Computation of Nearly Exact 3D Electrostatic Field in Gas Ionization Detectors

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Abstract No.: 202 & 203

Introduction

Gas Ionization Detectors

Ionization chamber, Proportional counter, Geiger-Muller counter, MWPC, Drift chamber, TPC, GEM, RPC, MSGC

Devices capable of localizing particle trajectories

Nuclear experiments, Medical imaging, Space, Astrophysics

Energetic particle passing through gas causes ionization. Gain and collection effected through the application of high electric field.

- **Electrostatic forces play a major role in determining gas detector performance.**
- **Thorough understanding of electrostatic properties of gas detectors is critically important in design and operation phases.**

Computation of Electrostatic Field

BEM

- ✓ Reduced dimension
- ✓ Accurate for both potential and its gradient

- x Complex numerics
- x Numerical boundary layer
- x Numerical and physical singularities

Analytic

- ✓ Exact
- ✓ Simple interpretation

- x Restricted
- x 2D geometry
- x Small set of geo-metries

FEM / FDM

- ✓ Nearly arbitrary geometry
- ✓ Flexible

- x Interpolation for non-nodal points
- x Numerical differentia-tion for field gradient
- x Difficulty in unbounded domains

Analytic expression for simple geometries and their combination

Computation of charge densities

x Realistic geometries
x Multiple dielectrics

Summation of the contributions at any position using computed charges

MAXWELL
(Ansoft FEM)

TOSCA
(Vector Fields)

- **Potential is calculated as a low order polynomial. Electric field being even low order tend to be very inaccurate in the regions where the field changes rapidly (as log r or $1/r^2$).**
- **Potential is continuous, but field may not be.**
- **Time consuming**

BEM Basics

- **Numerical implementation of boundary integral equation based on Green's function by discretization of boundary**
- **Boundary elements endowed with distribution of sources, doublets, dipoles, vortices**

Electrostatics BIE

$$\Phi(\vec{r}) = \int_S G(\vec{r},\vec{r}')\rho(\vec{r}')dS'$$

Potential at r

Charge density at r'

Green's function

$$G(\vec{r},\vec{r}') = \frac{1}{4\pi\varepsilon\left|\vec{r}-\vec{r}'\right|}$$

ε - permittivity of medium

discretization

$$[A]\{\rho\} = \{\Phi\}$$

Influence Coefficient Matrix

accuracy

Major Approximations

- While computing the influences of the singularities, the singularities modeled by a sum of known basis functions with constant unknown coefficients.
- The strengths of the singularities solved depending upon the boundary conditions, modeled by shape functions.

Constant element approach

Singularities assumed to be concentrated at centroids of the elements, except for special cases such as self influence.

Sufficient to satisfy the boundary conditions at centroids of the elements.

Numerical boundary layer

Difficulties in modeling physical singularities

geometric singularity

boundary condition singularity

Present Approach

Analytic expressions of potential and force field due to a uniform distribution of source on a flat rectangular surface

Restatement of the approximations

- Singularities distributed uniformly on the surface of boundary elements
- Strength of the singularity changes from element to element.
- Strengths of the singularities solved depending upon the boundary conditions, modeled by the shape functions

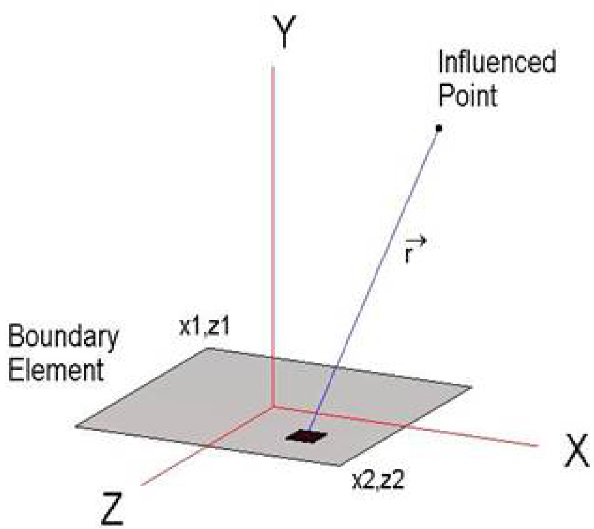
Nearly Exact BEM Solver (NEBEM)

Foundation expressions are analytic and valid for the complete physical domain

NEBEM

Basic formulation

Influence of a flat boundary element



$$\Phi(X,Y,Z) = \frac{1}{2}\times\left\{ \begin{array}{l} 2\times(X\,|\,Z\,|\,x_i\,|\,z_j)\times\ln\left(\dfrac{D_{i,j}-(X\,|\,Z-x_i\,|\,z_j)}{D_{m,n}-(X\,|\,Z-x_m\,|\,z_n)}\right) \\ +iS_j|Y|\times\left[\tanh^{-1}\left(\dfrac{R_j-iI_i}{D_{i,j}|Z-z_j|}\right)-\tanh^{-1}\left(\dfrac{R_j+iI_i}{D_{i,j}|Z-z_j|}\right)\right] \end{array} \right\} - 2\pi Y$$

