# Missing Mass Measurement Using Kinematic Cusp

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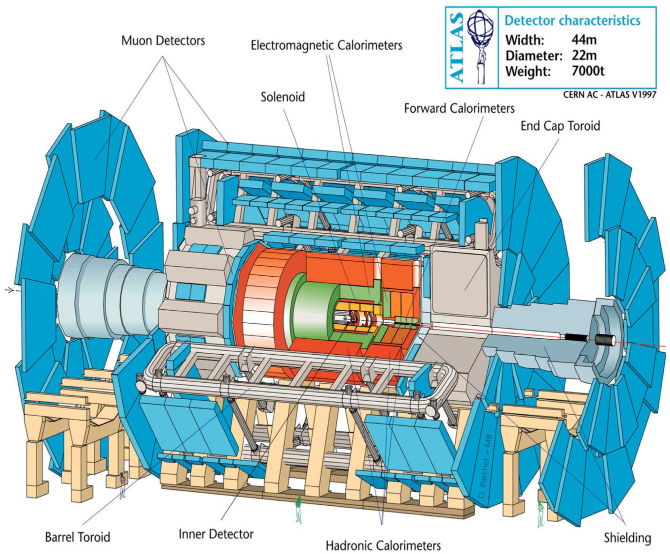
# Outline

- Introduction : antler decay
- Kinematic Cusp in invariant mass distribution
- Kinematic Cusp in angular distribution
- Effect of Spin correlation and finite decay width
- Conclusion

# Introduction

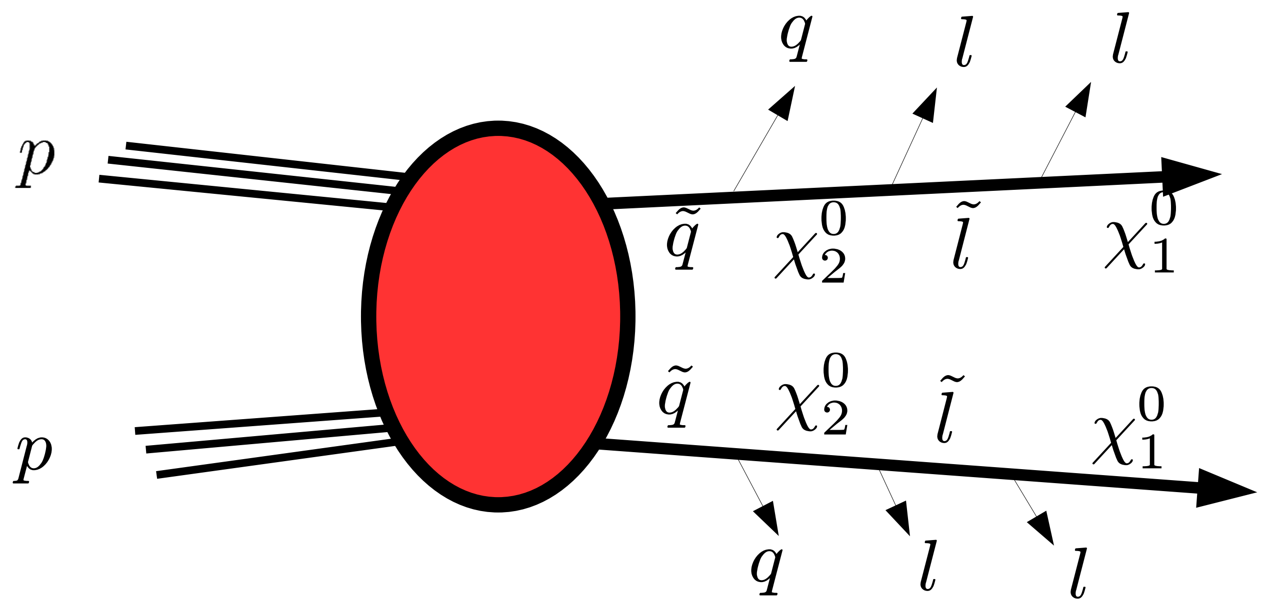
LHC may reveal the dark side of the universe.




Pair produced Dark Matter : R-parity, KK-parity, T-parity.

# Hard to identify, Hard to measure

SUSY-like cascade decay has been studied throughly.




- invariant mass distribution
- $M_{T2}$
- wedge box
- statistical on-shell mass solution
- $\sqrt{\hat{s}_{\rm min}}$ : generalization

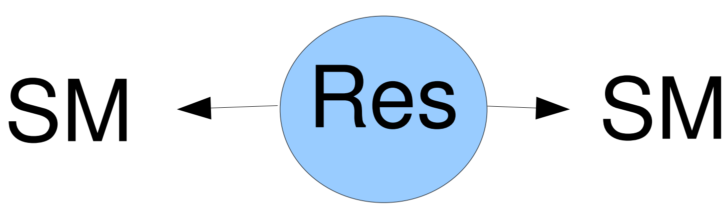
Use heavy resonant particle.

