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The Effective Long Range Interaction and Resonances in $n\alpha\alpha$- Systems at Astrophysical Energies

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The features of $n\alpha\alpha$ - system for astrophysical energies ($\ll 1$ MeV) have been investigated on the basis of Faddeev's equations within the framework of the study of resonance fusion possibility in stellar matter [1]. Attention has been given to the determination and analysis of resonance states of the system.

It was found that the series of resonance states appear in the $n\alpha\alpha$ system at very low energies under certain conditions. The lifetimes of these three body resonances proved to be close to the lifetime of unstable ground state of ^8Be.

Simple forms of $\alpha\alpha$- and $n\alpha$ - potentials are taken in order to satisfy scattering data at very low energies. The α-particles are considered as elementary.

It is shown that the effective long range interaction acting as well-known two body potential $\sim r^{-2}$ can appear in this model of the $n\alpha\alpha$ system. It leads to resonance states in the system. Thomas's and Efimov's effects in three body systems can be cited as typical examples of influence of effective long range interaction [2]. Moreover, the resonance phenomena can take place in systems composed of one neutron and three or more α-particles within the low energy region.

The sharp resonance in a system consisting of a neutron and few α-particles is considered as a stimulus to resonance fusion, i.e. this can be a new mode of fusion. Furthermore, the resonance fusion can give results in many astrophysical phenomena.

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It's known that Coulomb forces lead to serious difficulties in a three-body problem, especially at positive energies [3]. We follow by the popular method of screening of Coulomb forces. The set of Faddeev's equations can be represented as a mathematically strict statement of a three-body problem where two-body forces have finite radii of acting [4].

1. Simple forms of two-body interactions.

As known, there are no stable nuclei consisting of 5 or 8 nucleons. At present a determination of effective potentials between a neutron and an α-particle or between two α-particles is an unsolvable few-body problem. However, it is possible to describe these two-body problems at very low energies within the framework of simple potential models.

Interactions between two particles are chosen in the form of separable potentials:

$$\lambda V = |\nu > \lambda < \nu|, \tag{1.1}$$

where a coupling constant λ is a real quantity. Obviously the amplitude of scattering is expressed in the following analytical form:

$$f(k,k';Z) = < k|\hat{f}|k' >, \ \ \hat{f} = -|\nu > \eta(Z) < \nu|, \tag{1.2}$$

where

$$\eta^{-1} = \lambda^{-1} - A(Z), \ \ A = < \nu|G_0(Z)|\nu >, \tag{1.3}$$

$G_0(Z) = (Z - H_0)^{-1}$, in physical limit $Z \to E + i \cdot 0_+$. Hereinafter indexes of states, recoupling coefficients, etc. are omitted for simplicity of notes. Formalism and details of two- and three-body scattering theory can be found, for instance, in [5].

It's suitable to normalize $A(0) = -1$. For S-wave, for example, the values $\lambda \leq -1$ correspond to the area of bound states, $-1 < \lambda \leq 0$ to virtual states and $\lambda > 0$ to quasi stationary ones. In the last case the amplitude has poles in k-complex plane in points of $k = k_{Res} = \pm k_R + ik_I$, where $E_{Res} = k_{Res}^2/2\mu$, $E_R = (k_R^2 - k_I^2)/2\mu$, $\Gamma = -4k_R k_I/2\mu$, μ - the reduced mass and $k_I < 0$.

To describe $n\alpha$-scattering at low energies, $E \ll 10 MeV$, it's enough to take into account S- and P-wave potentials only. In S-wave the $n\alpha$ effective potential is characterized by repulsive force. Attractive forces act already in P-waves and give resonances in partial amplitudes although they have higher energies and typical lifetimes for nuclear interactions [6].

For S-wave the $n\alpha$-potential is chosen in the form of (1.1) with form-factor $\nu(k) = < \nu|k >= Const/(1+x^2)$, where $Const = \sqrt{8\pi/(2\mu\beta)}$, $x = k/\beta$ and β is the inverse radius of S-wave nuclear force. Here the nuclear coupling constant $\lambda = x_R^2 = (k_R/\beta)^2 \gg 1$, i.e. the amplitude is almost unvaried at low energies, where $x \ll 1$. So, the parameters fixed by experimental $n\alpha$ scattering data are equal to: $\beta = 0.751 fm^{-1}$ and $\lambda = 14.94$. In turn they give the resonance energy and width such as: $E_R \simeq \Gamma \simeq 200\, MeV$.

