FCC-ee parameters optimization at high and low energies, different schemes of crossings

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Outline

- Basic equations and limitations
- The optimization procedure
- Head-on collision at 175 GeV
- Crossing angle at 175 GeV
- Crabbing of the bunches due to beam-beam
- Large Piwinski angle at 45.5 GeV with and w/o CW
- Different schemes of crossings
- Conclusions

Luminosity

For flat beams (both head-on and crossing angle collision):

$$L = \frac{\gamma}{2er_e} \cdot \frac{I\xi_y}{\beta_y^*} \cdot R_H$$

I – total beam current (defined by SR power of 50 MW)

ξ_y – vertical betatron tune shift, its limit depends on the collision scheme

R_H – hour-glass factor: $R_H \approx$ [0.86, 0.71, 0.6] for L_i/β_y^* = [1, 2, 3]

L_i – length of the interaction area:

$$L_i = \frac{\sigma_z}{\sqrt{1+\phi^2}} \qquad \phi = \frac{\sigma_z}{\sigma_x} tg\left(\frac{\theta}{2}\right) \text{ – Piwinski angle}$$

β_y^* should be minimized as much as possible, but there are restrictions:

- beta-function at the final quads raises as $1/\beta_y^*$ that affects dynamic aperture and can create problems with chromaticity corrections
- L_i should be squeezed to $L_i \sim \beta_y^*$, for head-on it means $\sigma_z \sim \beta_y^*$

Beamstrahlung

At high energies (tt, H) the luminosity is limited by the beamstrahlung lifetime, which is proportional to:

$$\tau_{bs} \sim \exp\left(\frac{2\eta\alpha\rho}{3r_e\gamma^2}\right)\cdot\frac{\rho^{3/2}}{L_i\gamma^2}$$

α – fine structure constant
η – energy acceptance
ρ – average bending radius of a particle's trajectory at IP

Obviously, the major tool for reducing beamstrahlung is making ρ larger. For flat beams, ρ is inversely proportional to the surface charge density of the opposing beam:

$$\frac{1}{\rho} \sim \frac{N_p}{\sigma_x\sigma_z} \sim \frac{\xi_y}{L_i}\sqrt{\frac{\varepsilon_y}{\beta_y^*}}$$

It follows that the vertical emittance should be minimized, and L_i should be not small. As a consequence, β_y^* also should be not too small, the optimum value is about 2 mm (at high energies only!).

Dinamical emittance & beta

The effect depends on the lattice features, but can be estimated as:

$$\varepsilon = \frac{1+2\pi\xi\cot\mu_0}{\sqrt{1+4\pi\xi\cot\mu_0-4\pi^2\xi^2}}\varepsilon_0 \qquad \beta = \frac{1}{\sqrt{1+4\pi\xi\cot\mu_0-4\pi^2\xi^2}}\beta_0$$

When ν_x is shifted closer to half-integer:

- The horizontal emittance ε_x increases and β_x decreases.
- The factor for β_x is larger, so σ_x slightly decreases.
- The actual tune shift $\Delta\nu_x$ decreases and we get more room for the footprint, so normally (e.g. KEKB) the bunch current can be raised to boost the luminosity.
- At high energy FCC-ee the bunch current is limited by the beamstrahlung lifetime, so we need to [slightly] reduce it since σ_x decreases.
- Dynamical increase of ε_x leads to ε_y increase due to coupling, and luminosity drops.

This issue needs to be addressed when performing optimization!

Optimization

For better understanding we perform optimization and comparison of 4 collision schemes (head-on, θ = 11 mrad, θ = 30 mrad with and w/o CW) for all 4 energies (work in progress).

What is fixed for all schemes at a given energy:

- Energy spread, energy loss per turn
- Energy acceptance
- Damping decrements
- Total beam current
- Transverse apertures
- Minimal emittances and momentum compaction
- Betatron coupling: $\varepsilon_y = \varepsilon_x \cdot 0.002 + \varepsilon_{y0}$

What is optimized:

- RF voltage (i.e. bunch length)
- Betatron tunes
- Beta functions at IP
- Bunch population

What is the model:

- Beam-beam tracking code `Lifetrac`, quasi-strong-strong simulations
- Beamstrahlung: direct simulation of photons emission
- Linear lattice between IPs (can be replaced by the real nonlinear lattice when DA is obtained)
- Dynamical emittances and beta-functions

What is the goal:

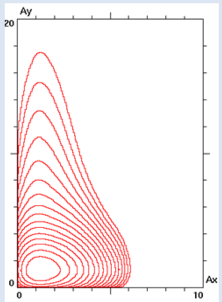
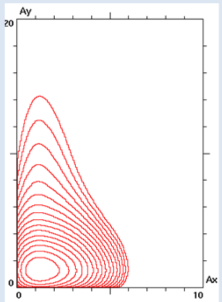
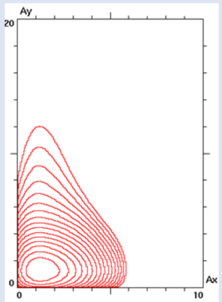
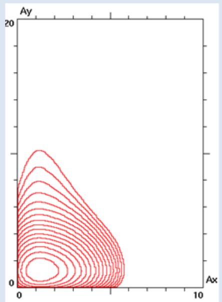
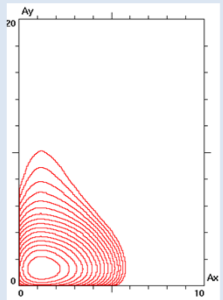
- Maximum luminosity
- Lifetime > 10 min
- Tolerance to 10% difference in bunch population (critical due to beamstrahlung!)
- Tolerance to offsets of ~ 0.05 σ_{xy} between colliding bunches at IPs
- Tolerance to asymmetry of ~ 0.002 in betatron tune advances between IPs

List of main lattice parameters @ 175 GeV

Perimeter, P [km]	100
Momentum compaction, α	$5.7 \cdot 10^{-6}$
Emittance, ε_x / ε_y [nm]	1.3 / 0.0026
Energy spread, σ_E	$1.6 \cdot 10^{-3}$
Energy loss per turn, U_0 [GeV]	8.48
RF frequency [MHz]	400
Damping time, τ_x / τ_y /τ_z [turns]	40 / 40 / 20
Energy acceptance, η	0.02
Beta functions at IP, β_x / β_y [mm]	500 / 2
Number of IPs, N_{IP}	4

A. Bogomyagkov, "Status of crab waist IR version 6-14-3", presented at FCC-ee meeting № 13, February 9 (2015), https://indico.cern.ch/event/367430/

