

# TRACK FINDING TECHNIQUE IN THE MDT DETECTOR

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## Abstract

This paper addresses the problem of muon track finding in a monitored drift tube chamber, which is a part of the ATLAS detector. In the presence of high background noise, the commonly used track finding algorithms suffer from a relatively high fake track rate. We treat the track finding problem in a Bayesian framework, adding previously unused information. A new algorithm, based on the Hough transform, has been developed to make use of the additional information. We show that it improves the muon track detection performance significantly.

## INTRODUCTION

The LHC, the largest hadron collider ever built, presents new challenges for physicists and engineers. With the anticipated luminosity of the LHC, we expect as many as one billion total collisions per second, of which at most 10 to 100 per second might be of potential scientific interest. The track reconstruction algorithms applied at the LHC will therefore have to reliably reconstruct tracks of interest in the presence of background hits.

One of the two major general-purpose experiments at LHC is called ATLAS. Since muons are among the most important particles to be detected as a sign of new phenomena, a stand-alone muon spectrometer is being built for ATLAS [1]. This system is located in a large background environment which makes the muon tracking a very challenging task. Since the monitored drift tubes chambers (MDTs) will cover most of the spectrometer area, a muon track finding algorithm with high efficiency and low fake rate is crucial.

In the next sections we briefly describe the MDT detector and the track finding problem. We describe the potential improvement of using additional prior information and the new Bayesian approach for the tracking problem. Then, we describe the results of an algorithm, based on an extension of the Hough transform, and compare its performance to the common used [2] algorithm.

## THE MDT DETECTOR

The basic MDT detection element is a cylindrical aluminium drift tube of 30 mm diameter and length in the range 0.9 to 6.2 m [1]. On each chamber the tubes are arranged in two multi-layers, each formed by three or four layers of tubes. When an energetic particle passes through

the tube, it ionizes a local region of the gas that fills the tube. The ionized cluster of electrons drifts towards the anode wire and a charge avalanche develops. The output signal is the digitized drift time which is followed by a relatively long dead-time period.

Figure 1 describes the drift path of the ionized cluster to the wire and the MDT chamber geometry.

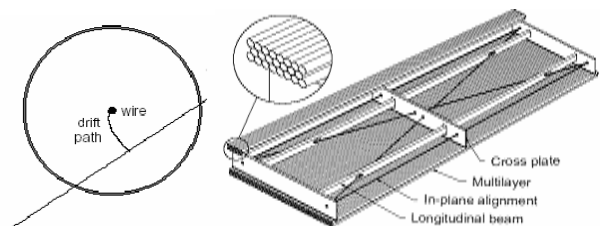


Figure 1: An electron avalanche through the anode wire in a single drift tube (left). Schematic drawing of a MDT chamber (right).

The particle hit position can be measured using a so called r-t relation. The time it takes the ionized cluster of electrons to reach the anode wire and generate an electric signal is proportional to the distance between the particle hit and the wire. Using the r-t relation it is possible to calculate the distance of the hit position to the wire. Given a set of particle hit radiuses (drift circles), one should find the muon track and estimate its parameters. Since the curvature of the muon track in a single chamber is negligible, we can state the local tracking problem as finding all the possible straight lines given a set of hit measurements.

Figure 2 describes an example of this tracking problem:

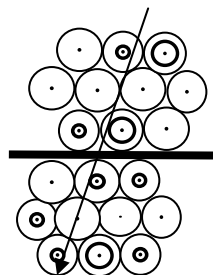


Figure 2: Example of a track, given a set of drift circles.

## THE ROLE OF PRIOR INFORMATION IN TRACK IDENTIFICATION

For a given set of drift circles, the common track finding approach tries to find all tracks tangent to a subset of the drift circles. As lower bound on the probability of track detection (track efficiency) for any algorithm that tries to find at least  $n$  collinear drift circles out of  $M$  layers is given by:

$$P_D(p_\mu) = \sum_{i=n}^M \binom{M}{i} \cdot p_\mu^i \cdot (1-p_\mu)^{M-i} \quad (1)$$

where  $p_\mu$  is the probability of getting a muon hit (i.e. muon drift circle) given that the tube was crossed by a muon. Due to the high efficiency of the detectors,  $p_\mu \approx 1$  unless there is an interfering physical effect. The probability  $p_\mu$  is assumed to be the same for each tube and does not depend on the other tubes. In the presence of background,  $p_\mu$  becomes smaller (due to dead time from other particles traversing the detector, for example) and the probability of track detection decreases. For example, for  $p_\mu = 0.95$  any algorithm trying to identify a track based on at least 4 collinear hits out of 6 layers will have probability of track detection limited to 0.998, whereas for  $p_\mu = 0.9$  it will be limited to 0.984.

In order to find out the range of  $p_\mu$  for the LHC environment, we used the data taken from a test beam, experiment designed to simulate the real environment of the LHC. In the test beam a muon source is used to simulate the tracks of interest, where a photon source is used to simulate the background particle noise. The hits from the detector under test (the MDT) are compared to the real, known track parameters, given by an external precision detector called “telescope”. This test beam data with a background photon source are used throughout this paper.