For P-waves ($J = 3/2, 1/2$) the $n\alpha$-potentials may also be chosen in the form of (1.1) but with form-factors $\nu(k) = Const \cdot x/(1+x^2)$. For example, for $J = 3/2$ the parameters $\beta = 1.175 fm^{-1}$ and $\lambda = -0.97$ that correspond to $E_R \simeq 0.9 MeV$ and $\Gamma \simeq 0.6 MeV$ can be easily obtained [7].

A system of two α-particles has the very sharp resonance at very low positive energy. The resonance can be described by the sum of nuclear potential (1) and Coulomb potential - U.

Besides Coulomb scattering of charged nuclear particles, the Coulomb forces lead to modification of their nuclear transition matrixes [3]. In our case the pole of amplitude will correspond to $\eta_C^{-1}(E_{res}) = 0$, where

$$\eta_C^{-1} = \lambda^{-1} - A^C(Z), \quad A^C = < \nu^C|G_0(Z)|\nu^C > = < \nu|G_C(Z)|\nu >, \tag{1.4}$$

and $G_C(Z) = (Z - H_0 - U)^{-1}$. As $\lambda^{-1} = A^C(E_{res})$ and the nuclear coupling constant λ is independent of Coulomb forces, $A^C(E_{res})$ must be constant too. It means that position of resonance point E_{res} has to be changed if the quantity of $A^C(Z)$ is varied with increasing of Coulomb coupling constant.

Coulomb shifts for nuclear parameters can be small or large depending on the energy of resonance and the dimensionless constant ζ. For two α-particles $\zeta = a_B \cdot \beta$, where $a_B \approx 3.6 fm^{-1}$ - Bohr radius of system. Using these parameters and the ratio of $|k_I/k_R| \approx 1.85 \cdot 10^{-5}$, it is possible to determine the nuclear parameters λ and β [7].

Calculations give two sets of parameters: with $\lambda > 0$ and $\lambda < -1$. The case of attractive force is espesially interesting because it gives nuclear parameters close to typical nuclear characteristics: $\beta = 0.6385 fm^{-1}$ and $\lambda = -3.2389$.

2. Resonances of $n\alpha\alpha$ system

Following Faddeev's equations and omitting mathematical details and intermediate expressions we can write down the resulting equation for transition matrix between the selected channels. In our case these channels correspond to the same $n(\alpha\alpha)$-state which contains a free neutron and a pair of interacting α-particles.

As usual, we introduce indices to mark three-body values with a sort of particle that first outgoing area of interactions for asymptotic, and two-body values with a sort of third particle that is absent in the pair. Total potential of our model is equal to the sum of two-body potentials $V = \sum V_i$, where $i = n, \alpha, \alpha'$.

It's convenient to separate the pole part of the two-body $\alpha\alpha'$ t-matrix. Similar methods are widely used in three-body problems to extract the main characteristics of considering processes. The rest, i.e. a nonpole part of $\alpha\alpha'$ t-matrix, is considered as a correction member. Its contribution is calculated with ordinary perturbation theory.

The equation for three-body amplitude $f_{n,n'}$ with accuracy up to a member and a factor which have no influence on the pole features can be written in the following form:

$$f^0_{n,n'} = -V^0_{n,n'} + \sum V^0_{n,n''} \; \eta_C(Z_{n''}) \; f^0_{n'',n'} \ . \tag{2.1}$$

Here the effective potential corresponds to a triangle diagram where t_α (t-matrix of $n\alpha'$-subsystem) is situated at the top of the diagram:

$$V^0_{n,n'} = \sum_{\vec{k}_\alpha} < \nu^C_n|G_0(Z) \; t_\alpha \; G_0(Z)|\nu^C_{n'} > . \tag{2.2}$$

Coulomb forces can be taken into account to accurately obtain the modified $\alpha\alpha$ nuclear form-factor which is written as ν^C_n. All corrections for $f^0_{n,n'}$ can be calculated by ordinary perturbation theory.