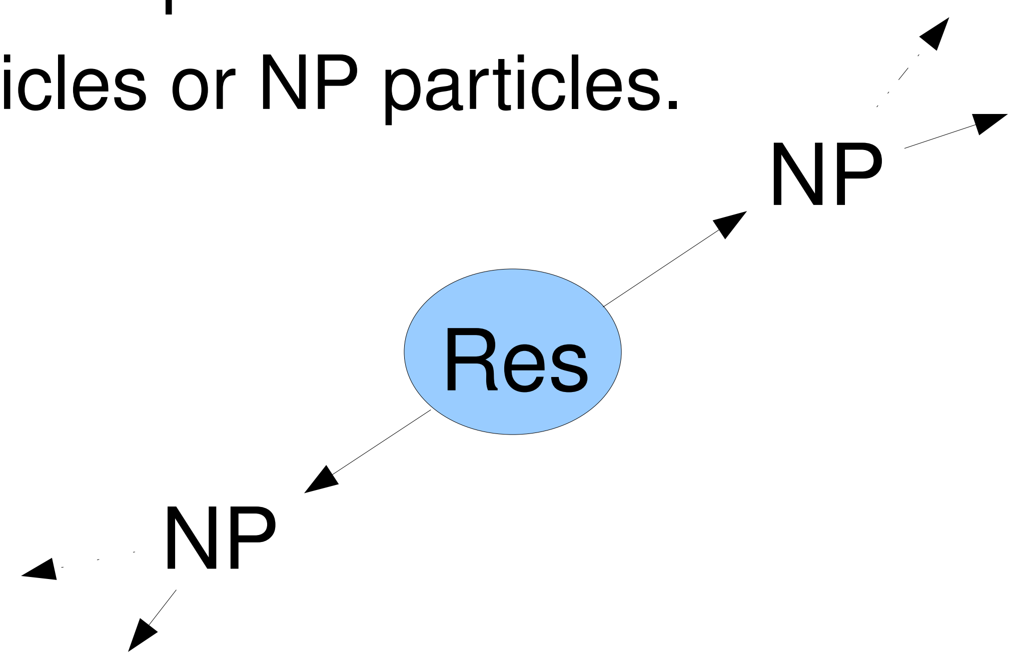
$Z'$ new neutral gauge boson

$V_\mu^{(2)}$ higher KK modes

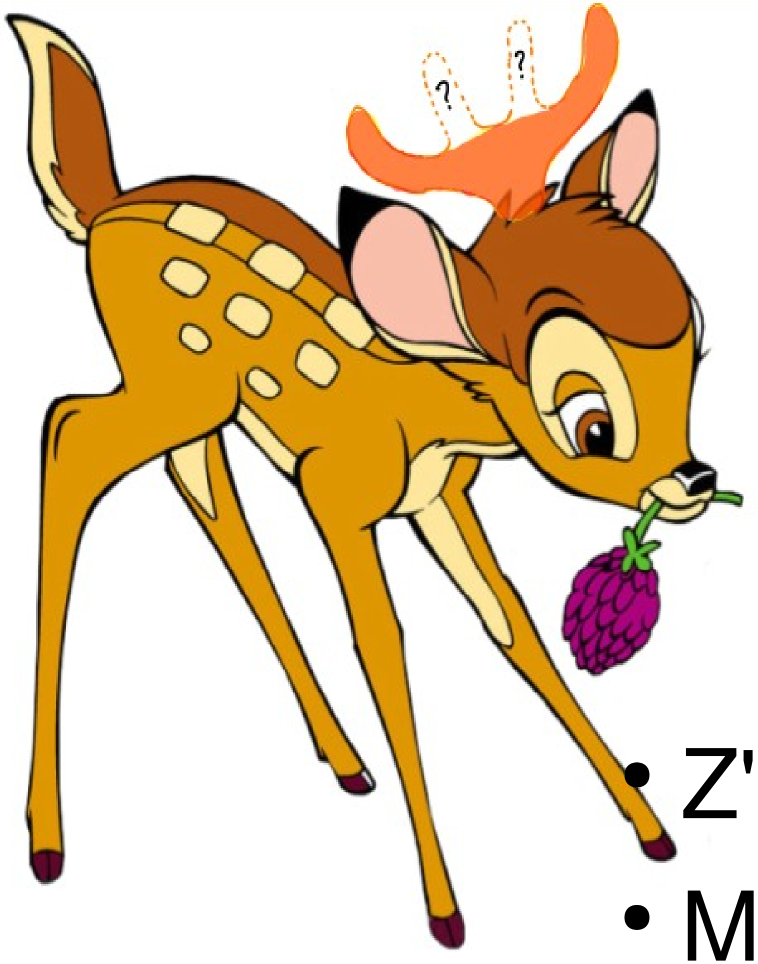
$H$ Heavy Higgs

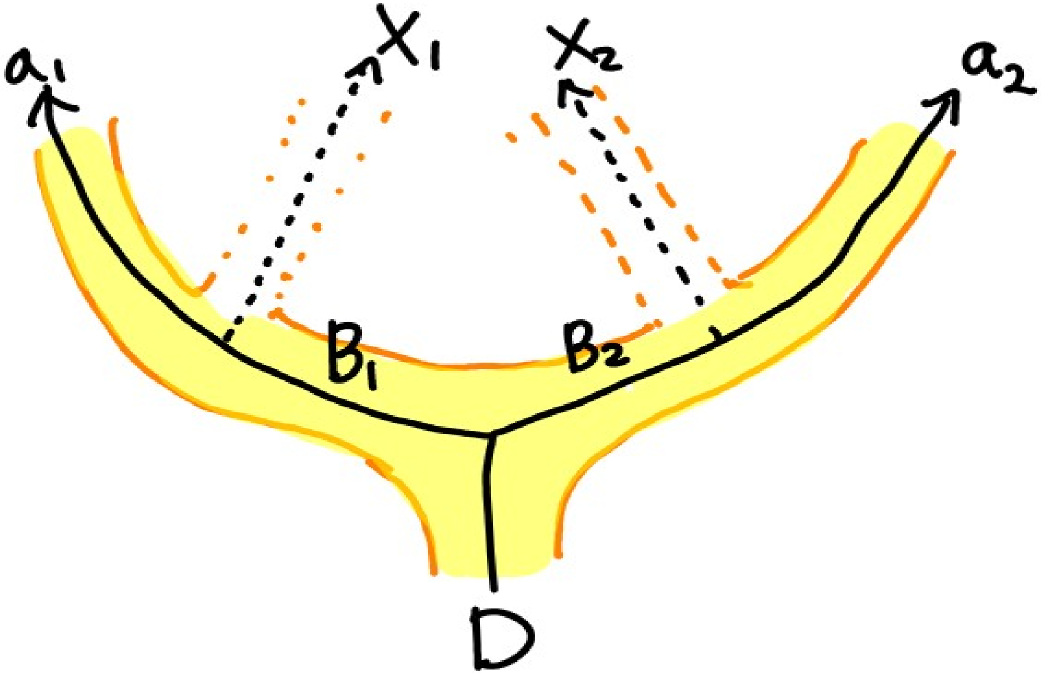
Many models have resonance particle
that can decay to SM particles or NP particles.



# Antler decay




- Z' SUSY: $Z' \to l^+ l^- \to l^+ \chi_1^0 l^- \chi_1^0$
- MSSM: $H \to \chi_2^0 \chi_2^0 \to Z \chi_1^0 Z \chi_1^0$
- UED: $Z_\mu^{(2)} \to L^{(1)} L^{(1)} \to l^+ \gamma^{(1)} l^- \gamma^{(1)}$
- LHwT: $H \to T\bar{T} \to tA\bar{t}A$

