Viscous hydrodynamics with shear and bulk viscosity

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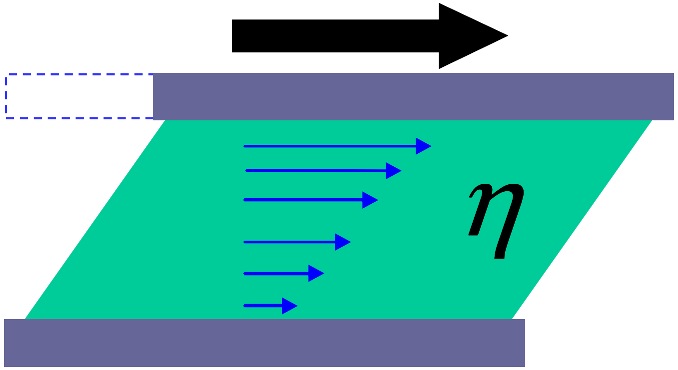
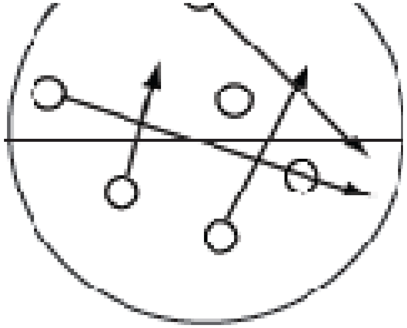
What is viscosity

Shear viscosity –measures the resistance to flow

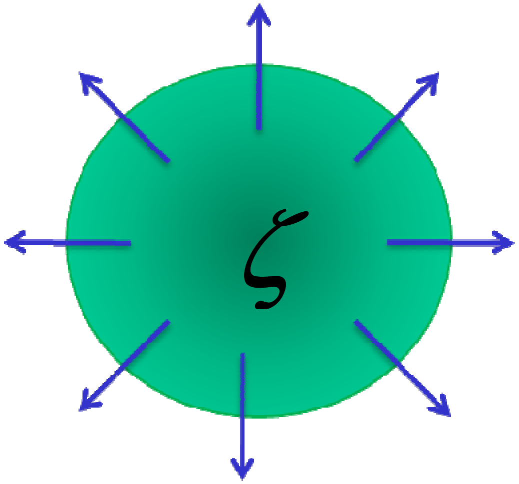


the ability of momentum transfer



Bulk viscosity –measure the resistance to expansion

-volume viscosity



Determines the dynamics of compressible fluid

The QGP viscosity

Kubo formulas:

shear viscosity: $\eta = \frac{1}{20} \lim_{\omega \to 0} \int d^4x e^{i\omega t} < [T^{ij}(x) T^{ij}(0)] > \theta(t)$

bulk viscosity: $\zeta = \frac{1}{18} \lim_{\omega \to 0} \int d^4x e^{i\omega t} < [T_i^i(x) T_i^i(0)] > \theta(t)$

Shear viscosity: uncertainty principle requires a lower limit for η / s

- weakly coupled QCD: $\eta / s \sim 1$ Arnold, Moore & Yaffe, 00,03
- lattice SU(3) gluon dynamics : $\eta / s < 1$ Meyer, PRD 07
- strongly coupled AdS/CFT prediction : $\eta / s \geq 1/4\pi \sim 0.08$ D.T. Son et al. ‘01,’05

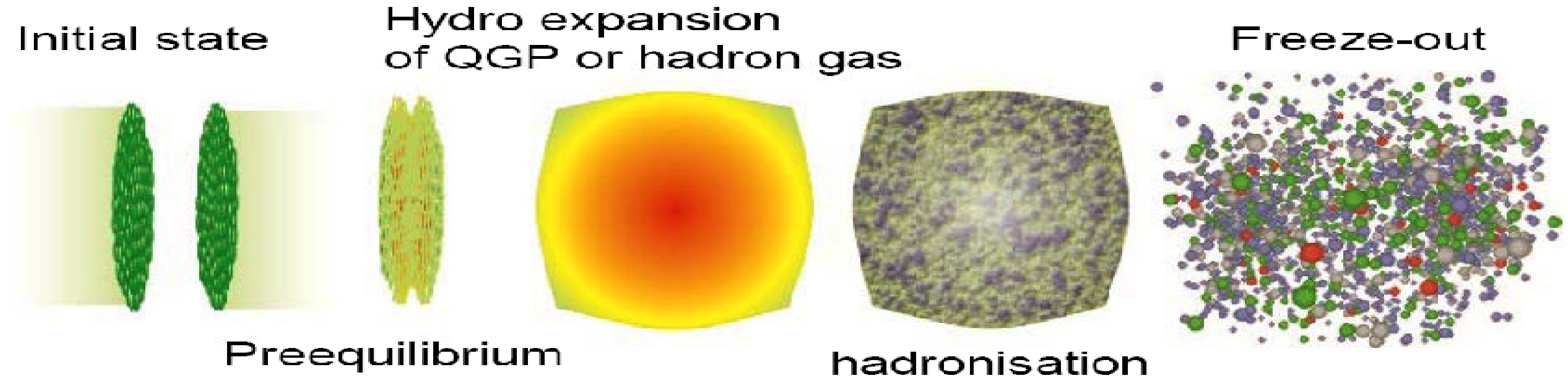
Bulk viscosity: zero for classical massless particles, ζ / s reaches a peak near T_c

- weakly coupled QCD prediction: $\zeta / s << 1$ Arnold, Dogan & Moore, PRD06
- lattice SU(3) gluon dynamics : $\zeta / s \big|_{\sim T_c} = 0.73 \left[^{2.0}_{0.5} \right]$ Meyer, PRL08
- LET+ assum. of spectral fun. + Lattice data: $\zeta / s \big|_{\sim T_c} \sim 0.8$ Kharzeev, et al. 07-08
- strongly coupled AdS/CFT prediction: $\zeta / s > 2\eta / s (1/3 - c_s^2)$ Buchel, 07

  $\zeta / s \big|_{\sim T_c} \sim 0.05$ Gubser, et al . 0806 ..

To extract the QGP viscosity from experimental data, we need viscous hydrodynamics

Viscous hydro with shear & bulk viscosity



Conservation laws:

$$\partial_\mu T^{\mu\nu}(x) = 0 \qquad T^{\mu\nu} = (e + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

Evolution equations for shear pressure tensor $\pi^{\mu\nu}$ and bulk presurre:

$$\tau_\pi \Delta^{\alpha\mu}\Delta^{\beta\nu}\dot{\pi}_{\alpha\beta} + \boxed{\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}} - \frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\lambda\left(\frac{\tau_\pi}{\eta T}u^\lambda\right)$$

$$\tau_\Pi \dot{\Pi} + \boxed{\Pi = -\zeta(\partial \cdot u)} - \frac{1}{2}\Pi\frac{\zeta T}{\tau_\Pi}\partial_\lambda\left(\frac{\tau_\Pi}{\zeta T}u^\lambda\right)$$