4 log terms

4+4 complex $\tanh^{-1}$ terms

$$D_{i,j} = \sqrt{(X-x_i)^2 + Y^2 + (Z-z_j)^2}$$

$$R_i = Y^2 + (Z-z_i)^2$$

$$I_i = (X-x_i)|Y|$$

$$S_i = Sign(Z-z_i)$$

$$\Phi(X,Y,Z) = \int_{x1}^{x2}\int_{z1}^{z2} \frac{dxdz}{\sqrt{(X-x)^2+(Y-y)^2+(Z-z)^2}}$$

Value of multiple dependent on strength of source and other physical consideration

$$F_X(X,Y,Z) = \ln\left(\frac{D_{i,j} - (Z - z_j)}{D_{m,n} - (Z - z_n)}\right)$$ → 2 terms

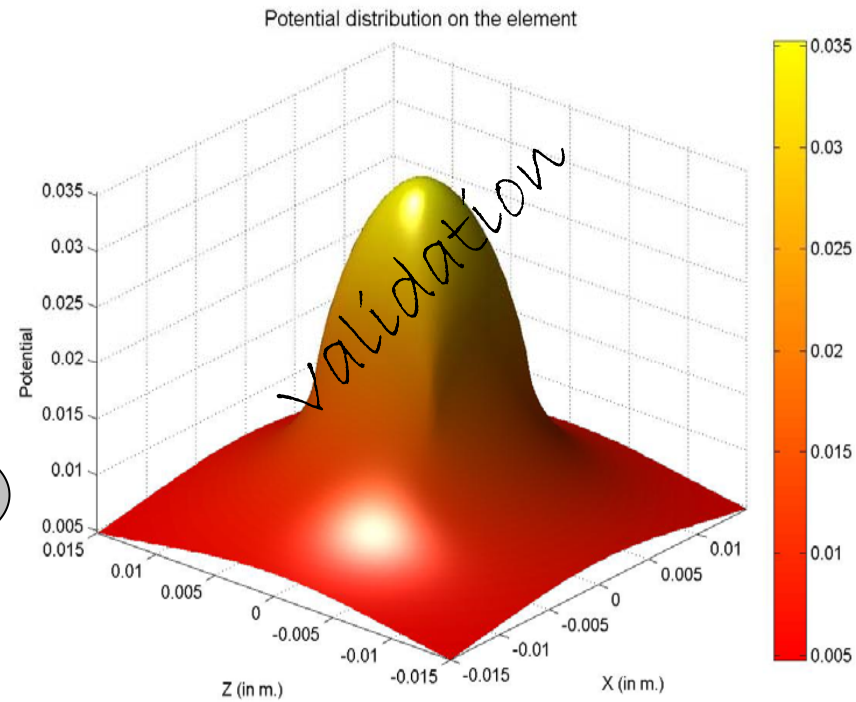
$$F_Y(X,Y,Z) =$$

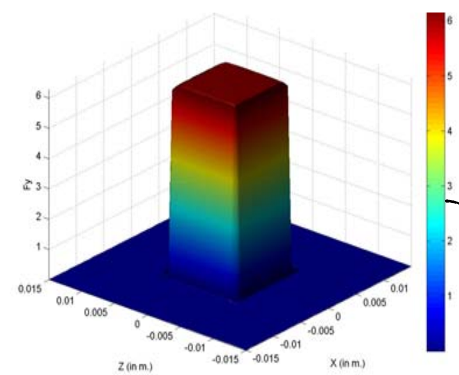
$$-\frac{i}{2}\,Sign(Y) \times \left\{ \begin{aligned} & S_j \tanh^{-1}\left(\frac{R_j + iI_i}{D_{i,j}\left|Z - z_j\right|}\right) \\ & + S_j \tanh^{-1}\left(\frac{R_j - iI_i}{D_{i,j}\left|Z - z_j\right|}\right) \end{aligned} \right\} + C$$ → 4+4 terms

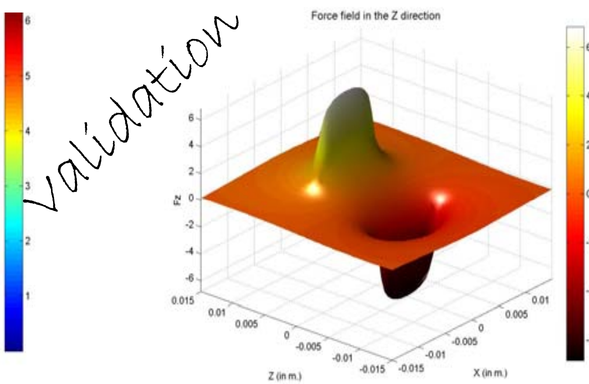
$$F_Z(X,Y,Z) = \ln\left(\frac{D_{i,j} - (X - x_i)}{D_{m,n} - (X - x_m)}\right)$$ → 2 terms