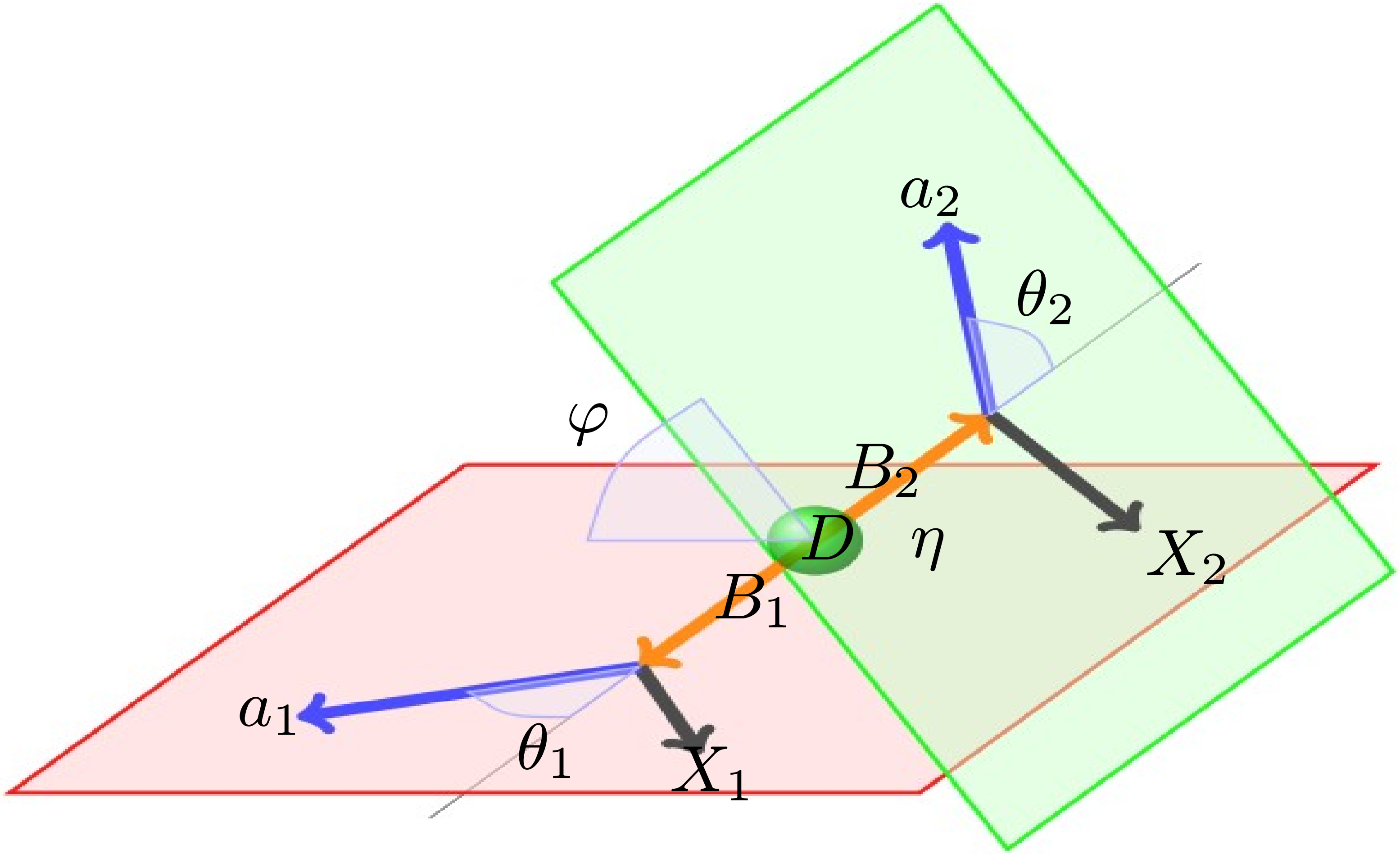
We observe kinematic distributions have cusp peaks.

Mass ratio of two adjacent particles in the decay chain can be factorized in cusp and end point.

Invariant mass and angular distribution have observational complementarity.