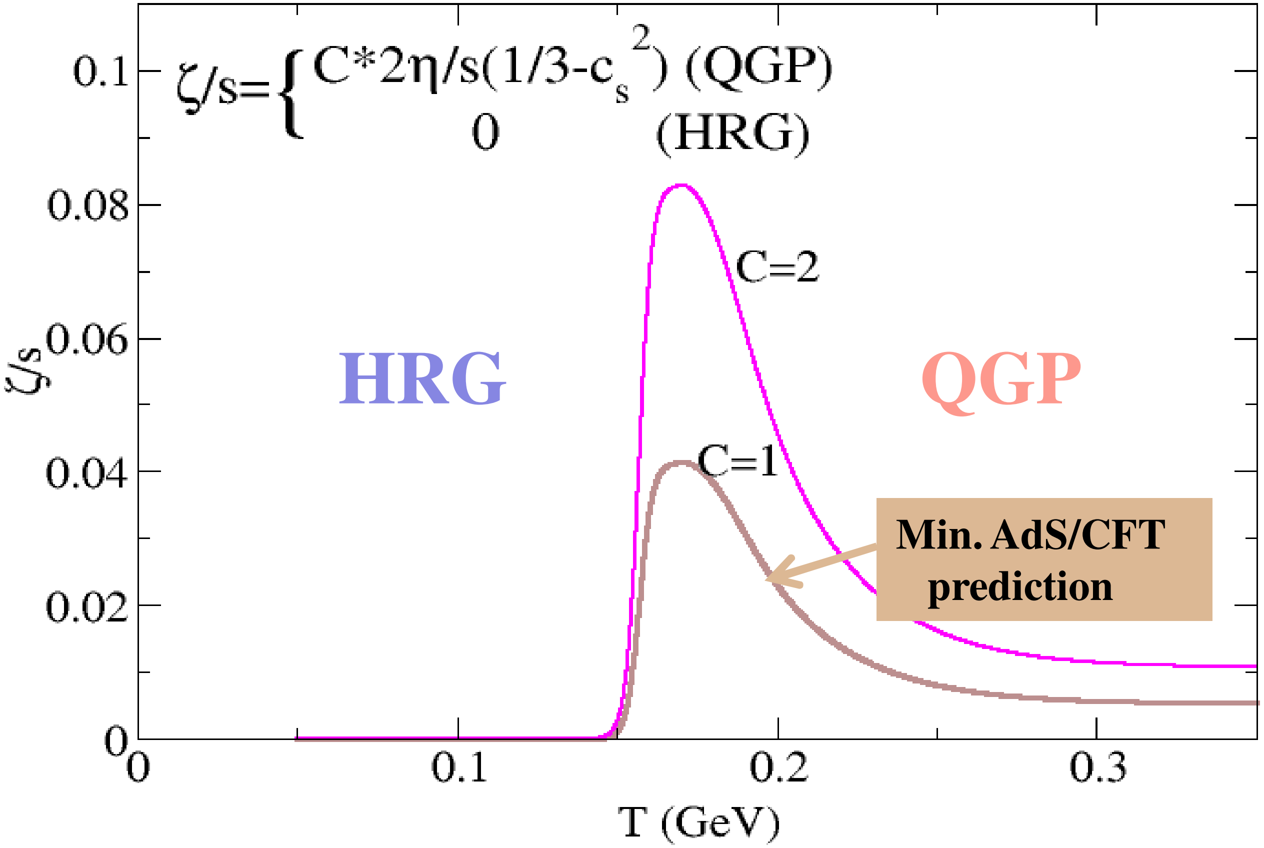
(2nd order shear-bulk -mixing term (Muronga, Rischke) not included.)

Numerical Results

Shear viscosity:

$$\eta / s = 0.08 \approx 1/4\pi, \quad \text{or} \quad \eta / s = 0$$

Bulk viscosity:



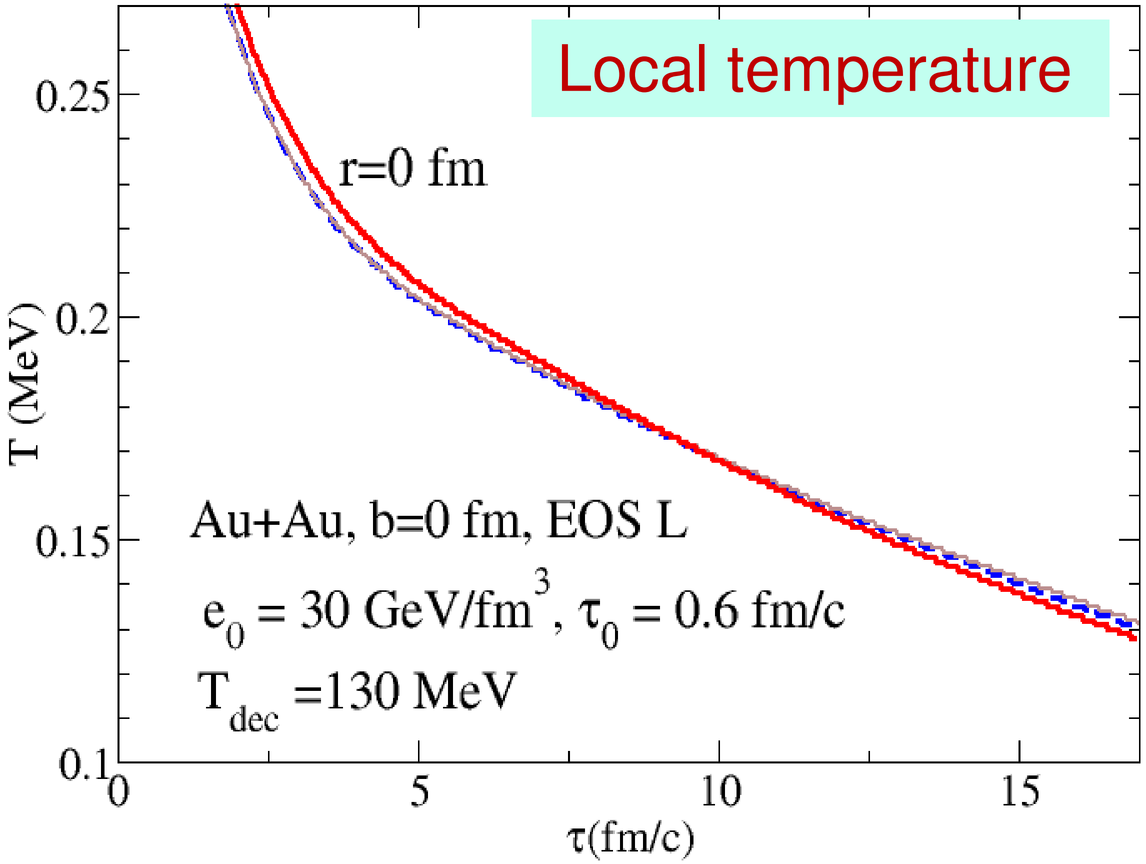
Min. AdS/CFT prediction

Relaxation times: (see later)

Shear viscosity vs. bulk viscosity (I)

Same initial & final conditions

- - - ideal hydro **— viscous hydro-shear only** **—viscous hydro-bulk only**



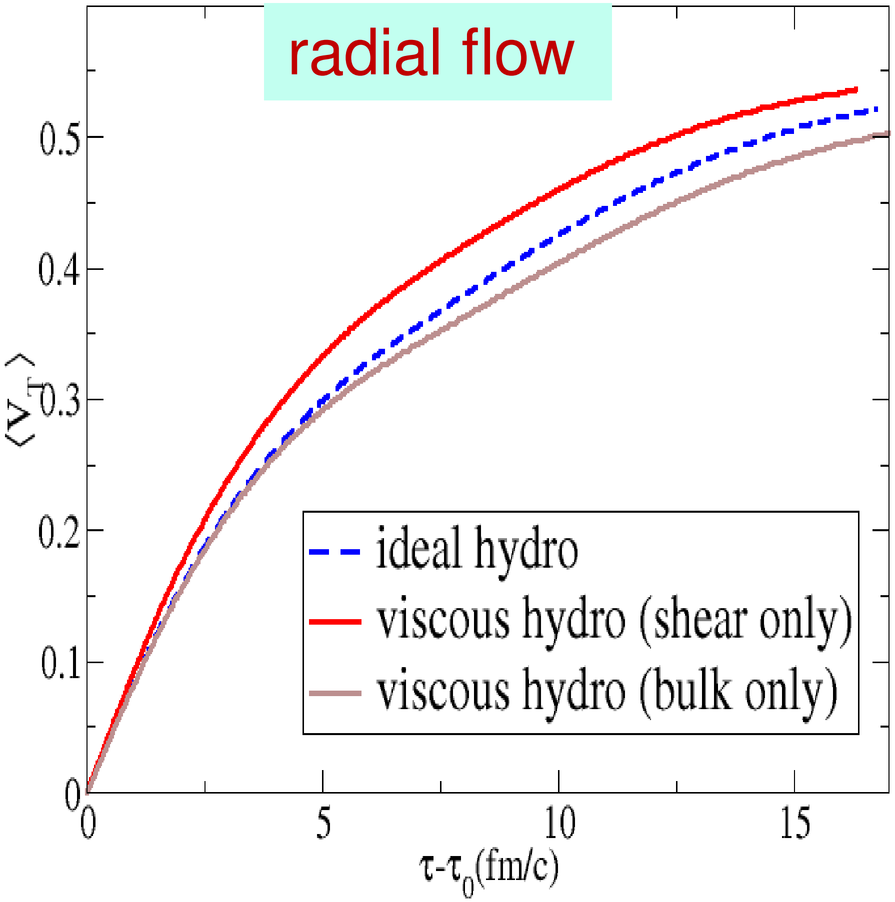
-Shear viscosity: decelerate cooling process in early stage
accelerate cooling process in middle and late stages