C is a constant of integration as follows:

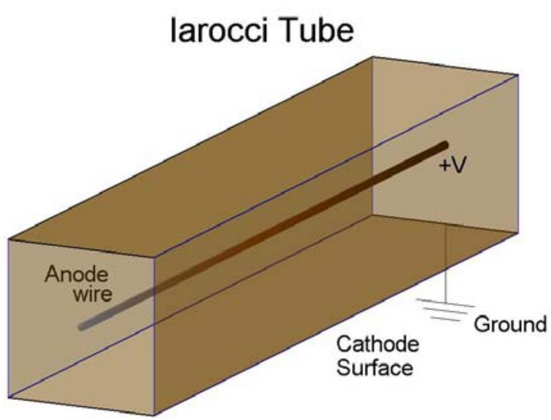
$$C = \begin{cases} 0, & \text{if outside the XZ extent of the element} \\ 2\pi, & \text{if within, and } Y > 0 \\ -2\pi, & \text{if within and } Y < 0 \end{cases}$$

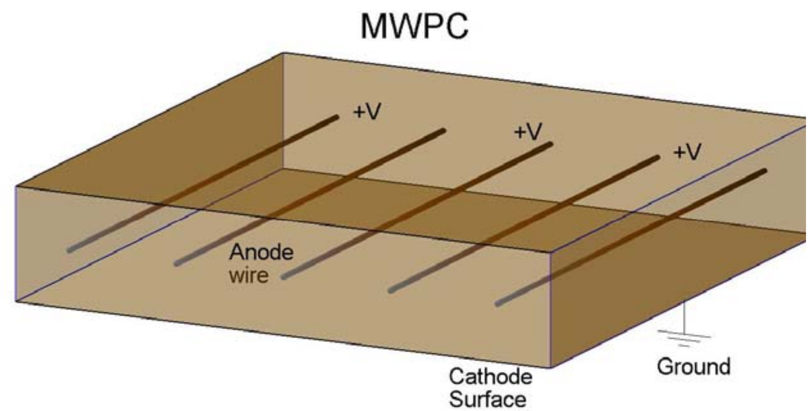