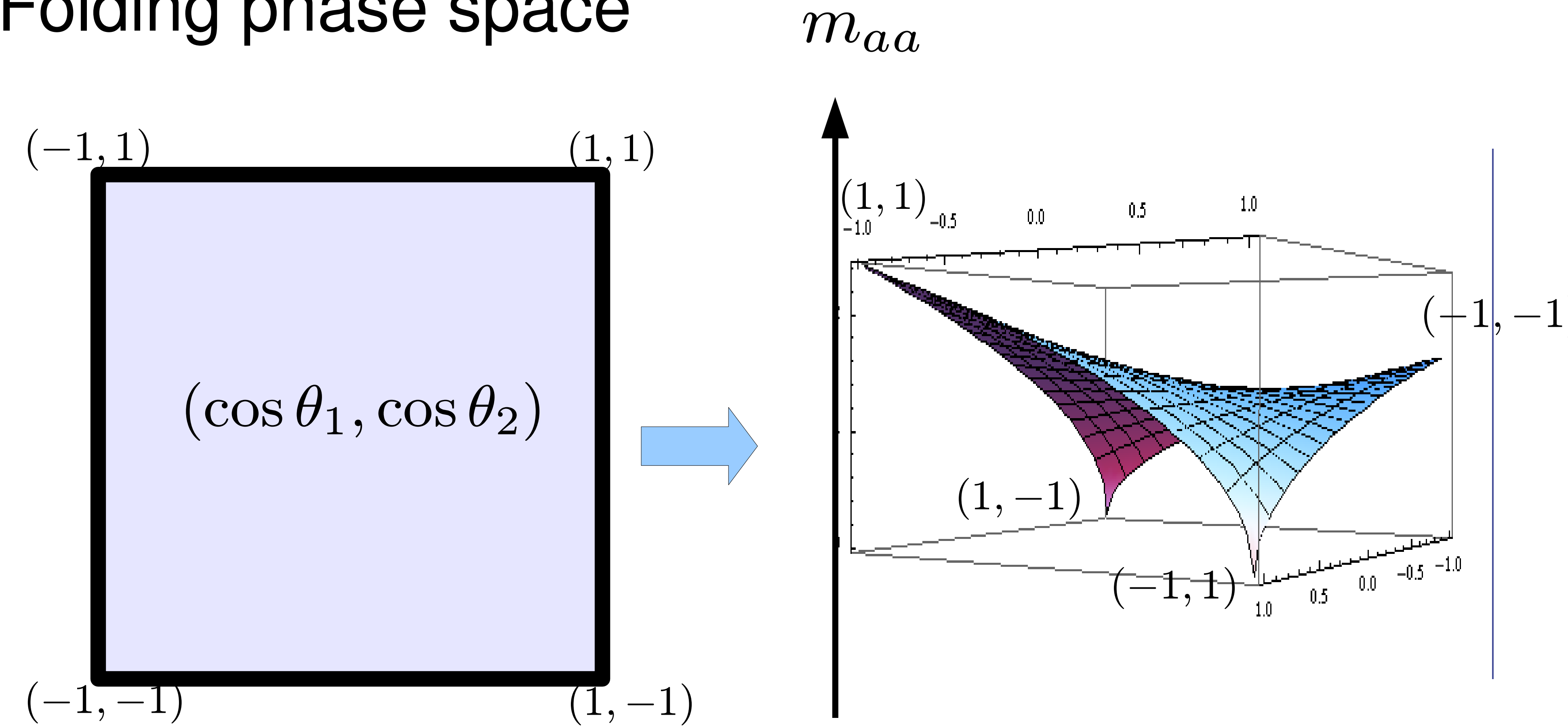
# Kinematics in antler decay

$$D \to B_1 B_2 \to a_1 X_1 a_2 X_2$$



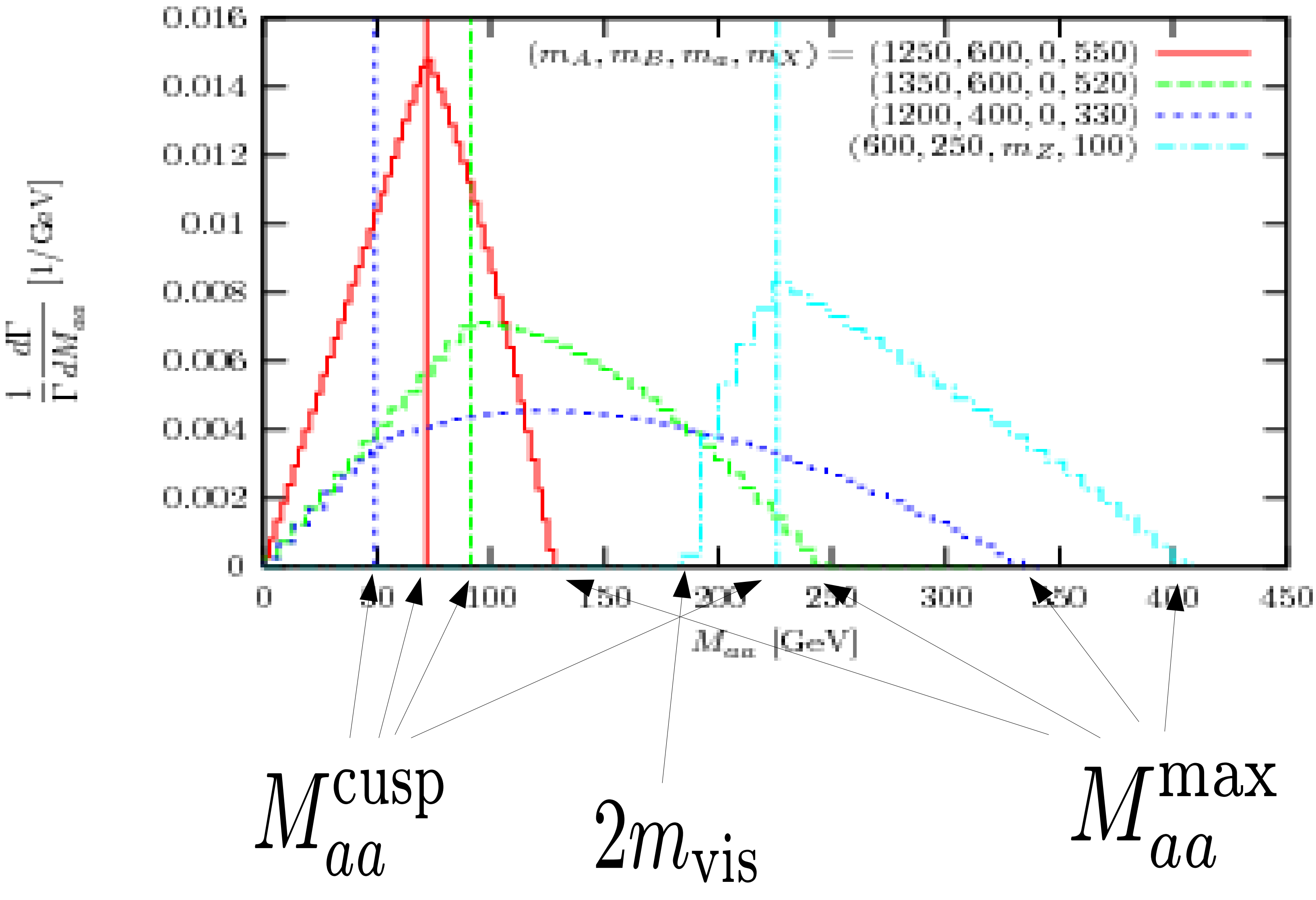

rapidity of B in the D rest frame: $\cosh \eta = \dfrac{m_D}{2m_B}$

# Folding phase space




$$
\begin{aligned}
m_{aa}^2 = \quad & \cosh 2\eta + \sinh 2\eta \cos\theta_1 \\
& + \cosh 2\eta \cos\theta_2 + \cosh 2\eta \cos\theta_1 \cos\theta_2 \\
& + \sin\theta_1 \sin\theta_2 \cos\varphi
\end{aligned}
$$

# Invariant Mass Distribution



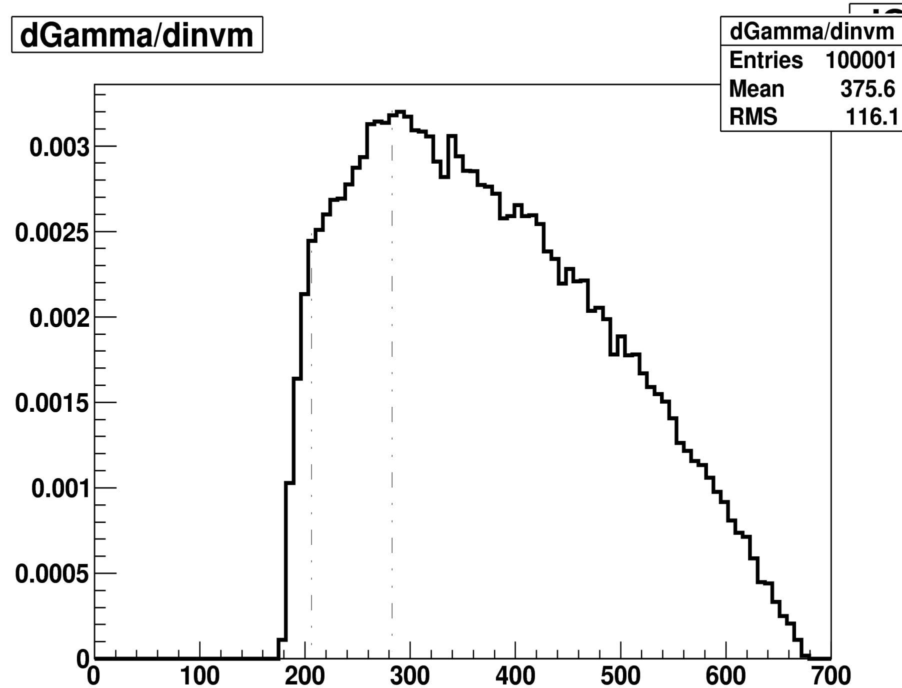
# Analytic Formula (massless visible particle)

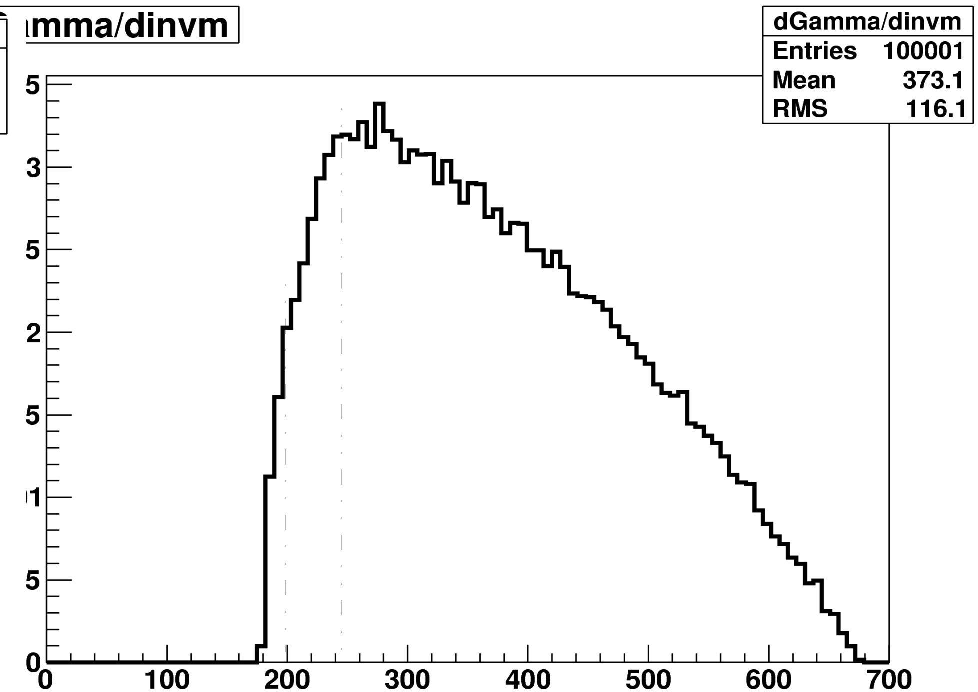
$$\frac{d\Gamma}{dM_{aa}} \propto \begin{cases} M_{aa} \log \dfrac{M_{aa}^{\max}}{M_{aa}^{\text{cusp}}}, & \text{if } M_{aa} \le M_{aa}^{\text{cusp}}, \\ \\ M_{aa} \log \dfrac{M_{aa}^{\max}}{M_{aa}}, & \text{if } M_{aa} > M_{aa}^{\text{cusp}} \end{cases}$$

