Appilcation of Maximum Likelihood Method for the Analysis of Spin Density Matrix Elements of Vector Meson Production at hermes

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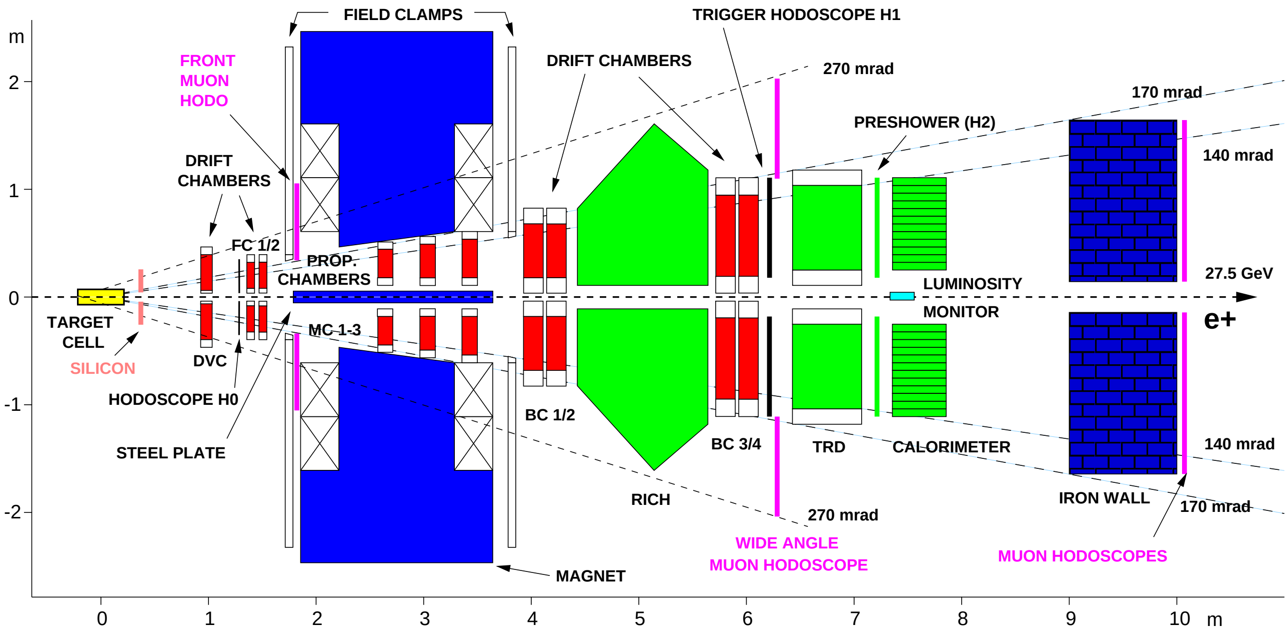
HERMES Spectrometer



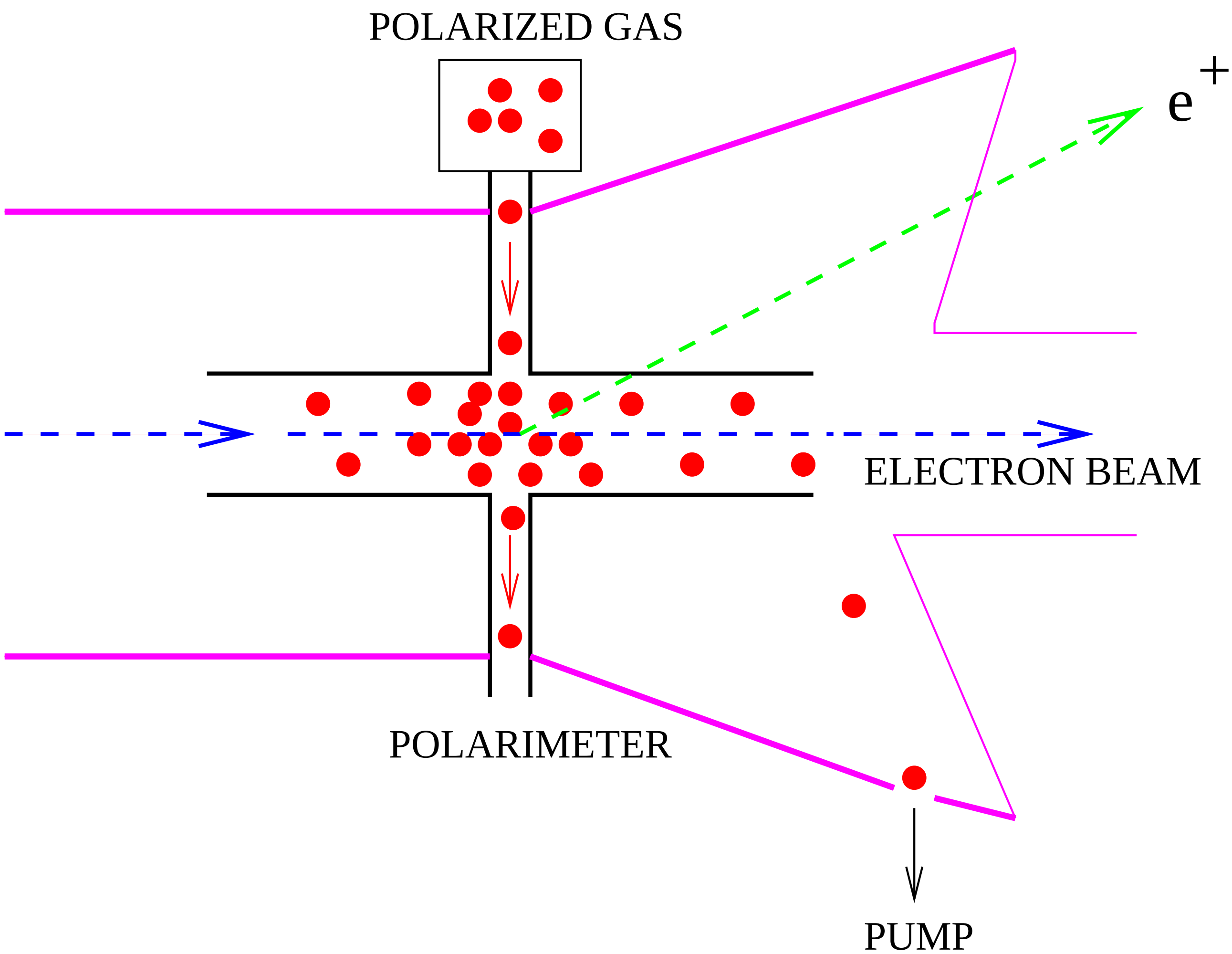
- Two identical halves of forward spectrometer having acceptance $40 < \Theta < 220$ mrad with momentum resolution $\leq 1\%$