Head-on, 175 GeV, RF voltage

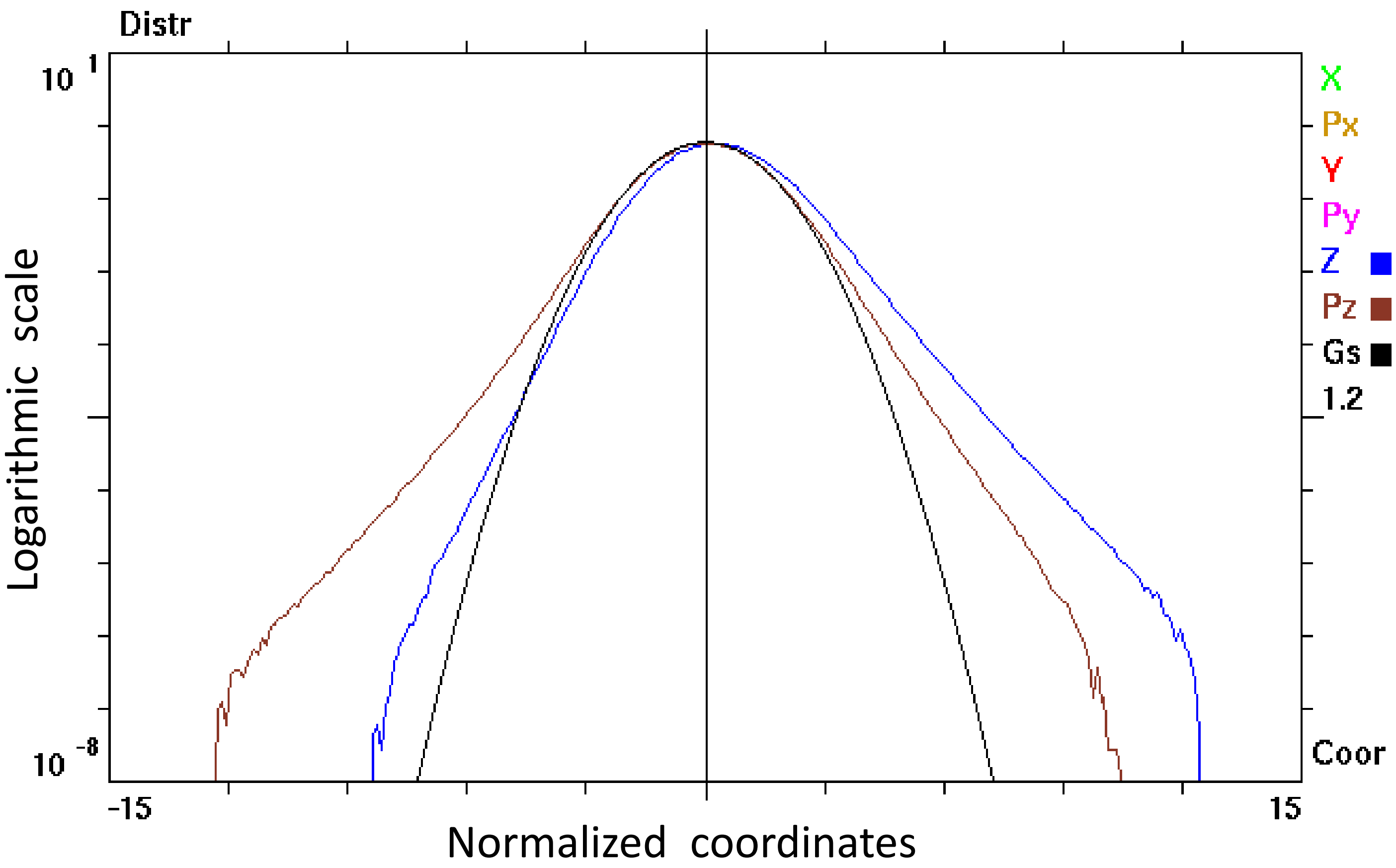
URF [GV]	9.5	10	10.5	11	11.5
ν_s	0.0136	0.0151	0.0164	0.0174	0.0183
N_p	$1.2\cdot10^{11}$	$1.15\cdot10^{11}$	$1.1\cdot10^{11}$	$1.05\cdot10^{11}$	$1.05\cdot10^{11}$
N_b	100	104	109	114	114
σ_{zo} / σ_{zbs} [mm]	2.67 / 3.31	2.4 / 2.99	2.22 / 2.77	2.09 / 2.59	1.98 / 2.49
ε_x / ε_y [nm·rad]	1.79 / 0.0044	1.76 / 0.0042	1.74 / 0.0040	1.71 / 0.0039	1.71 / 0.0039
$\Delta\nu_x$ /$\Delta\nu_y$	0.0986 / 0.1604	0.0959 / 0.1506	0.0931 / 0.1423	0.0902 / 0.1348	0.0903 / 0.1327
L [$cm^{-2}s^{-1}$]	$1.34\cdot10^{34}$	$1.39\cdot10^{34}$	$1.41\cdot10^{34}$	$1.40\cdot10^{34}$	$1.42\cdot10^{34}$
τ [min]	15	25	25	25	20
Density Contour plots $10\sigma_x \times 20\sigma_y$					

9.5 GV is too small: hour-glass and long vertical tails affect the luminosity and the lifetime.

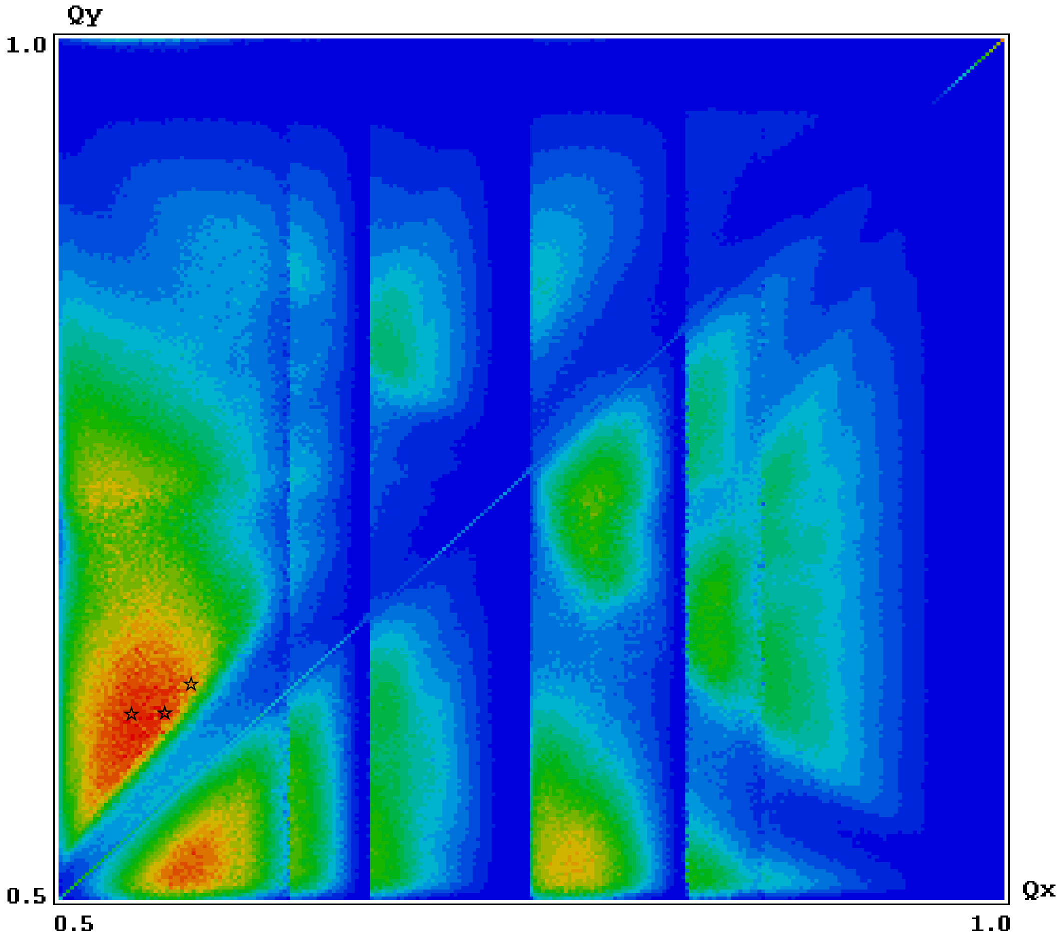
The optimum is about 10.5 ÷11 GV. When increasing the voltage further, the lifetime drops.

