frac{1}{E}f(p),$$

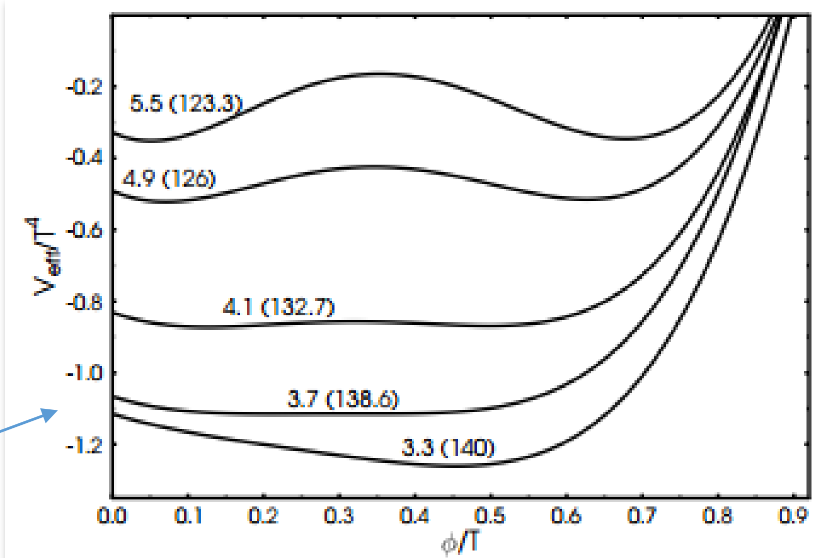
$$\partial_\mu\partial^\mu\vec{\pi} + \frac{\delta U}{\delta\vec{\pi}} = -g\langle\bar{q}\gamma_5\vec{\tau}q\rangle = -g\vec{\rho}_{ps} \qquad \vec{\rho}_{ps} = g\vec{\pi}d_q\int\frac{d^3p}{(2\pi)^3}\frac{1}{E}f(p)$$

Quark fluid:

$$\partial_\mu T^{\mu\nu} = g\rho_s\partial^\nu\sigma + g\vec{\rho}_{ps}\partial^\nu\vec{\pi}$$

- Quarks and antiquarks: the heat bath (fluid), which interact with the chiral field via effective mass $g\sigma$.
- σ field: order parameter for chiral phase transition.

the order of the phase transition is in charged by the coupling g.



Chiral Hydrodynamics

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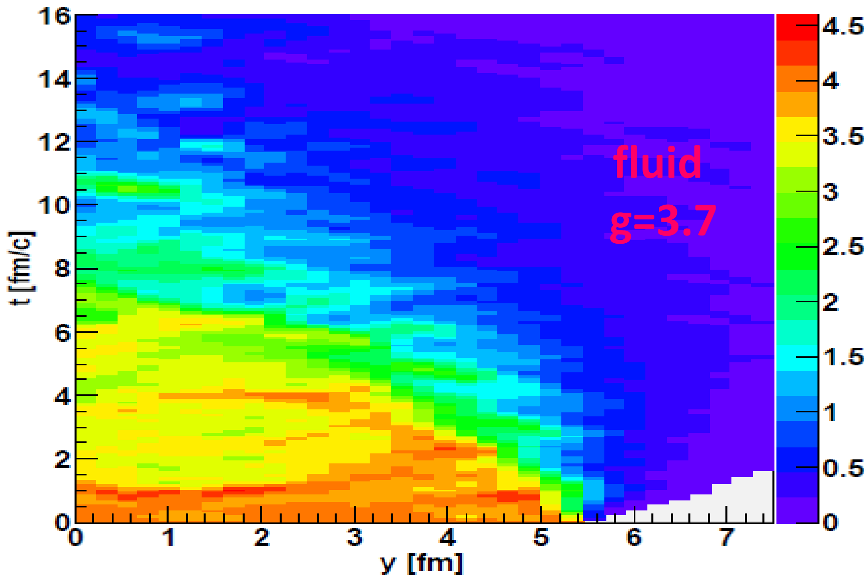
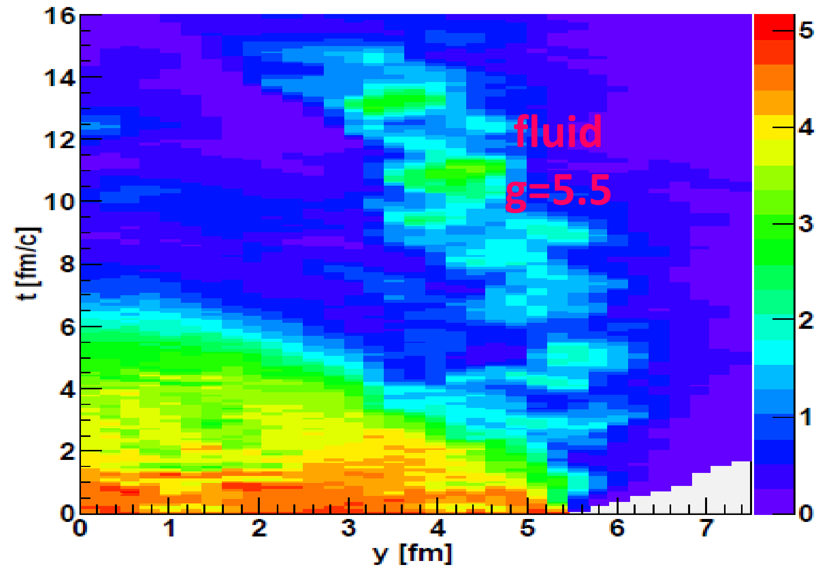
$$\mathcal{L} = \bar{q}\left[i\gamma - m - g\left(\sigma + i\gamma_5\tau\pi\right)\right]q + \frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi\partial^\mu\pi\right] - U\left(\sigma,\pi\right)$$

$$\begin{cases} \partial_\mu\partial^\mu\sigma + \frac{\delta U_{eff}}{\delta\sigma} + g\left\langle\bar{q}q\right\rangle = 0 \\ \partial_\mu T^{\mu\nu}_{fluid} = S^\nu, \;\; S^\nu = -\left(\partial^2\sigma = +\frac{\delta U_{eff}}{\delta\sigma}\right)\partial^\nu\sigma \end{cases}$$

(fluctuation is introduced by initial condition)