- Electron identification with efficiency $\geq 98\%$ at hadron contamination $\leq 1\%$

- Calorimeter with resolution $\Delta E/E[\%] = 1.5_{\pm 0.5} + 5.1_{\pm 1.1}/\sqrt{E[GeV]}$

- RICH from 1998 with π^{+-} vs K^{+-} and P separation over all kinematic region

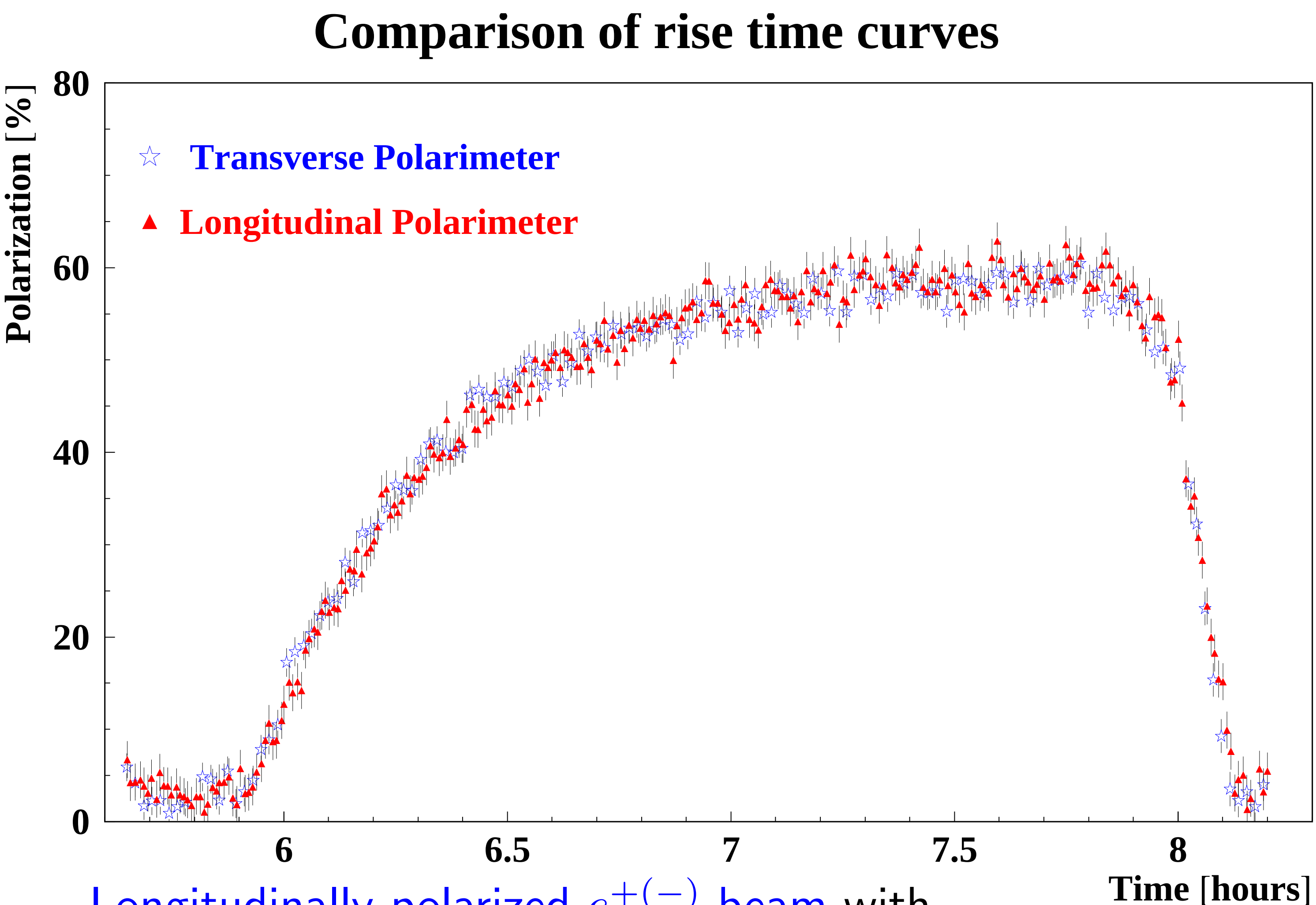
The Target



Internal storage cell gas target with high density

- polarised: $\sim 10^{14}$ nucl/cm^2, polarization: $\sim 88\%$ for H/D
- unpolarized: $\sim 10^{15}$ nucl/cm^2
- high density runs at the end of the fill in 2000 with ^{1}H, ^{2}D, ^{4}He and ^{20}Ne gases at $\sim 5 \cdot 10^{15}$ nucl/cm^2