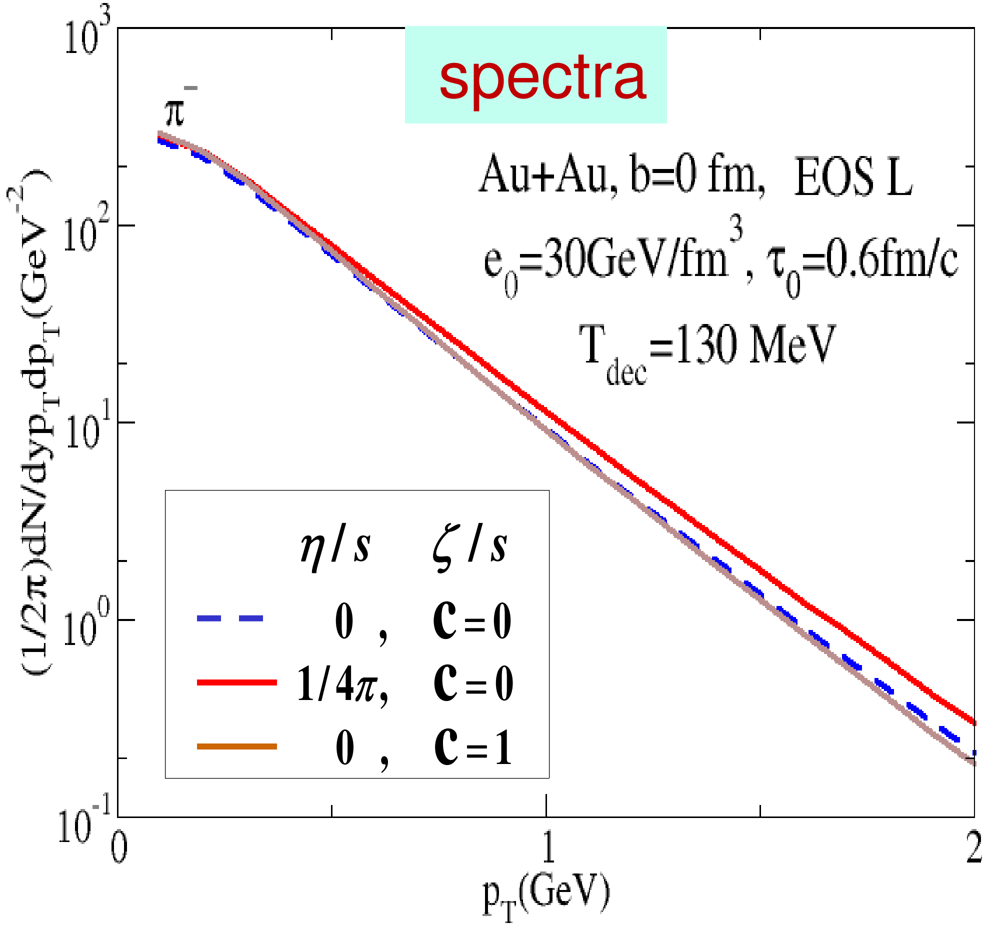
-Bulk viscosity: decelerate cooling process

Shear viscosity vs. bulk viscosity (II)

Same Initial & final conditions

- - - ideal hydro **—— viscous hydro-shear only** **—— viscous hydro-bulk only**





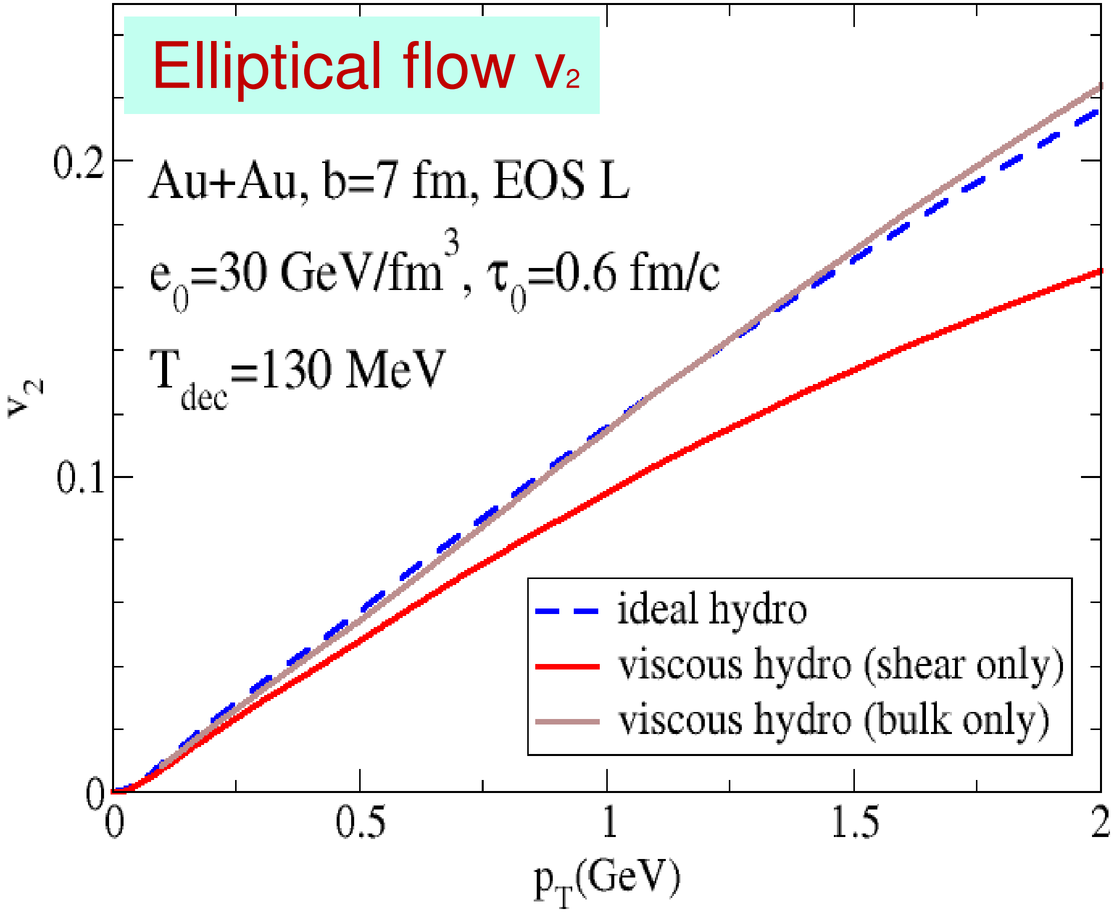
-shear viscosity: increases radial flow, results in flatter spectra

-bulk viscosity: decreases radial flow, results in steeper spectra

Shear viscosity vs. bulk viscosity (III)