Table 1 describes the empirical probability of  $n$  valid muon hits per track using the test beam data. A valid muon hit is defined as a drift circle whose closest distance to the track is less than  $5\sigma_r$ , where  $\sigma_r$  is the drift radius standard deviation error.

We use each row in table 1 to calculate  $p_\mu$  in (1). One can verify that for each number of muon-hits  $n = 2, 3, \dots, 6$ ,  $p_\mu \approx 0.86$  gives the same probability in (1) as the actual empirical in Table 1. Thus, we conclude that the assumptions we used in (1) are correct.

| Number of valid hits per track | Empirical probability |
|--------------------------------|-----------------------|
| 6                              | 0.41                  |
| 5                              | 0.37                  |
| 4                              | 0.15                  |
| 3                              | 0.05                  |
| 2                              | 0.01                  |

Table 1 – the empirical probability for different number of muon-hits per real muon track using the test beam data in the presence of background photon source

Using (1) with  $p_\mu = 0.86$  and  $n = 4$ , we can see that any algorithm trying to detect at least 4 collinear hits out of 6 layers loses about 6% of the tracks in case of noisy environment. Trying to increase the detection probability by using  $n < 4$  will result in an unacceptably large fake track rate.

The cases where the muon crosses the tubes but no muon-hits are produced can be explained with the help of additional information. For example, a background particle crossing the tube before the muon will cause dead time in the tube, during which a muon hit cannot be detected. Since we almost always have additional information that explains the lack of muon hit, we can define an “effective” muon hit probability  $\bar{p}_\mu$  as the probability to get any valid tube signal given that a muon track crossed the tube, using the prior information  $\theta_i$ :

$$\bar{p}_\mu = \sum_{i=1}^K p(\mu | \theta_i) p(\theta_i) \quad (2)$$

where  $K$  represents the different hit cases (number of priors), a detailed in the next section.

Since the probability of having any valid tube signal is equal to or bigger than the probability of having a muon hit,  $\bar{p}_\mu \geq p_\mu$  and  $P_D(\bar{p}_\mu) \geq P_D(p_\mu)$  correspondingly.

Practically, a requirement of having a valid tube signal in all layers ( $M = 6$ ) results in  $P_D(\bar{p}_\mu) \approx 100\%$ .

## THE BAYESIAN APPROACH FOR TRACK FINDING

The probability density function (PDF) of the drift circle radius for a given track with line parameters  $\phi$ , is defined as  $p(r_i | \phi, H)$ , where  $r_i$  is the drift circle radius derived from the r-t relationship and  $H$  is the hypothesis that muon track traverses the tube. The distance from the tube center to the track is defined as  $r_{track}(\phi)$ .

Figure 3 depicts the different physical scenarios and the corresponding geometrical representation of the PDF regime component.

| Hit case  | Time diagram | Spatial pdf regime |
|---|--------------|--------------------|
| Muon hit in a valid time window<br>$p_\mu = p(\theta_1   H)$                  |              | a                  |
| Prior background hit in a valid time window<br>$p_{bkg} = p(\theta_2   H)$    |              | b                  |
| Prior background hit in a negative time window<br>$p_{neg} = p(\theta_3   H)$ |              | c                  |
| The muon crosses the tube wall<br>$p_w = p(\theta_4   H)$                     |              | d                  |

Figure 3- the different signal possibilities when a muon traverses the tube. The left column describes the different cases. The middle column describes the corresponding cases in the time domain and the right column describes the spatial pdf regime.

Case (a) describes the possibility of having a valid muon drift radius. In this case, assuming a Gaussian drift radius error, the hit radial probability is given by:

$$p(r_i | \phi, \theta_1, H) = \frac{1}{\sigma_r \sqrt{2\pi}} \exp\{- (r_i - r_{track}(\phi))^2 / 2\sigma_r^2\}.$$

Cases (b) and (c) describe the possibilities of having a background particle hit before the muon hit. In these cases the background particle causes a masked (or dead-time) period of about 700ns corresponding to the maximum drift time. In case (b) the background particle comes within the valid time window, where in case (c) it comes before the valid time window. In these cases, the hit PDF regime is  $p(r_i | \phi, \theta_2, H)$  and  $p(r_i | \phi, \theta_3, H)$ , respectively. Since the muon can come anytime during the masked period, both  $p(r_i | \phi, \theta_2, H)$  and  $p(r_i | \phi, \theta_3, H)$  are assumed to be uniform distributions. The probabilities  $p_{bkg} = p(\theta_2 | H)$  and  $p_{neg} = p(\theta_3 | H)$  are the probability of having a background hit within or before the valid window, given that a muon crossed the tube. Case (d) describes the possibility that the muon crosses the tube wall. In this case, the PDF regime  $p(r_i | \phi, \theta_4, H)$  is

assumed a uniform distribution along the tube wall, where  $p_w = p(\theta_4 | H)$  is the probability that the muon crosses the tube wall.