It can be raised again by decrease of the bunch current, but at the expense of luminosity.

An example of equilibrium distribution in the longitudinal dimension
(beamstrahlung + nonlinearity of RF voltage)



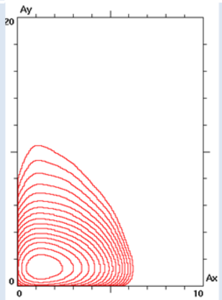
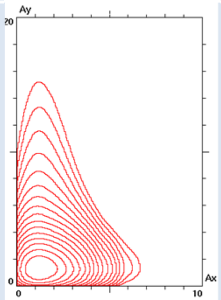
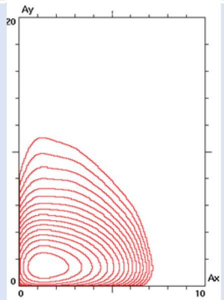
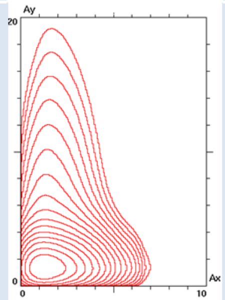
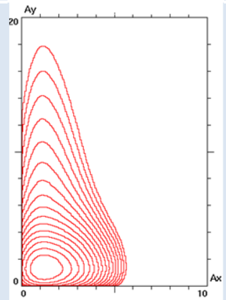
Head-on, 175 GeV, scan of betatron tunes



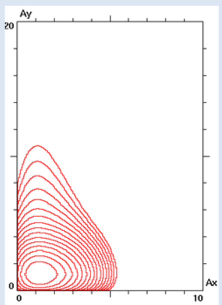
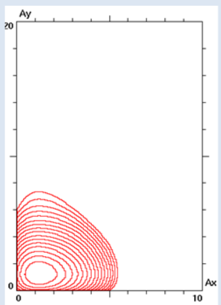
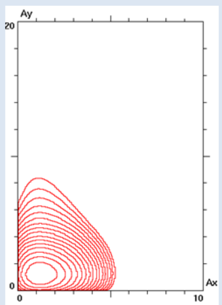
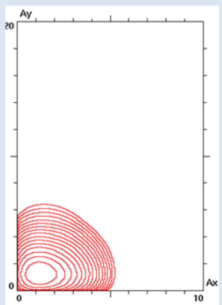
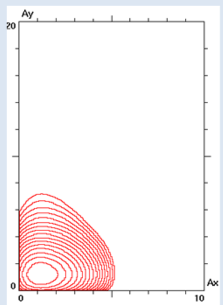
$$F = \frac{Lumi}{(\varepsilon_x \varepsilon_y)^{1/4}}$$

The markers: (0.54, 0.61), (**0.56, 0.61**), (0.575, 0.63).

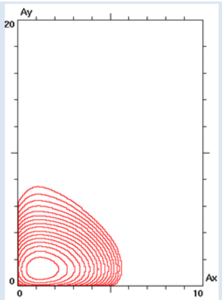
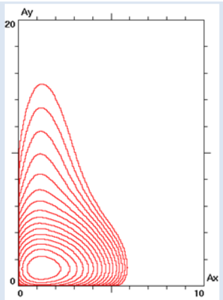
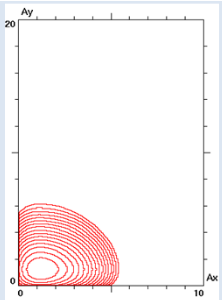
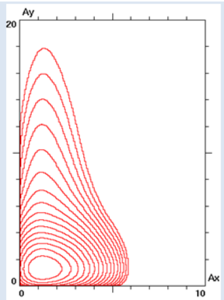
Head-on, 175 GeV, variation of parameters

ν_x / ν_y	0.54 / 0.61	0.575 / 0.63	0.54 / 0.61	0.56 / 0.61	0.56 / 0.61
β_x / β_y [mm]	500 / 2	500 / 2	1000 / 2	1000 / 2	500 / 1
N_p	$1.05 \cdot 10^{11}$	$1.15 \cdot 10^{11}$	$1.5 \cdot 10^{11}$	$1.5 \cdot 10^{11}$	$1.0 \cdot 10^{11}$
N_b	114	104	80	80	120
σ_{zbs} [mm]	2.78	2.77	2.78	2.72	2.70
$\varepsilon_x / \varepsilon_y$ [nm·rad]	1.93 / 0.0044	1.69 / 0.0041	2.21 / 0.0051	1.96 / 0.0046	1.69 / 0.0044
β_{xd} / σ_x [mm]	213 / 0.0202	302 / 0.0226	379 / 0.0289	498 / 0.0312	272 / 0.0214
$\Delta\nu_x / \Delta\nu_y$	0.0846 / 0.1380	0.0967 / 0.1397	0.1066 / 0.1303	0.1130 /0.1259	0.0872 / .1357
L [$cm^{-2}s^{-1}$]	$1.35 \cdot 10^{34}$	$1.36 \cdot 10^{34}$	$1.26 \cdot 10^{34}$	$1.22 \cdot 10^{34}$	$1.28 \cdot 10^{34}$
τ [min]	25	20	25	25	20
Density contour plots $10\sigma_x \times 20\sigma_y$					

