HXSWG 11, CERN

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Status Update: HH Production @ NLO

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MAX-PLANCK-GESELLSCHAFT

higgstools

Overview

1. HEFT Results & Motivation

2. Virtual MEs **In Progress**

Tool Chain

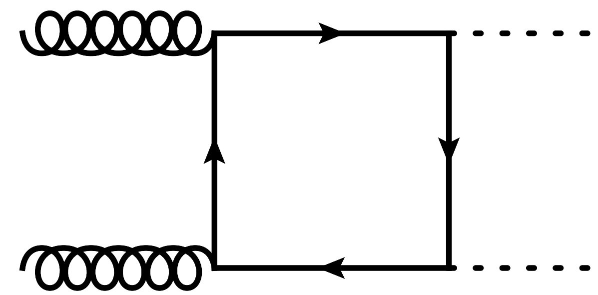
Integral Reduction

Numerical Computation of Master Integrals

3. Real Radiation & Cross-checks

Gluon Fusion

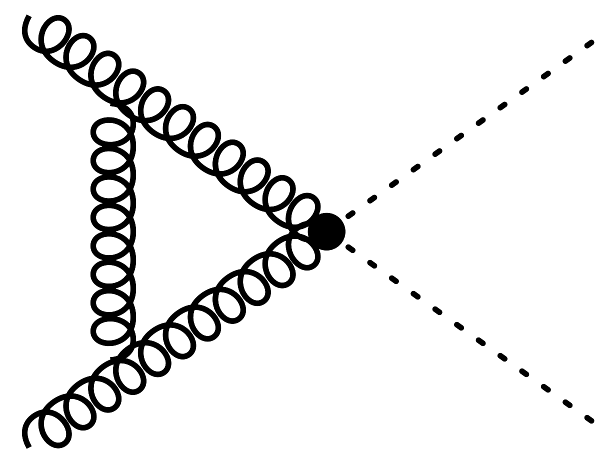
1. LO (1-loop), Dominated by top (bottom contributes ~1%)
   Glover, van der Bij 88



2. Born Improved NLO H(iggs)EFT $m_T \to \infty$ **K≈ 2**
   Plehn, Spira, Zerwas 96, 98; Dawson, Dittmaier, Spira 98

   A. Including m_T in Real radiation **-10%**
   Maltoni, Vryonidou, Zaro 14

   B. Including $\mathcal{O}(1/m_T^{12})$ terms in Virtual MEs **±10%**
   Grigo, Hoff, Melnikov, Steinhauser 13; Grigo, Hoff 14; Grigo, Hoff, Steinhauser 15

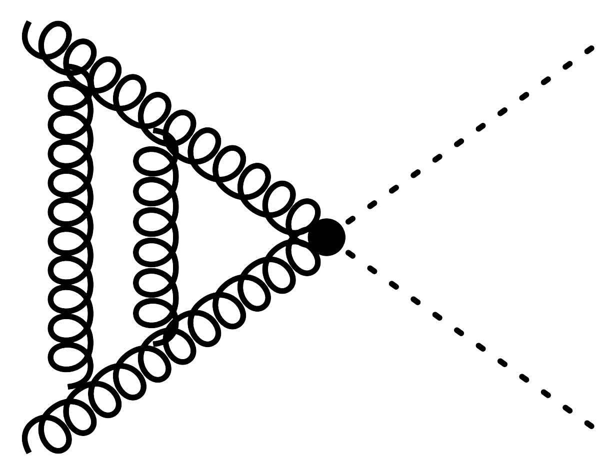


3. Born Improved NNLO HEFT +20%
   De Florian, Mazzitelli 13

   Including matching coefficients
   Grigo, Melnikov, Steinhauser 14

   Including terms $\mathcal{O}(1/m_T^{12})$ in Virtual MEs
   Grigo, Hoff, Steinhauser 15

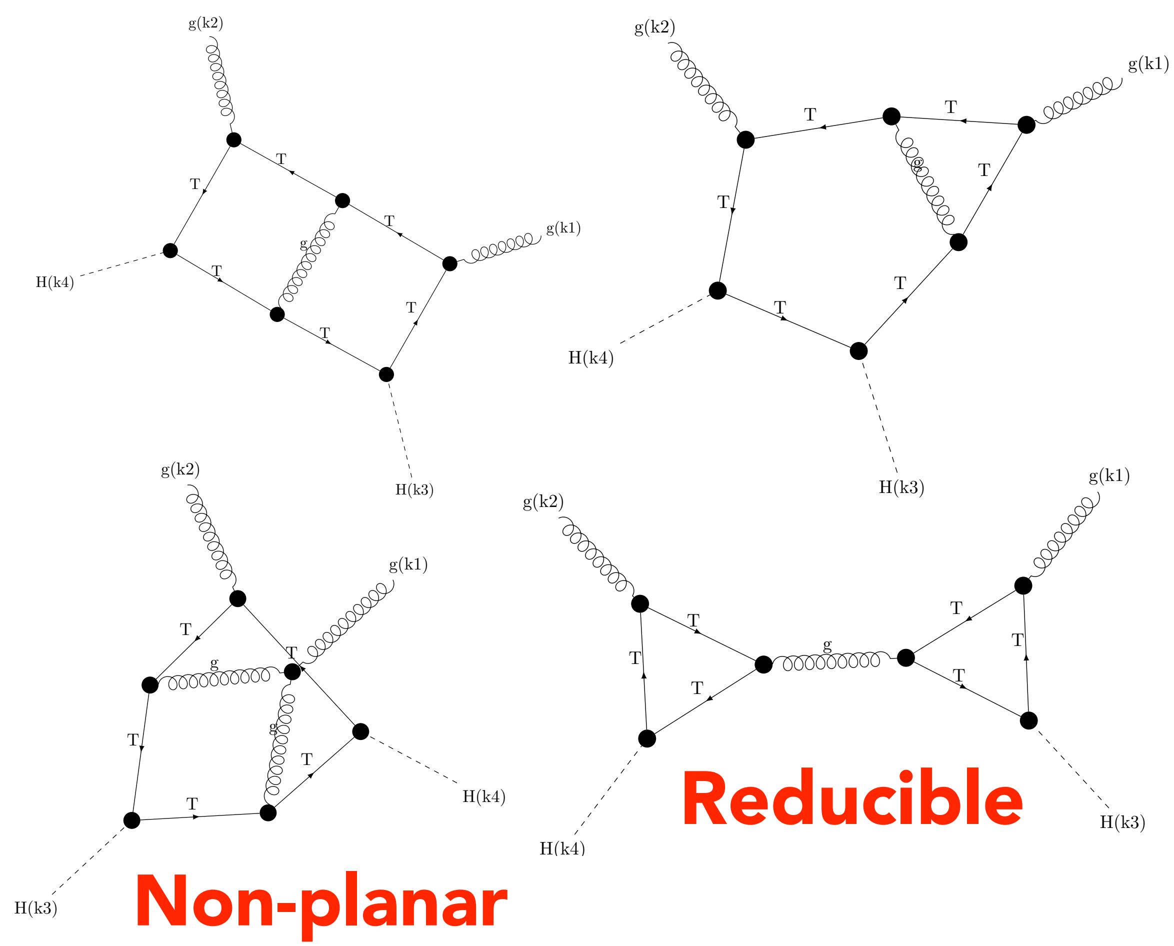
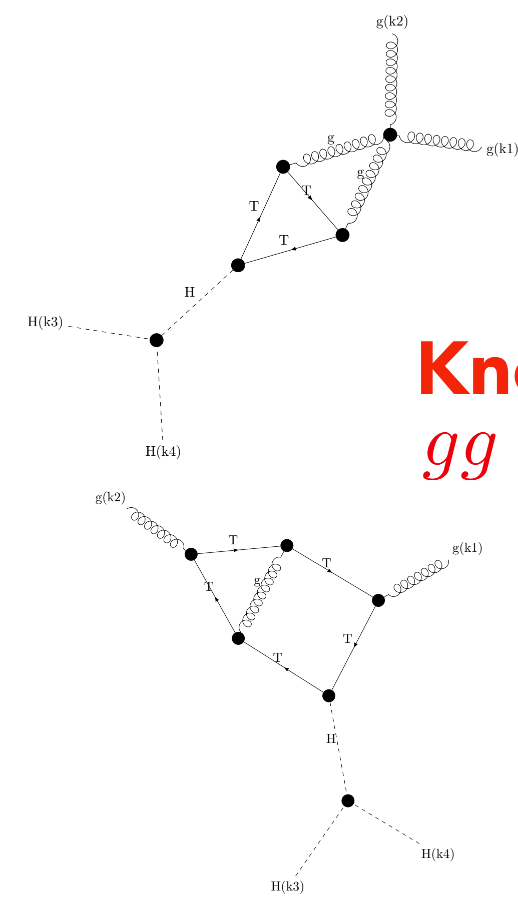
   NNLL + NNLO Matching +9%
   de Florian, Mazzitelli 15



Virtual MEs

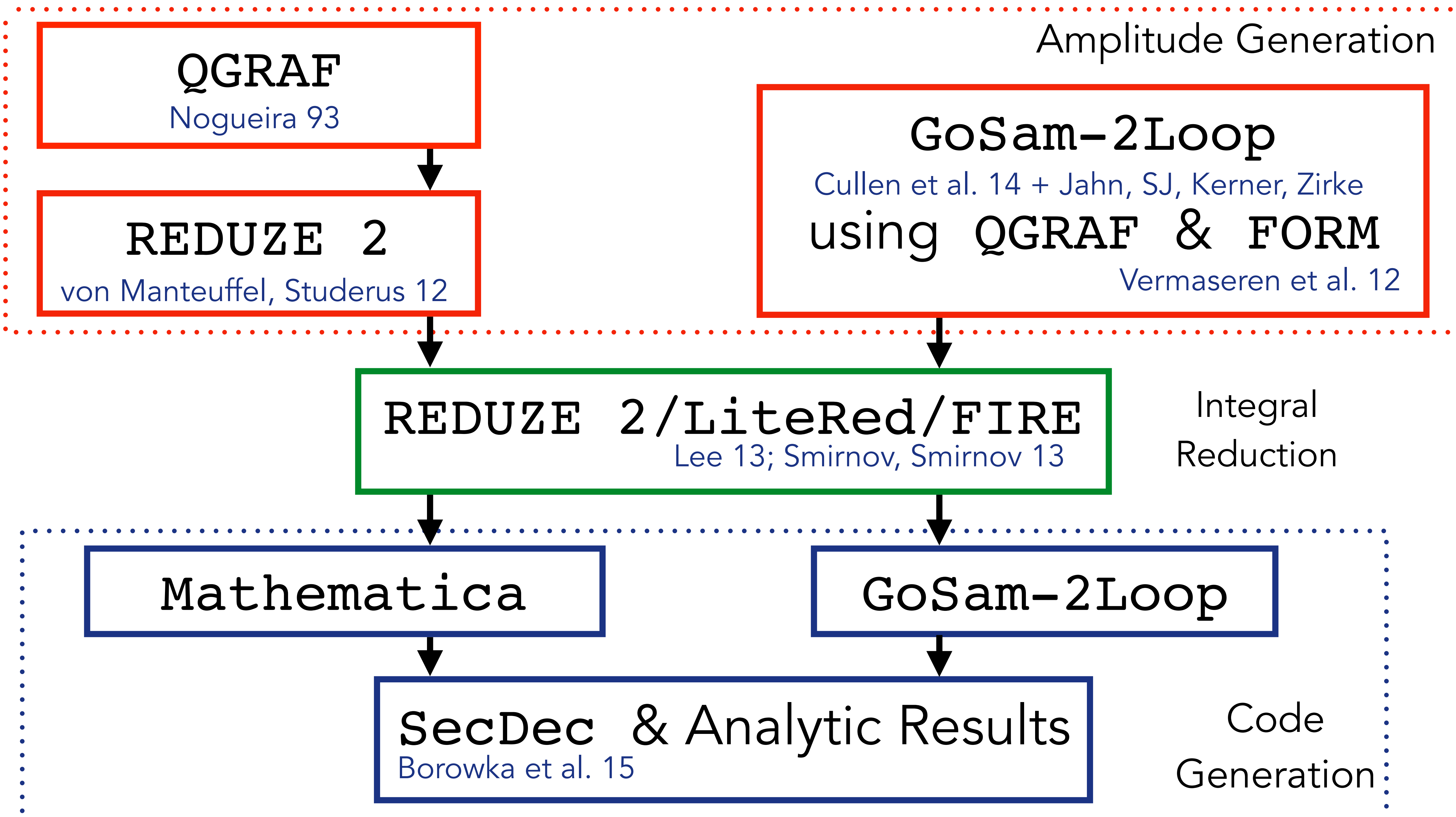
Goal: Compute $gg \to HH$ @ NLO (2-Loop) including m_T