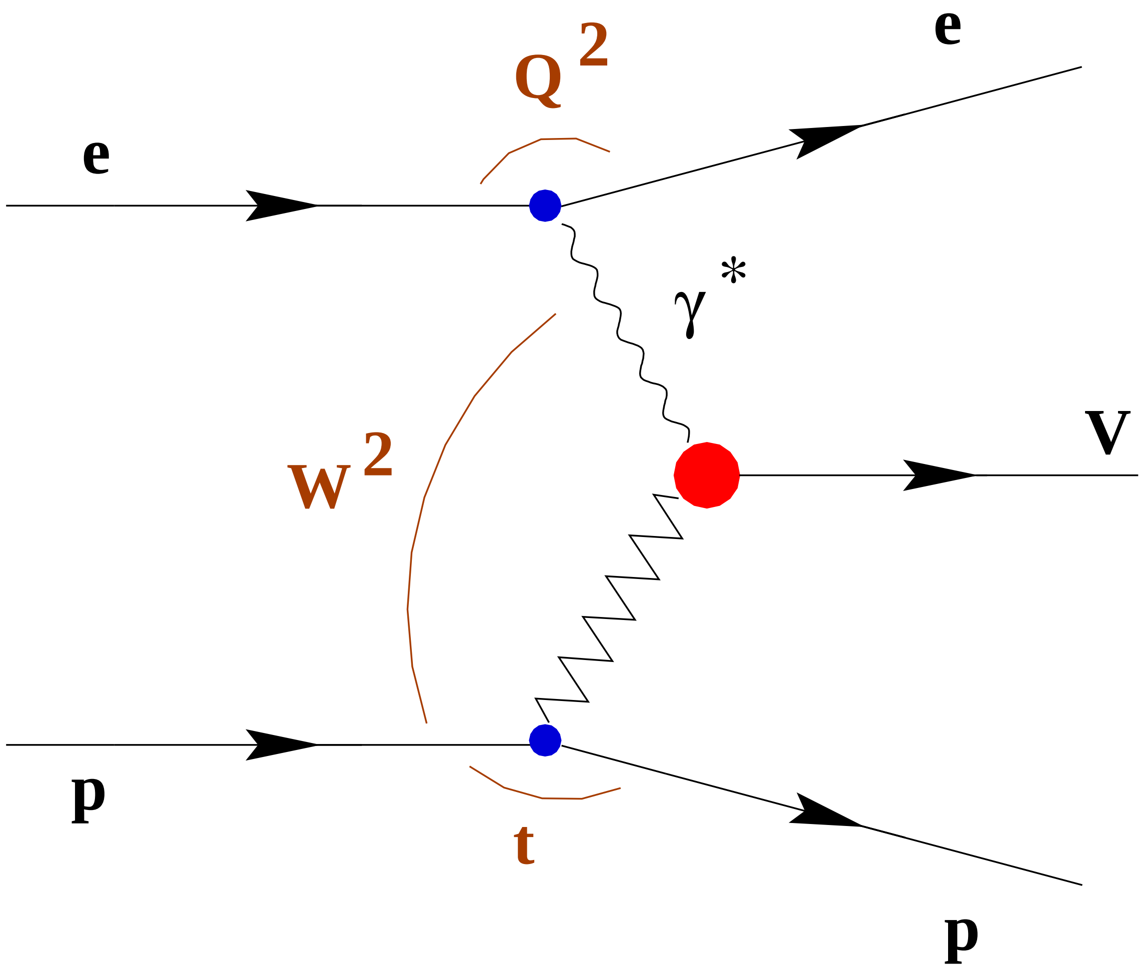
The Beam



Longitudinally polarized $e^{+(-)}$ beam with

- $P = 27.56$ GeV/c,
- current $50 \ldots 10$ mA,
- polarization $40 \ldots 60\%$