$$\frac{M_{aa}^{\text{cusp}}}{M_{aa}^{\max}} = \exp(-2\eta) = \frac{m_D^2 - 2m_B^2}{2m_B^2} - \frac{m_D}{m_B}\sqrt{\frac{m_D^2}{4m_B^2} - 1}$$

$$M_{aa}^{\text{cusp}} M_{aa}^{\max} = m_B^2 \left(1 - \frac{m_X^2}{m_B^2}\right)^2$$

# Analytic Formula (Massive)



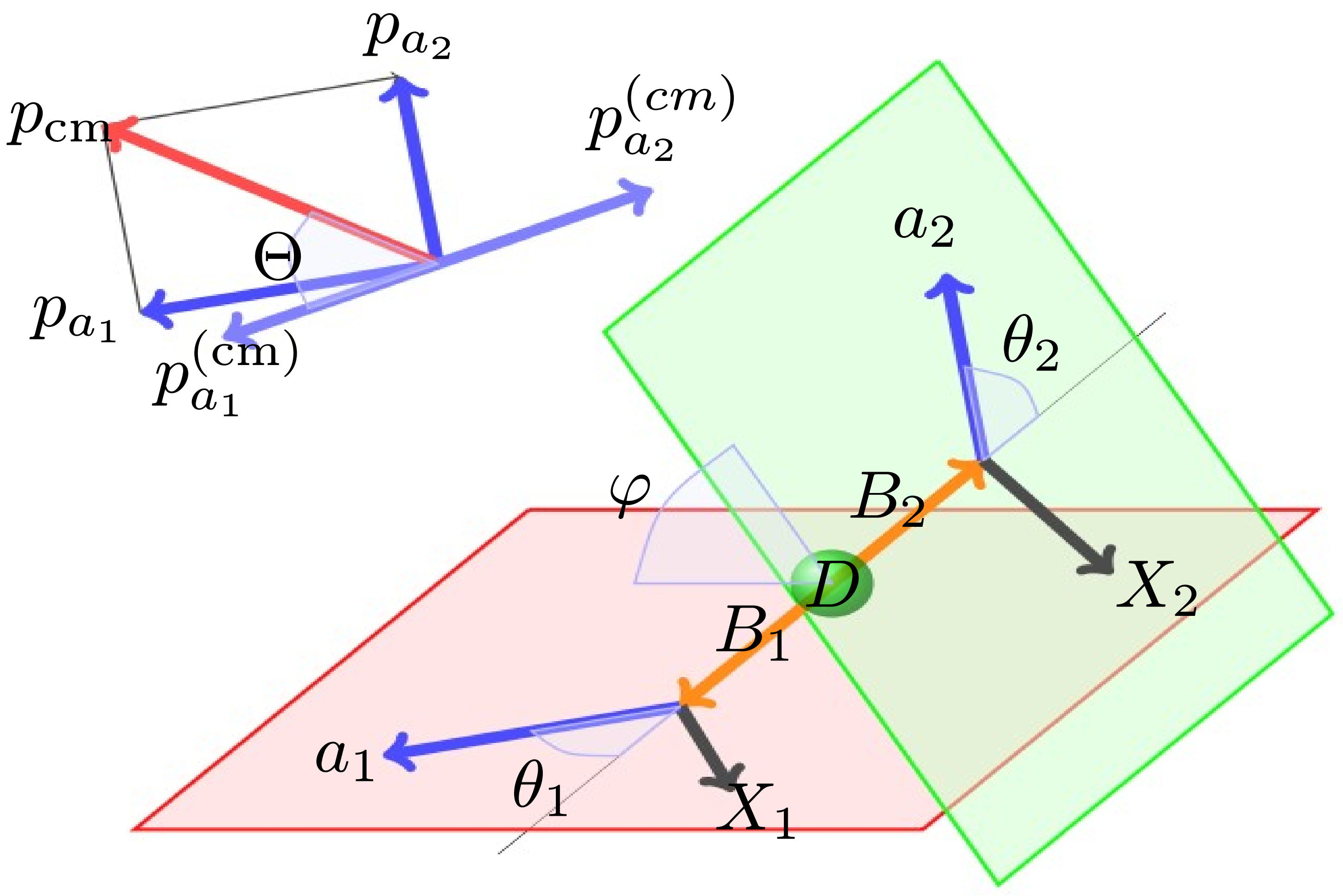

$$\chi \equiv \frac{M_{aa}^2}{(2m_a^2)} - 1$$

$$\left.\frac{d\Gamma}{d\chi}\right|_{\eta \le \frac{\zeta}{2}} \propto \begin{cases} 2\cosh^{-1}\chi, & \text{if } 1 \le \chi \le c_\eta; \\ 4\eta, & \text{if } c_\eta \le \chi \le c_-; \\ 2\zeta + 2\eta - \cosh^{-1}\chi, & \text{if } c_- \le \chi \le c_+, \end{cases}$$

$$\left.\frac{d\Gamma}{d\chi}\right|_{\frac{\zeta}{2} < \eta < \zeta} \propto \begin{cases} 2\cosh^{-1}\chi, & \text{if } 1 \le \chi \le c_-; \\ 2\zeta - 2\eta + \cosh^{-1}\chi, & \text{if } c_- \le \chi \le c_\eta; \\ 2\zeta + 2\eta - \cosh^{-1}\chi, & \text{if } c_\eta \le \chi \le c_+, \end{cases}$$

$$c_+ = \cosh 2(\eta + \zeta) \qquad c_- = \cosh 2(\eta - \zeta) \qquad c_\eta = \cosh 2\eta$$