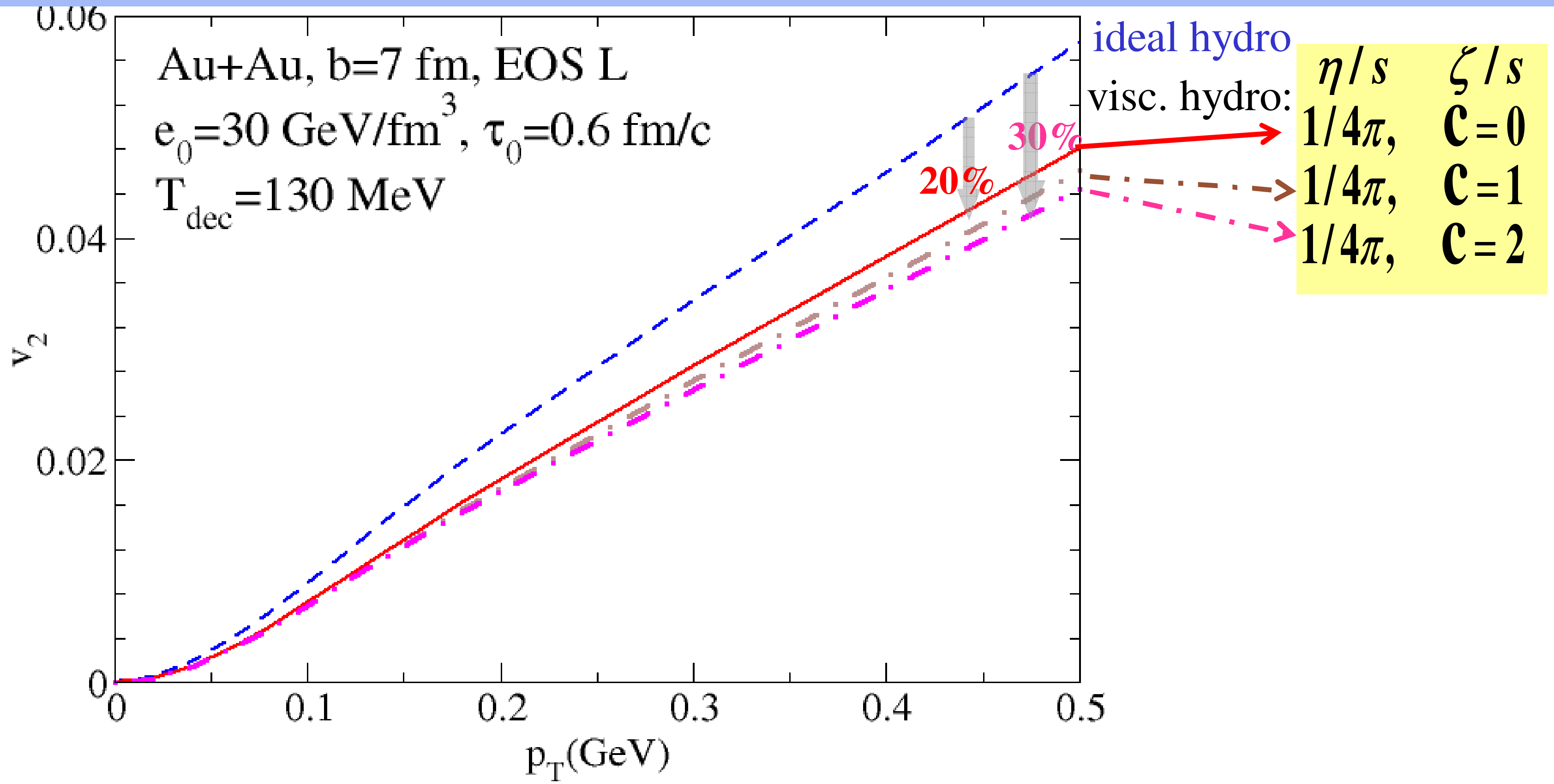
Same Initial & final conditions

– – – ideal hydro **—— viscous hydro-shear only** **——viscous hydro-bulk only**


Elliptical flow v_2

-v_2 is sensitive to both shear and bulk viscosity

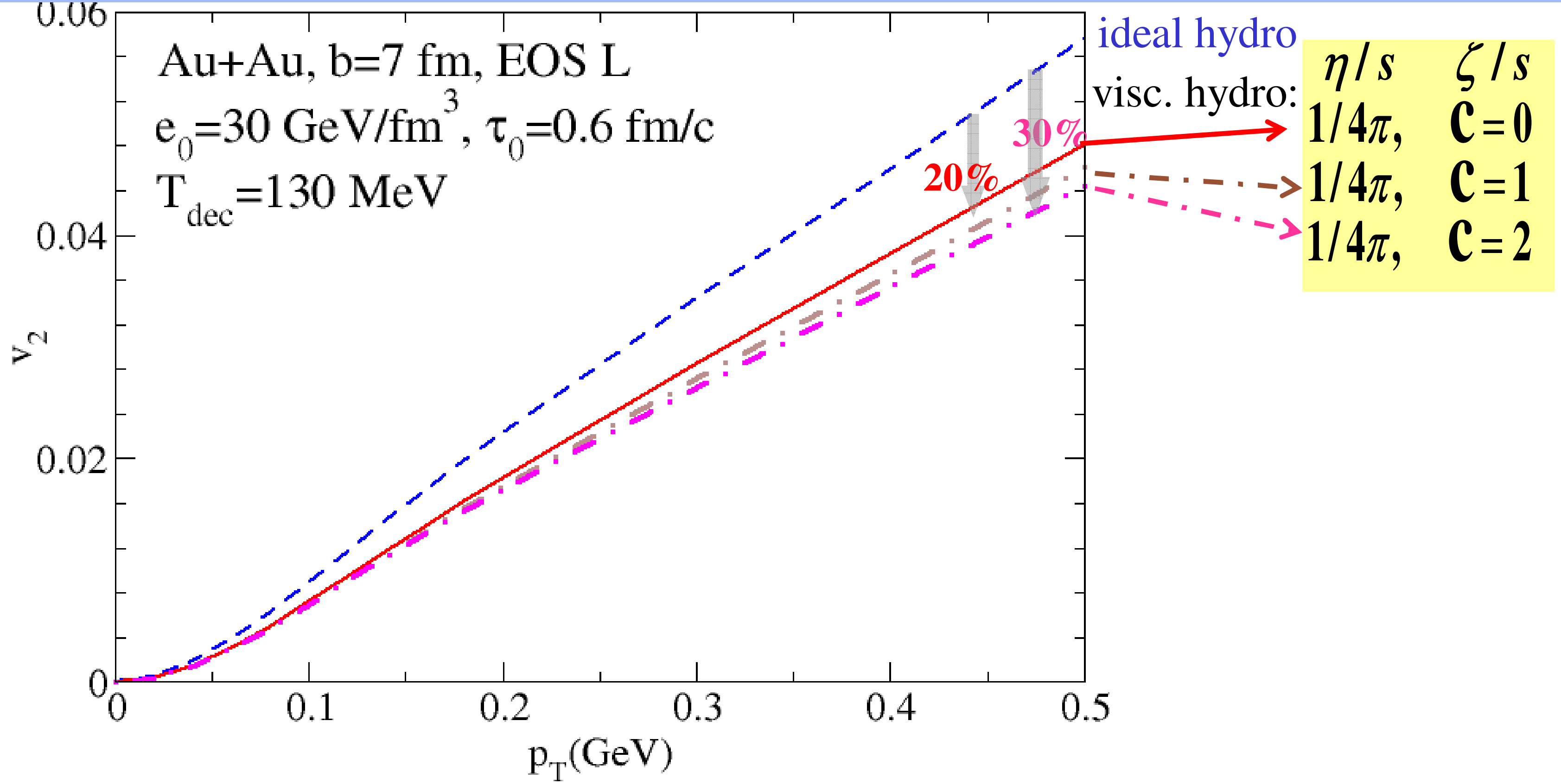
Viscous v2 suppression: shear and bulk viscosity



-at RHIC, **2 x min. bulk viscosity** could result in **~50%** additional v_2 suppression

-when extracting the η / s from RHIC data, bulk viscous effects cannot be neglected

Viscous v2 suppression: shear and bulk viscosity



-at RHIC, **2 x min. bulk viscosity** could result in **~50%** additional v_2 suppression

-when extracting the η/s from RHIC data, bulk viscous effects cannot be neglected

bulk viscosity effects: (a) Change the flow profile during hydro evolution
(b) Additional spectra correction δf along freeze-out surface $(\zeta \neq 0)$

Song & Heinz: v2 will **decrease**, **flow corrections only (a),** $\zeta = 0$, at freeze-out

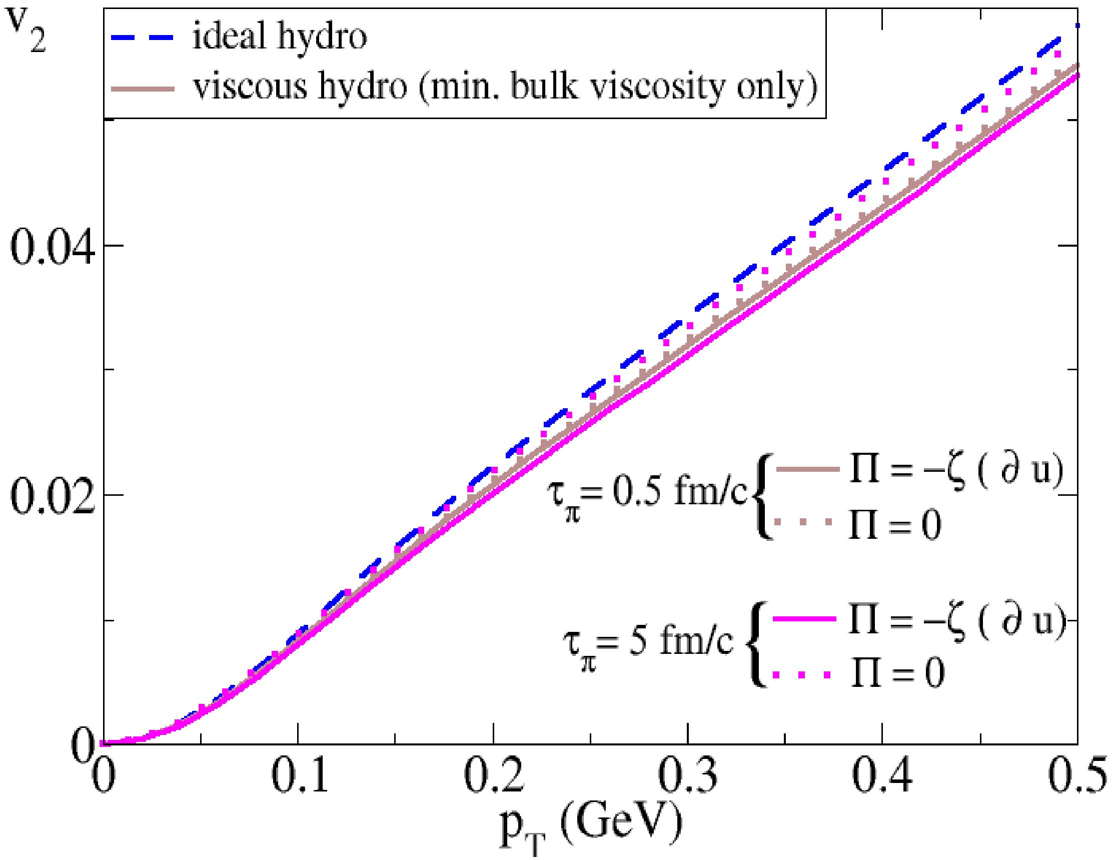
Monnai & Hirano: v2 will **increase**, **spectra corrections only(b),** ideal hydro for evolution