Numerical Implementation





Discretization

Surfaces: 21 x 21

Polygon: 12 x 21

Wire: 1 x 21

→

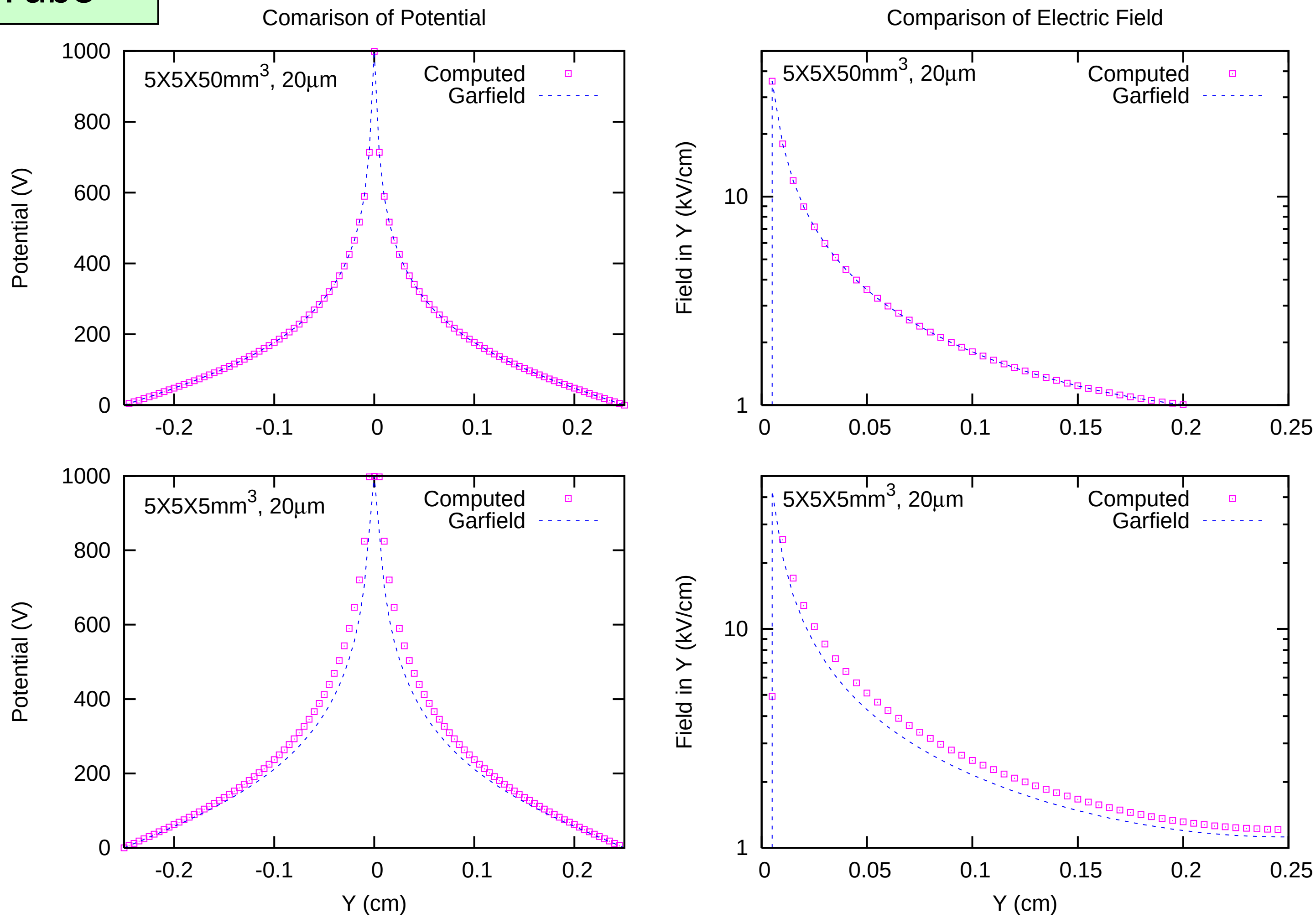
Computation time: 15 to 45 mins user

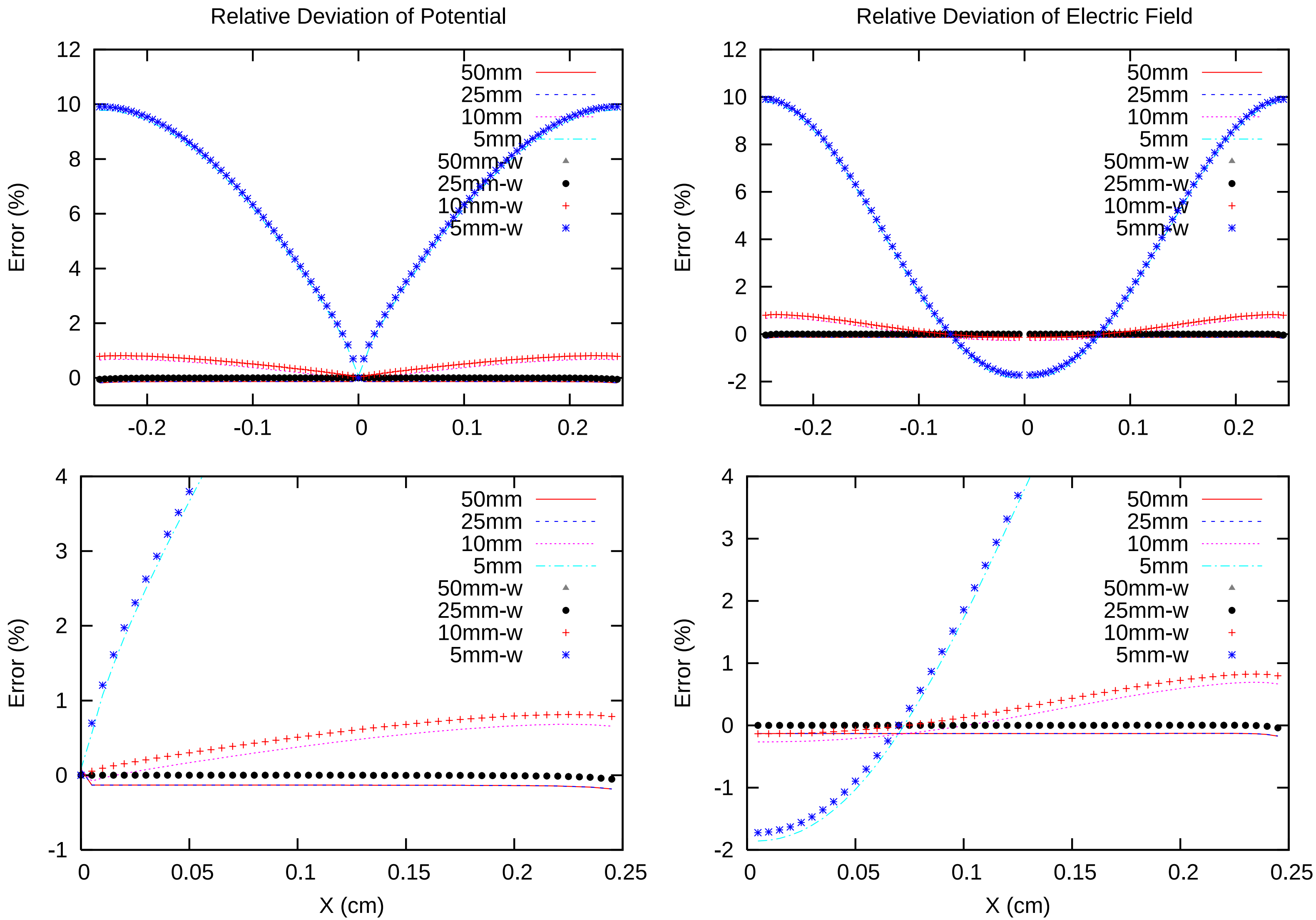
Memory: ~100 MB

OS: Linux Fedora Core 3 running on Pentium IV, 2GB RAM, gcc version 3.4.2