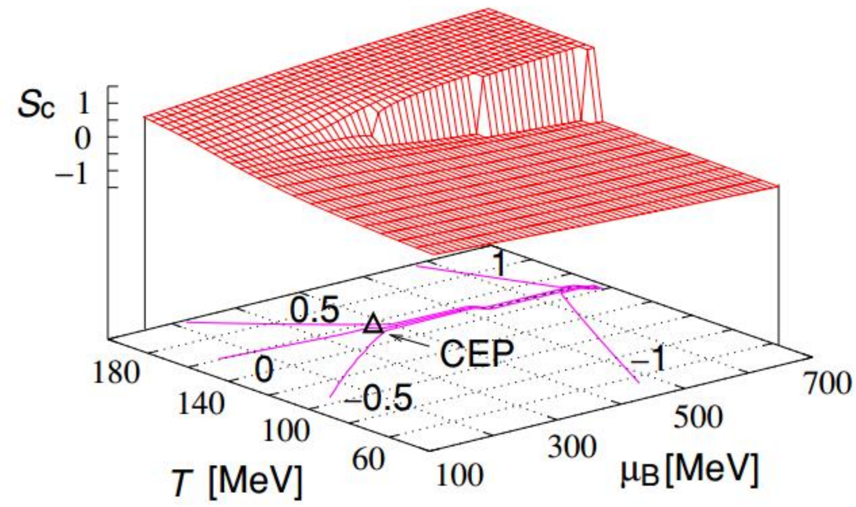


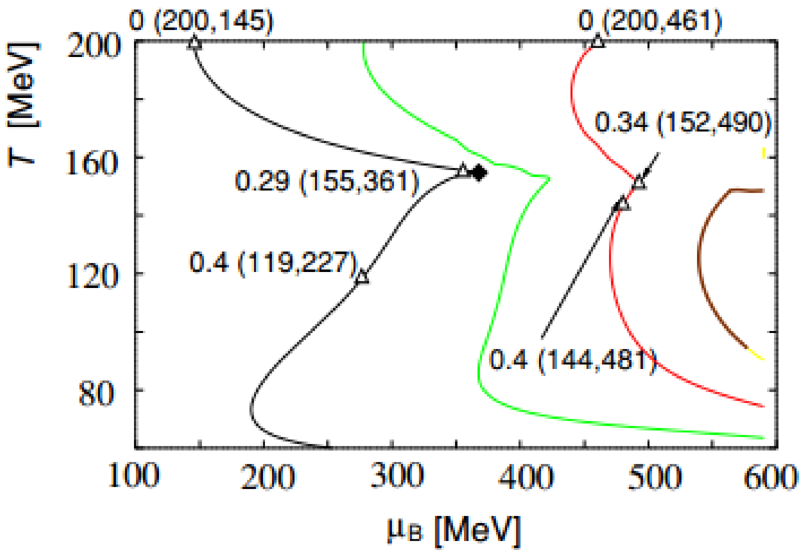
- Chiral fluid dynamics with dissipation & noise — Nahrgang,et al., PRC 84, 024912 (2011)
- Chiral fluid dynamics with a Polyakov loop (PNJL) — Herold, et al., PRC 87, 014907 (2013)

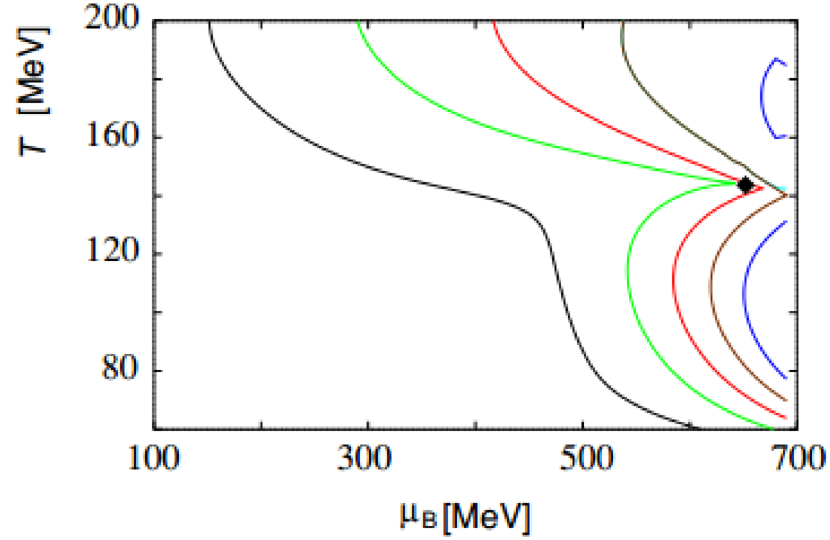
EOS with CEP employed in hydrodynamics

C.Nonaka, M. Asakawa, PRC 71, 044904 (2005)

$$s(T,\mu_B)=\frac{1}{2}(1-\tanh[S_c(T,\mu_B)])s_H(T,\mu_B)+\frac{1}{2}(1+\tanh[S_c(T,\mu_B)])s_Q(T,\mu_B),$$

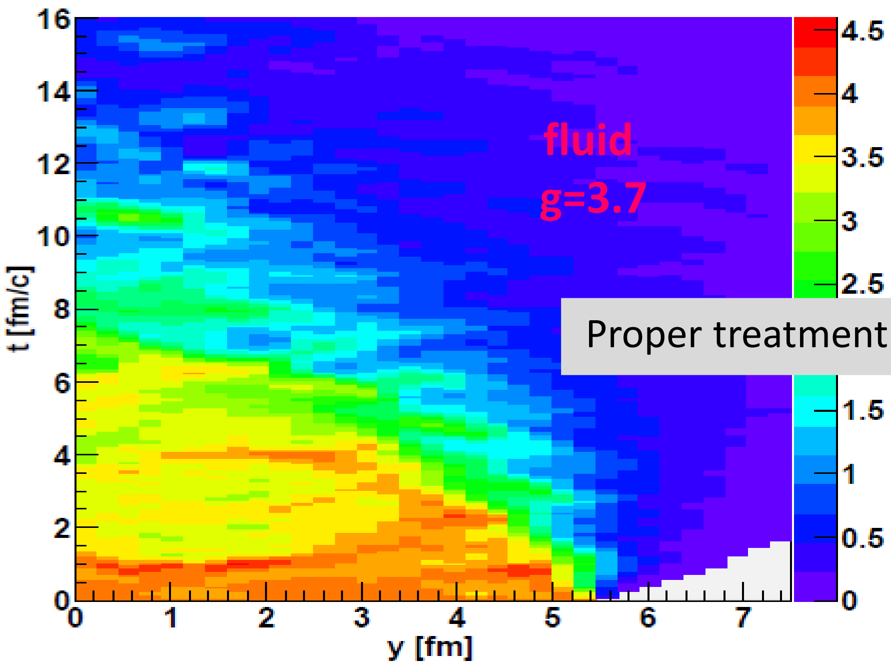




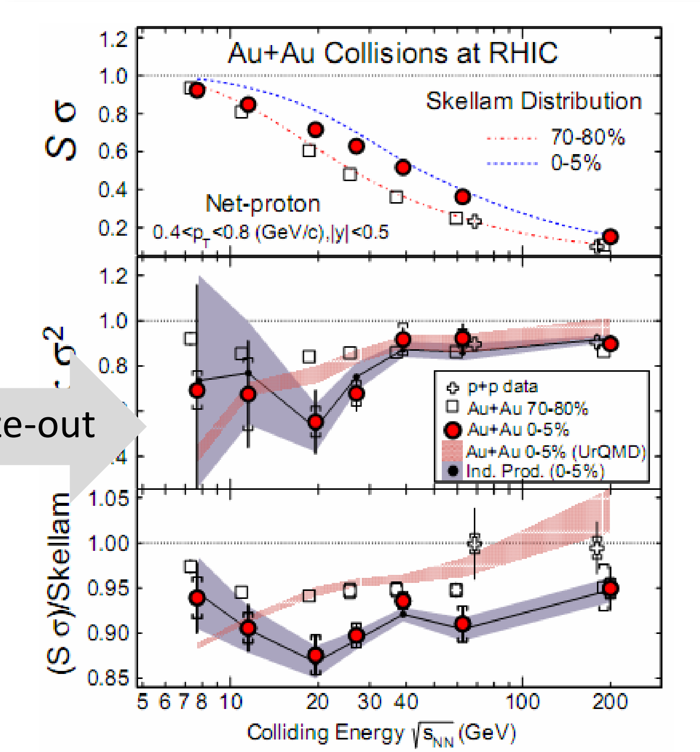


Essential ingredients for dynamical modeling:

1. Evolution of bulk matter with external field √
2. EOS with CEP √
3. A Proper treatment of freezeout scheme ?



Proper treatment of freeze-out



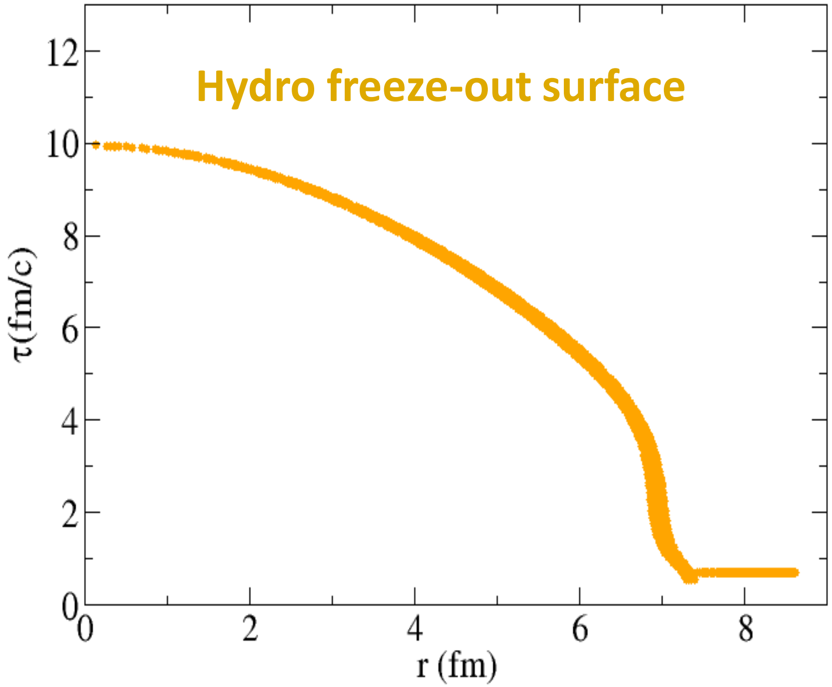
To connect the dynamical evolution with experimental observables, it is important to properly treat the freeze-out procedure with external field.

Lijia Jiang, Pengfei Li & Huichao Song in preparation

Correlated fluctuations along the freeze-out surface near Tc

-- theoretical model

Particle emissions near Tc with external field



Jiang, Li & Song in preparation

Particle emissions in traditional hydro

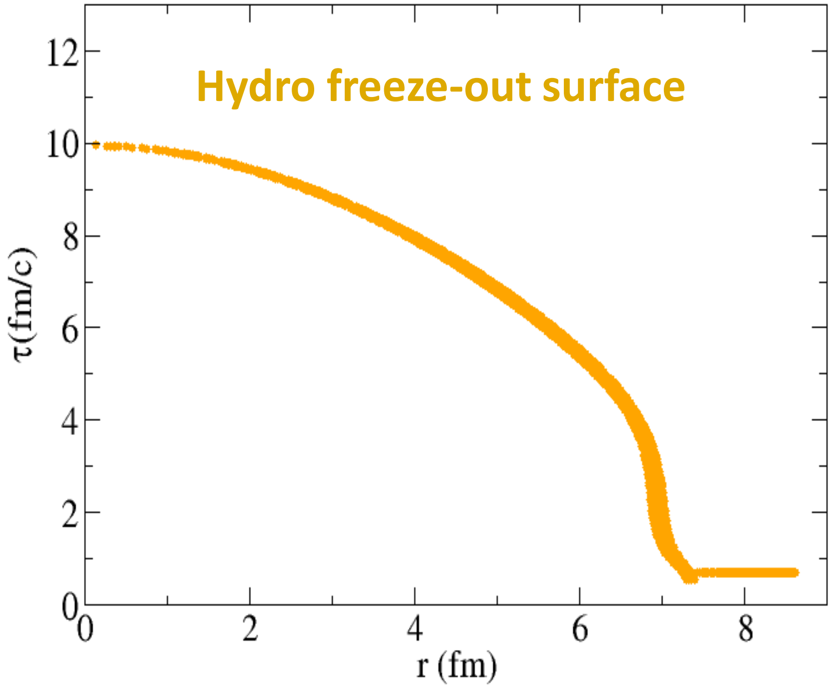
$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p)$$

Particle emissions with external field

$$M \to g\left(\bar{\sigma} + \sigma(x)\right)$$

$$\begin{aligned} f(x,p) &= f_0(x,p)\left[1 - g\sigma(x)/(\gamma T)\right] \\ &= f_0 + \delta f \end{aligned}$$

Particle emissions near Tc with external field



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$$\begin{aligned} \langle \delta f_1 \delta f_2 \rangle_\sigma &= f_{01} f_{02} f_{03} \left(\frac{g^2}{\gamma_1 \gamma_2} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \rangle_c \,, \\ \langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma &= f_{01} f_{02} f_{03} \left(-\frac{g^3}{\gamma_1 \gamma_2 \gamma_3} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \,, \\ \langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma &= f_{01} f_{02} f_{03} f_{04} \left(\frac{g^4}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{1}{T^4} \right) \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c \,. \end{aligned}$$

$$\langle \delta f_1 \delta f_2 \rangle_\sigma = f_{01} f_{02} f_{03} \left(\frac{g^2}{\gamma_1 \gamma_2} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma = f_{01} f_{02} f_{03} \left(-\frac{g^3}{\gamma_1 \gamma_2 \gamma_3} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma = f_{01} f_{02} f_{03} f_{04} \left(\frac{g^4}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{1}{T^4} \right) \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c .$$

$$\Downarrow \quad \frac{\left(\int d^3x\right)^i \left\langle (\delta f)^i \right\rangle}{\left(\int d^3x\right)^i 1}$$

[M. Stephanov, PRD (1999) & PRL (2009).]

$$\langle \delta n_p \delta n_q \rangle_\sigma = \frac{G^2}{VT} \frac{n_p n_q}{\omega_p \omega_q} \frac{1}{m^2}$$

$$\langle \delta n_p \delta n_q \delta n_k \rangle_\sigma = \frac{2\lambda_3}{V^2 T} \frac{n_p n_q n_k}{\omega_p \omega_q \omega_k} \left(\frac{G}{m^2} \right)^3$$

$$\langle \delta n_{p_1} \delta n_{p_2} \delta n_{p_3} \delta n_{p_4} \rangle_\sigma = \frac{6}{V^3 T} \frac{n_{p_1} n_{p_2} n_{p_3} n_{p_4}}{\omega_{p_1} \omega_{p_2} \omega_{p_3} \omega_{p_4}} \left(\frac{G}{m^2} \right)^4 \left[2 \left(\frac{\lambda_3}{m} \right)^2 - \lambda_4 \right]$$

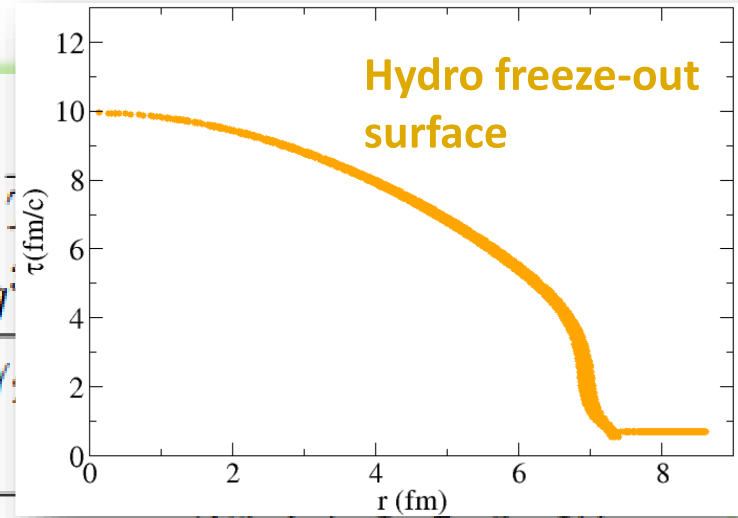
- **For stationary & infinite medium, integrate over coordinate space, the results in Stephanov PRL09 are reproduced.**

Jiang, Li & Song in preparation

$$\langle\delta f_1\delta f_2\rangle_\sigma = f_{01}f_{02}f_{03}\left(\frac{g^2}{\gamma_1\gamma_2}\cdots\right.$$

$$\langle\delta f_1\delta f_2\delta f_3\rangle_\sigma = f_{01}f_{02}f_{03}\left(-\frac{g^3}{\gamma_1\gamma_2\cdots}\cdots\right.$$

$$\langle\delta f_1\delta f_2\delta f_3\delta f_4\rangle_\sigma = f_{01}f_{02}f_{03}f_{04}\left(\frac{\cdots}{\gamma_1\gamma_2\gamma_3\gamma_4 T^4}\right)\cdots$$