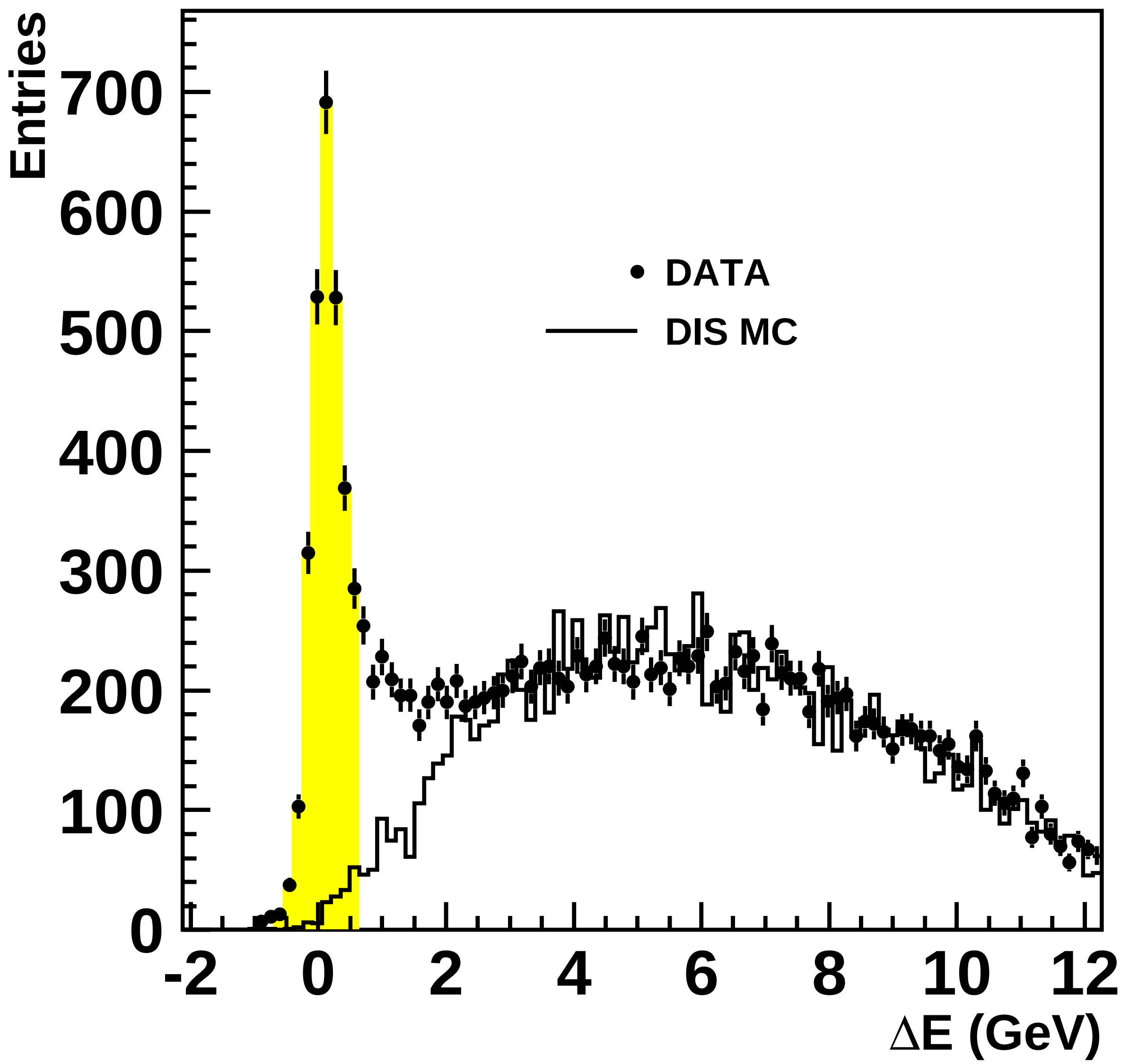
Exclusive ρ^0 pruduction



$$e^+ + p \rightarrow e^{+'} + p + \rho^0(\rightarrow \pi^+\pi^-)$$

$e^{+'}$ and $\pi^+\pi^-$ detected

Selection of Exclusive ρ^0 Events

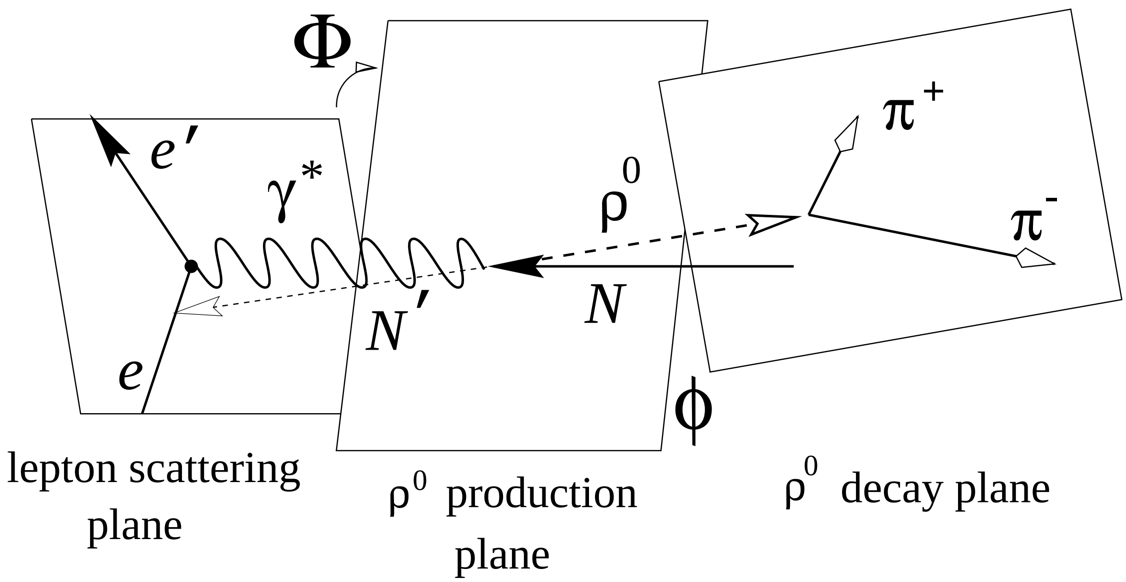


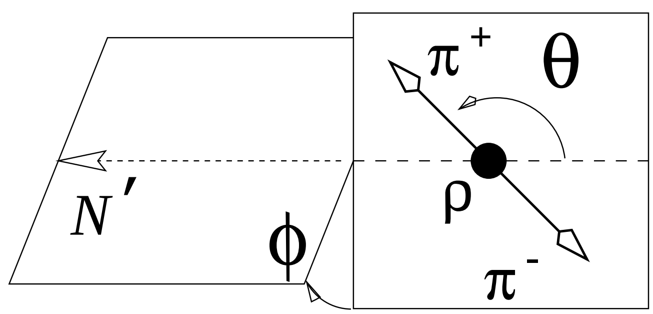
$$\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$$