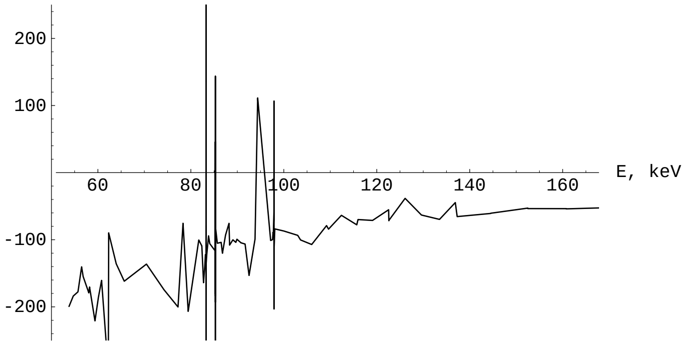

Figure 1: Real part of $f^0_{n,n}$ calculated with permanent step between points $\approx 50\ eV$ or more.

It should be noted that the resonance behavior of η_C leads to irregularities of the kernel of the integral equation (2.1). These irregularities within a certain region can be compensated by oscillatory behavior of $f^0_{n,n'}$, i.e. oscillations of solutions (2.1) near this region. Physically these oscillations represent themselves as resonances of a three-body system. In correction member irregularities do not already exist.

In coordinate space the Shredinger equation corresponding to (2.1) contains the potential $\sim \gamma/r^2$ in the region of irregularities (r - distance between neutron and $\alpha\alpha$-subsystem). As known this potential leads to oscillatory solutions if dimensionless constant $\gamma > 1/4$ [2].

For effective potential (2.2), the integration on intermediate states smooths out singularities of Green functions and gives a regular result. It is happened even in the case of positive energies in a three-body system.

The effective potential (2.2) has been calculated numerically with definite relative accuracy $\sim 10^{-4}$ under the fixed x_n and x'_n. The set of points x_n is determined by a special auxiliary subprogram at every step of energy E. The number of points depends on the determination accuracy. The density of points is larger near of irregularities of kernel of integral equation (2.1).

The Faddeev's equations give solutions for amplitudes which are much more regular than the kernels of these equations. The method of point set determination is the same one which is used to calculate the resonance levels of three-body problem at $E < 0$ [2].

3. Results and conclusions

The $n\alpha\alpha$ resonances are located in the region surrounded by the $\alpha\alpha$-resonance point (Fig. 1). It is important that the widths of the resonances can be proved to be smaller than the width of the $\alpha\alpha$-resonance (Fig. 2). As the energy of the $\alpha\alpha$ subsystem is only part of energy of $n\alpha\alpha$ system, the α-particles of stellar matter can be involved with nuclear synthesis at lower temperatures than in the absence of neutrons. For instance, Fig. 1 and 2 show the curves calculated at energy of the $\alpha\alpha$ subsystem $E_{\alpha\alpha} \approx 50 keV$.

This situation becomes more interesting if the results from atomic physics will be taken into account where the long range interactions in four particle systems lead to numbers of resonances

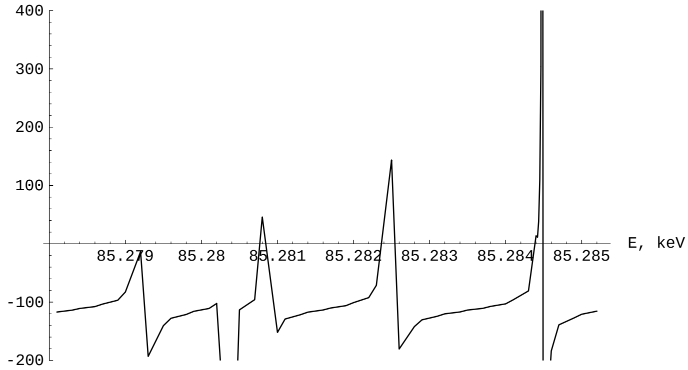

Figure 2: Real part of $f^0_{n,n}$ calculated with permanent step between points ≈ 0.05 eV.

then in three-body systems [8]. Moreover, the system of three α-particles has a sharp resonance at low energy [6]. So, the systems consisted of one neutron and few a-particles can have a very rich spectrum with sharp resonances at low energy region.

As a result, light nuclei can be produced during fusion via few-body resonance states with mass several times heavier than α-particle mass. Neutrons can be involved into nuclear few-body synthesis or can be thrown out with high energy. Then the neutron can form the long lifetime resonance again and stimulate the nuclear synthesis. Such mechanism of neutron catalysis is similar to well-known μ-catalysis via μdt-molecules.

Free neutrons in stellar matter can appear as a result of nuclear reactions, in particular, reactions of proton cycle. It is suggested that there is helium core inside of star. The above few-body resonance fusion takes place on the boundary area of the core.

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