CORRELATED particle emissions along the freeze-out surface

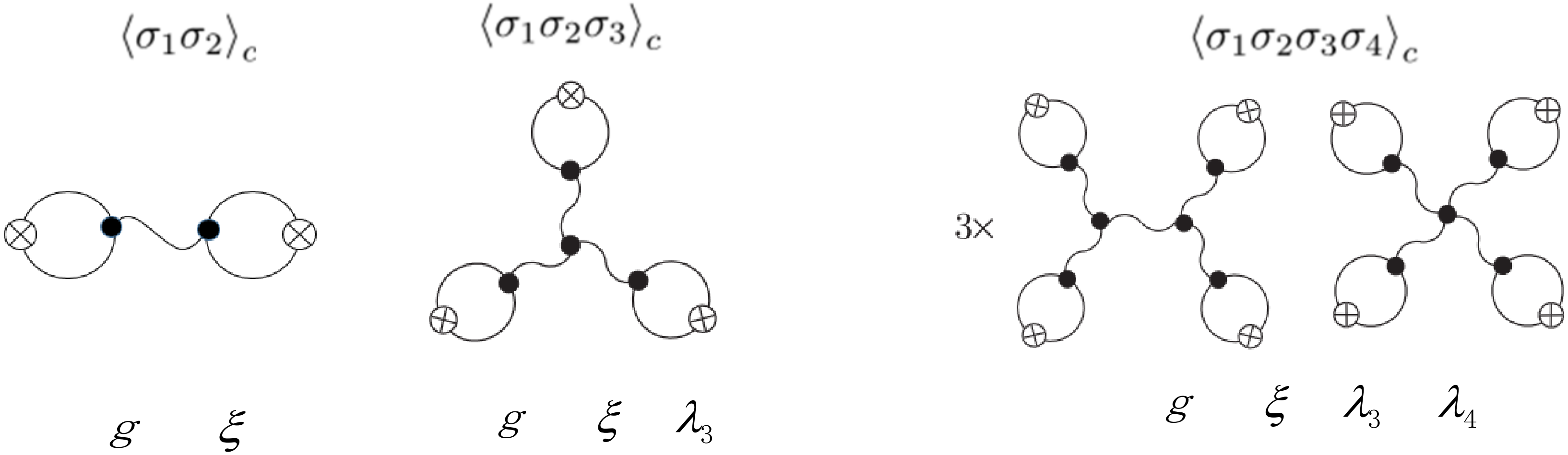
$$\left\langle(\delta N)^2\right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2\left(\prod_{i=1,2}\left(\frac{1}{E_i}\int d^3p_i\int_{\Sigma_i}p_{i\mu}d\sigma_i^\mu d\eta_i\right)\right)\frac{f_{01}f_{02}}{\gamma_1\gamma_2}\frac{g^2}{T^2}\langle\sigma_1\sigma_2\rangle_c\,,$$

$$\left\langle(\delta N)^3\right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3\left(\prod_{i=1,2,3}\left(\frac{1}{E_i}\int d^3p_i\int_{\Sigma_i}p_{i\mu}d\sigma_i^\mu d\eta_i\right)\right)\frac{f_{01}f_{02}f_{03}}{\gamma_1\gamma_2\gamma_3}\left(-\frac{g^3}{T^3}\langle\sigma_1\sigma_2\sigma_3\rangle_c\right),$$

$$\left\langle(\delta N)^4\right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4\left(\prod_{i=1,2,3,4}\left(\frac{1}{E_i}\int d^3p_i\int_{\Sigma_i}p_{i\mu}d\sigma_i^\mu d\eta_i\right)\right)\frac{f_{01}f_{02}f_{03}f_{04}}{\gamma_1\gamma_2\gamma_3\gamma_4}\frac{g^4}{T^4}\langle\sigma_1\sigma_2\sigma_3\sigma_4\rangle_c$$

--partially include the evolution effects and volume effects

The choice of input parameters



- $g \sim (0, 10)$

  phenomenological model
  in vacuum: $m_p \sim$ 900 MeV -> $g \sim 10$; large T: non-interacting, $g \sim 0$

- $\xi \sim (0.5, 5)\mathrm{fm}$

  volume effects, critical slowing down
  ξ increases when the CEP is approaching. (maximum ξ at 27 GeV)

- $\lambda_3 \sim (0, 8)$, $\lambda_4 \sim (4, 20)$

  lattice simulation of the effective potential around critical point.
  increase from the crossover side to the 1st order phase transition side

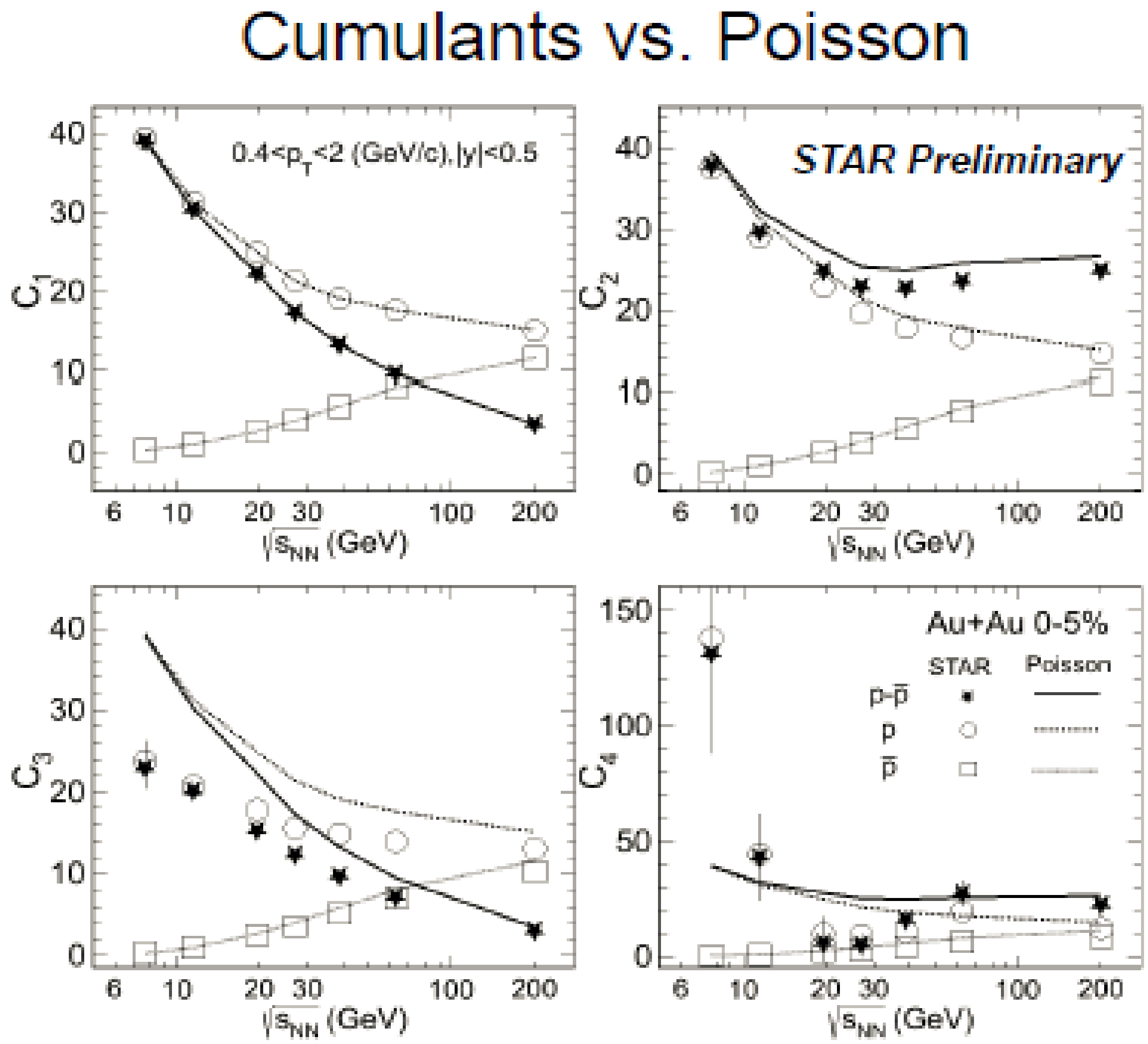
A. Andronic, et al. NPA (2006); M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009); S. P. Klevansky, Rev. Mod. Phys, Vol, 64, No.3 (1992); W. Fu, Y-x, Liu, Phys. Rev. D 79, 074011(2009); M. M. Tsypin, Phys. Rev. Lett. 73, 2015 (1994); M. M. Tsypin, Phys. Rev. B 55, 8911 (1997).; B. Berdnikov and K. Rajagopal, Phys. Rev. D 61, 105017 (2000).