with $M_X^2 = (p + q - v)^2$

Measured Φ, ϕ, Θ Angles of ρ^0 production

Photon-Nucleon CMS





ρ^0 Rest Frame

Maximum Likelihood Method in MINUIT

Binned Likelihood Method with Poisson distribution for the number of events in each bin. $8 \times 8 \times 8$ bins of $\cos(\Theta), \phi, \Phi$ was used.

Input of data:
3-dimensional matrix of data and
normalized background to be subtracted from the data before fitting

Monte Carlo Events:
3-dimensional matrix of fully reconstructed MC events at uniform angular distribution

Extraction procedure: simultaneous fit of 23 SDME, with one set of parameters for negative and positive beam helicity.

Function 1: Fit of 23 SDME r_{ij}^{α}

W$(\cos\Theta, \phi, \Phi) = W^{unpol} + W^{long.pol.}$

W$^{unpol}(\cos\Theta, \phi, \Phi) = \frac{3}{4\pi}\Big[\frac{1}{2}(1 - r_{00}^{04}) + \frac{1}{2}(3r_{00}^{04} - 1)\cos^2\Theta$

$- \sqrt{2}Re(r_{10}^{04})\sin 2\Theta\cos\phi - r_{1-1}^{04}\sin^2\Theta\cos 2\phi$

$- \epsilon\cos 2\Phi\Big(r_{11}^{1}\sin^2\Theta + r_{00}^{1}\cos^2\Theta - r_{1-1}^{1}\sin^2\Theta\cos 2\phi\Big)$

$- \epsilon\sin 2\Phi\Big(\sqrt{2}Im(r_{10}^{2})\sin^2\Theta\sin\phi + Im(r_{1-1}^{2})\sin 2\Theta\sin 2\phi\Big)$

$+ \sqrt{2\epsilon(1+\epsilon)}\cos\Phi$
$\Big(r_{11}^{5}\sin^2\Theta + r_{00}^{5}\cos^2\Theta - \sqrt{2}Re r_{10}^{5}\sin 2\Theta\cos\phi - r_{1-1}^{5}\sin^2\Theta cos2\phi\Big)$

$+ \sqrt{2\epsilon(1+\epsilon)}\sin\Phi$
$\Big(\sqrt{2}Im(r_{10}^{6})\sin 2\Theta\sin\phi + Im(r_{1-1}^{6})\sin^2\Theta\sin 2\phi\Big)\Big]$

W$^{long.pol.}(\cos\Theta, \phi, \Phi) = \frac{3}{4\pi}P_{beam}\Big[$

$\sqrt{1-\epsilon^2}\Big(\sqrt{2}Im(r_{10}^{3})\sin 2\Theta\sin\phi + Im(r_{1-1}^{3})\sin^2\Theta\sin 2\phi\Big)$

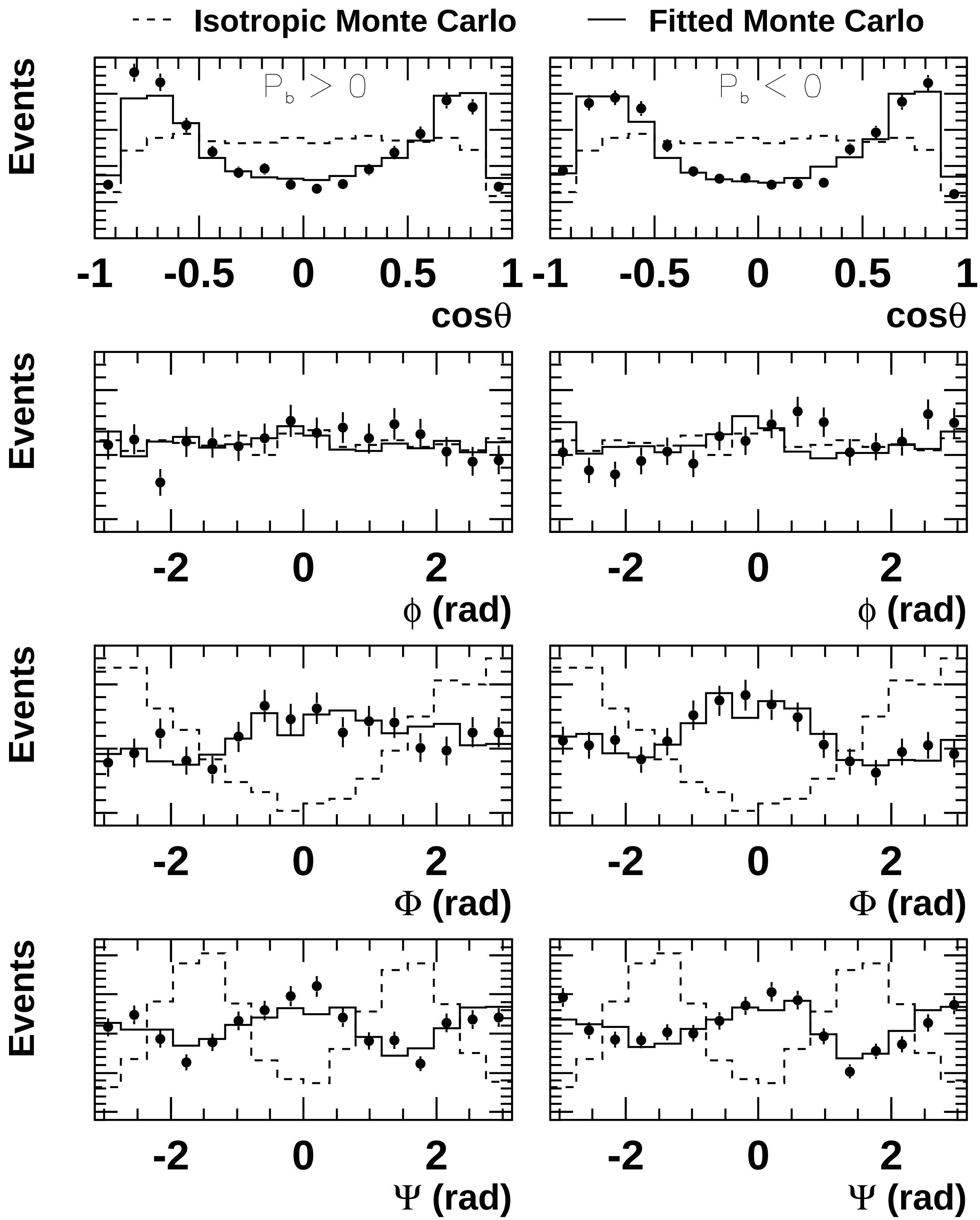
$+ \sqrt{2\epsilon(1-\epsilon)}\cos\Phi$
$\Big(\sqrt{2}Im(r_{10}^{7})\sin 2\Theta\sin\phi + Im(r_{1-1}^{7})\sin^2\Theta\sin 2\phi\Big)$