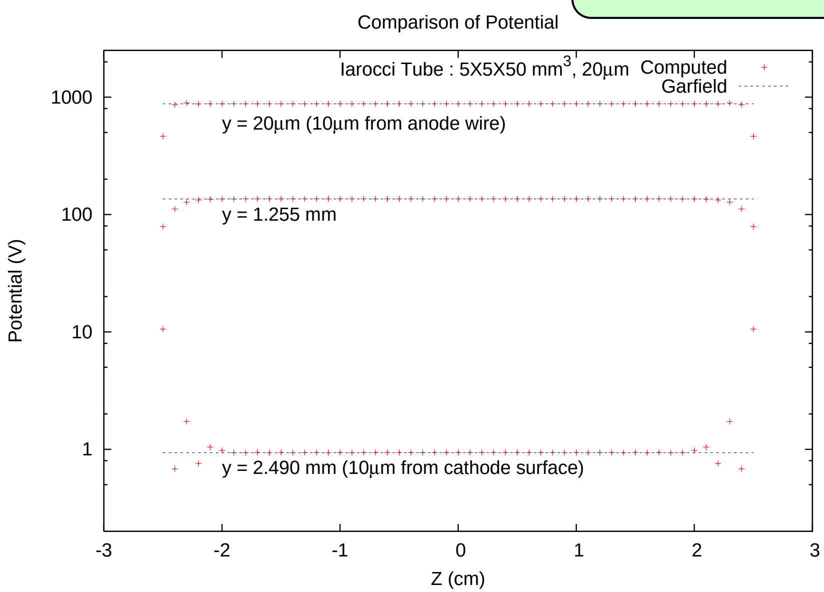
Results

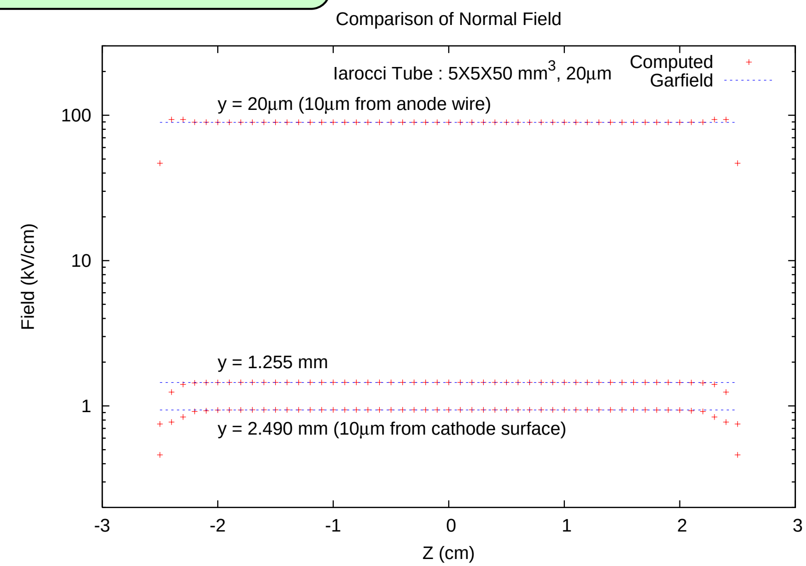
Iarocci Tube



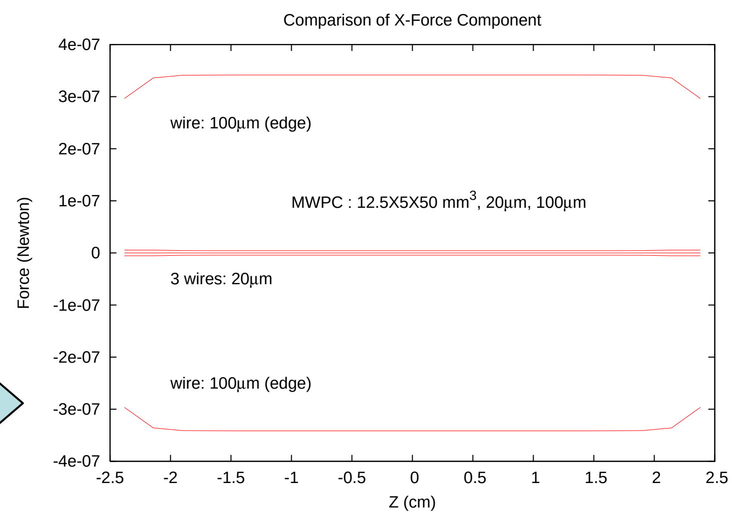

Relative Deviation of Potential
Relative Deviation of Electric Field
Error (%)
X (cm)
50mm
25mm
10mm
5mm
50mm-w
25mm-w
10mm-w
5mm-w