Bulk Viscosity

-relaxation time effects

Bulk viscous v2 suppression:

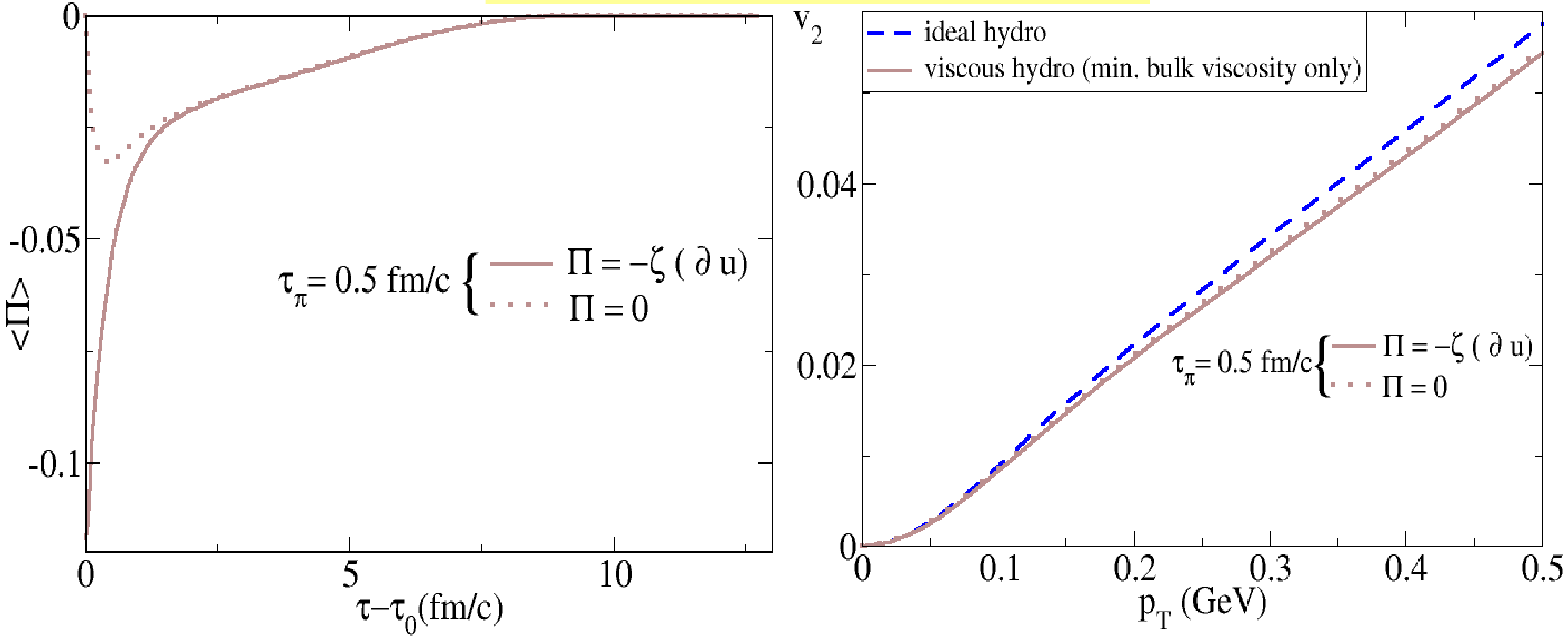
-- **Smaller vs. larger** relaxation time



-viscous effects from bulk viscosity strongly depend on relaxation time and the initialization for bulk pressure

Effects from initialization of Π (I)

Smaller relaxation time

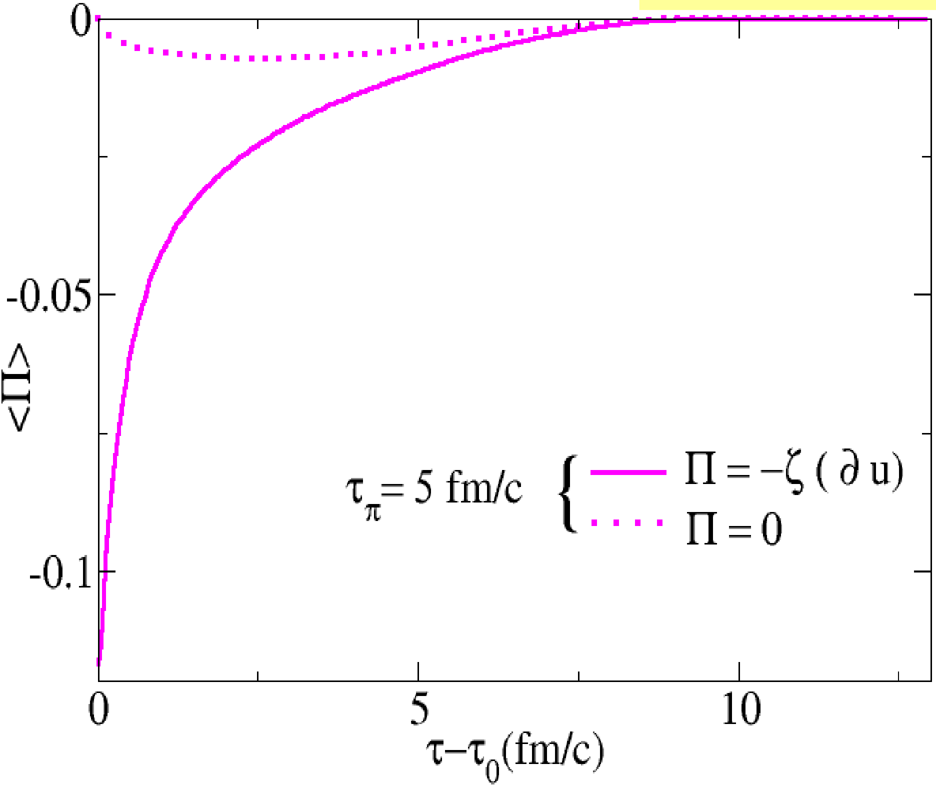


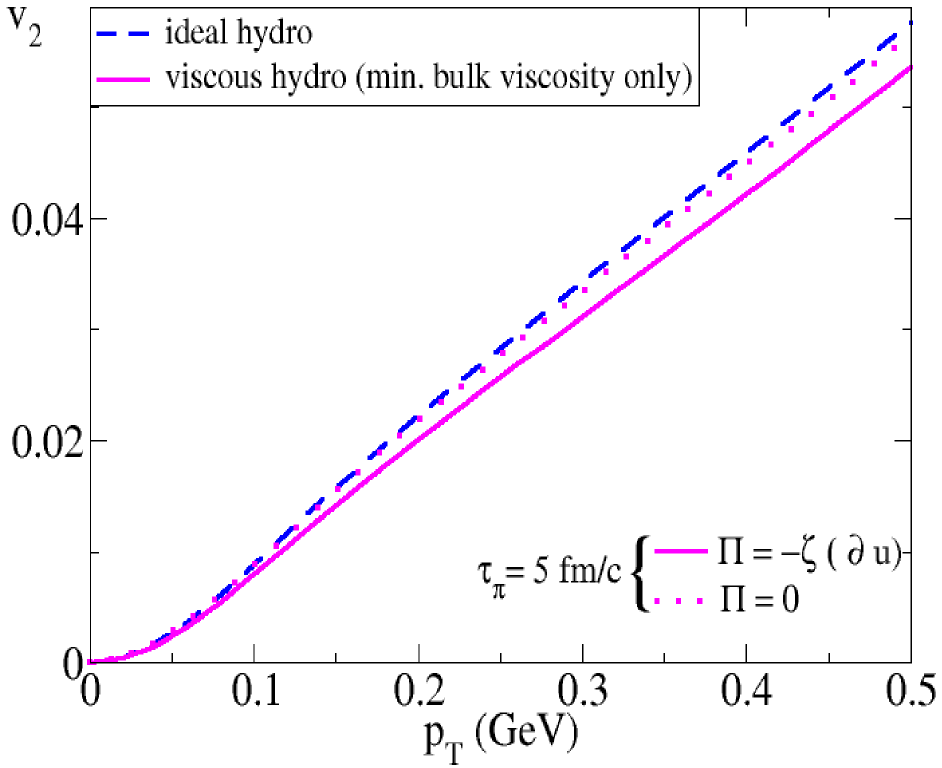
-When τ_π is small $(\tau_\pi = 0.5 \text{ fm/c})$, v_2 is **insensitive** to different initializations of Π

-after (several relaxation times), viscous pressure loses memory of initial cond.