Virtual MEs: $gg \to HH$ ~~$q\bar{q} \to HH$~~ ← **NNLO**

	Yukawa only (≤ 4-point)	Self-coupling (3-point)
NLO	 101 Diagrams	 21 Diagrams

Virtual MEs: Tool Chain

Partial cross-check: 2 Implementations



Integral Reduction

Tensor integrals rewritten as inverse propagators

Scalar products:

$$S = \frac{l(l+1)}{2} + lm$$

$l = 2$ # Loops

$m = 3$ # L.I External momenta

$S = 9$

Choose 8 Integral families with 9 propagators each

Integrals	1-loop	2-loop
Direct	63	9865
+ Symmetries	21	1601
+ IBPs	8	~260-270 (currently 327)

Reduction with:

`REDUZE`, `LiteRed`, `FIRE`

Simplification, fix:

$m_T = 173\ \mathrm{GeV},\ m_H = 125\ \mathrm{GeV}$

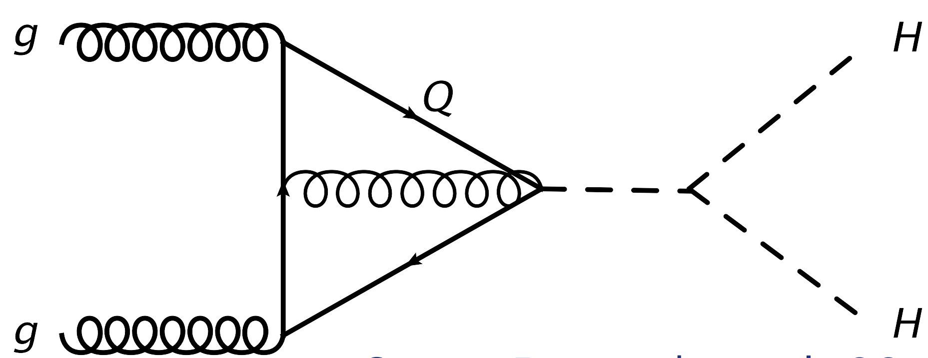
(Mostly) Finite Basis

Panzer 14; von Manteuffel, Panzer, Schabinger 15

Complete

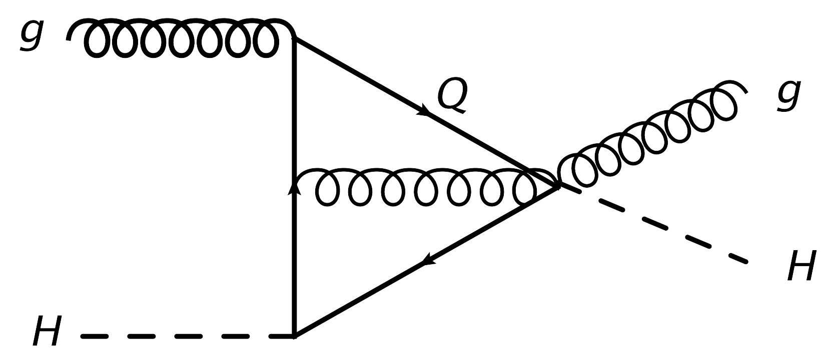
Master Integrals

Known Analytically:



Spira, Djouadi et al. 93, 95;
Bonciani, P. Mastrolia 03,04;
Anastasiou, Beerli et al. 06;

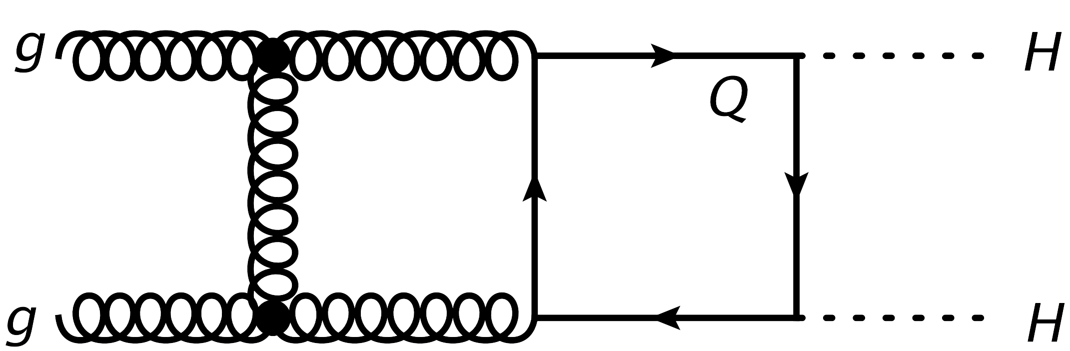
3-point, 1 off-shell leg
HPLs



Gehrmann, Guns, Kara 15

3-point, 2 off-shell legs
Generalized HPLs, 12 Letters

Numeric Evaluation:



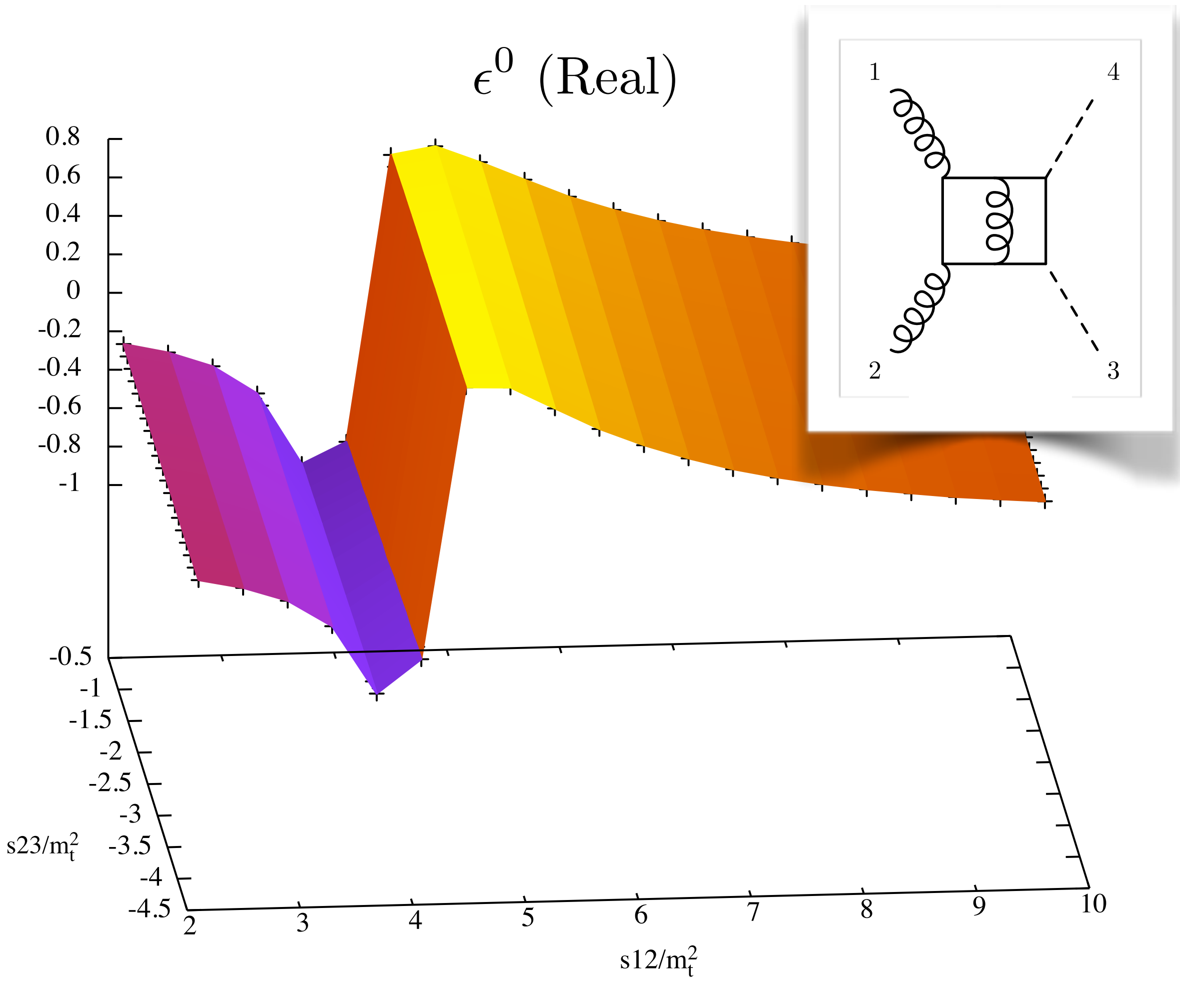
Up to 4-point,
4 scales s, t, m_T^2, m_H^2
`SecDec`

Thanks: Matthias Kerner

Master Integrals: Numerics

`SecDec` used at amplitude level:

- Avoid reevaluation of integrals
- Target accuracy of integrals based on contribution to amplitude + time/evaluation
- Continue integration until desired amplitude precision reached



Thanks: Gudrun Heinrich

Current Status: Cross-checks

- Single Higgs Part vs `Sushi` - OK
  Harlander, Liebler, Mantler 13
- Pole cancellation - OK (4+ digits)
- H & HH vs HEFT - Underway

Next Step:
Run on cluster
(Hydra, Garching)

Check & Run

Real Radiation

Real Radiation (HH + j...):

Huge simplification!