Jiang, Li & Song in preparation

Correlated fluctuations along the freeze-out surface near Tc

---- the experimental data VS thermal baselines

STAR data vs Thermal fluctuation baselines (I)



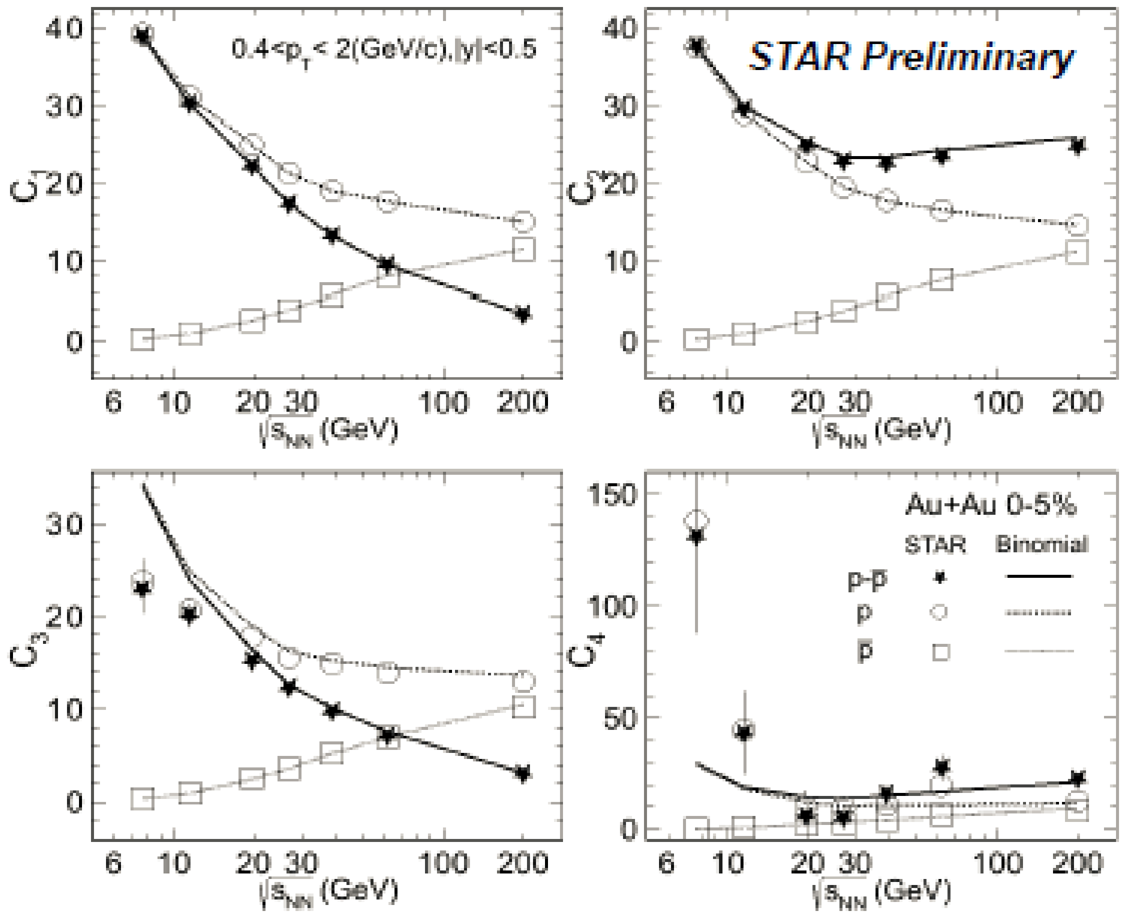
[Xiaofeng Luo(for the STAR Collaboration), PoS(CPOD2014)019]

Fluctuations measured in Experiment: critical fluct. + thermal fluct. + …

The higher order cumulants shows large deviations from Poisson expectations

STAR data vs Thermal fluctuation baselines (II)

Cumulants vs. Binomial



[Xiaofeng Luo(for the STAR Collaboration), PoS(CPOD2014)019]

Fluctuations measured in Experiment: critical fluct. + thermal fluct. + …

The binomial distributions (BD) better describes the data than Poisson, but still show large deviations for C3 and C4 at lower collision energies.

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Correlated fluctuations along the freeze-out surface near Tc

---- comparison with the experimental data

A) Model + Poisson baseline

B) Model + Binomal baseline

$\kappa\sigma^2$, $S\sigma$: (Model + Poisson baseline)

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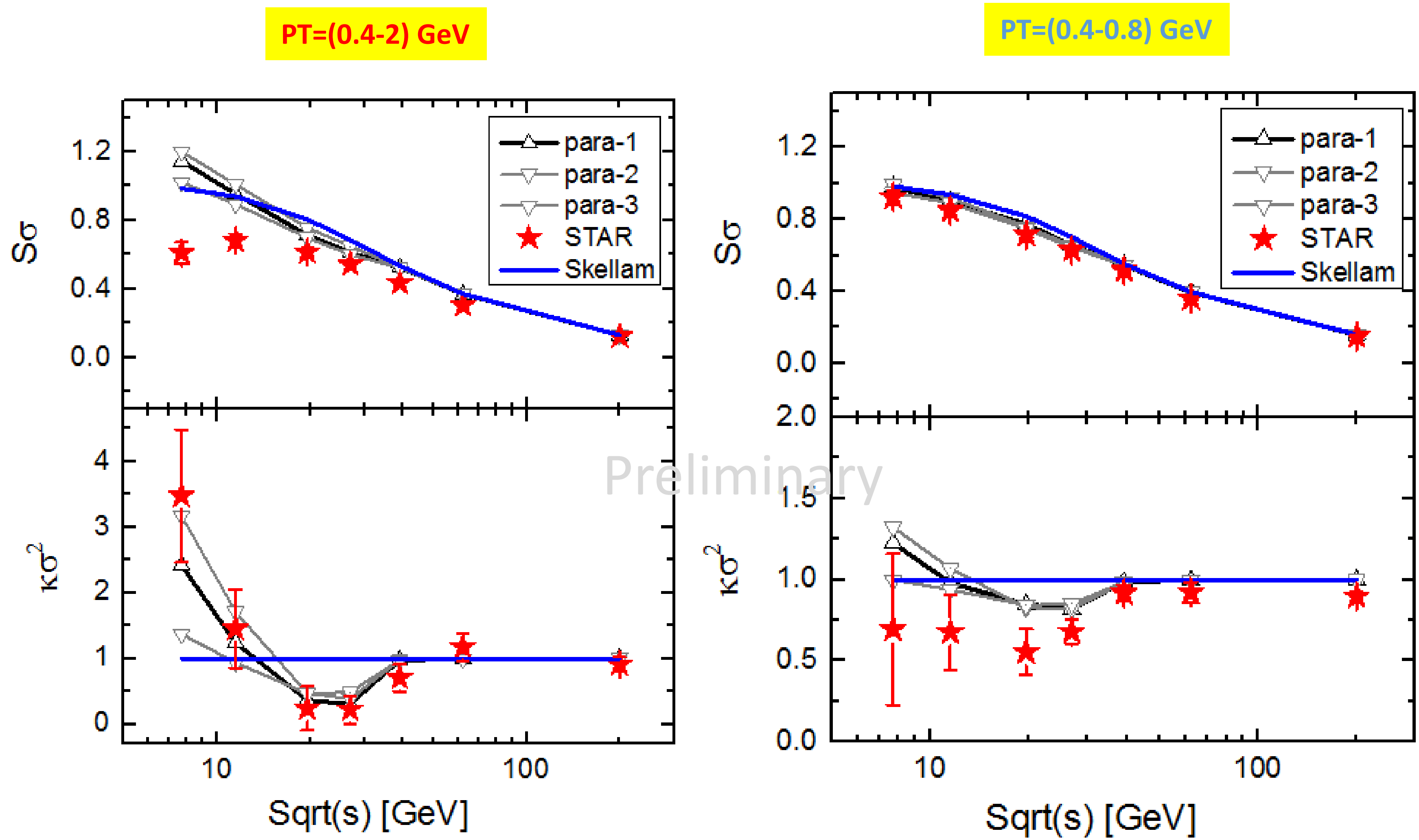
Net Protons: 0-5%