Effects from initialization of Π (II)

larger relaxation time



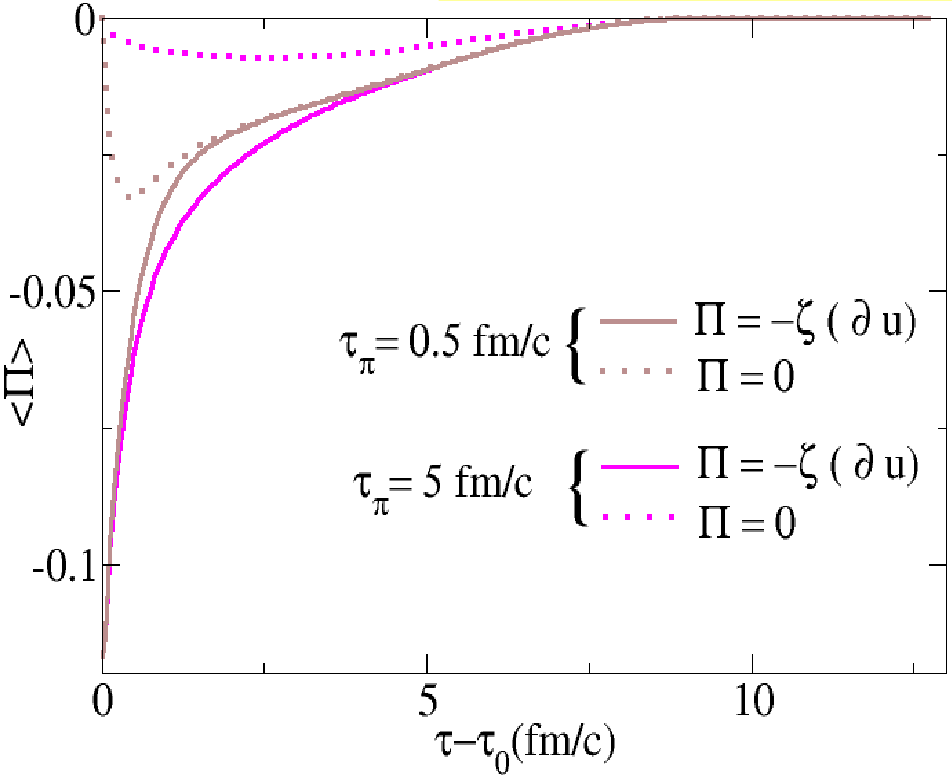


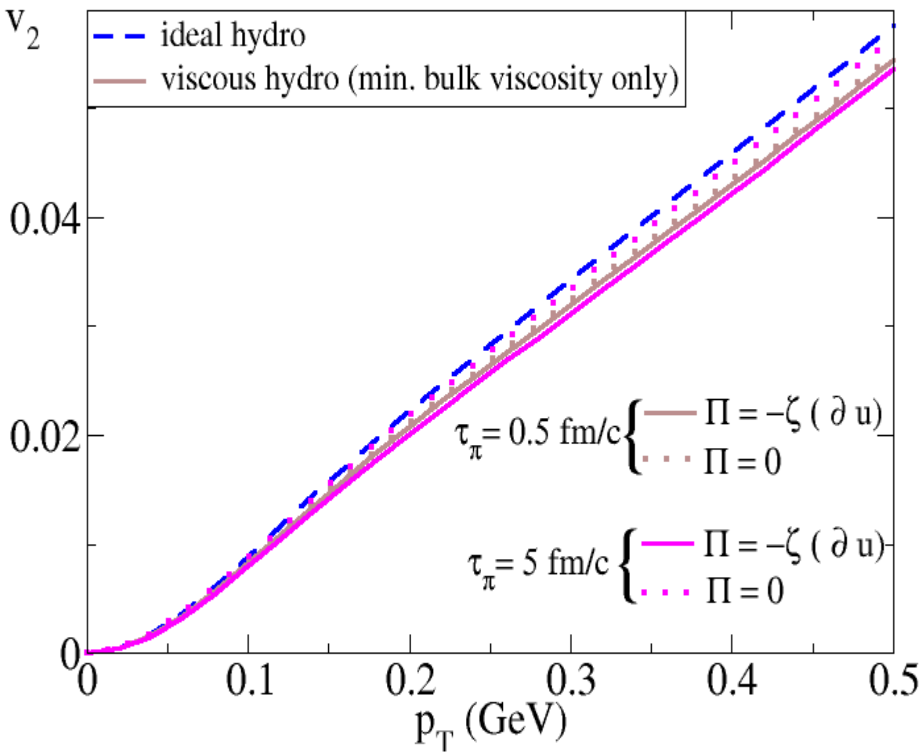
-When τ_π is larger $(\tau_\pi = 5\ \mathrm{fm/c})$, v_2 is **sensitive** to different initializations of Π

-after (several relaxation times), viscous pressure loses memory of initial cond.

Effects from initialization of Π (III)

Smaller vs. larger relaxation time





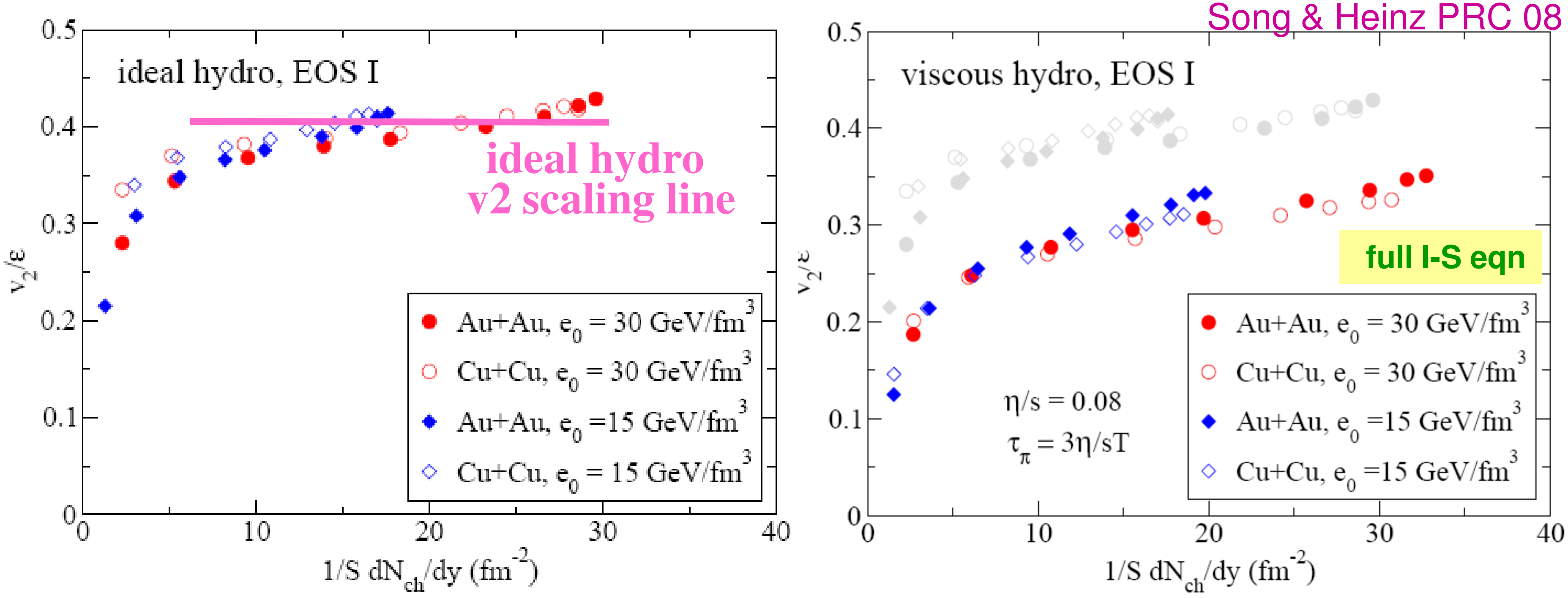
-viscous effects from bulk viscosity strongly depend on **relaxation time** and the **initialization for bulk pressure**

Multiplicity scaling of v_2/ε

--Effects from system size
and collision energy

Multiplicity scaling of v_2/ε EOS I

Song & Heinz PRC 08



Ideal hydrodynamics: multiplicity scaling of v_2/ε is weakly broken:

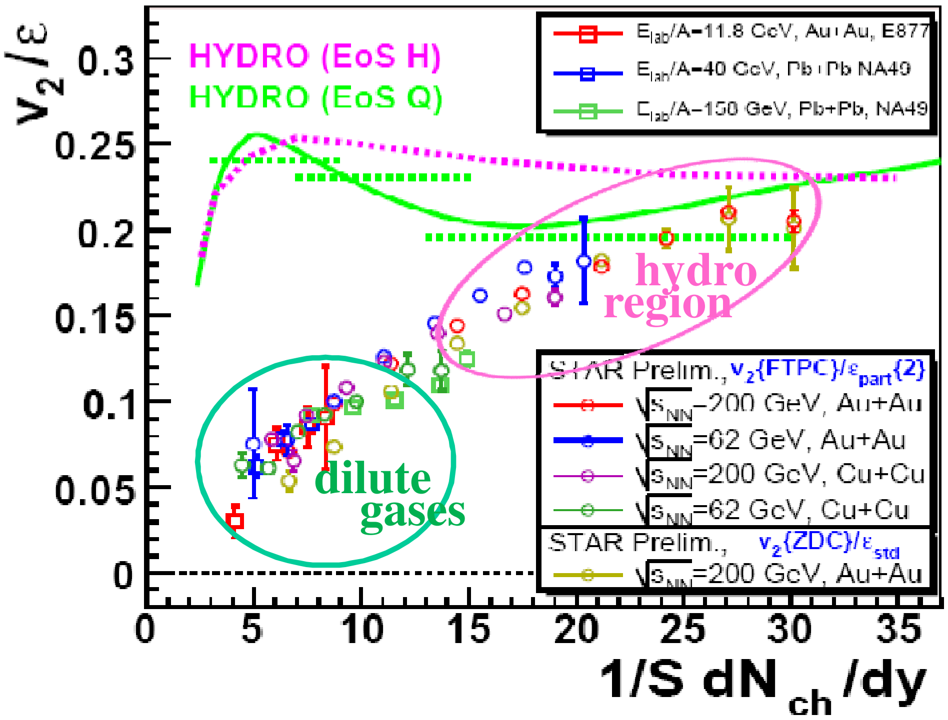
- freeze-out condition introduces time scale, breaking scale invariance of id. hydro eqns.
- Initial profiles for Cu+Cu and Au+Au systems are not identical after a rescaling

Viscous hydrodynamics: additional scale breaking by shear viscosity, resulting in fine structure of v_2/ε:

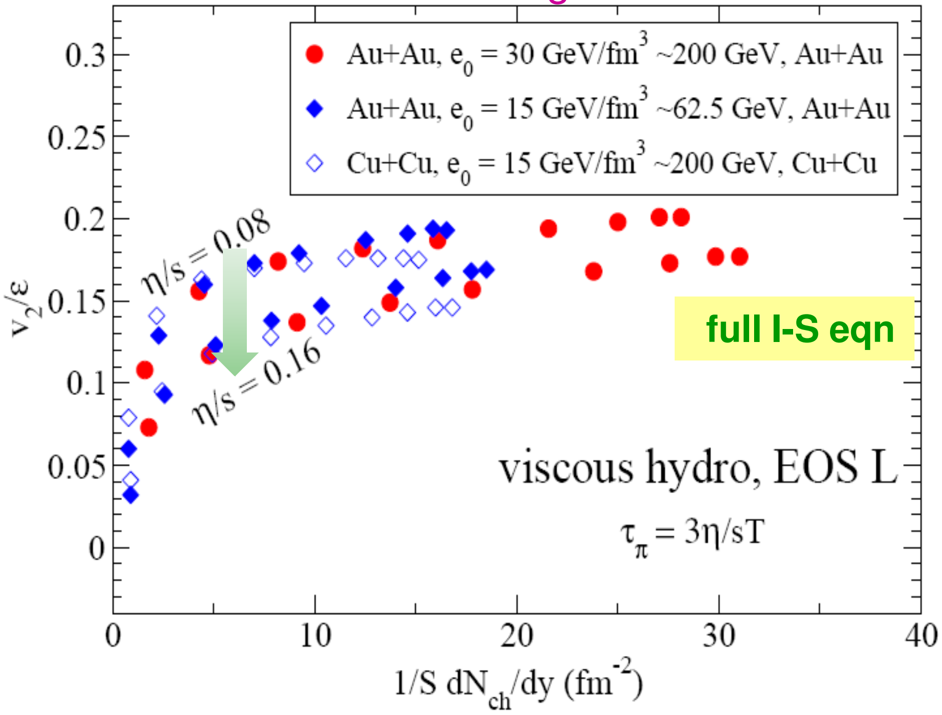
- for similar initial energy density, Cu+Cu curves are slightly below the Au+Au curves
- at fixed $\frac{1}{S}\frac{dN_{ch}}{dy}$, the $e_0 = \mathbf{15GeV/fm}^3$ curves are slightly above the $e_0 = \mathbf{30GeV/fm}^3$ ones

Viscous effects are larger for smaller systems and lower collision energies

Multiplicity scaling of v_2/ε EOS L



Song & Heinz PRC 08



- experimental data show qualitatively similar fine ordering as viscous hydro prediction
- to reproduce slope of v_2/ε vs. (1/S)dN/dy, a better description of the highly viscous hadronic stage is needed: *T*-dependent η/s, viscous hydro + hadron cascade
- the experimental v_2/ε vs. (1/S)dN/dy scaling (slope and fine structure) is another good candidate to constrain) η/s
- this requires, however, experimental and theoretical improvements: reduced error bars, accounting for *T*-dependence of $\eta/s, \zeta/s$ near T_c, modeling hadronic phase with realistic cascade

A Short Summary

- v_2 is sensitive to η/s

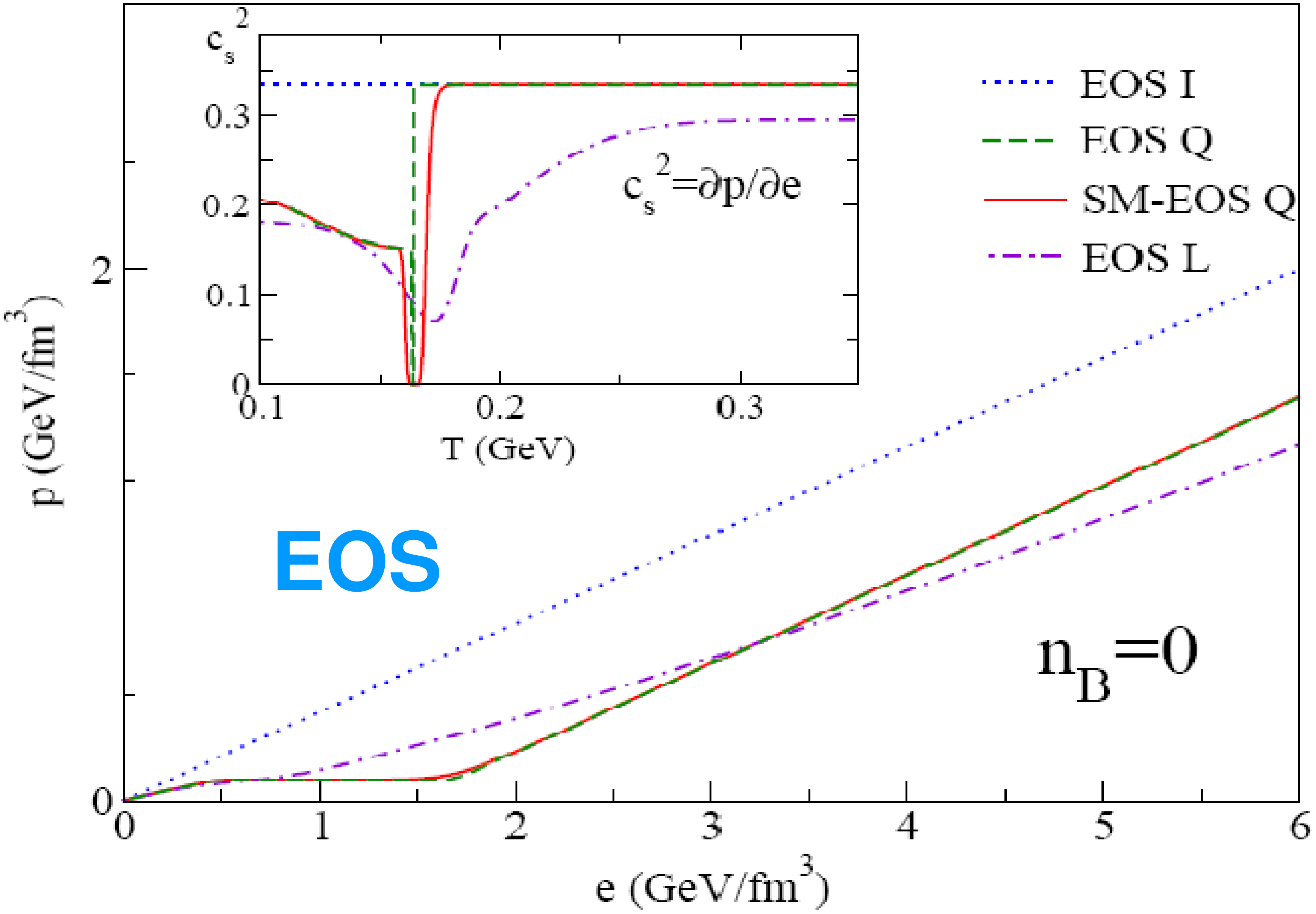
- multiplicity scaling of v_2/ε is a good candidate to extract the QGP viscosity:
  - larger viscous effects in smaller systems and at lower collision energies