1-j Channels:

$$gg \to HH + g$$

$$gq \to HH + q \qquad g\bar{q} \to HH + \bar{q}$$

$$q\bar{q} \to HH + g$$

	Diagrams
Tree ⊗ Double	0
1-loop ⊗ Single	54+8+8+8

GoSam for MEs + Catani-Seymour Dipole Subtraction

Cullen et al. 14 Catani, Seymour 96

Checks:

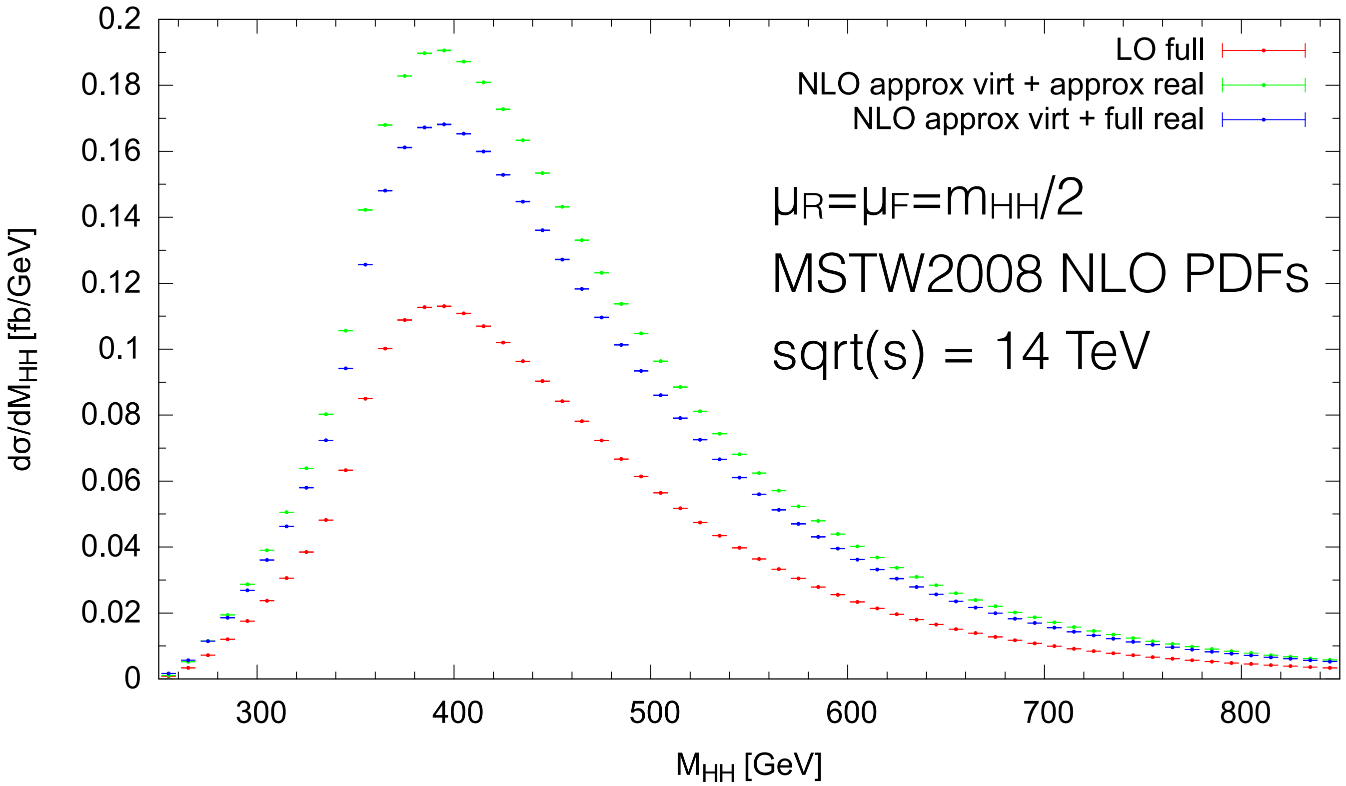
$gg \to Hg$ etc. reproduced & compared to Sushi

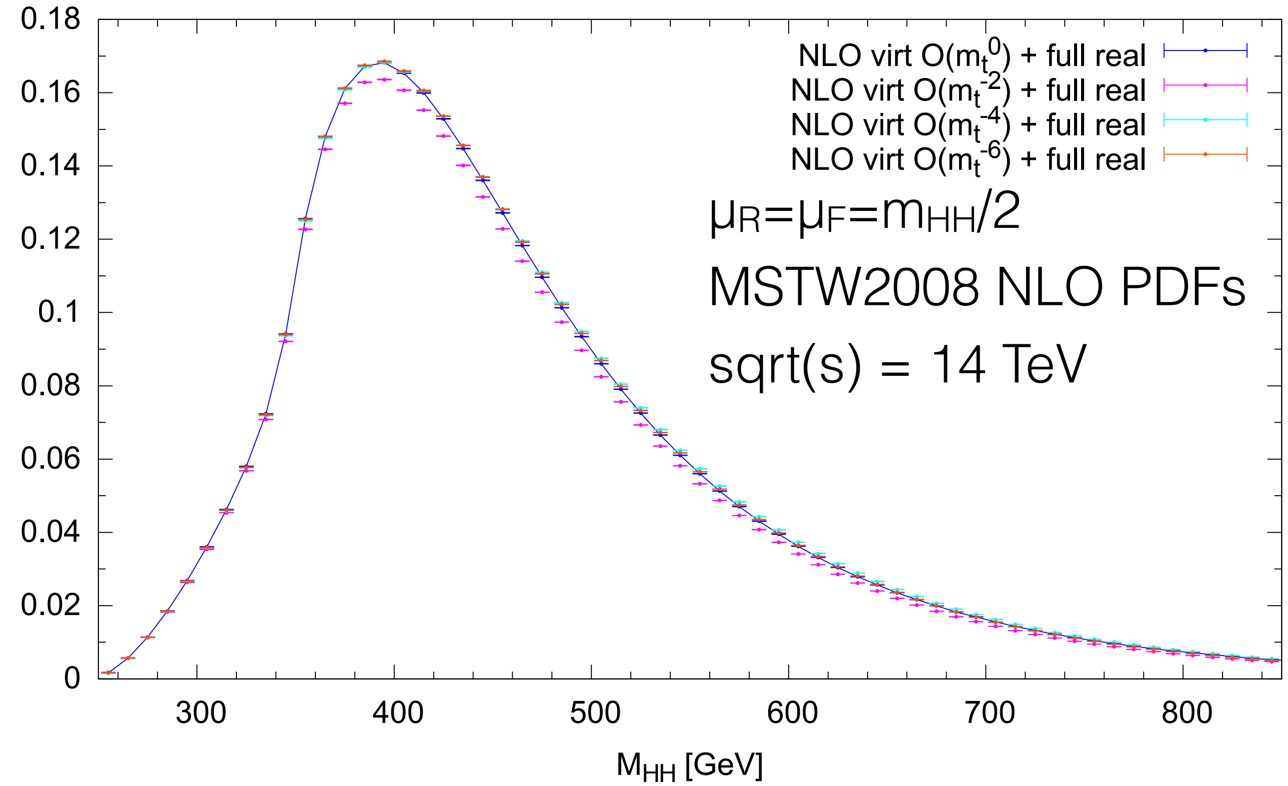
Independence of dipole-cut α parameter

Nagy 03

Complete

Real Radiation + HEFT





Approx/Full Reals + Virtuals as asymptotic expansion in $1/m_T^2$ (**q2e/exp** + **Reduze** + **matad**)

Harlander, Seidensticker, Steinhauser 97,99; Steinhauser 00

PDF4LHC15_nlo_30_pdfas
$m_H = 125$ GeV
$m_T = 172.5$ GeV
Uncertainty:
$\mu_R = \mu_F = \frac{m_{HH}}{2}$
$\mu_R = \mu_F \in \left[\frac{\mu_0}{2}, 2\mu_0\right]$
PDF with (without) α_s variation

	$gg \to HH$ total cross section [fb]	
$\sqrt{s}$ [TeV]	NLO HEFT + Full Reals	Maltoni, Vryonidou, Zaro
7	$5.939\,^{+17\%}_{-16\%} \pm 4.6\% (\pm 4.1\%)$	$5.82\,^{+18\%}_{-16\%} (\pm 4.0\%)$
8	$8.590\,^{+17\%}_{-15\%} \pm 4.2\% (\pm 3.8\%)$	$8.63\,^{+17\%}_{-15\%} (\pm 3.6\%)$
13	$28.83\,^{+15\%}_{-13\%} \pm 3.3\% (\pm 2.8\%)$	$28.4\,^{+16\%}_{-13\%} (\pm 2.7\%)$
14	$34.15\,^{+15\%}_{-13\%} \pm 3.2\% (\pm 2.6\%)$	$34.0\,^{+15\%}_{-13\%} (\pm 2.6\%)$

Thanks: Tom Zirke

Conclusion

HH Production

- Key measurement for probing the self coupling (HL-LHC era)
- HEFT implies that the NLO K factor for gluon fusion is large
   Unknown top mass effects give large uncertainty
   Full NLO corrections important

Ongoing/Future

- Complete checks of virtual amplitude
- Run on cluster, obtain enough phase-space points for accurate total cross-section prediction

Thank you for listening!

Backup

Slide: Tom Zirke

Approximate top-mass effects at NLO

$$\sigma^{NLO}(p) = \int d\phi_3 \left[\left(d\sigma^R(p)\right)_{\epsilon=0} - \left(\sum_{\text{dipoles}} d\sigma^{LO}(p) \otimes dV_{\text{dipole}} \right)_{\epsilon=0} \right] ✅$$
$$+ \int d\phi_2 \left[d\sigma^V(p) + d\sigma^{LO}(p) \otimes \mathbf{I} \right]_{\epsilon=0}$$
$$+ \int_0^1 dx \int d\phi_2 \left[d\sigma^{LO}(xp) \otimes (\mathbf{P} + \mathbf{K})(x) \right]_{\epsilon=0} ✅$$

$$d\sigma^V + d\sigma^{LO}(\epsilon) \otimes \mathbf{I} \approx d\sigma^V_{\text{exp},N} \frac{d\sigma^{LO}(\epsilon)}{d\sigma^{LO}_{\text{exp},N}(\epsilon)} + d\sigma^{LO}(\epsilon) \otimes \mathbf{I}$$
$$= \left(d\sigma^V_{\text{exp},N} + d\sigma^{LO}_{\text{exp},N}(\epsilon) \otimes \mathbf{I} \right) \frac{d\sigma^{LO}(\epsilon)}{d\sigma^{LO}_{\text{exp},N}(\epsilon)}$$
$$= \left(d\sigma^V_{\text{exp},N} + d\sigma^{LO}_{\text{exp},N}(\epsilon) \otimes \mathbf{I} \right) \frac{d\sigma^{LO}(\epsilon = 0)}{d\sigma^{LO}_{\text{exp},N}(\epsilon = 0)} + \mathcal{O}(\epsilon)$$

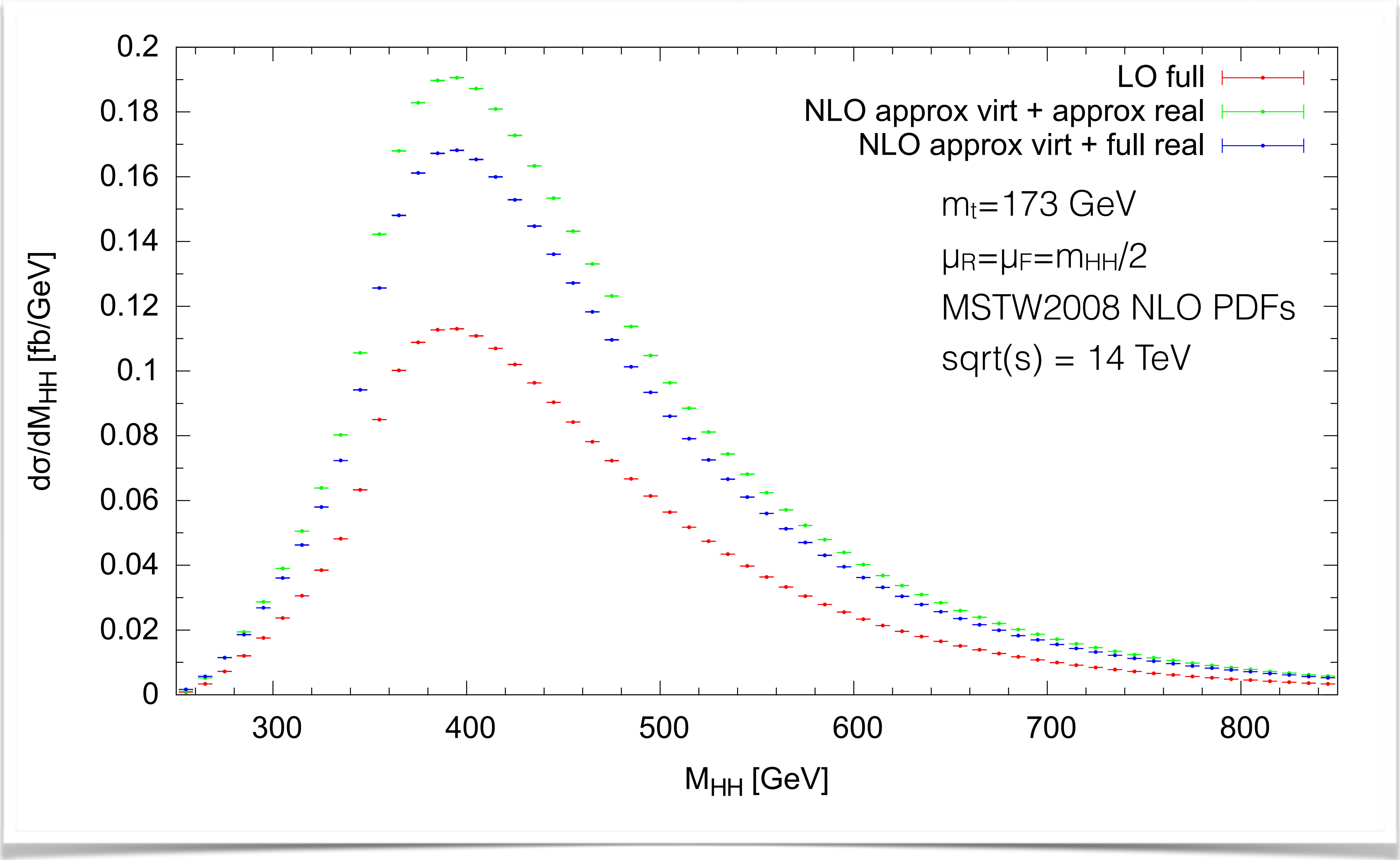
$$d\sigma_{\text{exp},N} = \sum_{k=0}^{N} d\sigma^{(k)} \left(\frac{\Lambda}{m_t} \right)^{2k}$$