C1 C2 C3 C4 (Model + Poisson baseline)

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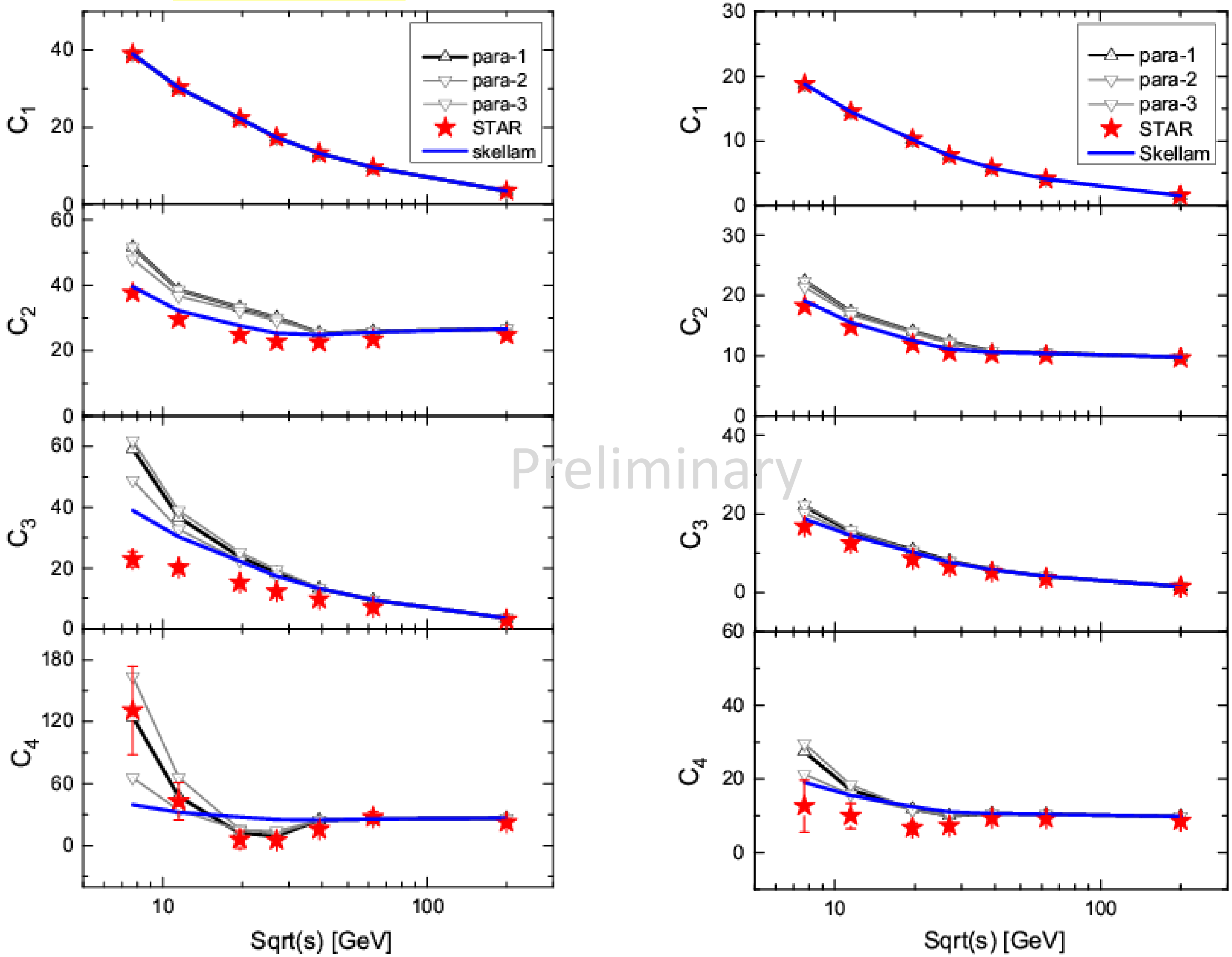
PT=(0.4-2) GeV

Net Protons 0-5%

PT=(0.4-0.8) GeV



Critical fluctuations give positive contribution to C2 , C3; well above the poisson baselines, can NOT explain/describe the C2 , C3 data

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Correlated fluctuations along the freeze-out surface near Tc

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B) Model + Binomal baseline

$\kappa\sigma^2, S\sigma$: (Model + Binomial baseline)

Jiang, Li & Song in preparation

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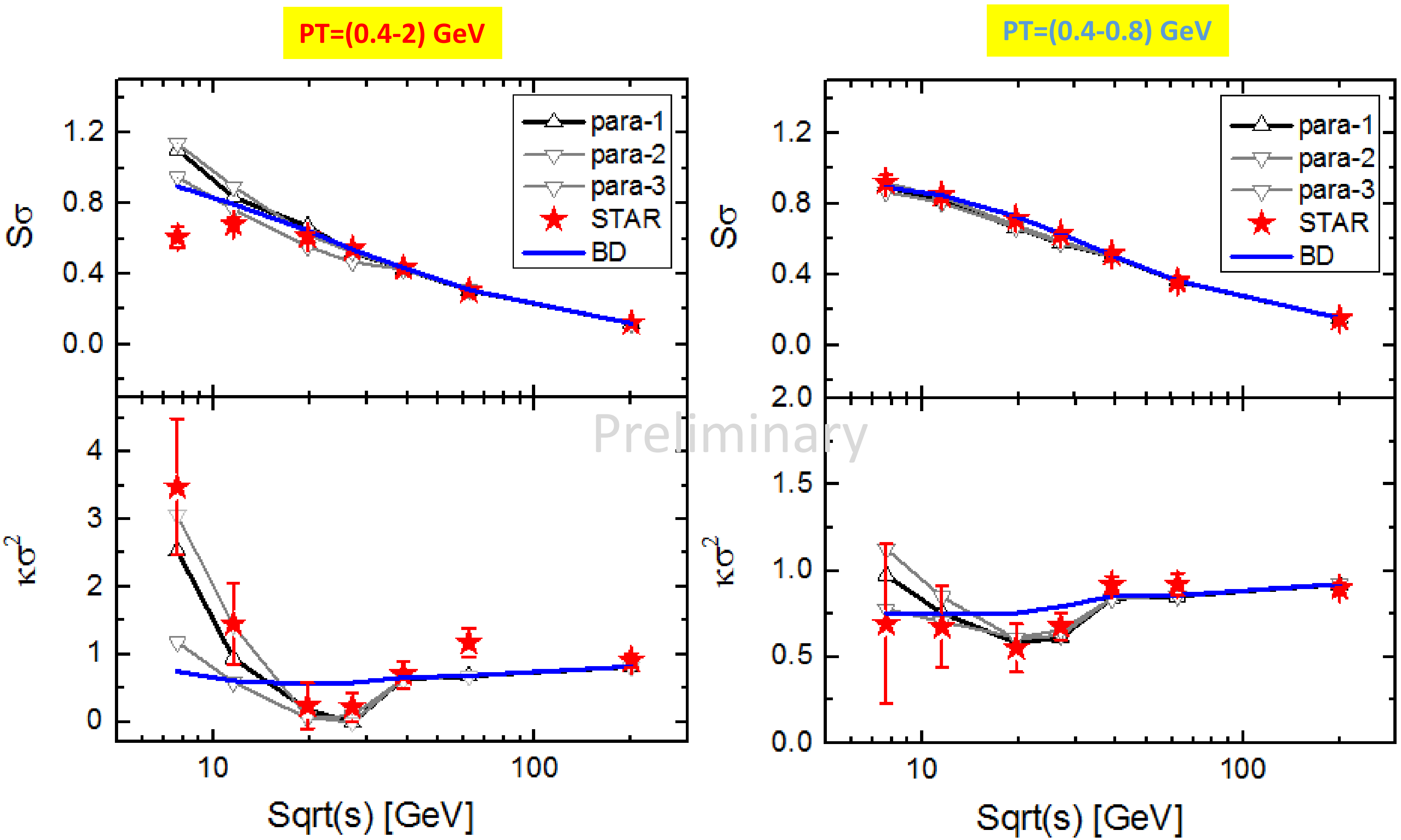