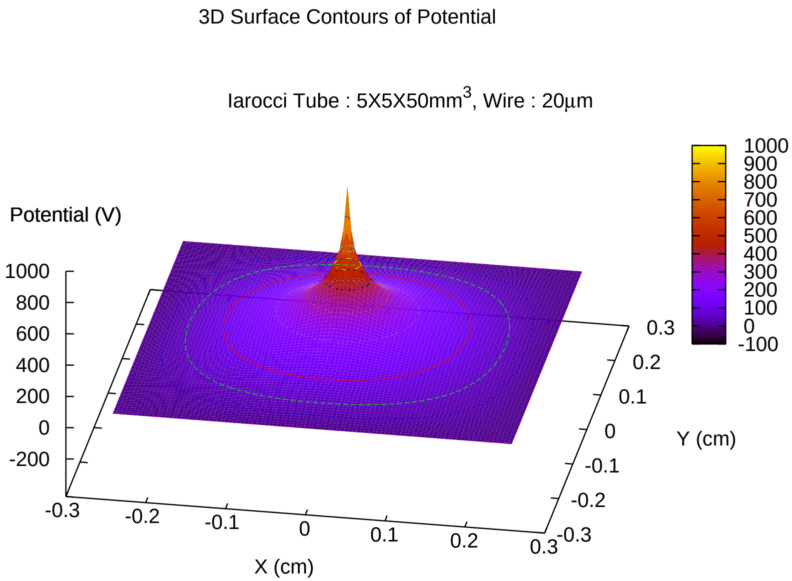
Accuracy where it matters most!