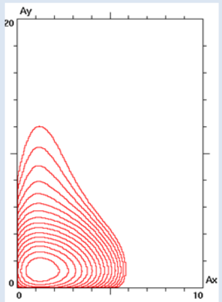
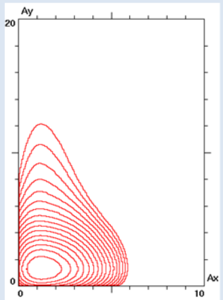
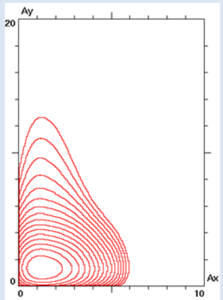
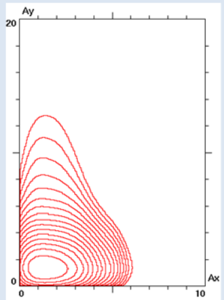
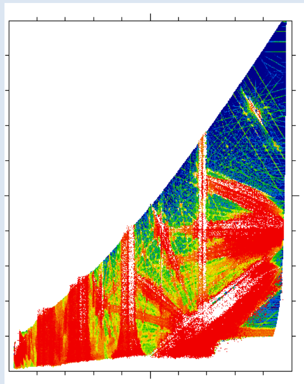
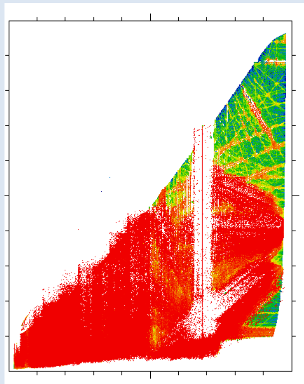
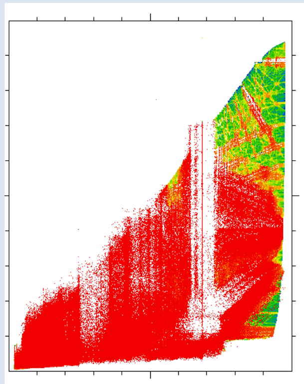
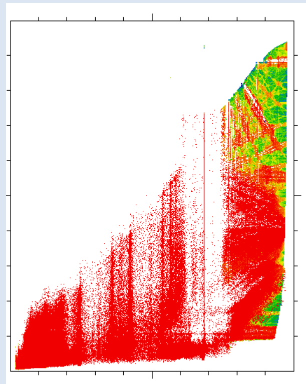
Head-on, 175 GeV, RF 800 MHz (shorter bunch)

URF [GV]	9.5	9.5	10.5	10.5	11.5
β_y [mm]	1	2	1	2	1
ν_s	0.0192	0.0192	0.0231	0.0231	0.0246
N_p	$0.85 \cdot 10^{11}$	$0.90 \cdot 10^{11}$	$0.80 \cdot 10^{11}$	$0.80 \cdot 10^{11}$	$0.75 \cdot 10^{11}$
N_b	141	133	150	150	160
σ_{zo} / σ_{zbs} [mm]	1.89 / 2.25	1.89 / 2.29	1.57 / 1.90	1.57 / 1.90	1.40 / 1.70
ε_x / ε_y [nm·rad]	1.61 / 0.0037	1.64 / 0.0036	1.59 / 0.0035	1.59 / 0.0035	1.56 / 0.0034
$\Delta\nu_x$ / $\Delta\nu_y$	0.0773 / 0.1119	0.0808 / 0.1160	0.0738 / 0.0981	0.0738 / 0.1007	0.0702 / 0.0882
L [$cm^{-2}s^{-1}$]	$1.30 \cdot 10^{34}$	$1.30 \cdot 10^{34}$	$1.36 \cdot 10^{34}$	$1.23 \cdot 10^{34}$	$1.35 \cdot 10^{34}$
τ [min]	25	25	20	25	20
Density contour plots $10\sigma_x \times 20\sigma_y$					

Head-on, 175 GeV, asymmetry in bunch currents

Asymmetry	10 %		20 %	
Bunch	e-	e+	e-	e+
N_p	$1.1 \cdot 10^{11}$	$0.99 \cdot 10^{11}$	$1.1 \cdot 10^{11}$	$0.88 \cdot 10^{11}$
σ_{zbs} [mm]	2.61	2.85	2.49	2.91
Δs [mm]	0.63	0.89	0.48	0.96
$\varepsilon_x / \varepsilon_y$ [nm·rad]	1.68 / 0.0037	1.74 / 0.0041	1.63 / 0.0035	1.75 / 0.0043
$\Delta\nu_x / \Delta\nu_y$	0.0860 / 0.1299	0.0937 / 0.1433	0.0787 / 0.1166	0.0938 / 0.1434
L [$cm^{-2}s^{-1}$]	$1.29 \cdot 10^{34}$		$1.14 \cdot 10^{34}$	
τ [min]	~200	10	> 2000	5
Density contour plots $10\sigma_x \times 20\sigma_y$				

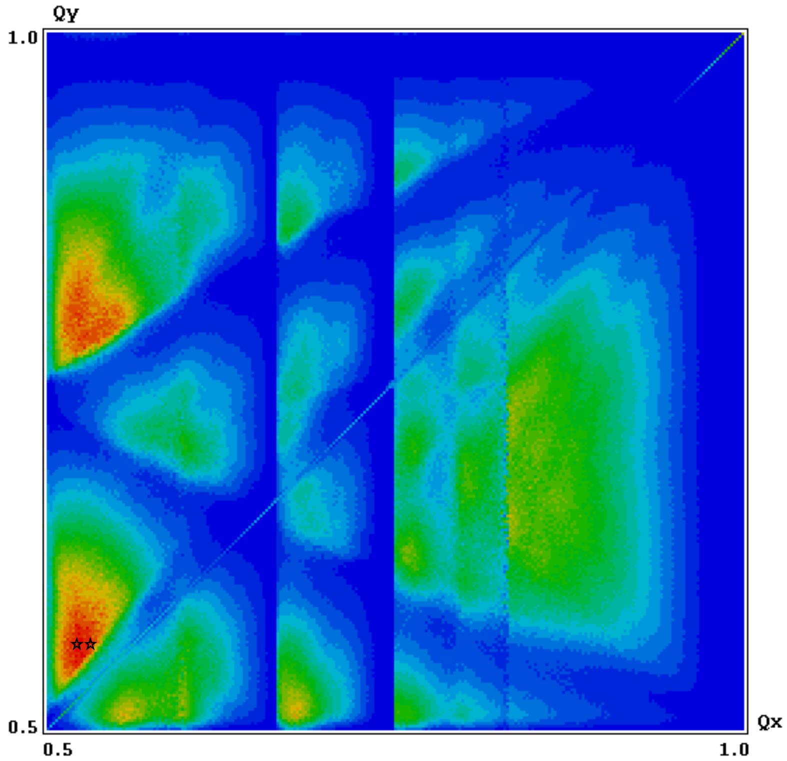
Asymmetry in phase advances + offsets

Asymmetry	none	$\Delta_{x,y}$ ~0.05 σ $\Delta\nu_{x,y}$ ~0.001	$\Delta_{x,y}$ ~0.10 σ $\Delta\nu_{x,y}$ ~0.002	$\Delta_{x,y}$ ~0.10 σ $\Delta\nu_{x,y}$ ~0.005
ε_x / ε_y [nm·rad]	1.74 / 0.0040	1.79 / 0.0041	1.83 / 0.0042	1.98 / 0.0043
L [$cm^{-2}s^{-1}$]	$1.41\cdot10^{34}$	$1.40\cdot10^{34}$	$1.37\cdot10^{34}$	$1.33\cdot10^{34}$
Density contour plots $10\sigma_x \times 20\sigma_y$				
FMA footprints ν_x: 0.24 ÷ 0.62 ν_y: 0.44 ÷ 1.0				