# $\cos\Theta$ angular distribution

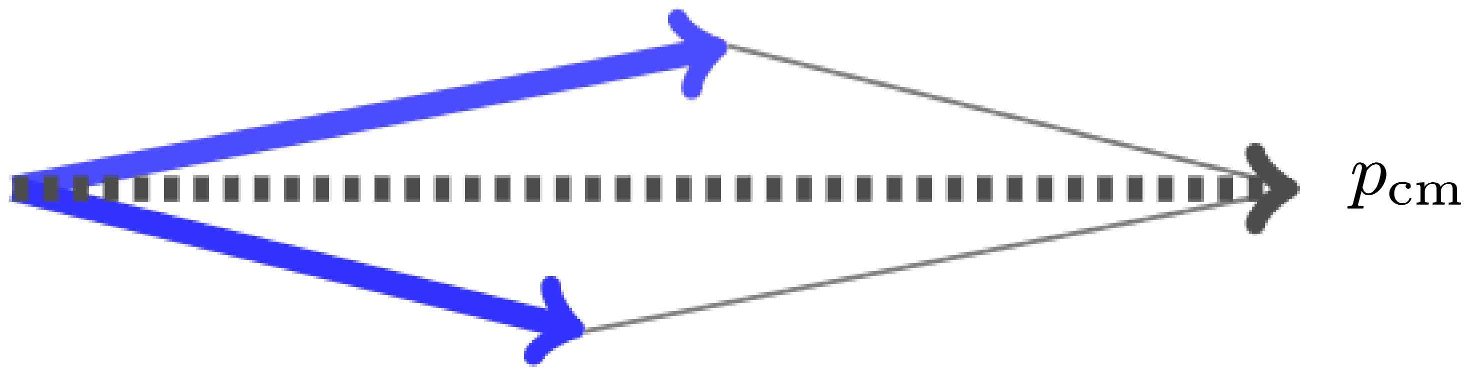



Frame dependent : depending

on Mother particle velocity in the lab frame

In the lab frame,

$$\begin{array}{ccccc} & E & p_{\parallel} & p_{\perp} & \\ p_1 = ( & \epsilon\cosh z_1, & \epsilon\sinh z_1, & \epsilon & ) \end{array}$$




$$p_2 = (\epsilon\cosh z_2, \epsilon\sinh z_2, \quad -\epsilon \quad )$$

$$\cos\Theta = \tanh(z_1 - z_2)/2$$

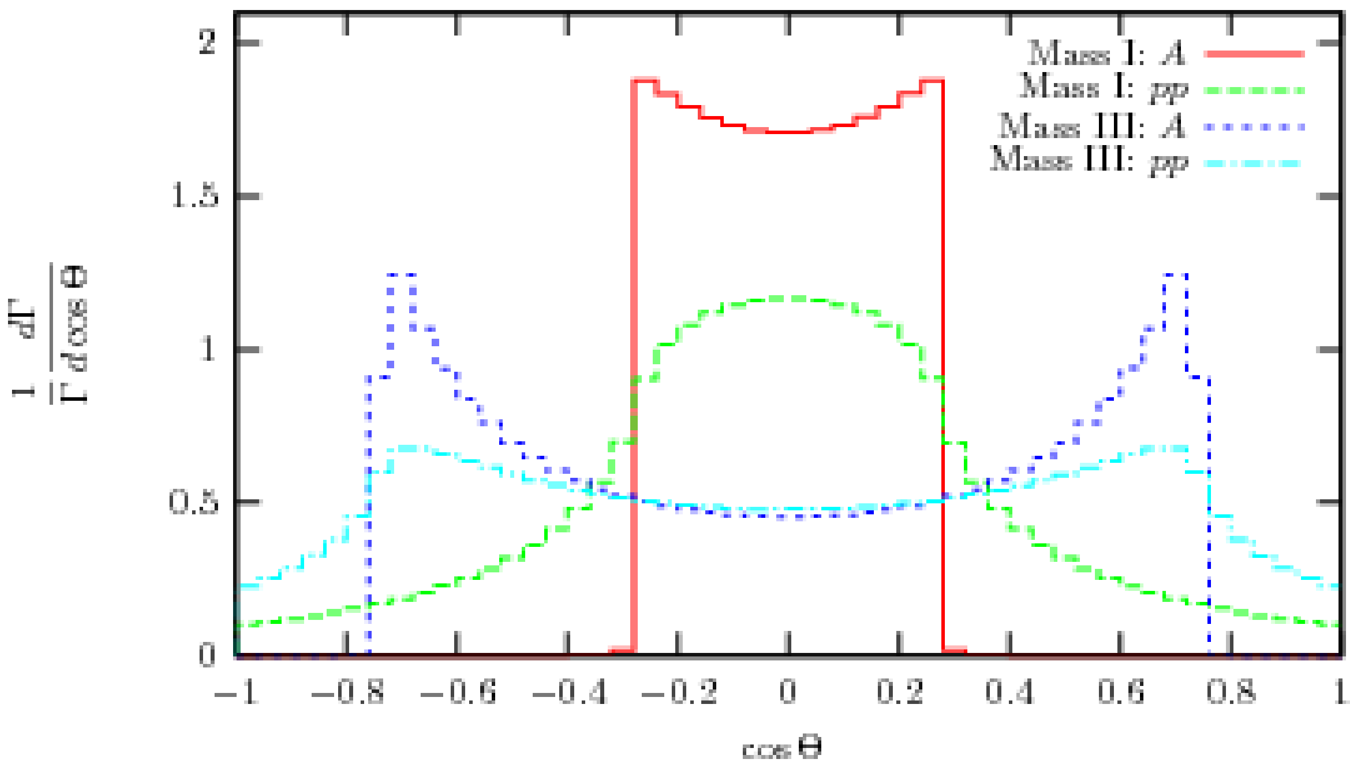
As $\epsilon$ goes to zero, $z_1$ and $z_2$ have bigger difference, $\cos\Theta$ becomes larger.