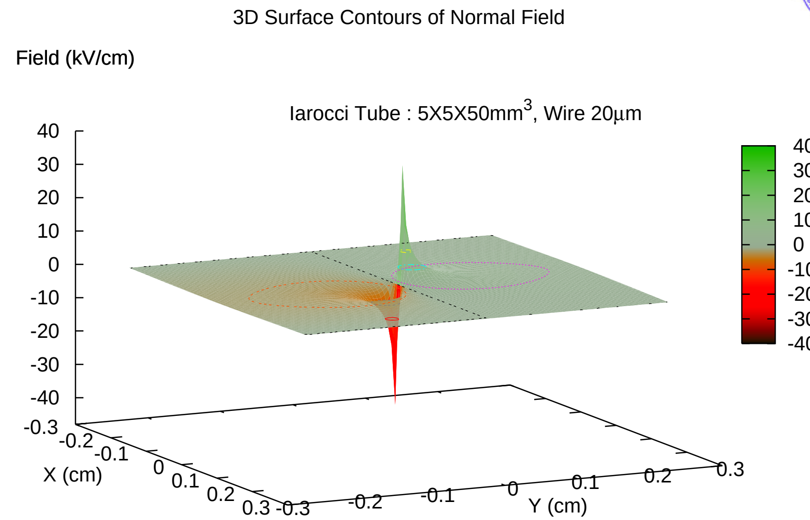


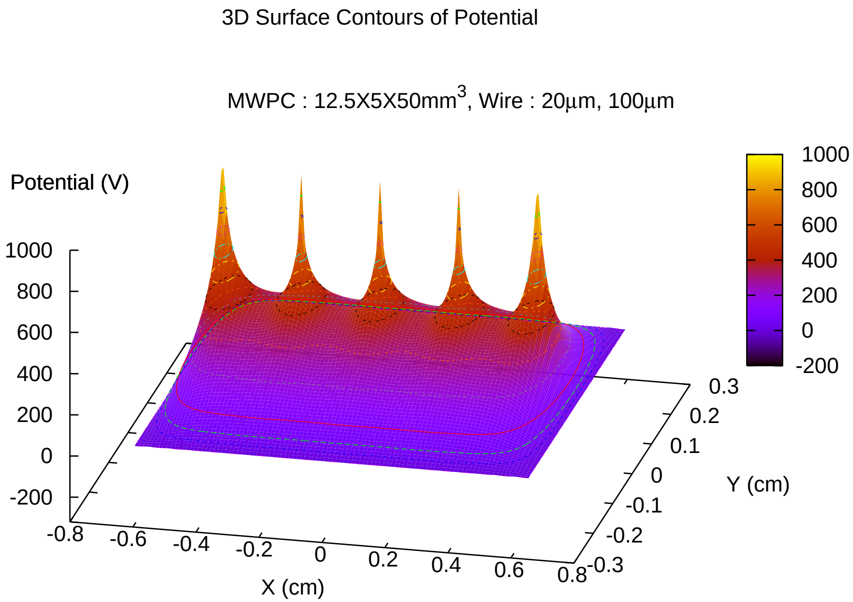


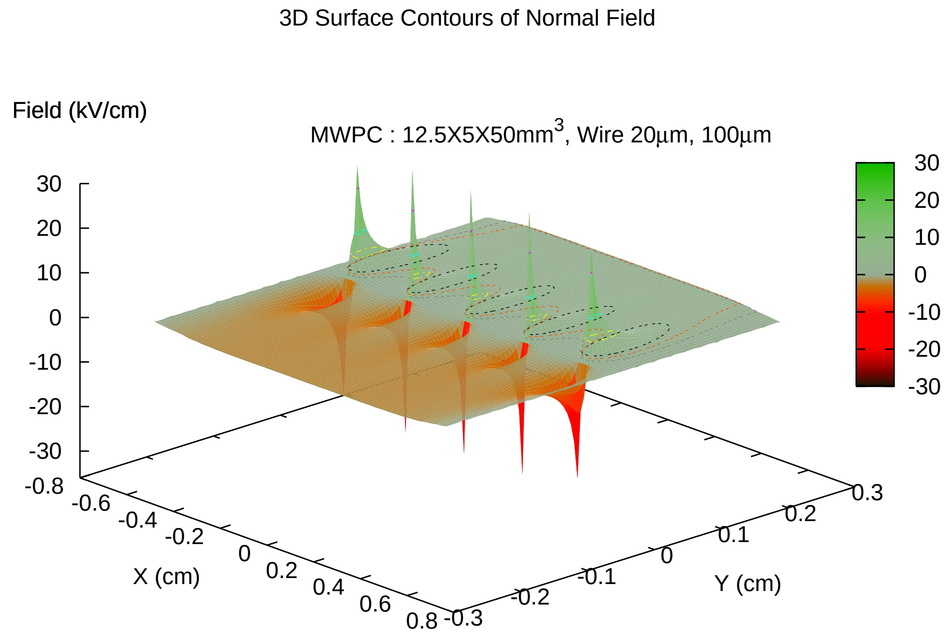
Electrostatic force on anode wires in an MWPC in X direction



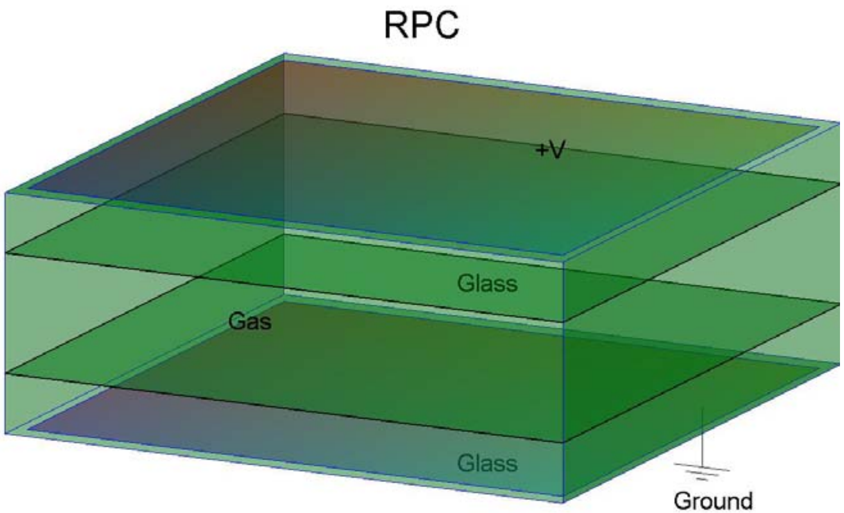

3D Surface Contours of Potential
Iarocci Tube : 5X5X50mm3, Wire : 20µm
Potential (V)
1000
800
600
400
200
0
-200
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
X (cm)
Y (cm)
1000
900
800
700
600
500
400
300
200
100
0
-100

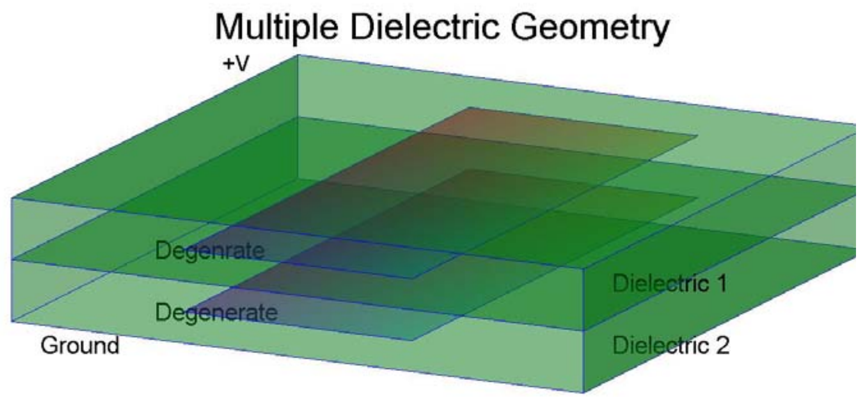

3D Surface Contours of Normal Field
Field (kV/cm)
Iarocci Tube : 5X5X50mm3, Wire 20µm
40
30
20
10
0
-10
-20
-30
-40
X (cm)
Y (cm)
-0.3
-0.2
-0.1
0
0.1
0.2
0.3