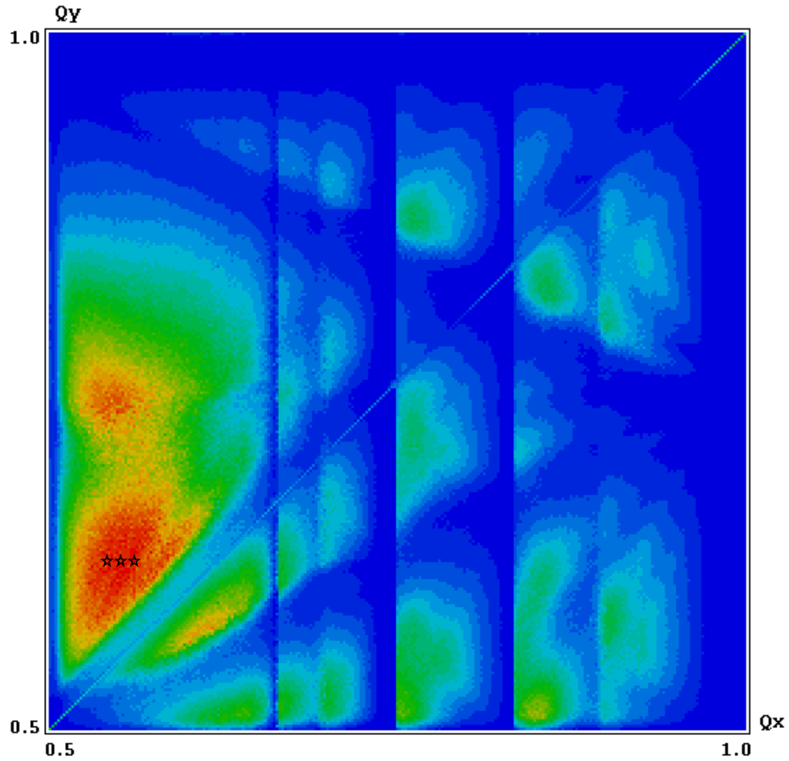
Head-on, 175 GeV, Conclusions

- Optimization is determined by a compromise between hour-glass and beamstrahlung lifetime.
- Optimal RF parameters: 400 MHz, 10.5 ÷11 GV
- Optimal vertical beta-function: 2 mm
- The difference between bunch currents should be within 10%
- Asymmetry of ~ 0.002 in betatron tune advances between IPs is acceptable
- Peak luminosity per IP: $1.4 \cdot 10^{34}$

Crossing 11 mrad, 175 GeV, scan of betatron tunes



The same crossing angles

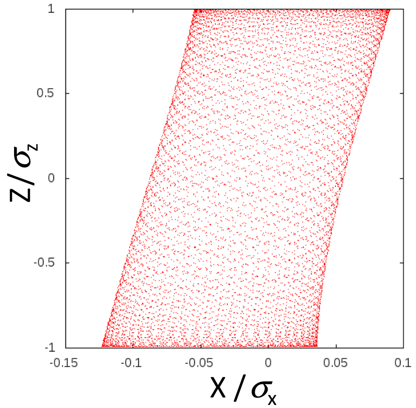


Alternate signs of crossing angles

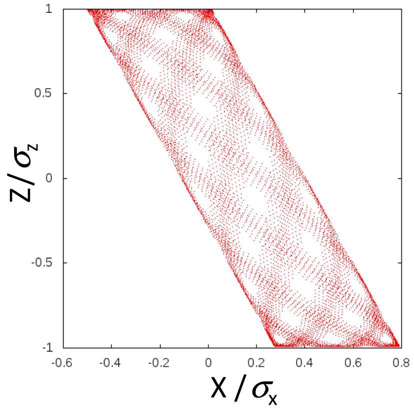
Bunch crabbing due to beam-beam

Particle tracking, 10000 turns, initial coordinates (normalized): (0, 0, 0, 0, 1, 0)

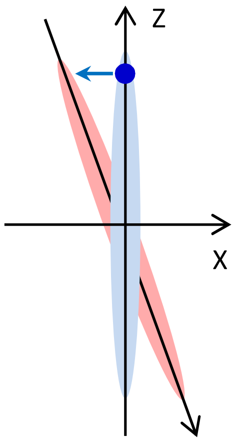
The same angles

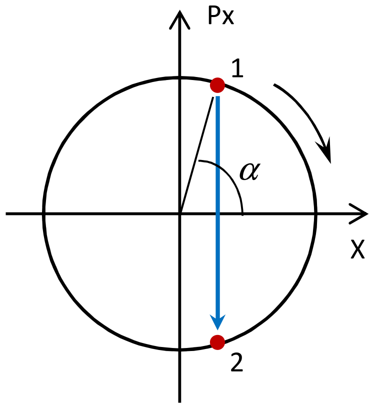


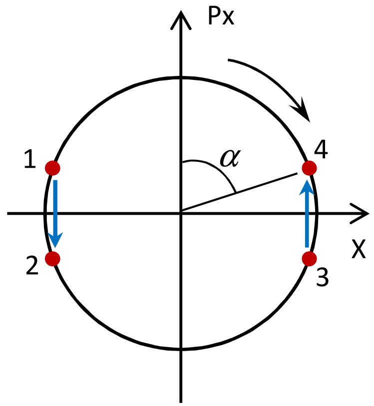
Alternate signs of angles



Simple scheme to find equilibrium horizontal coordinate X for $Z = \sigma_z$ and small synchrotron tune (frozen longitudinal motion), betatron tune above half-integer



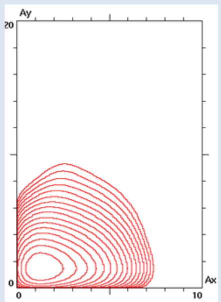
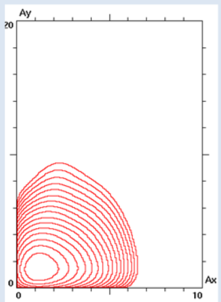
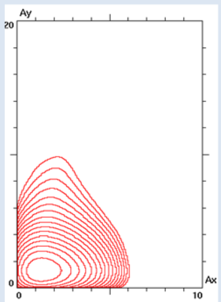
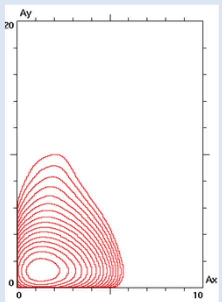
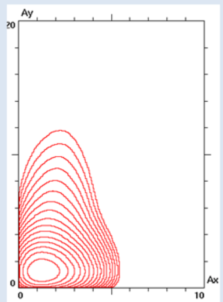




$$\alpha = \pi \cdot (1 - \nu_x)$$

$$x = \frac{\Delta p_x}{2} ctg(\pi \nu_x) \qquad x = -\frac{\Delta p_x}{2} tg(\pi \nu_x)$$

Crossing 11 mrad, 175 GeV

Crossing	11 mrad		± 11 mrad		
ν_x / ν_y	0.52 / 0.56	0.53 / 0.56	0.54 / 0.62	0.55 / 0.62	0.56 / 0.62
N_p	$1.20 \cdot 10^{11}$	$1.20 \cdot 10^{11}$	$1.15 \cdot 10^{11}$	$1.15 \cdot 10^{11}$	$1.15 \cdot 10^{11}$
N_b	100	100	104	104	104
σ_{zbs} [mm]	2.93	2.91	2.83	2.80	2.78
ε_x / ε_y [nm·rad]	2.20 / 0.0057	1.90 / 0.0056	1.98 / 0.0046	1.83 / 0.0043	1.73 / 0.0042
β_{xd}/ σ_x [mm]	188 / 0.0203	224 / 0.0206	231 / 0.0214	261 / 0.0219	290 / 0.0224
$\Delta\nu_x$ /$\Delta\nu_y$	0.0510 / 0.1022	0.0582 / 0.1018	0.0823 / 0.1389	0.0858 / 0.1391	0.0876 / 0.1365
Tilt x-z [mrad]	-0.114	-0.173	4.38	4.07	3.79
L [$cm^{-2}s^{-1}$]	$1.15 \cdot 10^{34}$	$1.14 \cdot 10^{34}$	$1.37 \cdot 10^{34}$	$1.39 \cdot 10^{34}$	$1.38 \cdot 10^{34}$
τ [min]	20	20	15	20	25
Density contour plots $10\sigma_x \times 20\sigma_y$					

In a scheme with alternate crossing angles and ν_x above half-integer, we have [almost] crab-crossing for free!