If D is at rest in the lab frame,

$$|\cos\Theta|_{\rm max} = \tanh\eta = \sqrt{1 - \frac{4m_B^2}{m_D^2}}$$

In the D rest frame,

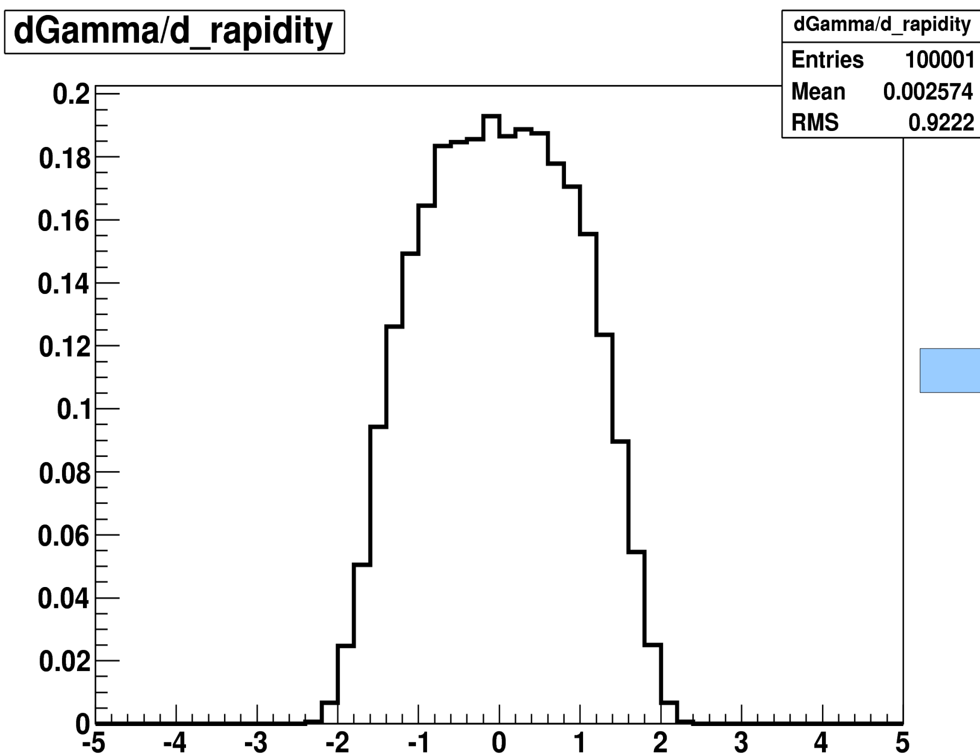
$$\frac{d\Gamma}{d\cos\Theta} \propto \begin{cases} (\sin\Theta)^{-3}, & \text{if } |\cos\Theta| \le \tanh\eta, \\ 0, & \text{otherwise,} \end{cases}$$

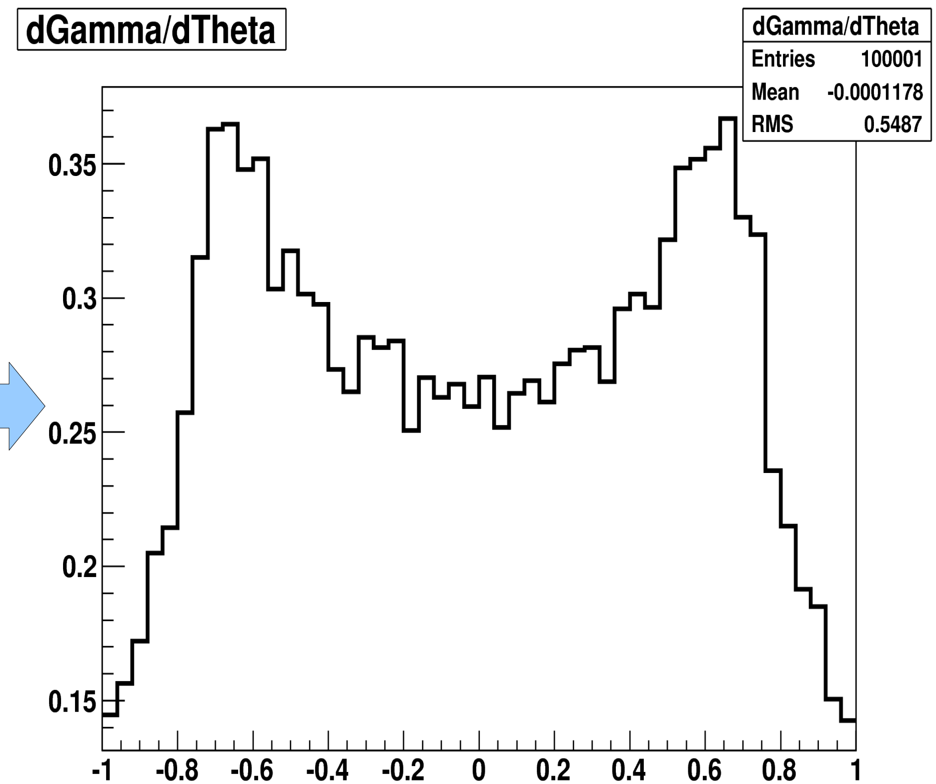



Due to moving mother particle D, we need to convolute $\cos\Theta$ with velocity distribution of D.

Experimentally we can measure D particle velocity distribution.

For example: s-channel production of $q\bar{q}$

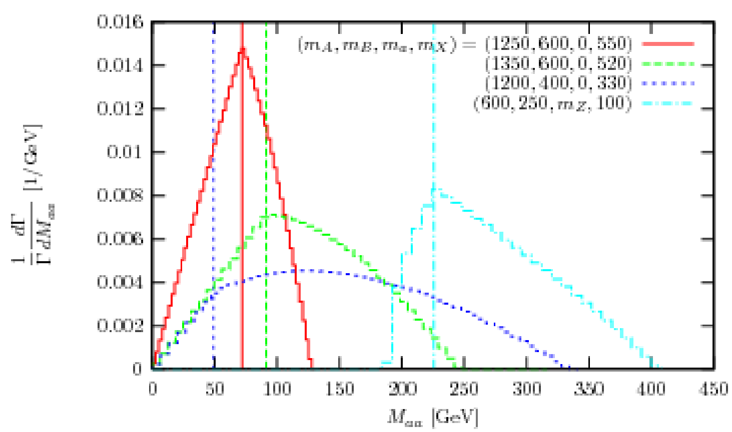


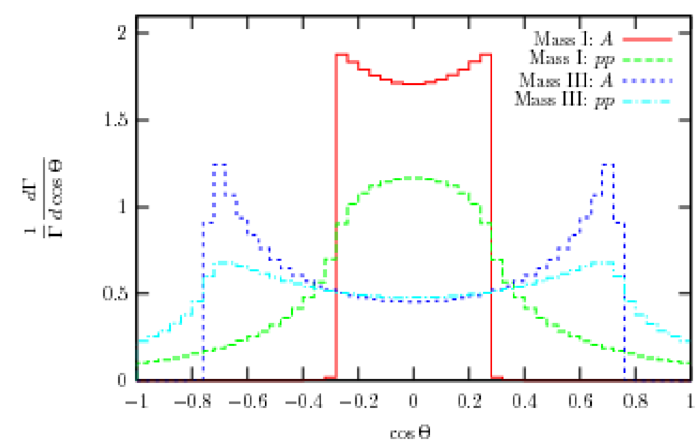




Cusp peak still appear at $|\cos\Theta| = \tanh\eta = \sqrt{1 - \frac{4m_B^2}{m_D^2}}$