$+ \sqrt{2\epsilon(1-\epsilon)}\sin\Phi$
$\Big(r_{11}^{8}\sin^2\Theta + r_{00}^{8}\cos^2\Theta - \sqrt{2}Re(r_{10}^{8})\sin 2\Theta\cos\phi - r_{1-1}^{8}\sin^2\Theta\cos 2\phi\Big)\Big]$
(1)

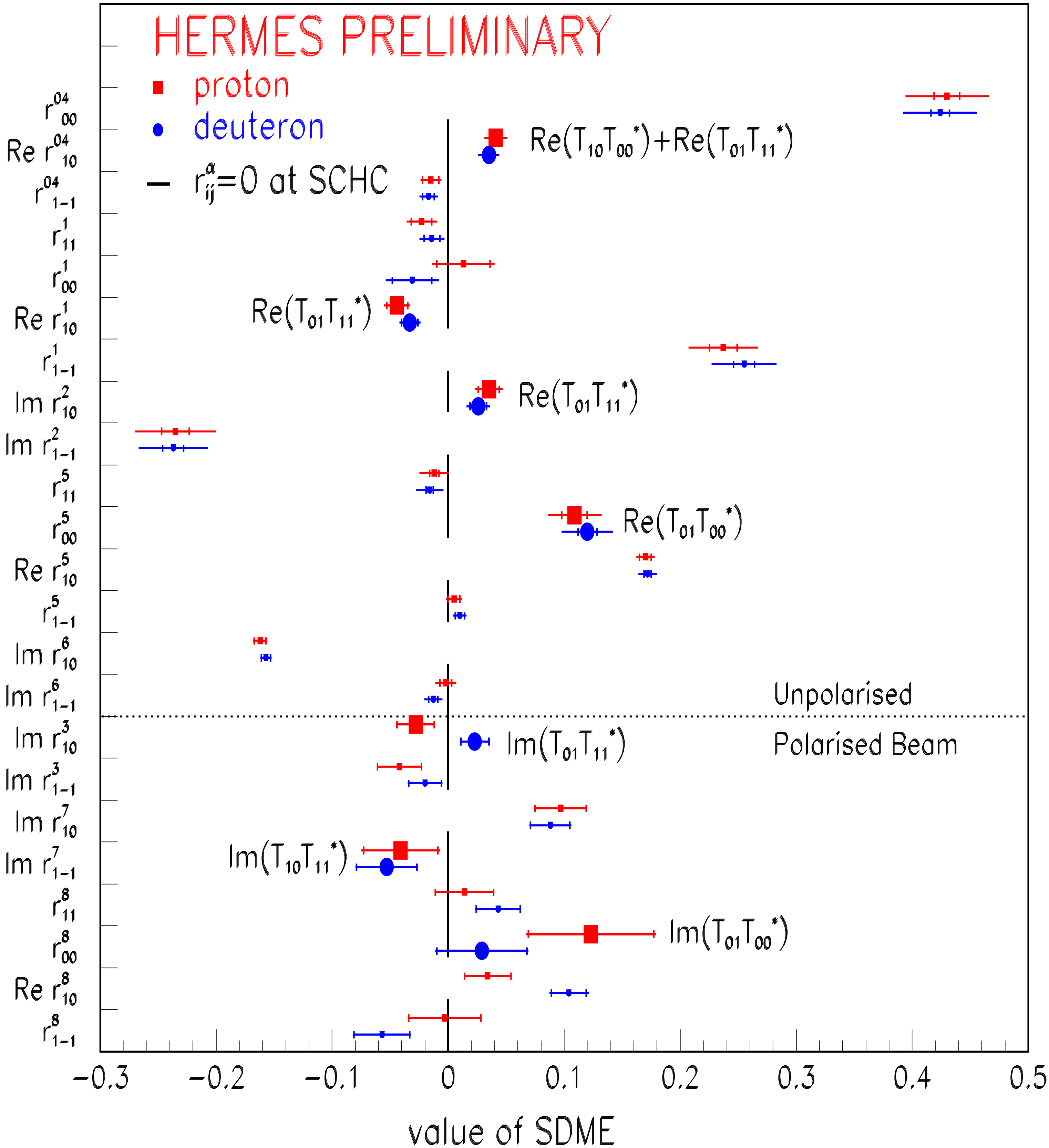
Data, Initial and Fitted Angular Distributions