- At 175 GeV everything is limited by beamstrahlung lifetime. There is no need in crab waist, and crossing angle reduces the length of interaction area L_i. In this case crab-crossing is helpful.

- At 45.5 GeV bunch lengthening due to beamstrahlung is larger, damping is much weaker, so we need large Piwinski angle to make $L_i << \sigma_z$ in order to avoid hour-glass and to implement crab waist. In this case crab-crossing kills the CW scheme!

Which schemes of crossings are allowed for crab waist ?

45.5 GeV, choice of parameters

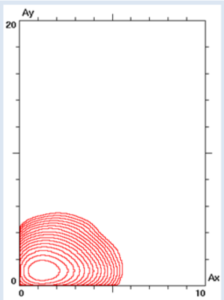
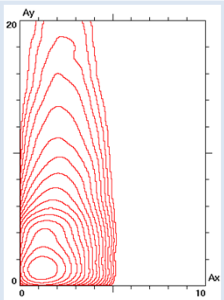
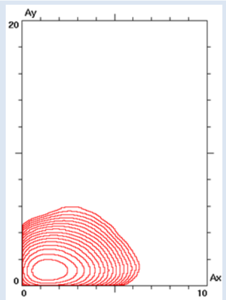
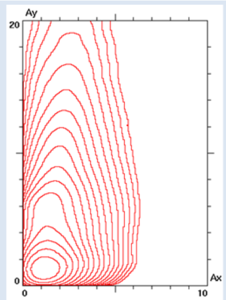
- If the lattice of arcs not changed and $\varepsilon_x \sim \gamma^2$, then for θ = 30 mrad we have $L_i \approx 0.5$ mm – too small, should be doubled to get $L_i \sim \beta_y$. It means doubling of σ_x which can be achieved by increase of ε_x or β_x, or both.
- The vertical emittance at this energy is defined mainly by the detector solenoid, second term in: $\varepsilon_y = \varepsilon_x \cdot 0.002 + \varepsilon_{y0}$
- It means that some increase of ε_x (e.g. by modifying the arc cell phase advances) does not affect ε_y. In addition, momentum compaction also raises: $\alpha \propto \varepsilon_x{}^{2/3}$. As a result bunch length increases too, that is helpful for CW.
- Increase of β_x has a drawback: $\xi_x \propto \beta_x$. On the other hand, ξ_x is small for large Piwinski angles, so we can allow its doubling.
- Finally, we double both ε_x and β_x (from 0.5 to 1.0 m).

List of parameters @ 45.5 GeV

Perimeter, P [km]	100
Momentum compaction, α	$9 \cdot 10^{-6}$
Emittance, ε_x / ε_y [pm]	170 / 1
Energy spread, σ_E	$4.16 \cdot 10^{-4}$
Energy loss per turn, U_0 [MeV]	38.75
RF voltage, URF [MV]	80
RF frequency [MHz]	400
Bunch length (SR), σ_z [mm]	3.5
Damping time, τ_x / τ_y /τ_z [turns]	2300 / 2300 / 1150
Beta functions at IP, β_x / β_y [mm]	1000 / 1
Number of IPs, N_{IP}	4
Crossing angle, θ [mrad]	30

45.5 GeV with and w/o CW

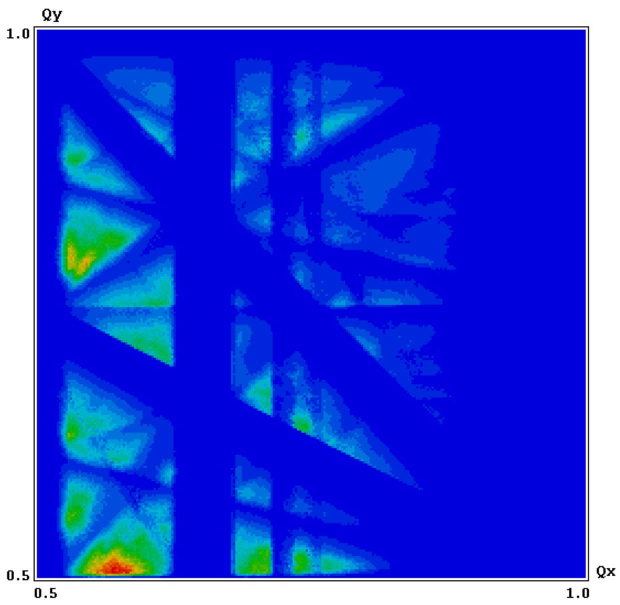
4-fold symmetry, same signs of crossing angle

Crab waist	ON	OFF	ON	OFF
ν_x / ν_y / ν_s	0.54 / 0.57 / 0.017			
N_p	$5.0 \cdot 10^{10}$		$1.0 \cdot 10^{11}$	
N_b	52 000		26 000	
σ_{zbs} [mm]	5.35	5.46	7.60	7.80
ε_x / ε_y [pm·rad]	220 / 1.1	220 / 2.3	260 / 1.2	260 / 2.8
$\Delta\nu_x$ / $\Delta\nu_y$	0.0284 / 0.0784	0.0277 / 0.0576	0.0290 / 0.1053	0.0276 / 0.0706
L [$cm^{-2}s^{-1}$]	$1.2 \cdot 10^{36}$	$7.3 \cdot 10^{35}$	$1.7 \cdot 10^{36}$	$8.3 \cdot 10^{35}$
Density contour plots $10\sigma_x \times 20\sigma_y$				