For any given track candidate with line parameters  $\phi$  the likelihood of a track, for the n-th tube is:

$$p_n(r_{i_n} | \phi, H_1) = \sum_{i=1}^K p_n(r_{i_n} | \phi, \theta_i, H) p(\theta_i | H) \quad (3)$$

The conditional probability  $p(\theta_i | H)$  is the probability of the different hit cases given the muon crossed the tube. This probability depends on the level of background particles, and can be statistically calculated during the calibration process, for example.

Since usually a background particle effects only one tube, we assume that the signals in different tubes are independent. Thus, the PDF of the random vector  $R_i$  of a group of tubes radiuses is:

$$p(R_i | \phi, H_1) = \prod_{n=1}^M p_n(r_{i_n} | \phi, H) \quad (4)$$

Our problem is to find the track parameters  $\phi$  which maximize the likelihood function in (4). In order to have low fake track rate, we follow the Generalized Likelihood Ratio Test (GLRT) technique [3] equivalent to require the likelihood to be above a minimal value  $\lambda$  :

$$\hat{\phi} = \arg\{\max_{\phi} p(R_i | \phi, H)\} \quad ; \quad p(R_i | \phi, H) > \lambda \quad (5)$$

Figure 4 provides a geometric representation of the problem; one should find the most probable tracks above a certain likelihood threshold.

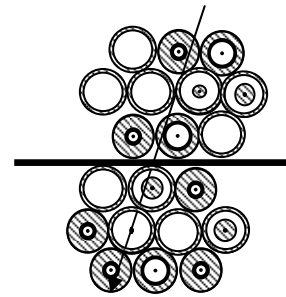


Figure 4- An illustration of the track finding problem. Each tube contributes four spatial pdf regimes. One should find all tracks with likelihood above a predefined threshold.

## RESULTS

Based on the above approach, we developed an algorithm [4] for track finding which makes use of the Hough transform [5,6,7]. While the common Hough transform transforms each hit to a line in the line parameter space, our extended Hough transform uses the different hit cases as a different inputs for the Hough space, and transforms the hits, for each hit case, independently. Thus, instead of having a single layer Hough space we get a multi layers Hough space. We quantize the Hough space into discrete cells; each contains several values, one for each hit case (a-d). For each Hough cell an equivalent to the track likelihood is calculated. Then, it is compared to a predefined threshold function.

The algorithm has been applied on the test beam muon data. Figure 5 depicts the average number of fake tracks per event vs. the track finding efficiency for the commonly-used algorithm, similar to [2], and for the new suggested algorithm. The common algorithm tries to find at least  $n$  collinear hits without using any additional information, whereas the new algorithm uses that information for improving the track detection performance. It can be easily seen that the use of the new approach reduces the number of fake tracks significantly, without reducing the probability of track detection. Figure 6 zooms on the new algorithm detection performance.

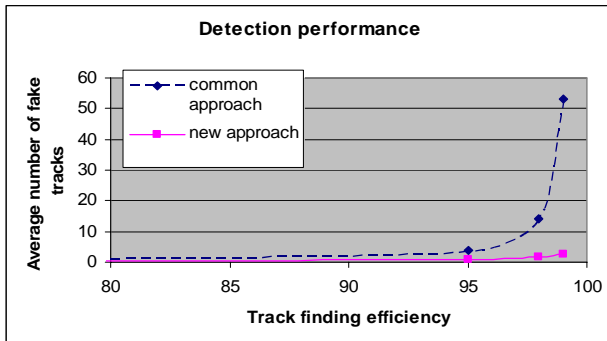


Figure 5: average number of fake tracks vs. track finding efficiency for the common approach (dashed line) and for the proposed Bayesian approach (solid line).

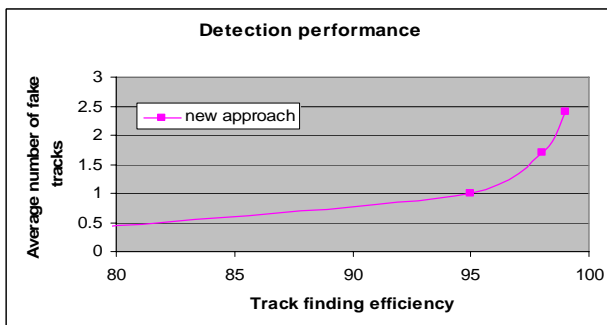


Figure 6: average number of fake tracks vs. track finding efficiency for the proposed Bayesian approach.

## CONCLUSION

The challenging noisy environment of the LHC makes the performance of the commonly-used tracking methods insufficient. A better understanding of the detector in the presence of background particles can be used to improve the tracking performance. Using available side information we define an effective muon hit probability which gives a very high probability of track detection, even if we require a valid signal in each tube. The requirement for having a valid signal in all (or almost all) tubes, leads to a significant reduction in the track fake rate.

A geometrical representation of the drift circle radius PDF for each tube can be used for implementing line finding algorithms (which are used, for example, in image processing). A novel extension of the Hough transform is used to detect the line and classify the hits associated with it. Using realistic data from a test beam experiment with a photon background source, we show that the Bayesian approach, together with the suggested extended Hough transform algorithm, result in significant improvement in the track finding performance.

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