$$\Lambda \in \{\sqrt{s}, \sqrt{t}, \sqrt{u}, m_h\}$$

- full real-emission matrix elements and dipoles
- virtual corrections as asymptotic expansion in $1/m_t^2$ with `q2e/exp` [Harlander, Seidensticker, Seidensticker] + `Reduze` [von Manteuffel, Studerus] + `matad` [Steinhauser]
- not directly comparable with [Grigo, Hoff, Steinhauser], (real radiation treated differently, expansion parameter $(m_H/m_t)^2$)

Slide: Tom Zirke

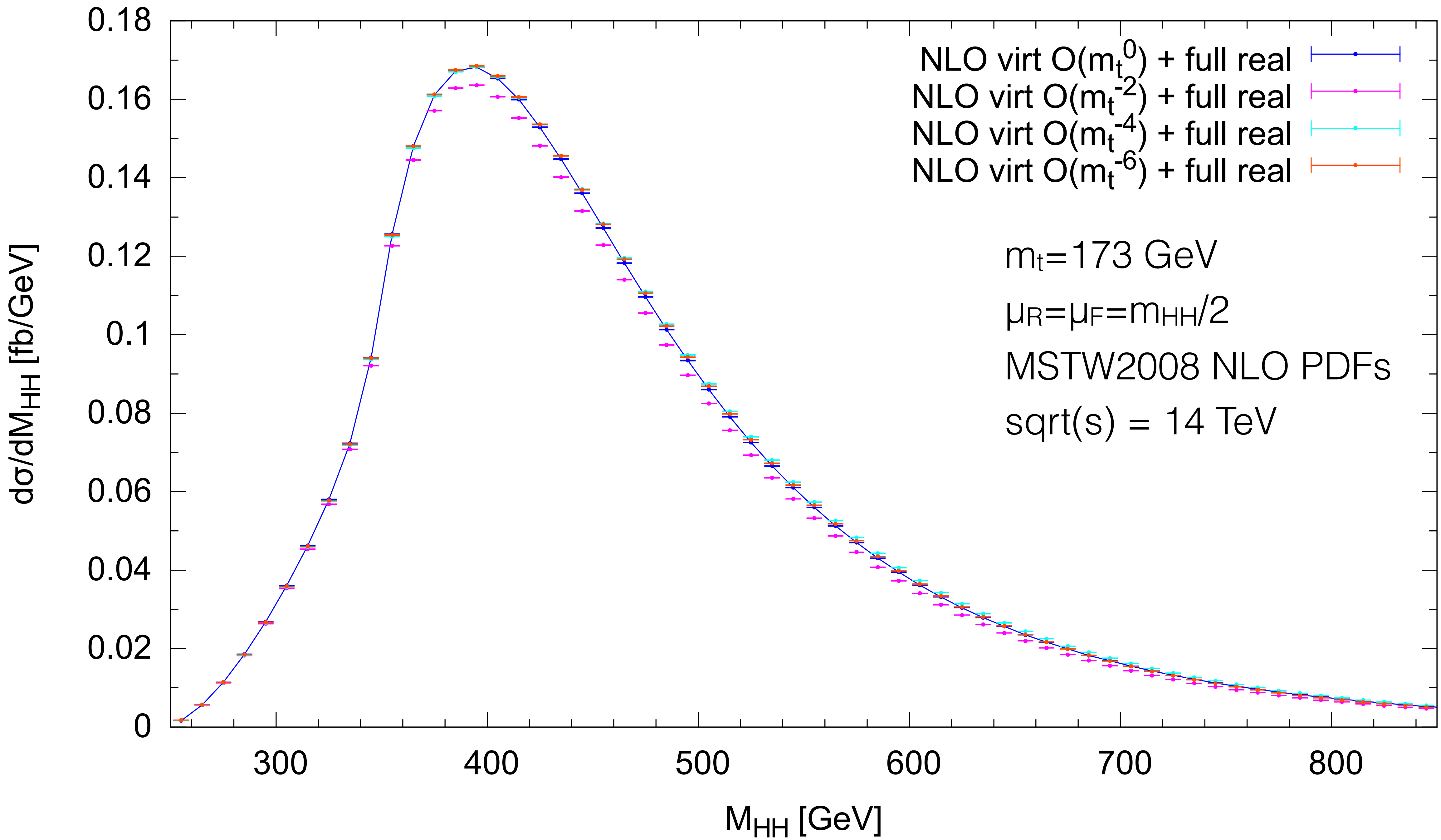
Mass effects in M_{HH} distribution (I)



- „approx“ ≙ rescaled expansion with N=0
- Known negative mass effects from real radiation

Slide: Tom Zirke

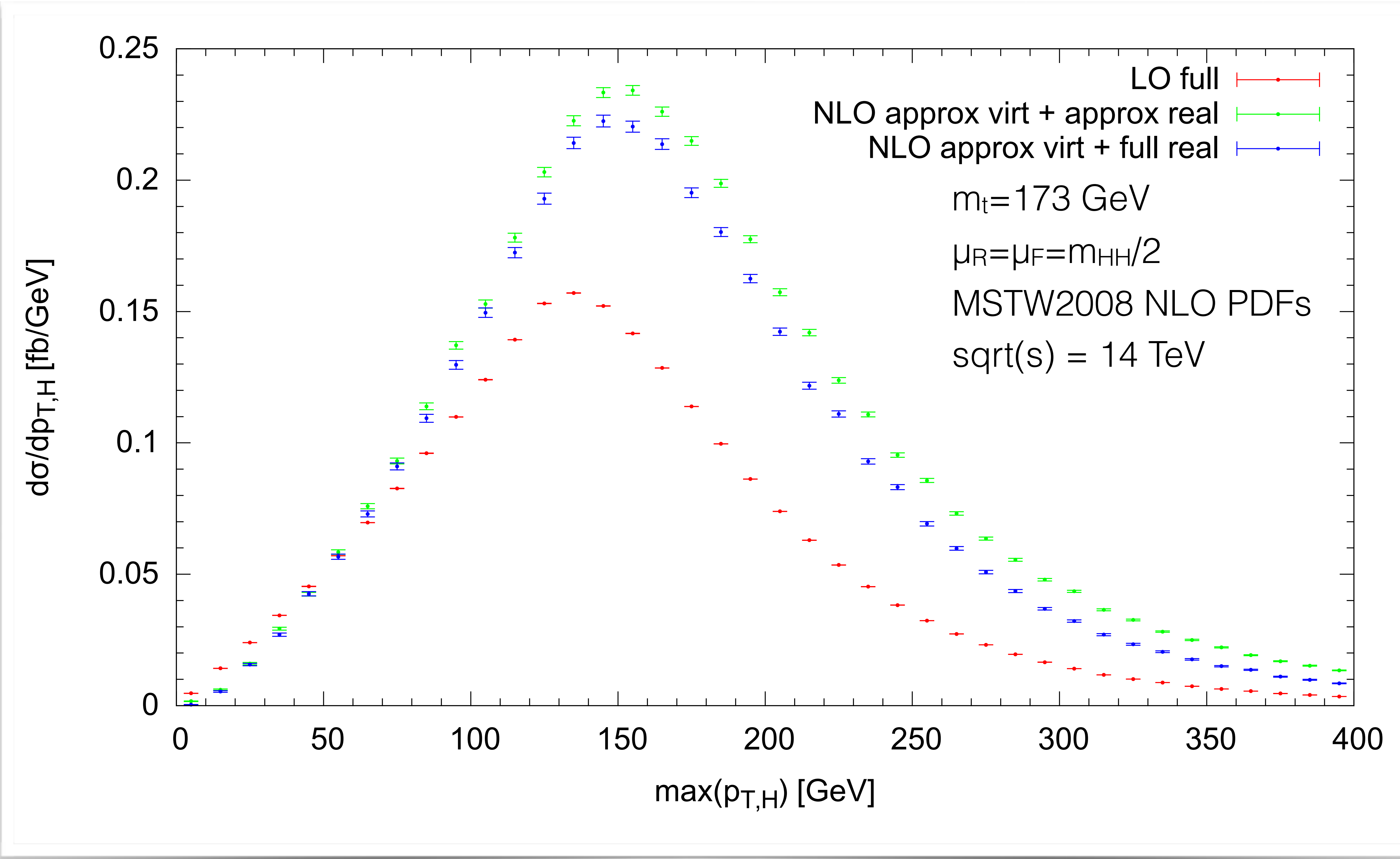
Mass effects in M_{HH} distribution (II)



- Slight tendency that -10% effect persists, but: spoilt cancellations? threshold effects?

Slide: Tom Zirke

Mass effects in p_T distribution (I)



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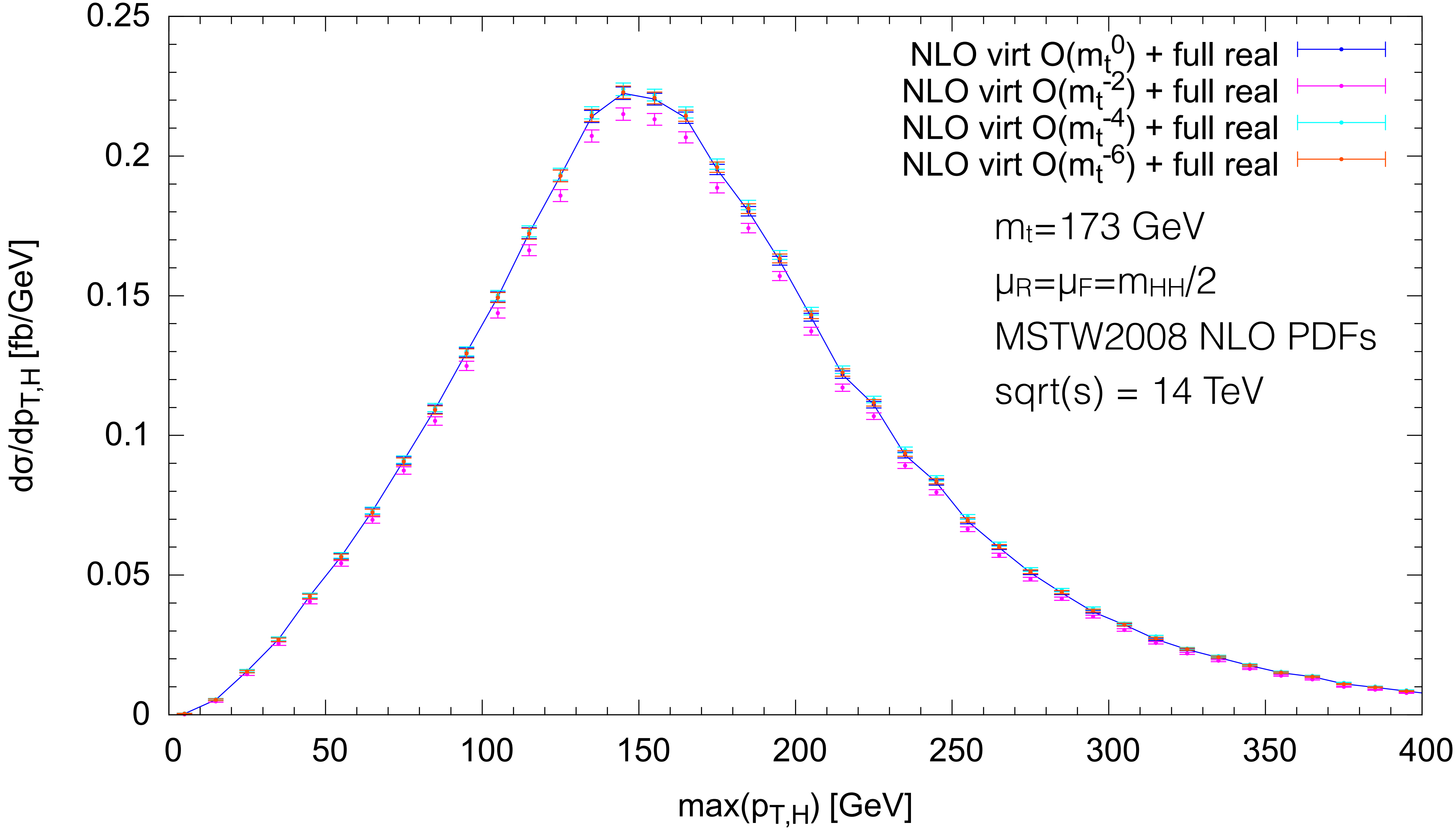
Mass effects in p_T distribution (II)