3D Surface Contours of Potential
MWPC : 12.5X5X50mm3, Wire : 20µm, 100µm
Potential (V)
1000
800
600
400
200
0
-200
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
X (cm)
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
Y (cm)


3D Surface Contours of Normal Field
Field (kV/cm)
MWPC : 12.5X5X50mm3, Wire 20µm, 100µm
30
20
10
0
-10
-20
-30
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
X (cm)
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
Y (cm)

Extension to Multi-Dielectric Systems





Discretization

Surfaces: 11 x 11

→

- Computation time: 5 to 10 mins user
- Memory: ~10 MB
- OS: Linux Fedora Core 3 running on Pentium IV, 2GB RAM, gcc version 3.4.2

Multiple Dielectric Systems

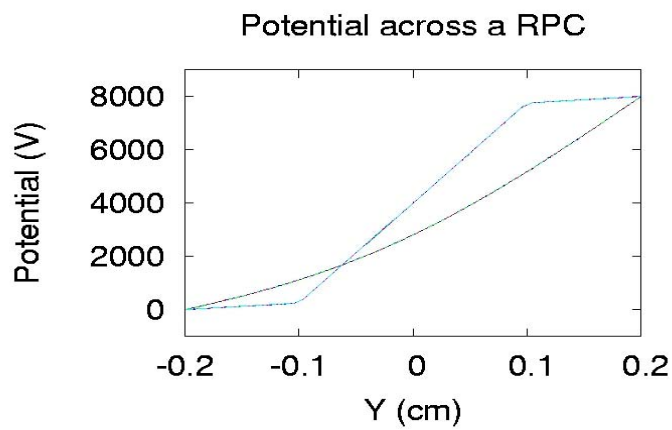
Two dimensional problem solved using the Dual BEM (DBEM) and compared with FEM in Chyuan et. al. [(Semicond. Sci. Technol. 19 (2004)

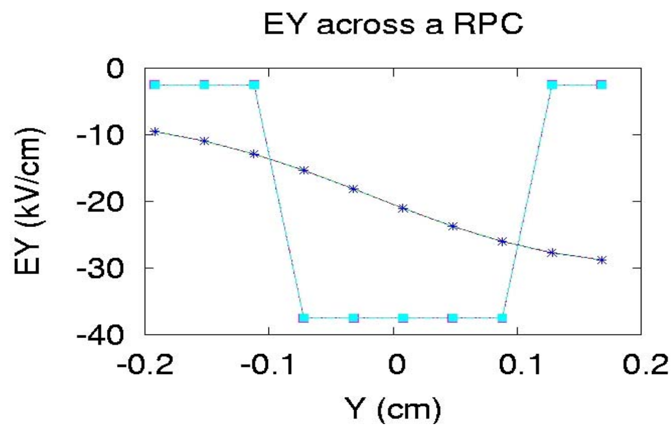
	Ratio of dielectrics = 10			Ratio of dielectrics = 0.1		
Location	FEM	DBEM	Present	FEM	DBEM	Present
18.0,3.0	0.1723103	0.17302	0.1740844	0.01741943	0.017302	0.01771752
4.0, 9.0	0.2809692	0.27448	0.2807477	0.0281006	0.027448	0.0286358
25.0, 16.0	0.6000305	0.59607	0.5991884	0.48833313	0.480640	0.4828946
5.0,17.0	0.679071	0.67492	0.6785017	0.5929200	0.589690	0.5926387

Multiple Dielectric Systems with degenerate surfaces

	Ratio of dielectrics = 10		
Location	FEM	DBEM	Present
24.0,16.5	0.514489	0.52181	0.5247903
6.5,12.0	0.2301575	0.23801	0.2398346
22.5,6.0	0.3638855	0.34638	0.3451232
4.0,3.5	0.1108643	0.10623	0.1058357

Computations of potential and field in an RPC





Conclusion

- **Precise computation of three dimensional surface charge density, potential and electrostatic field carried out for gas ionization detectors using Nearly Exact BEM (NEBEM) solver.**
- **The NEBEM yields precise results for a very wide range of realistic electrostatic configurations including multiple dielectric systems.**
- **The NEBEM, containing foundation expressions independent of special considerations, should be equally applicable to other areas of science and technology.**

- **Ongoing**
- ***Optimization being carried out.***
- ***Multi-physics nature of NEBEM being implemented.***
- ***Application to other areas being explored.***