C1 C2 C3 C4 (Model + Binomial baseline)

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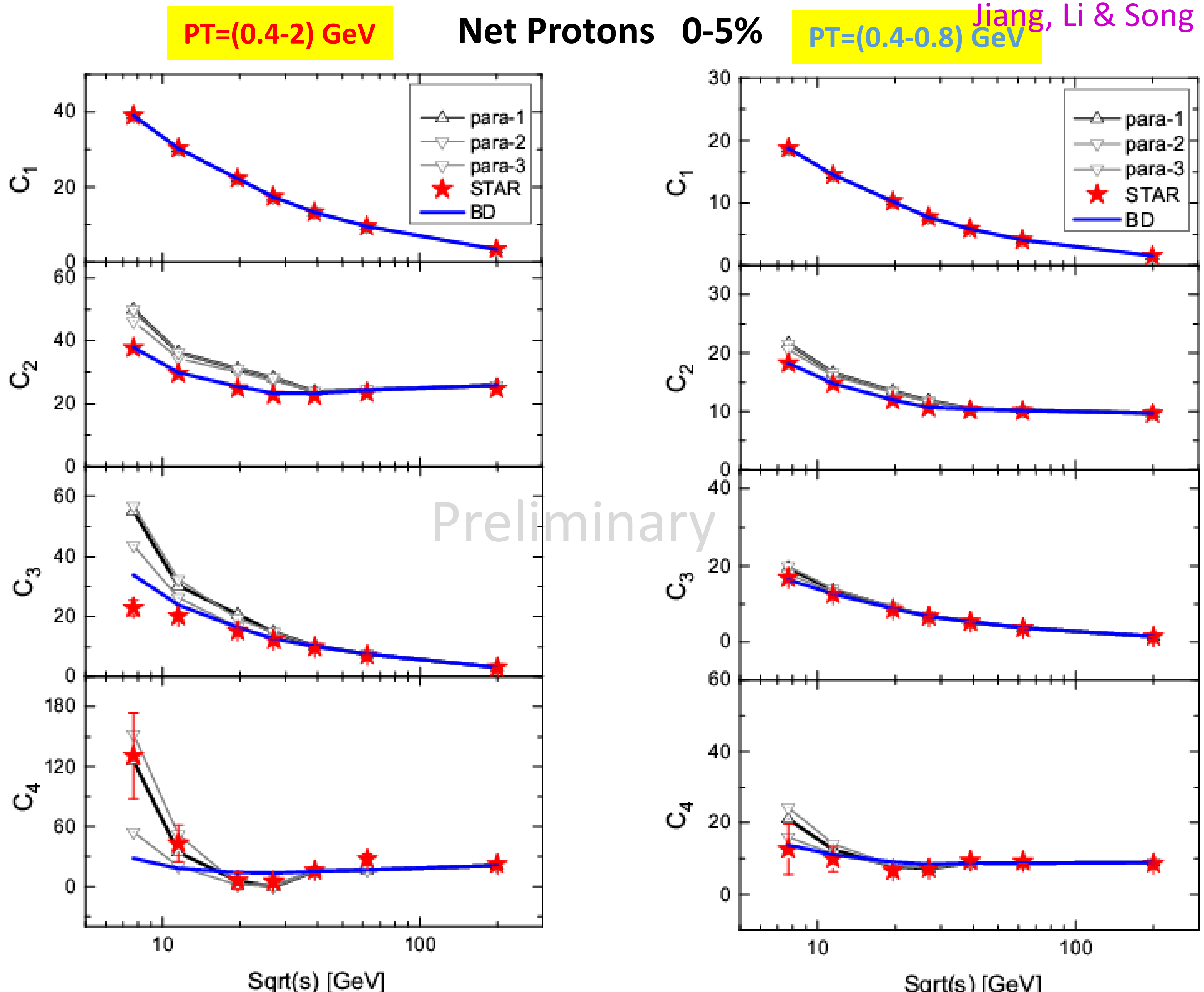

Critical fluctuations give positive contribution to C_2 , C_3; well above the binomial baselines, can NOT explain/describe the C_2 , C_3 data

Summary

- With the maximum pt increased from 0.8 to 2 GeV, STAR BES give exiting new measurements on cumulants for net protons, showing its potential to discover the QCD critical endpoint

- We calculate the correlated fluctuations along the hydro freeze-out surface, through coupling the emitted particles with the fluctuating sigma field

  - C_4 and $\kappa\sigma^2$ can be reproduced through tuning the parameters of the model

  - C_2 , C_3 are well above the poisson/Binomial baselines, which can NOT explain/describe data

  - Binomial baseline is better than poisson baseline, but still cannot describe the data of C_2, C_3

- To claim the discovery of critical endpoint, we need good theoretical models that describe both the cumulant ratios and the original cumulants at the same time.

- Factors need to be considered in the future:

  -- thermal fluctuation baselines: deviations from poisson

  -- dynamical evolution near the critical point

  -- …

Thank you for your attention!

Backup

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Correlated fluctuations along the freeze-out surface near Tc

---- comparison with the experimental data

Pt – (0.4, 2) GeV Non-Central Collisions

$\kappa\sigma^2, S\sigma$: Pt-(0.4-2) GeV (Model + Skellam/BD baseline)