Gluon Fusion (NLO HEFT)

HEFT Valid for: $\sqrt{s} \ll 2m_T$ HH Production for: $2m_H < \sqrt{s}$

Born Improved NLO QCD HEFT

$$\mathrm{d}\sigma^V_{\mathrm{NLO}}(m_T) \approx \mathrm{d}\bar{\sigma}^V_{\mathrm{NLO}}(m_T) \equiv \frac{\mathrm{d}\sigma^V_{\mathrm{NLO}}(m_T \to \infty)}{\mathrm{d}\sigma^V_{\mathrm{LO}}(m_T \to \infty)} \mathrm{d}\sigma^V_{\mathrm{LO}}(m_T)$$

$$\mathrm{d}\sigma^R(m_T \to \infty)$$

$$K \approx 2$$

A. Maltoni et al.14

$$\mathrm{d}\overline{\sigma}^V(m_T)$$

$$\mathrm{d}\sigma^R(m_T)$$

$$-10\%$$

B. Grigo, Hoff, Steinhauser 15

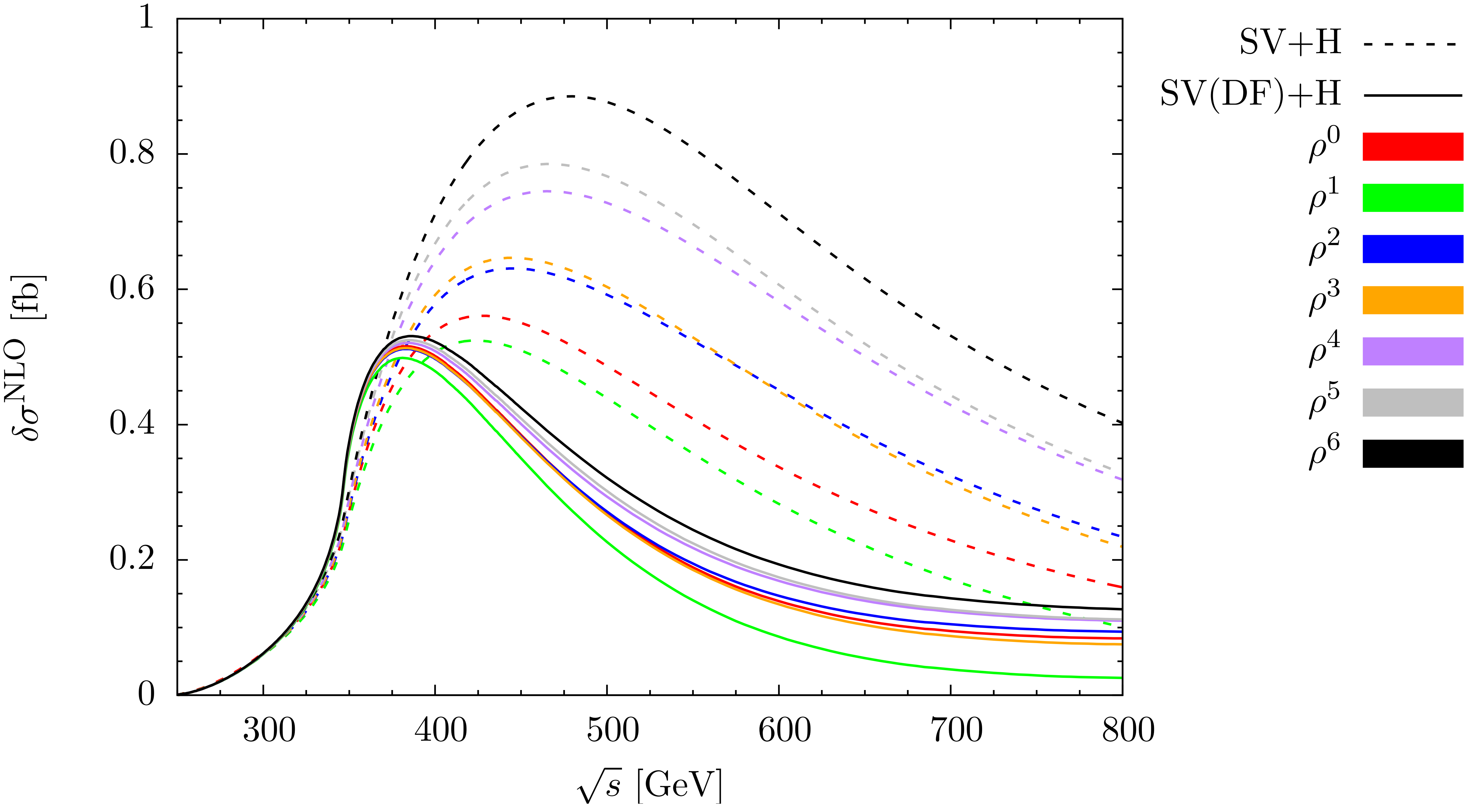
$$\mathrm{d}\hat{\sigma}(m_T) \equiv \mathrm{d}\sigma_0 + \mathrm{d}\sigma_1 \frac{m_H^2}{m_T^2} + \ldots + \mathrm{d}\sigma_6 \frac{m_H^{12}}{m_T^{12}}$$

$$\mathrm{d}\bar{\sigma}^{SV}_{\mathrm{NLO}}(m_T) \equiv \mathrm{d}\hat{\sigma}^{SV}_{\mathrm{NLO}}(m_T) \frac{\mathrm{d}\sigma^V_{\mathrm{LO}}(m_T)}{\mathrm{d}\hat{\sigma}^V_{\mathrm{LO}}(m_T)}$$

$$\mathrm{d}\bar{\sigma}^H_{\mathrm{NLO}}(m_T) \equiv \mathrm{d}\hat{\sigma}^H_{\mathrm{NLO}}(m_T) \frac{\sigma^V_{\mathrm{LO}}(m_T)}{\hat{\sigma}^V_{\mathrm{LO}}(m_T)}$$