HERMES PRELIMINARY

Diffractive ρ^0 Electroproduction (^{1}H)



Result: 23 Spin Density Matrix Elements



SDMEs in Terms of the Helicity Amplitudes

$$r^{04}_{00} = \frac{1}{1+\epsilon R}\left[\frac{1}{2N_T}\left(|T_{01}|^2 + |T_{0-1}|^2\right) + \frac{\epsilon R}{N_L}|T_{00}|^2\right] \quad (18)$$

$$\mathrm{Re}\, r^{04}_{10} = \frac{1}{1+\epsilon R}\mathrm{Re}\left[\frac{1}{2N_T}\left(T_{11}T^*_{01} + T_{1-1}T^*_{0-1}\right) + \frac{\epsilon R}{N_L}T_{10}T^*_{00}\right] \quad (19)$$

$$r^{04}_{1-1} = \frac{1}{1+\epsilon R}\mathrm{Re}\left[\frac{1}{2N_T}\left(T_{11}T^*_{-11} + T_{1-1}T^*_{-1-1}\right) + \frac{\epsilon R}{N_L}T_{10}T^*_{-10}\right] \quad (20)$$

$$r^{1}_{00} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\left(T_{0-1}T^*_{01} + T_{01}T^*_{0-1}\right) \quad (21)$$

$$r^{1}_{11} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\left(T_{1-1}T^*_{11} + T_{11}T^*_{1-1}\right) \quad (22)$$

$$\mathrm{Re}\, r^{1}_{10} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Re}\left(T_{1-1}T^*_{01} + T_{11}T^*_{0-1}\right) \quad (23)$$

$$r^{1}_{1-1} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\left(T_{1-1}T^*_{-11} + T_{11}T^*_{-1-1}\right) \quad (24)$$

$$\mathrm{Im}\, r^{2}_{10} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Re}\left(T_{1-1}T^*_{01} - T_{11}T^*_{0-1}\right) \quad (25)$$

$$\mathrm{Im}\, r^{2}_{1-1} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Re}\left(T_{1-1}T^*_{-11} - T_{11}T^*_{-1-1}\right) \quad (26)$$

$$\mathrm{Im}\, r^{3}_{10} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Im}\left(T_{11}T^*_{01} - T_{1-1}T^*_{0-1}\right) \quad (27)$$

$$\mathrm{Im}\, r^{3}_{1-1} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Im}\left(T_{11}T^*_{-11} - T_{1-1}T^*_{-1-1}\right) \quad (28)$$

$$r^{5}_{00} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\left(T_{00}T^*_{01} + T_{01}T^*_{00} - T_{00}T^*_{0-1} - T_{0-1}T^*_{00}\right) \quad (29)$$

$$r^{5}_{11} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\left(T_{10}T^*_{11} + T_{11}T^*_{10} - T_{10}T^*_{1-1} - T_{1-1}T^*_{10}\right) \quad (30)$$

$$\mathrm{Re}\, r^{5}_{10} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\mathrm{Re}\left(T_{10}T^*_{01} + T_{11}T^*_{00} - T_{10}T^*_{0-1} - T_{1-1}T^*_{00}\right) \quad (31)$$

$$r^{5}_{1-1} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\left(T_{10}T^*_{-11} + T_{11}T^*_{-10} - T_{10}T^*_{-1-1} - T_{1-1}T^*_{-10}\right) \quad (32)$$

$$\mathrm{Im}\, r^{6}_{10} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\mathrm{Re}\left(T_{10}T^*_{01} - T_{11}T^*_{00} + T_{10}T^*_{0-1} - T_{1-1}T^*_{00}\right) \quad (33)$$

$$\mathrm{Im}\, r^{6}_{1-1} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\mathrm{Re}\left(T_{10}T^*_{-11} - T_{11}T^*_{-10} + T_{10}T^*_{-1-1} - T_{1-1}T^*_{-10}\right) \quad (34)$$

$$\mathrm{Im}\, r^{7}_{10} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\mathrm{Im}\left(T_{10}T^*_{01} + T_{11}T^*_{00} + T_{10}T^*_{0-1} + T_{1-1}T^*_{00}\right) \quad (35)$$

$$\mathrm{Im}\, r^{7}_{1-1} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\mathrm{Im}\left(T_{10}T^*_{-11} + T_{11}T^*_{-10} + T_{10}T^*_{-1-1} + T_{1-1}T^*_{-10}\right) \quad (36)$$

$$r^{8}_{00} = \frac{\sqrt{R}}{1+\epsilon R}\frac{i}{\sqrt{2N_T N_L}}\frac{1}{2}\left(T_{00}T^*_{01} - T_{01}T^*_{00} - T_{00}T^*_{0-1} + T_{0-1}T^*_{00}\right) \quad (37)$$