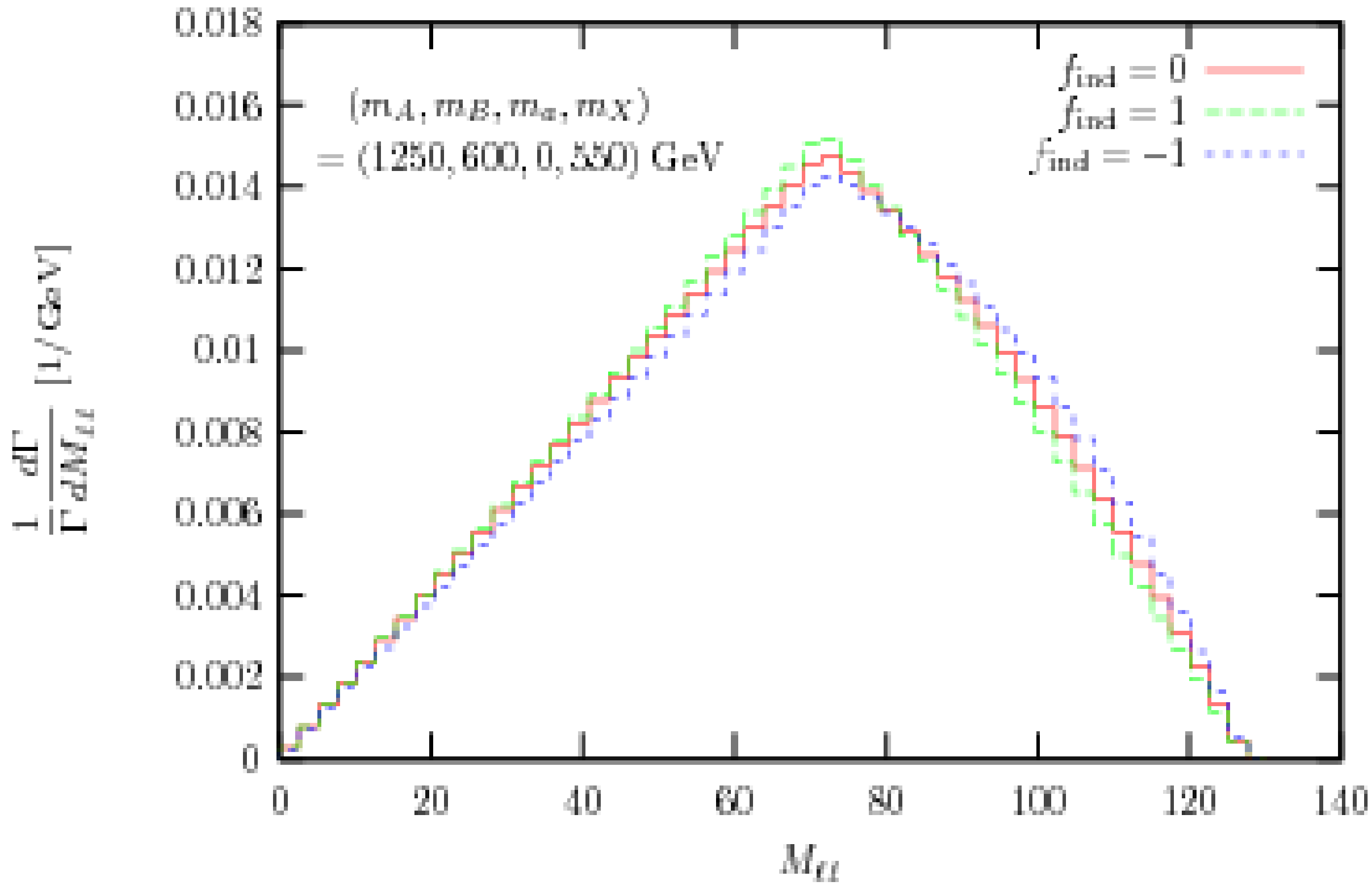
# Complementarity : $M_{aa}$ vs $\cos\Theta$

| | $M_{aa}$ | $\cos\Theta$ |
|---|---|---|
| Good for | Degenerate<br>$m_D \approx 2m_B$<br>Massive visible<br>$m_a \neq 0$ | large mass gap<br>$m_D \gg 2m_B$<br>Massless visible<br>$m_a = 0$ |



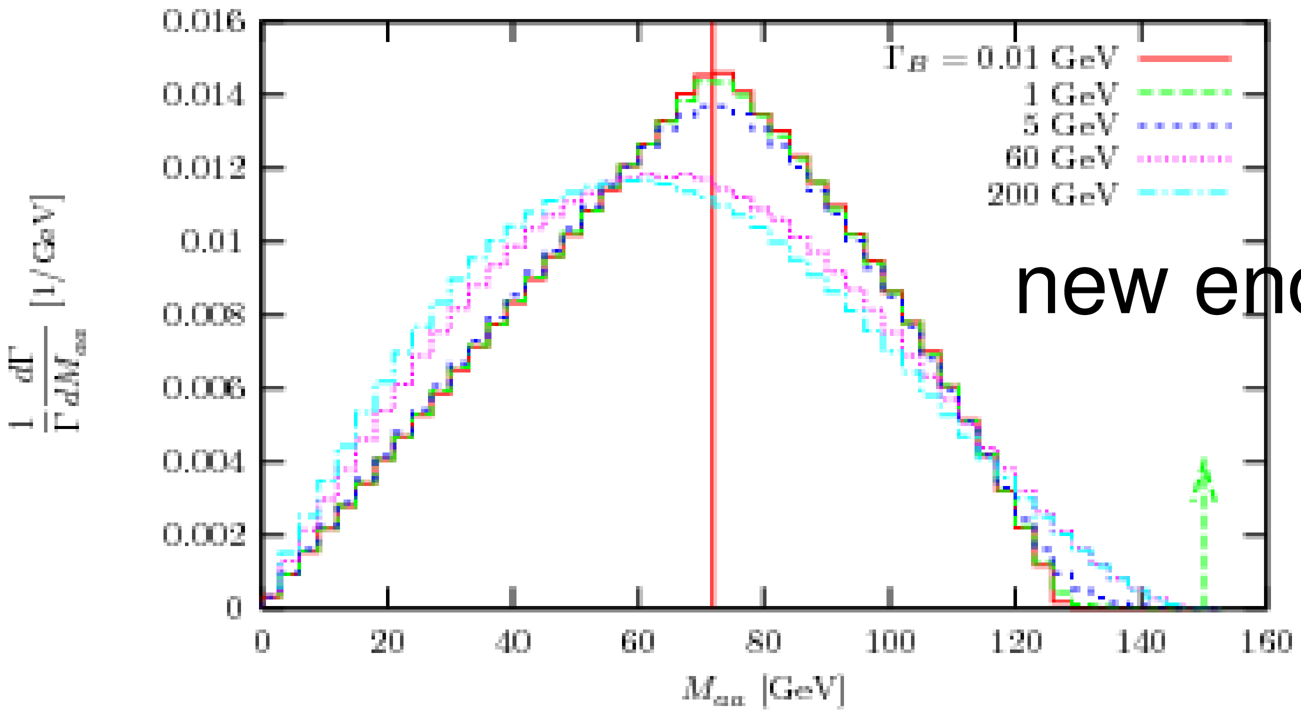



# Spin correlation effect




Negligible effect !

# Effect of Finite Decay Width




new end point appear

$$m_D - 2m_B$$

When $\dfrac{\Gamma}{m} \sim 1\%,$ negligible

When $\dfrac{\Gamma}{m} \sim 10\%,$ significant change

# Conclusion

- Antler decay can arise in various models.
- Cusp in $m_{aa}$ and $\cos\Theta$ appear
- Cusp observable factorizes mass ratio of adjacent particles in the decay chain.
- Complementarity in the cusp observables
- Spin effect is negligible, finite width effect is significant if $\Gamma/m \gtrsim 10\%$