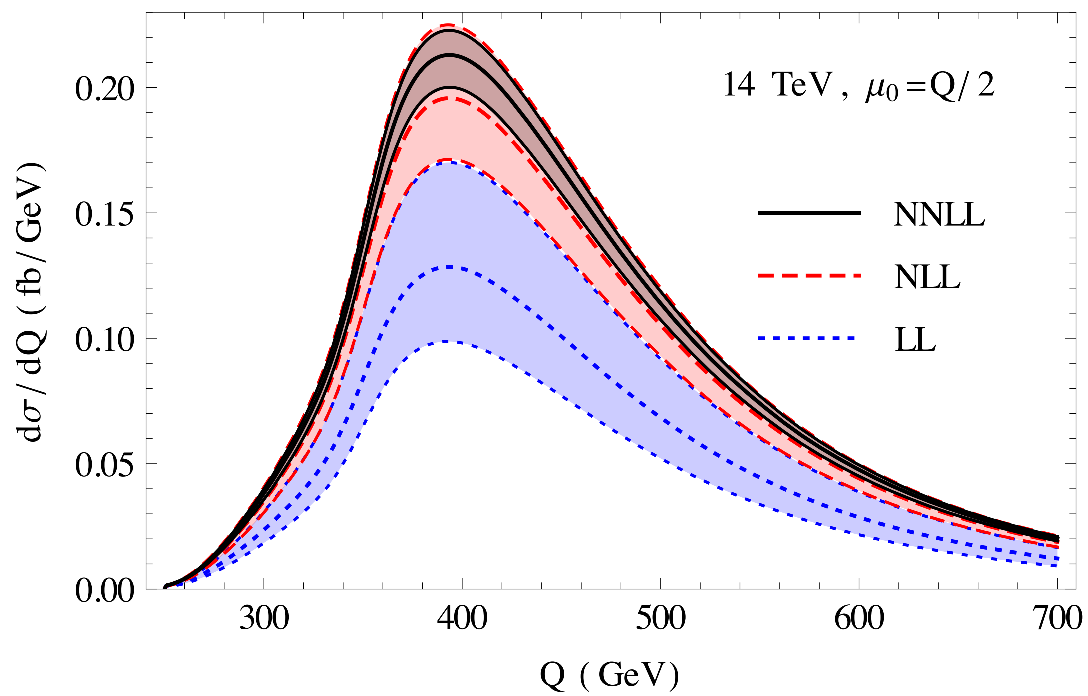
$$\pm 10\%$$

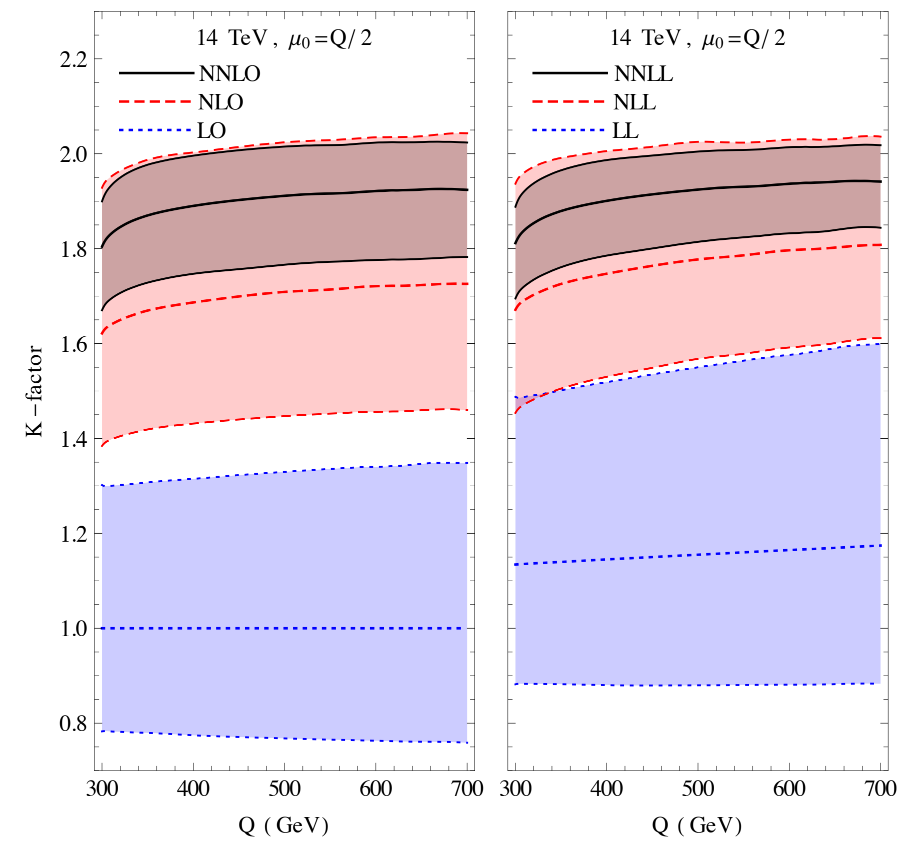
G.H.S Top Mass Expansion



Grigo, Hoff, Steinhauser 15

HEFT NNLO + NNLL

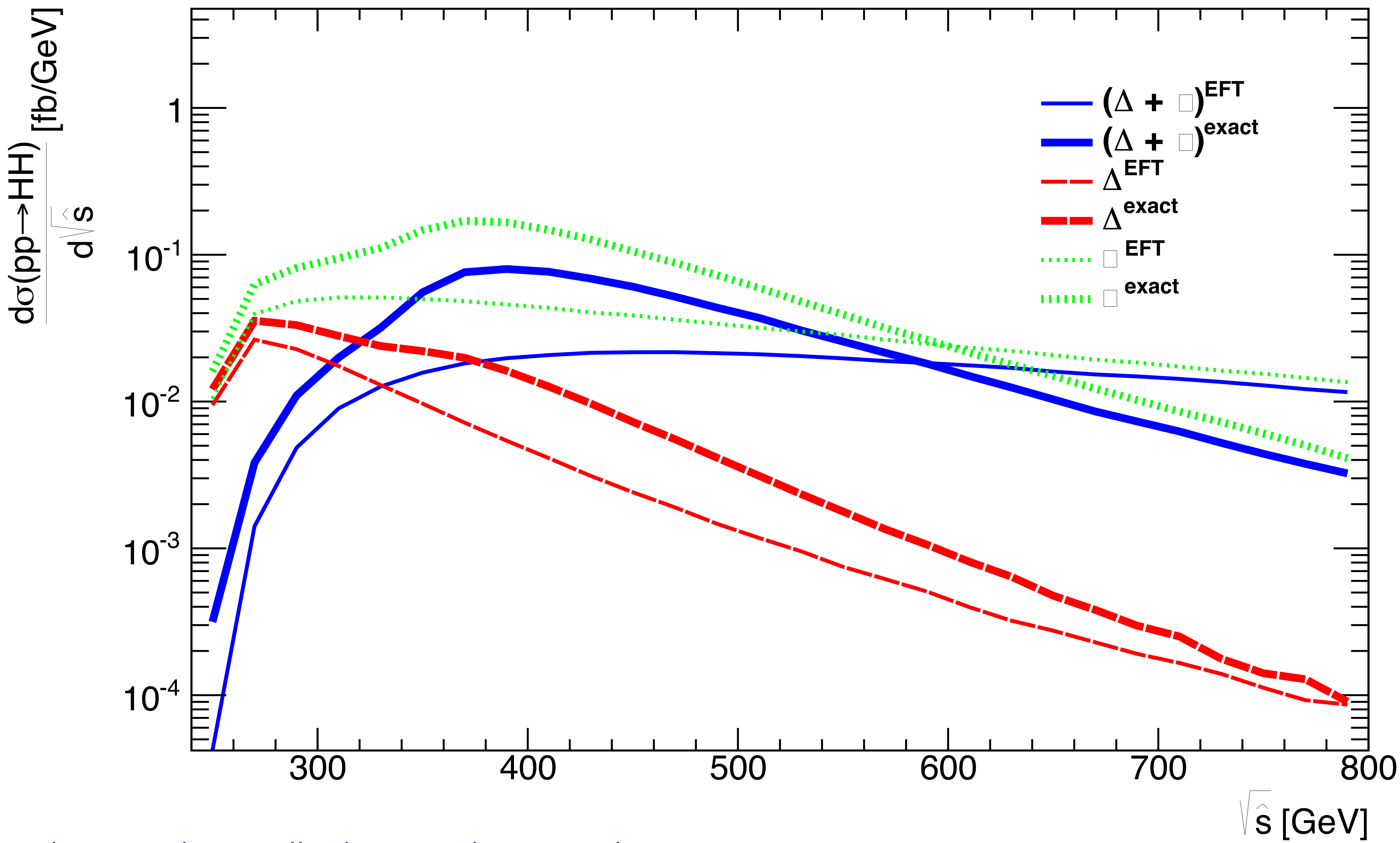




de Florian, Mazzitelli 15

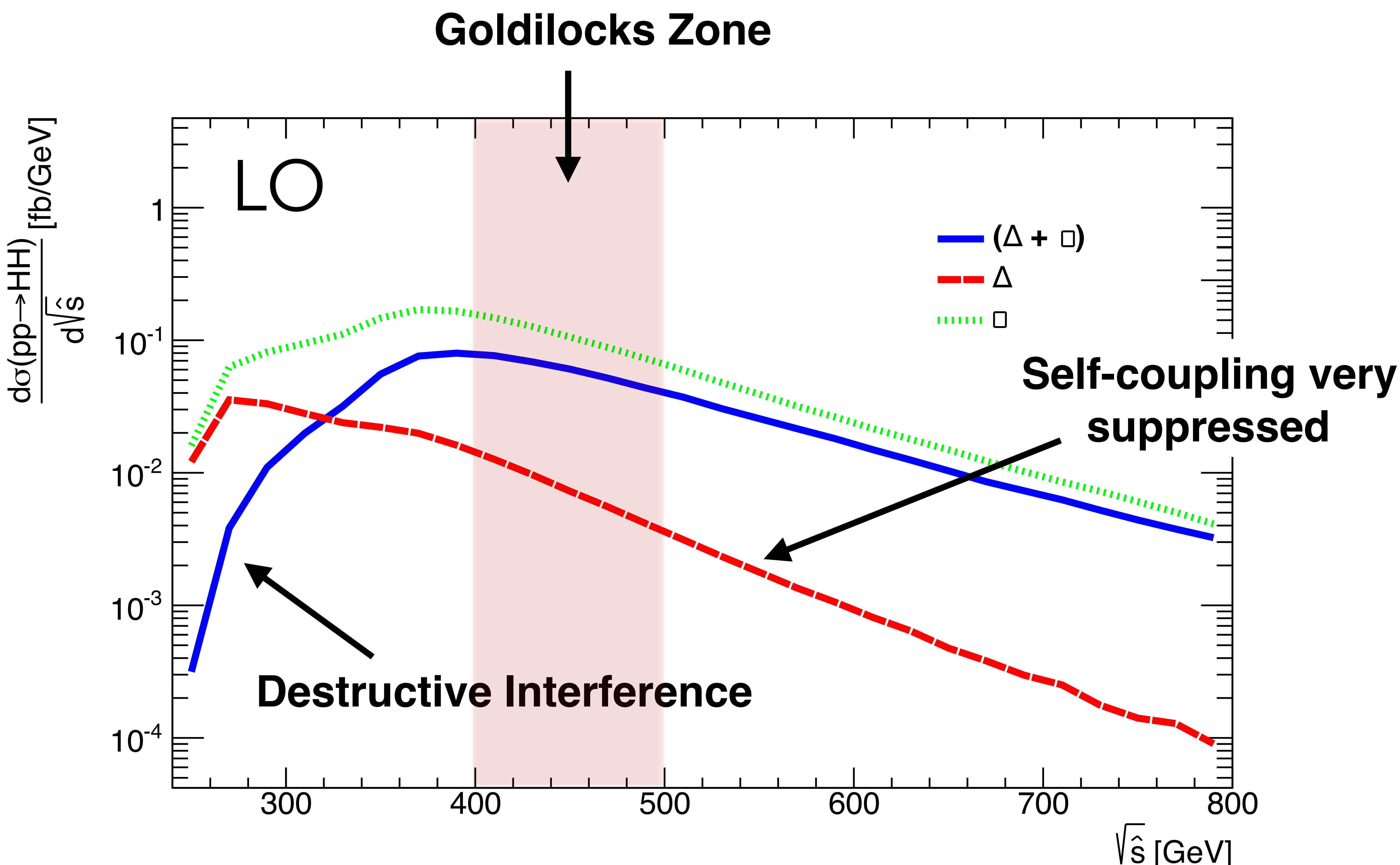
$\mu_0 = Q$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+α_S unc. (%)
8 TeV	9.92	$+9.3 - 10$	10.8	$+5.4 - 5.9$	$+5.6 - 6.0$	$+9.3 - 9.2$
13 TeV	34.3	$+8.3 - 8.9$	36.8	$+5.1 - 6.0$	$+4.0 - 4.3$	$+7.7 - 7.5$
14 TeV	40.9	$+8.2 - 8.8$	43.7	$+5.1 - 6.0$	$+3.8 - 4.0$	$+7.5 - 7.3$
33 TeV	247	$+7.1 - 7.4$	259	$+5.0 - 6.1$	$+2.2 - 2.8$	$+6.1 - 6.1$
100 TeV	1660	$+6.8 - 7.1$	1723	$+5.2 - 6.1$	$+2.1 - 3.0$	$+5.7 - 5.8$
$\mu_0 = Q/2$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+α_S unc. (%)
8 TeV	10.8	$+5.7 - 8.5$	11.0	$+4.0 - 5.6$	$+5.8 - 6.1$	$+9.6 - 9.3$
13 TeV	37.2	$+5.5 - 7.6$	37.4	$+4.2 - 5.8$	$+4.1 - 4.3$	$+7.8 - 7.6$
14 TeV	44.2	$+5.5 - 7.6$	44.5	$+4.2 - 5.9$	$+3.9 - 4.1$	$+7.6 - 7.4$
33 TeV	264	$+5.3 - 6.6$	265	$+4.6 - 6.1$	$+2.4 - 2.7$	$+6.3 - 6.1$
100 TeV	1760	$+5.3 - 6.7$	1762	$+4.9 - 6.4$	$+2.2 - 3.1$	$+6.2 - 7.0$