$$r^{8}_{11} = \frac{\sqrt{R}}{1+\epsilon R}\frac{i}{\sqrt{2N_T N_L}}\frac{1}{2}\left(T_{10}T^*_{11} - T_{11}T^*_{10} - T_{10}T^*_{1-1} + T_{1-1}T^*_{10}\right) \quad (38)$$

$$\mathrm{Re}\, r^{8}_{10} = -\frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\mathrm{Im}\left(T_{10}T^*_{01} - T_{11}T^*_{00} - T_{10}T^*_{0-1} + T_{1-1}T^*_{00}\right) \quad (39)$$

$$r^{8}_{1-1} = \frac{\sqrt{R}}{1+\epsilon R}\frac{i}{\sqrt{2N_T N_L}}\frac{1}{2}\left(T_{10}T^*_{-11} - T_{11}T^*_{-10} - T_{10}T^*_{-1-1} + T_{1-1}T^*_{-10}\right) \quad (40)$$

The ratio R of the longitudinal to transverse $\gamma^* p$ cross section and the two normalization factors N_L and N_T are given by

$$R = \frac{N_L}{N_T}, \quad (41)$$

$$N_L = |T_{00}|^2 + |T_{10}|^2 + |T_{-10}|^2, \quad (42)$$

$$N_T = \frac{1}{2}\left(|T_{11}|^2 + |T_{-1-1}|^2 + |T_{01}|^2 + |T_{0-1}|^2 + |T_{1-1}|^2 + |T_{-11}|^2\right). \quad (43)$$

Function 2: Fit of Amplitudes $T_{\lambda_{\rho^0}\ \lambda_{\gamma^*}}$

Input: 23 (15) SDME

Output: Helicity Transfer Amplitudes of $\gamma^* \to \rho^0$

$$
\begin{aligned}
&r^{04}_{00} = \frac{1}{1+\epsilon R}\left[\frac{1}{2N_T}\left(|T_{01}|^2 + |T_{0-1}|^2\right) + \frac{\epsilon R}{N_L}|T_{00}|^2\right] \\
&\mathrm{Re}\, r^{04}_{10} = \frac{1}{1+\epsilon R}\mathrm{Re}\left[\frac{1}{2N_T}\left(T_{11}T^*_{01} + T_{1-1}T^*_{0-1}\right) + \frac{\epsilon R}{N_L}T_{10}T^*_{00}\right] \\
&r^{04}_{1-1} = \frac{1}{1+\epsilon R}\mathrm{Re}\left[\frac{1}{2N_T}\left(T_{11}T^*_{-11} + T_{1-1}T^*_{-1-1}\right) + \frac{\epsilon R}{N_L}T_{10}T^*_{-10}\right] \\
&r^{1}_{00} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\left(T_{0-1}T^*_{01} + T_{01}T^*_{0-1}\right) \\
&r^{1}_{11} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\left(T_{1-1}T^*_{11} + T_{11}T^*_{1-1}\right) \\
&\mathrm{Re}\, r^{1}_{10} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Re}\left(T_{1-1}T^*_{01} + T_{11}T^*_{0-1}\right) \\
&r^{1}_{1-1} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\left(T_{1-1}T^*_{-11} + T_{11}T^*_{-1-1}\right) \\
&\mathrm{Im}\, r^{2}_{10} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Re}\left(T_{1-1}T^*_{01} - T_{11}T^*_{0-1}\right) \\
&\mathrm{Im}\, r^{2}_{1-1} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Re}\left(T_{1-1}T^*_{-11} - T_{11}T^*_{-1-1}\right) \\
&\mathrm{Im}\, r^{3}_{10} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Im}\left(T_{11}T^*_{01} - T_{1-1}T^*_{0-1}\right) \\
&\mathrm{Im}\, r^{3}_{1-1} = \frac{1}{1+\epsilon R}\frac{1}{2N_T}\mathrm{Im}\left(T_{11}T^*_{-11} - T_{1-1}T^*_{-1-1}\right) \\
&r^{5}_{00} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\left(T_{00}T^*_{01} + T_{01}T^*_{00} - T_{00}T^*_{0-1} - T_{0-1}T^*_{00}\right) \\
&r^{5}_{11} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\left(T_{10}T^*_{11} + T_{11}T^*_{10} - T_{10}T^*_{1-1} - T_{1-1}T^*_{10}\right) \\
&\mathrm{Re}\, r^{5}_{10} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\mathrm{Re}\left(T_{10}T^*_{01} + T_{11}T^*_{00} - T_{10}T^*_{0-1} - T_{1-1}T^*_{00}\right) \\
&r^{5}_{1-1} = \frac{\sqrt{R}}{1+\epsilon R}\frac{1}{\sqrt{2N_T N_L}}\frac{1}{2}\left(T_{10}T^*_{-11} + T_{11}T^*_{-10} - T_{10}T^*_{-1-1} - T_{1-1}T^*_{-10}\right)
\end{aligned}
$$

(2)

where

$R = N_L/N_T$

$N_L = |T_{00}|^2 + |T_{10}|^2 + |T_{-10}|^2$

$N_T = 0.5\left(|T_{11}|^2 + |T_{-1-1}|^2 + |T_{01}|^2 + |T_{0-1}|^2 + |T_{1-1}|^2 + |T_{-11}|^2\right)$ (3)

Result: Spin Transfer Amplitudes