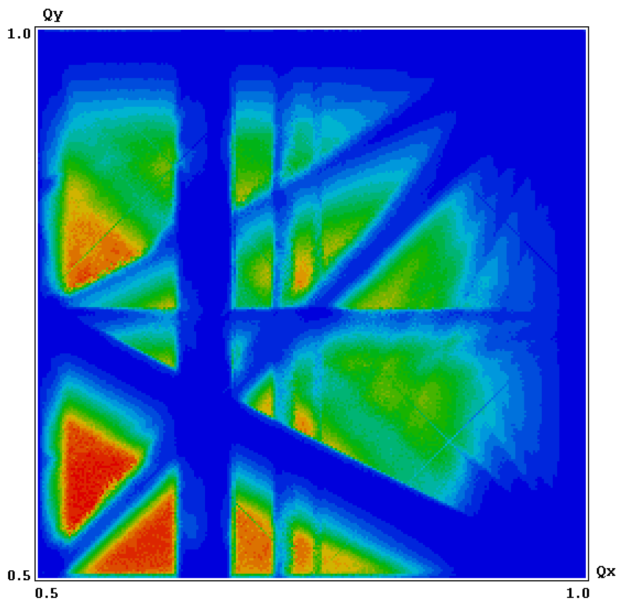
4-fold symmetry, angles: + + + +

Four additional crossings

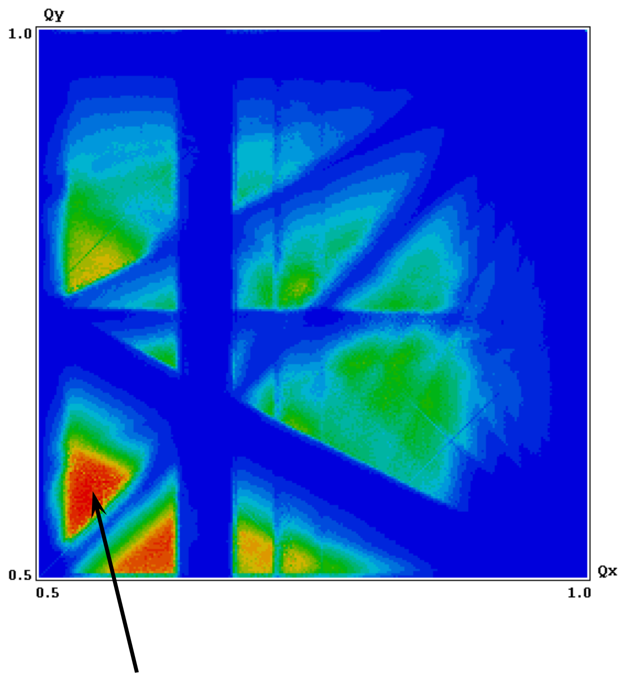
$N_p = 5 \cdot 10^{10}$, CW OFF



$N_p = 5 \cdot 10^{10}$, CW ON



$N_p = 10^{11}$, CW ON

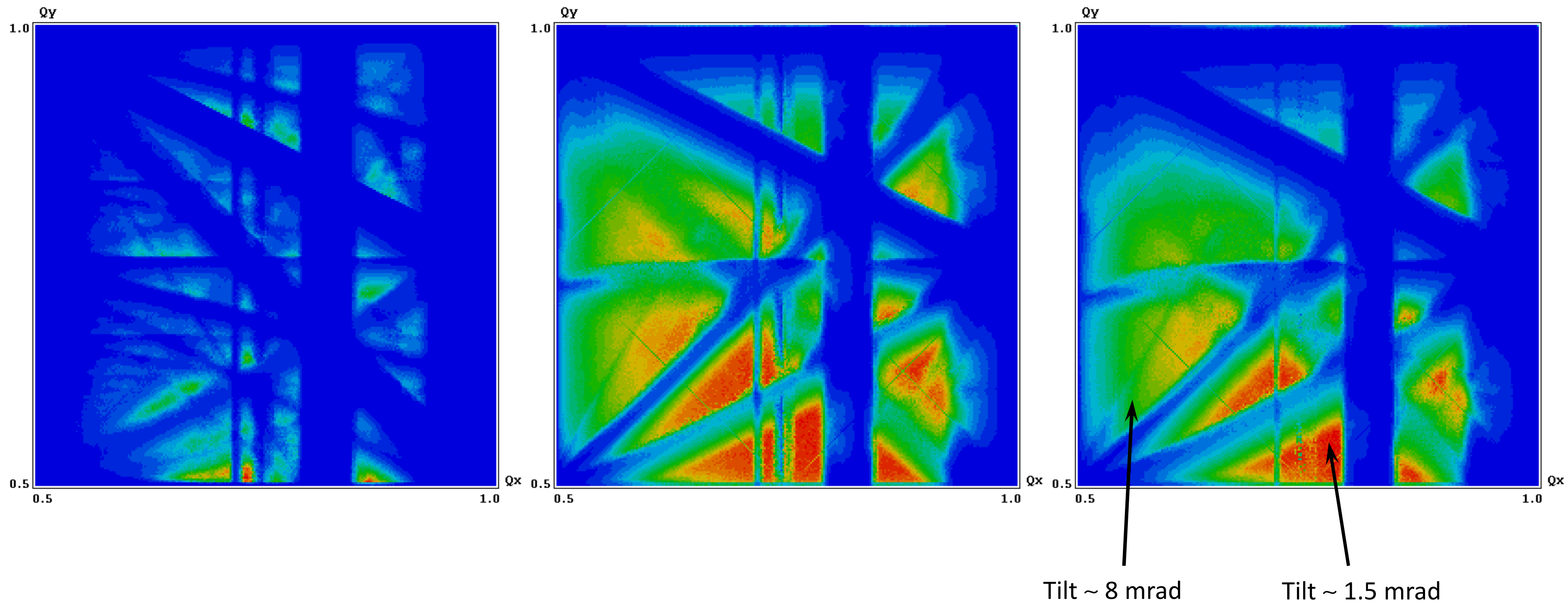


Tilt ~ – 0.3 mrad

2-fold symmetry, angles: + - + -

No additional crossings

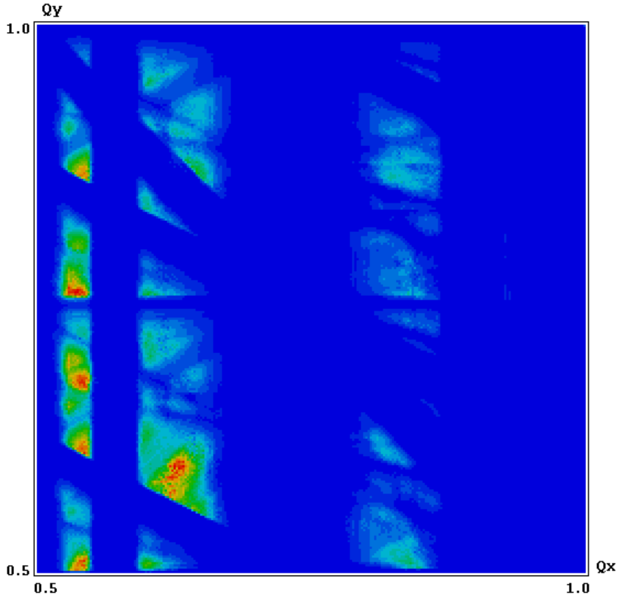
$N_p = 5 \cdot 10^{10}$, CW OFF $N_p = 5 \cdot 10^{10}$, CW ON $N_p = 10^{11}$, CW ON