LO vs LO HEFT



Slawinska, van den Wollenberg, Eijk, Bentvelsen 14

Self-Coupling Sensitivity

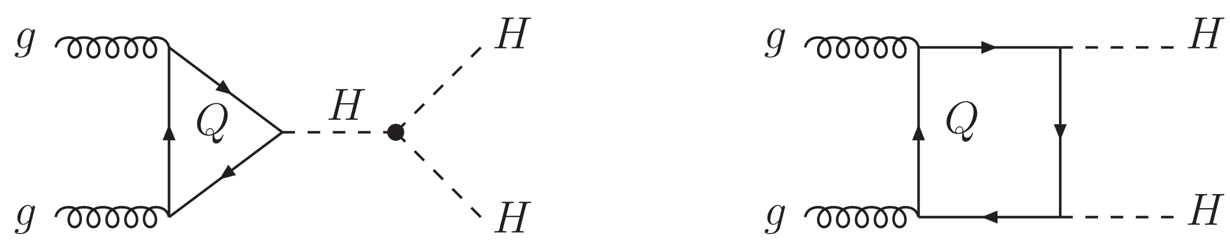


Slawinska, van den Wollenberg, Eijk, Bentvelsen 14

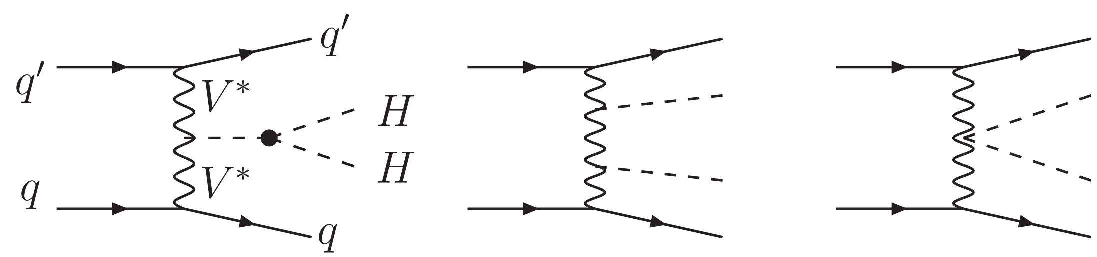
Production Channels

$\sigma(pp \to HH + X)$ @ 13TeV

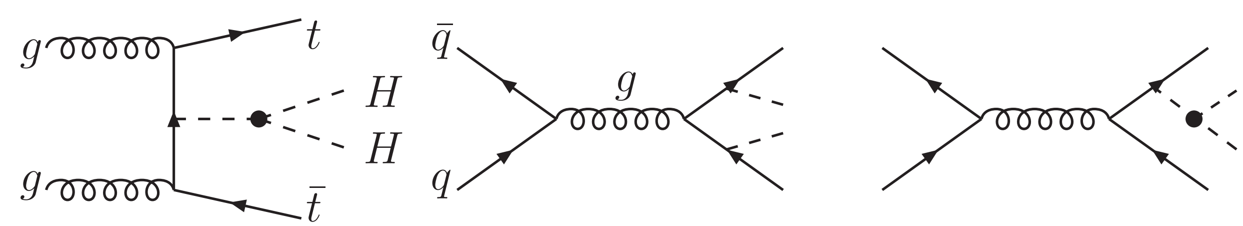
Gluon Fusion



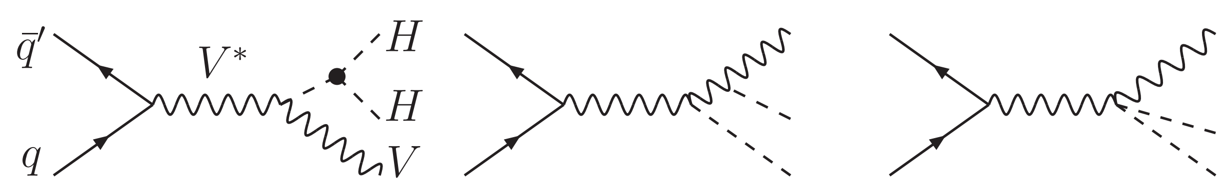
Vector Boson Fusion (VBF)



Associated top pair



Double Higgs-strahlung

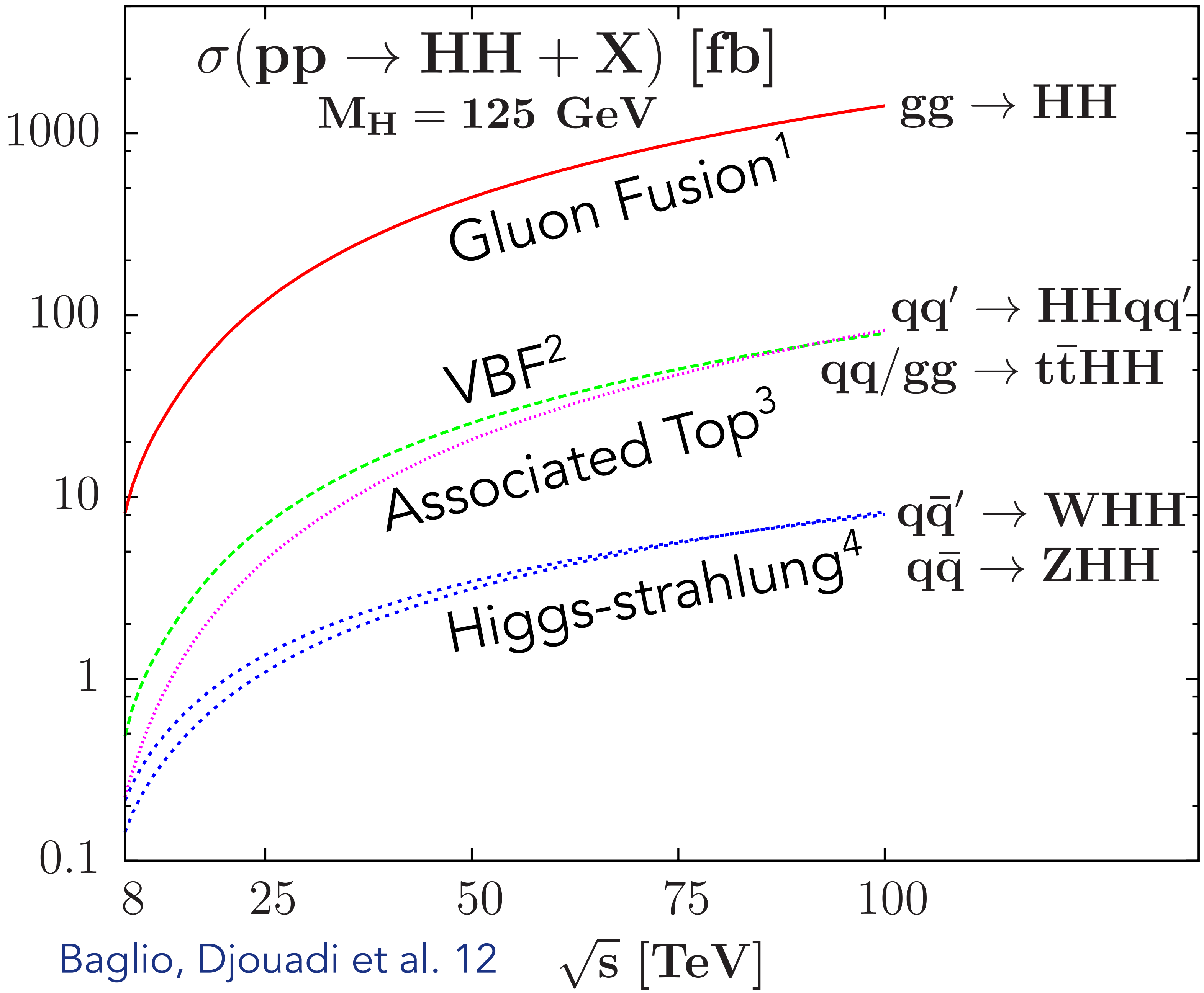


...

Baglio, Djouadi et al. 12

Production Channels

$$\sigma(pp \to HH + X) \sim \frac{1}{1000}\sigma(pp \to H + X)$$



Baglio, Djouadi et al. 12

[1] NLO QCD HEFT, `HPAIR`
Plehn, Spira, Zerwas 96, 98; Dawson et al. 98

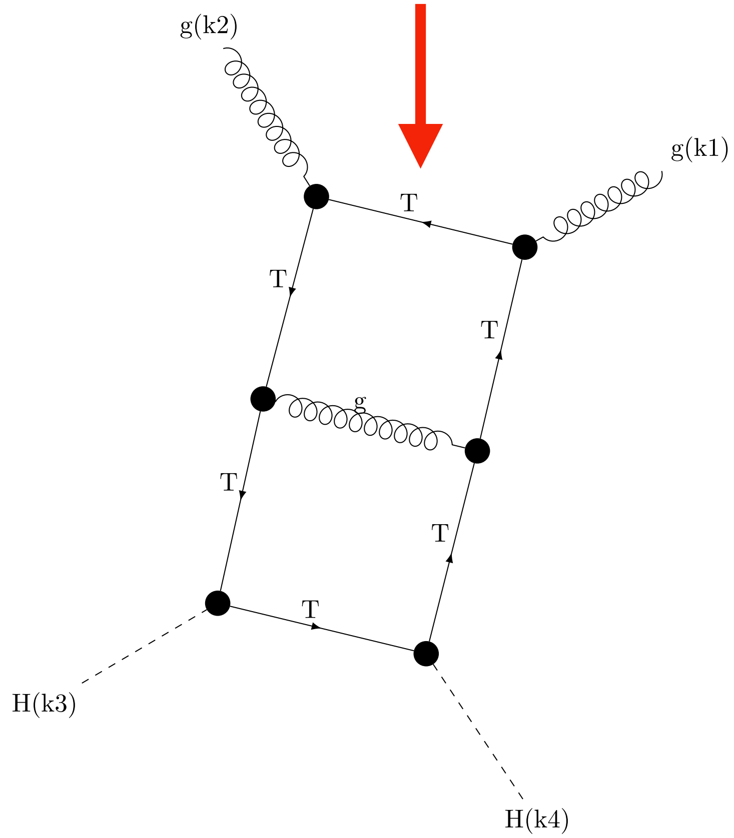
[2] NLO QCD, `VBFNLO`
Baglio, Djouadi et al. 12