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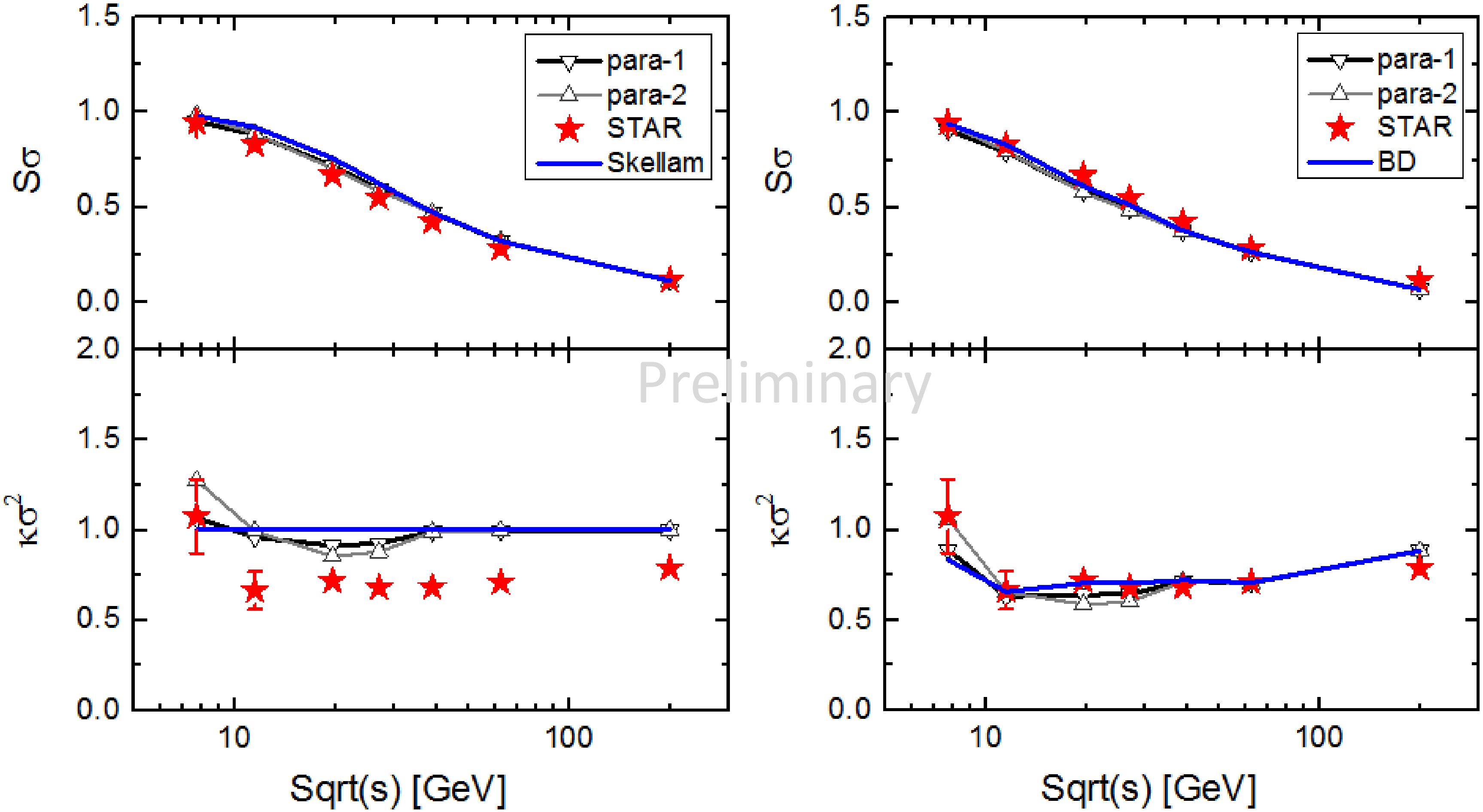
Net Protons 30-40%



C1 C2 C3 C4 : Pt-(0.4-2) GeV (Model + Skellam/BD baseline)

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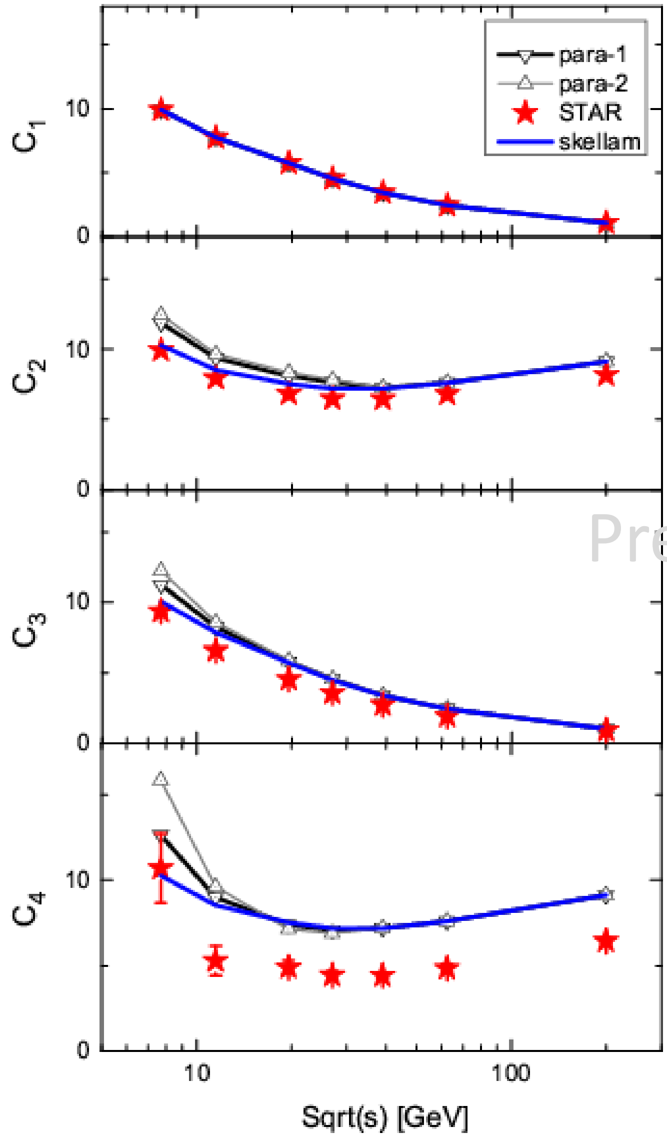
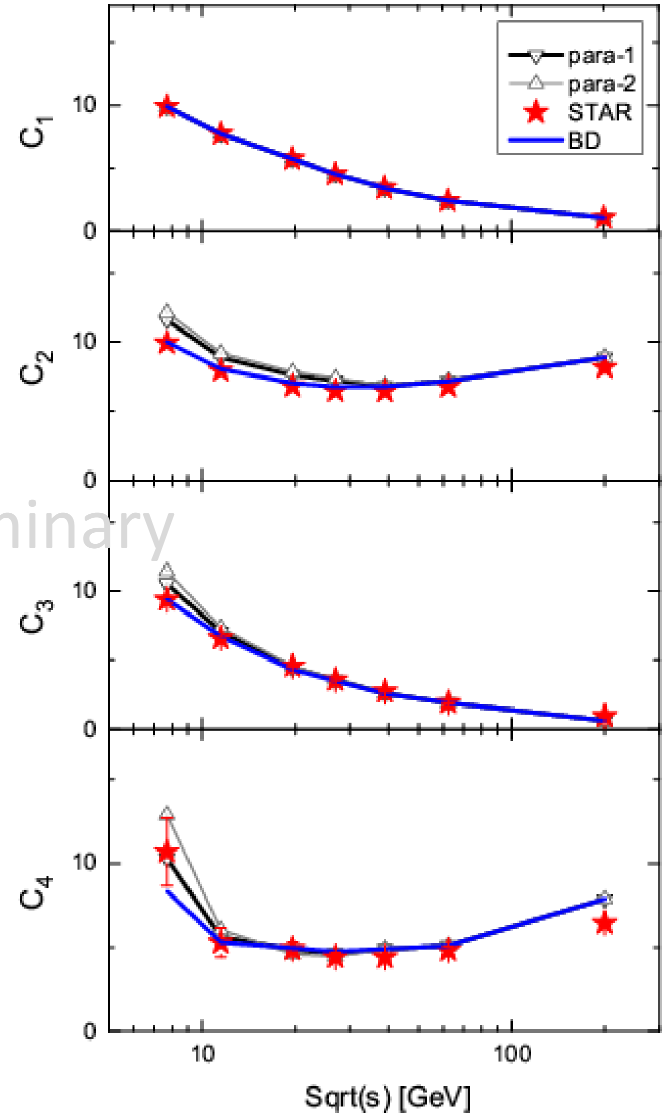
Net Protons 30-40%





Preliminary