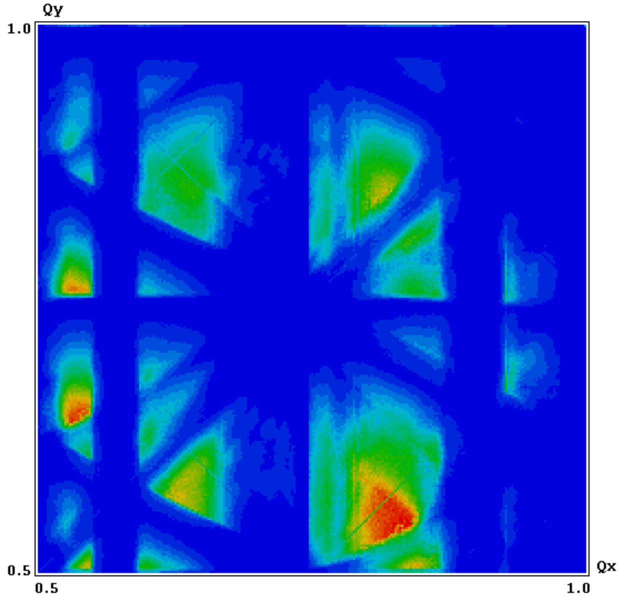
“2 + 2” IPs, angles: + - - +

Two additional crossings

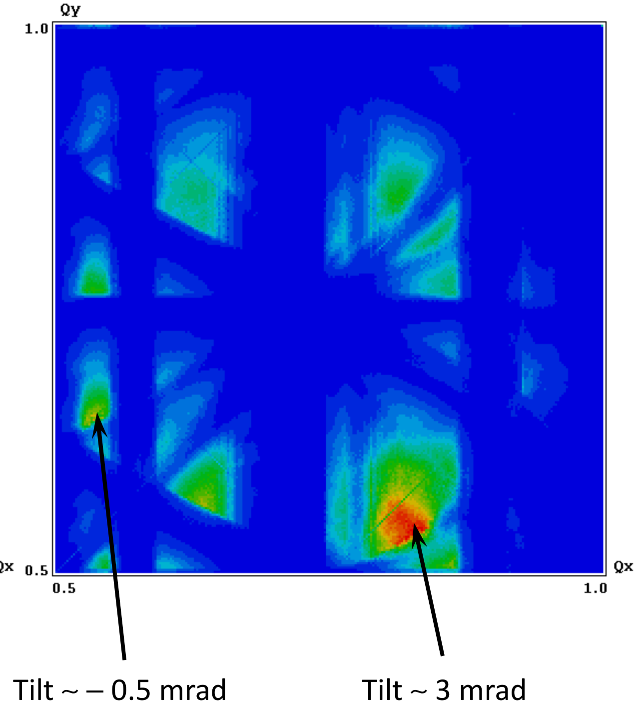
$N_p = 5 \cdot 10^{10}$, CW OFF



$N_p = 5 \cdot 10^{10}$, CW ON



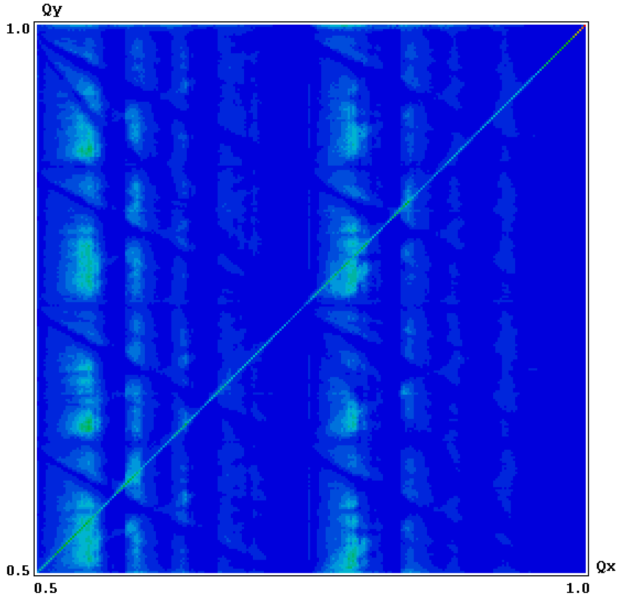
$N_p = 10^{11}$, CW ON



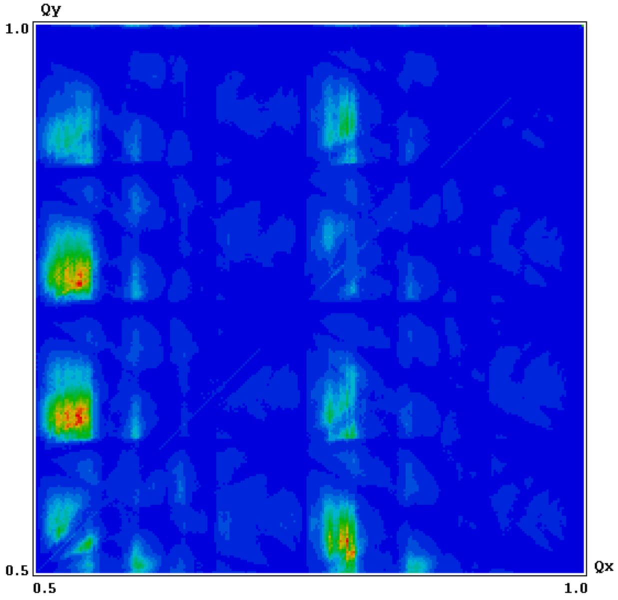
"1 + 3" IPs, angles: + + - +

Two additional crossings

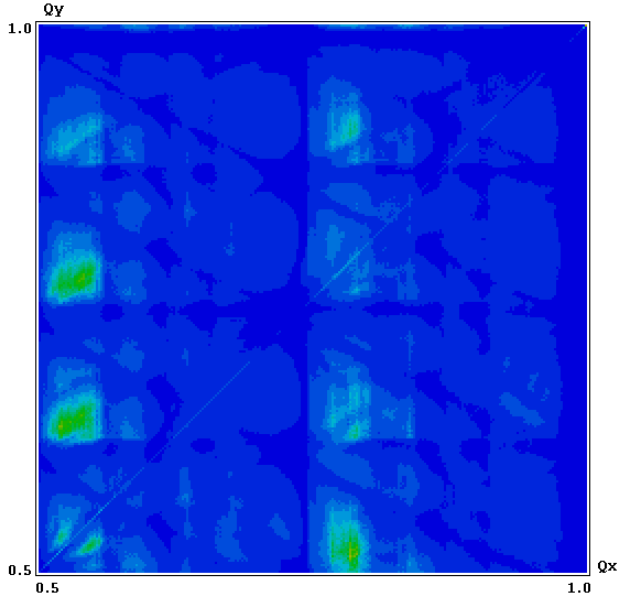
$N_p = 5 \cdot 10^{10}$, CW OFF



$N_p = 5 \cdot 10^{10}$, CW ON



$N_p = 10^{11}$, CW ON



Total luminosity with 4 IPs is smaller than with 2 IPs…

Conclusions

- The scheme "1 + 3" with two additional crossings (+ + - +) is unacceptable.
- Four additional crossings (+ + + +) is OK, but probably too complicated.
- The scheme "2 + 2" with two additional crossings (+ - - +) looks acceptable, but requires additional investigations.
- The best scheme with 4 IPs: no additional crossings (+ - + -) and working point at low energies (0.78, 0.55).
- Two additional crossings in 2 IPs scheme (+ +) is OK, working point at low energies (0.54, 0.57).
- No additional crossings in 2 IPs scheme (+ -) is also OK, working point at low energies (0.78, 0.55).
- Luminosity at 175 GeV without additional crossings is higher by 10÷20 %% (to be checked) due to "crabbing", working point (0.55, 0.62).