[3] LO QCD (NLO, `aMC@NLO`)
Frederix, Frixione et al. 14

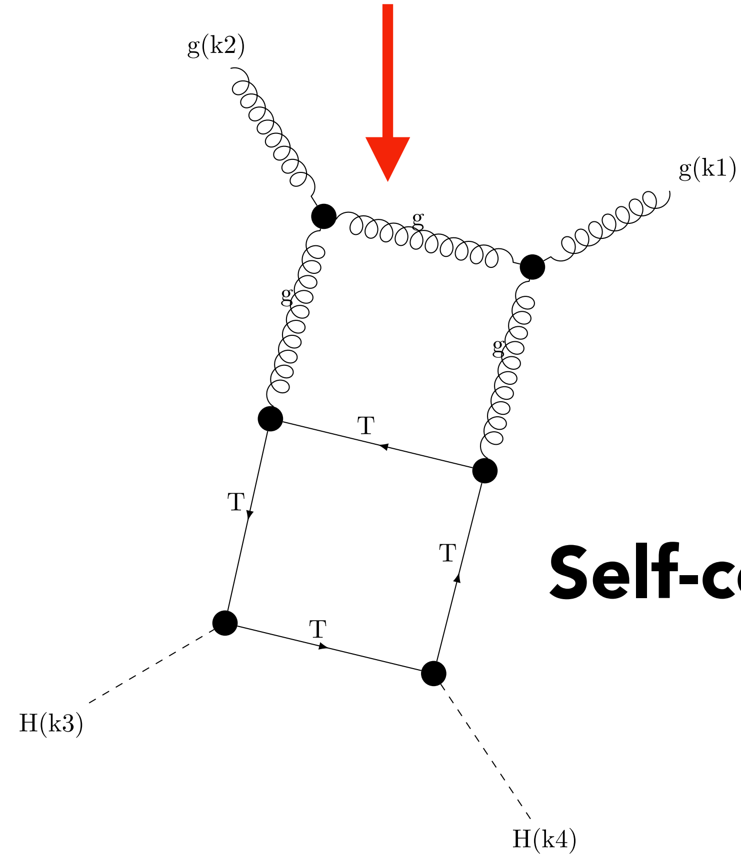
[4] NNLO QCD
Baglio, Djouadi et al. 12

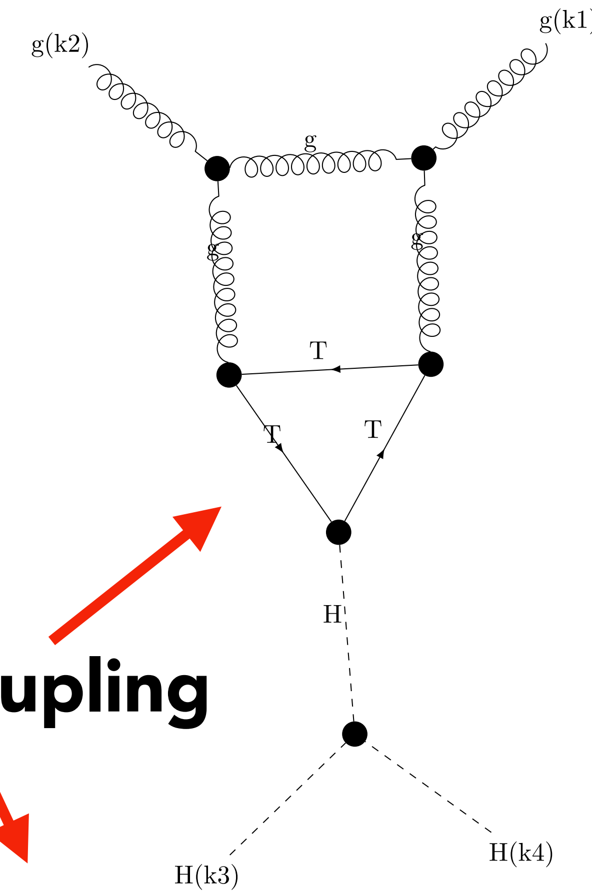
Diagrams

Massive Double Box

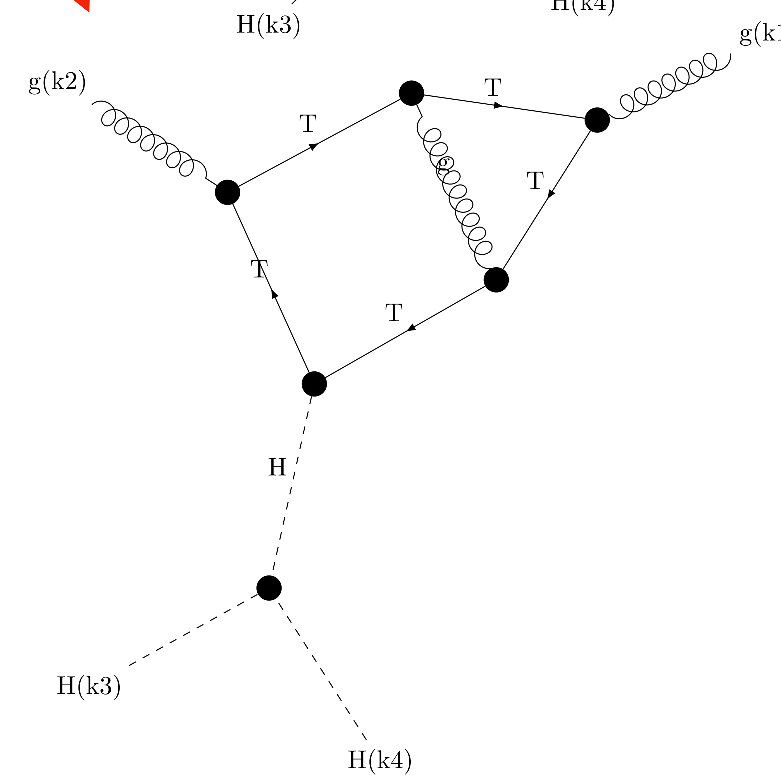


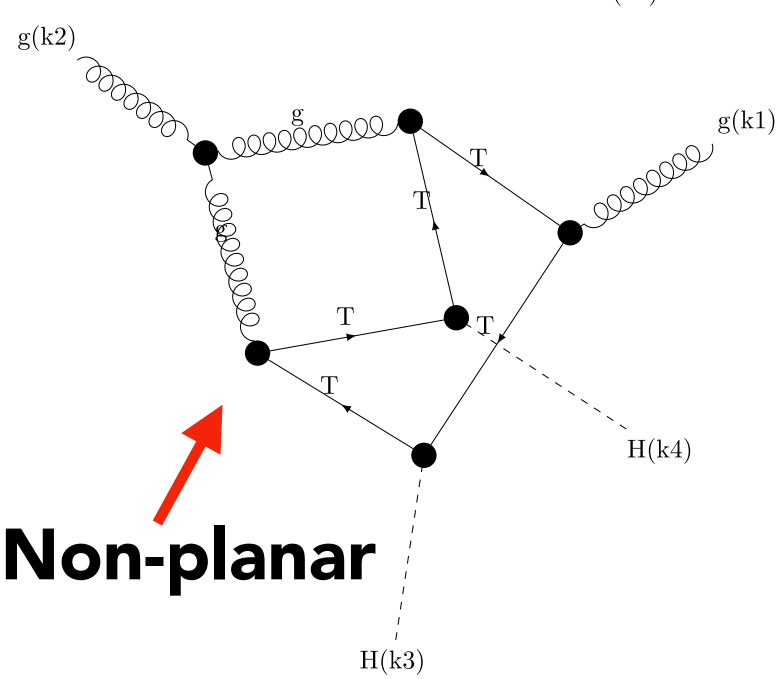
Massless/Massive Box





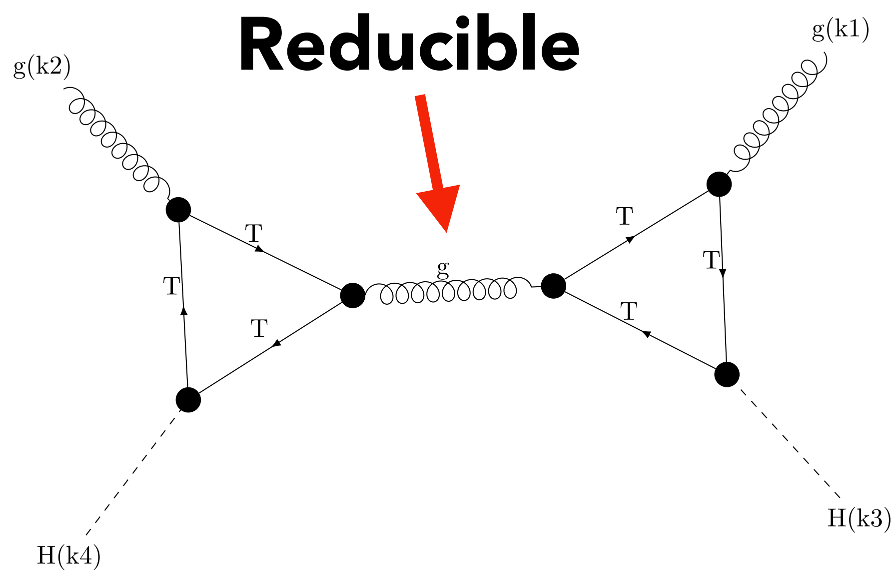
Self-coupling





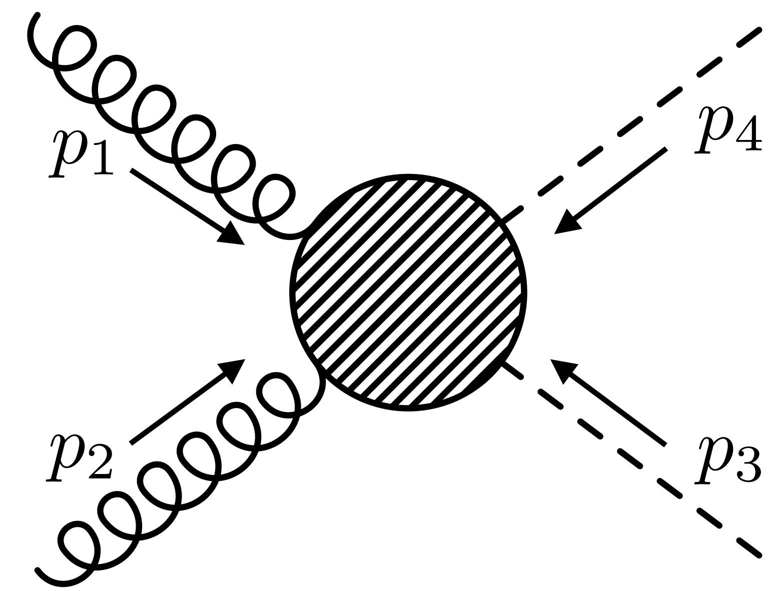
Non-planar

Reducible



Form Factor Decomposition

$$g(p_1)g(p_2) \to H(-p_3)H(-p_4) \qquad \sum_{i=1}^{4} p_i = 0$$



CDR (Dim = d)

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Decompose:
$$\mathcal{M}^{\mu\nu} = a_{00} g^{\mu\nu} + \cancel{a_{11} p_1^\mu p_1^\nu + a_{12} p_1^\mu p_2^\nu + a_{13} p_1^\mu p_3^\nu}$$
$$+ a_{21} p_2^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{23} p_2^\mu p_3^\nu$$
$$+ a_{31} p_3^\mu p_1^\nu + a_{32} p_3^\mu p_2^\nu + a_{33} p_3^\mu p_3^\nu$$

a_{ij} **functions of Mandelstams +** d

p_i **linearly indep.**

Transversity: $g(p_1):\ \epsilon^1_\mu p_1^\mu = 0$ $\quad$ $g(p_2):\ \epsilon^2_\nu p_2^\nu = 0$

Ward/Gauge: $p_{1\mu} \mathcal{M}^{\mu\nu} = 0$, $p_{2\nu} \mathcal{M}^{\mu\nu} = 0$ Gives further identities

Form Factor Decomposition

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Decomposition:

Form Factors (Contain integrals)

$$\mathcal{M}^{\mu\nu} \propto A_1(s,t,m_H^2,m_T^2,d) T_1^{\mu\nu} + A_2(s,t,m_H^2,m_T^2,d) T_2^{\mu\nu}$$

(Tensor) Basis

Choose: $\mathcal{M}^{++} = \mathcal{M}^{--} = -A_1$

$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -A_2$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_2^\mu p_1^\nu}{p_1 \cdot p_2} \qquad p_T^2 = \frac{ut - m_H^4}{s}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_2^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_2 \cdot p_3 p_3^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} + \frac{2 p_3^\mu p_3^\nu}{p_T^2}$$

Glover, van der Bij 88

Form Factor Decomposition

Construct Projectors:

No Integrals

$$P_j^{\mu\nu} = \sum_{i=1}^{2} B_{ji}(s, t, m_H^2, d) T_i^{\mu\nu}$$

Same Basis as amplitude

Such that:

$$P_{1\,\mu\nu}\mathcal{M}^{\mu\nu} = A_1$$

$$P_{2\,\mu\nu}\mathcal{M}^{\mu\nu} = A_2$$

Explicitly; separately calculate the contraction of each projector with $\mathcal{M}^{\mu\nu}$

Recall:

$$\mathcal{M}^{++} = \mathcal{M}^{--} = -A_1$$

$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -A_2$$

- Self-coupling diagrams are 1PR by cutting a scalar propagator
- By angular momentum conservation they contribute only to A_1

Integrals

k_i Loop momenta, p_i L.I. External momenta,

$N_i = (q_i^2 - a)$ Propagator-1, $q_i = \sum_{i=1}^{j} b_i k_i + \sum_{i=1}^{m} c_i p_i$

After Dirac algebra (Traces):

$$A_j \supset \int \mathrm{d}^d k_1 \int \mathrm{d}^d k_2 \frac{f(k_1 \cdot k_1, k_1 \cdot k_2, \ldots, k_2 \cdot p_3)}{N_1 \cdots N_7}$$

(Max) 7 Propagators in Diagram

$S >$ **#Propagators: Irreducible Numerators**

Number of Scalar products:

$$S = \frac{l(l+1)}{2} + lm$$

$l = 2$ # Loops

$m = 3$ # L.I External momenta

$S = 9$

Integral Reduction

Integral family: Add propagators s.t. all scalar products can be expressed in terms of (inverse) propagators

$$A_j \supset \int \mathrm{d}^d k_1 \int \mathrm{d}^d k_2 \frac{1}{N_1^{\alpha_1} \cdots N_9^{\alpha_9}} \equiv I(\alpha_1, \ldots, \alpha_9)$$

Encode all integrals by their propagator powers

Symmetries: $I(\alpha_1, \ldots, \alpha_9) = I(\sigma(\alpha_1), \ldots, \sigma(\alpha_9))$ ← **For some** $\alpha_i > 0$

Integration-by-parts (IBP) /Lorentz Invariance (LI) Identities
Tkachov 81; Chetyrkin, Tkachov 81 Gehrmann, Remiddi 99

Laporta/ S-Bases algorithms to automate application of these identities
Laporta 01; Smirnov, Smirnov 06