-When extracting QGP viscosity from experimental data, bulk viscosity effects should not be neglected

-More theoretical inputs are needed for bulk viscosity:
  - relaxation time
  - initialization for bulk pressure
  - bulk viscosity of hadronic phase, etc

Thank You

EOS


EOS I
EOS Q
SM-EOS Q
EOS L
c_s^2
$c_s^2 = \partial p/\partial e$
0
0.1
0.2
0.3
0.1
0.2
0.3
T (GeV)
$n_B = 0$
p (GeV/fm^3)
e (GeV/fm^3)
0
2
0
1
2
3
4
5
6

EOS

Viscous hydro in 2+1-dimension

$$\partial_\mu T^{\mu\nu}(x)=0 \qquad T^{\mu\nu}=(e+p+\Pi)u^\mu u^\nu-(p+\Pi)g^{\mu\nu}+\pi^{\mu\nu}$$

$$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta}=-\frac{1}{\tau_\pi}\left[\pi^{\mu\nu}-2\eta\sigma^{\mu\nu}\right]-\frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\lambda\left(\frac{\tau_\pi}{\eta T}u^\lambda\right)$$

$$D\Pi=-\frac{1}{\tau_\Pi}\left[\Pi+\zeta(\partial\cdot u)\right]-\frac{1}{2}\Pi\frac{\zeta T}{\tau_\Pi}\partial_\lambda\left(\frac{\tau_\Pi}{\zeta T}u^\lambda\right)$$

Bjorken approximation: (τ, x, y, η) coordinates 3+1⇨ 2+1

--the transport equations for energy momentum tensor are explicit written as:

$$\frac{1}{\tau}\partial_\tau(\tau T^{\tau\tau})+\partial_x(T^{\tau x})+\partial_y(T^{\tau y})=-\frac{p+\Pi+\tau^2\pi^{\eta\eta}}{\tau}$$

$$\frac{1}{\tau}\partial_\tau(\tau T^{\tau x})+\partial_x((T^{\tau x}-\pi^{\tau x})v_x)+\partial_y((T^{\tau x}-\pi^{\tau x})v_y)=-\partial_x(p+\Pi+\pi^{xx})-\partial_y\pi^{xy}$$

$$\frac{1}{\tau}\partial_\tau(\tau T^{\tau y})+\partial_x((T^{\tau y}-\pi^{\tau y})v_x)+\partial_y((T^{\tau y}-\pi^{\tau y})v_y)=-\partial_y(p+\Pi+\pi^{yy})-\partial_x\pi^{xy}$$

-shear tensor decelerate longitudinal expansion, but accelerate transverse expansion

-bulk pressure decelerates both longitudinal & transverse expansion (bulk pressure effectively softens the EoS near the QCD phase transition)

Viscous hydro: a short summary for shear viscosity

--shear viscosity only

-2+1-d viscous hydro code individually developed by different groups:

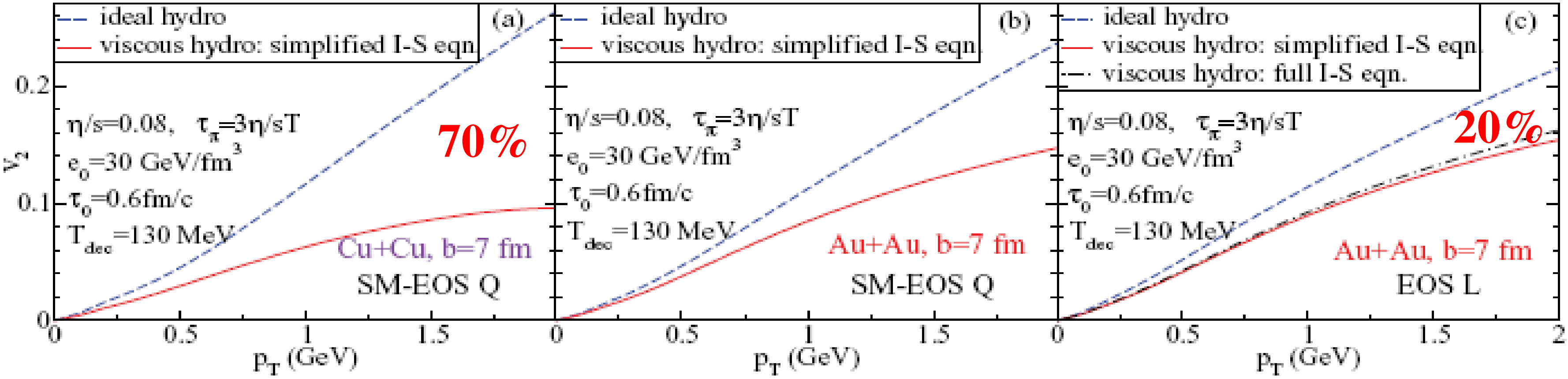
Romatsche & Romatschke (INT), Song & Heinz (OSU), Dusling & Teaney (Stony)
Huovinen & Molnar (Purdue), Chaudhuri (Kolkata, India)

-v_2 at RHIC is sensitive to even the minimum shear viscosity entropy ratio

-v_2 suppression from different groups ranges from 20% to 70%

-the above discrepancy was largely resolved by investigating effects from system size, EoS and different forms of I-S eqns. used

Song & Heinz, PRC 78 (2008)



-Code checking within the TECHQM collaboration:

TECHQM webpage

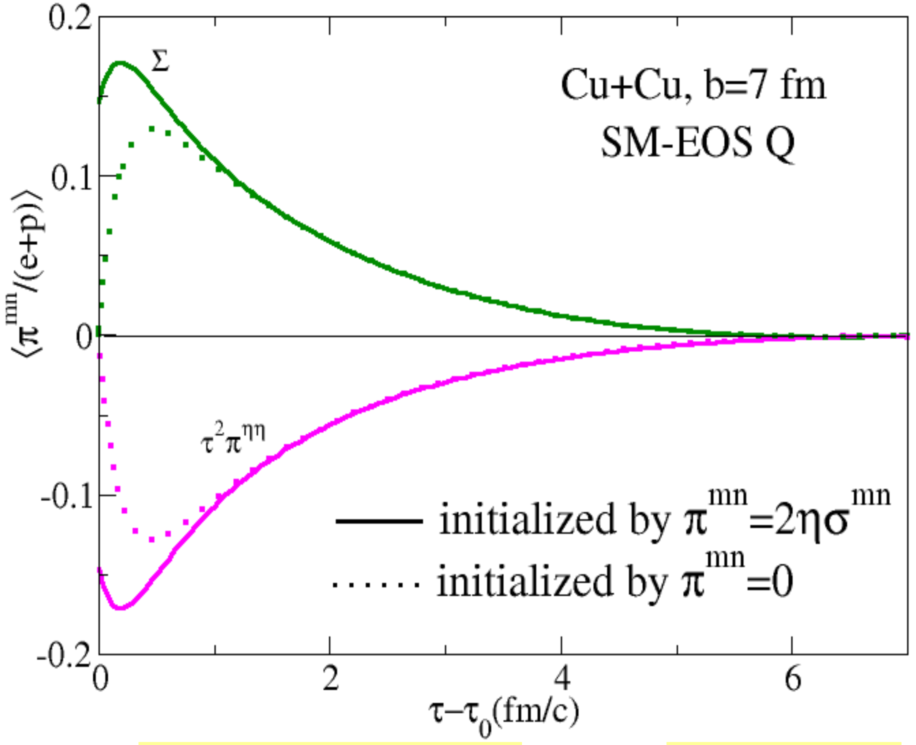
-The first attempt to extract QGP shear viscosity from RHIC data:

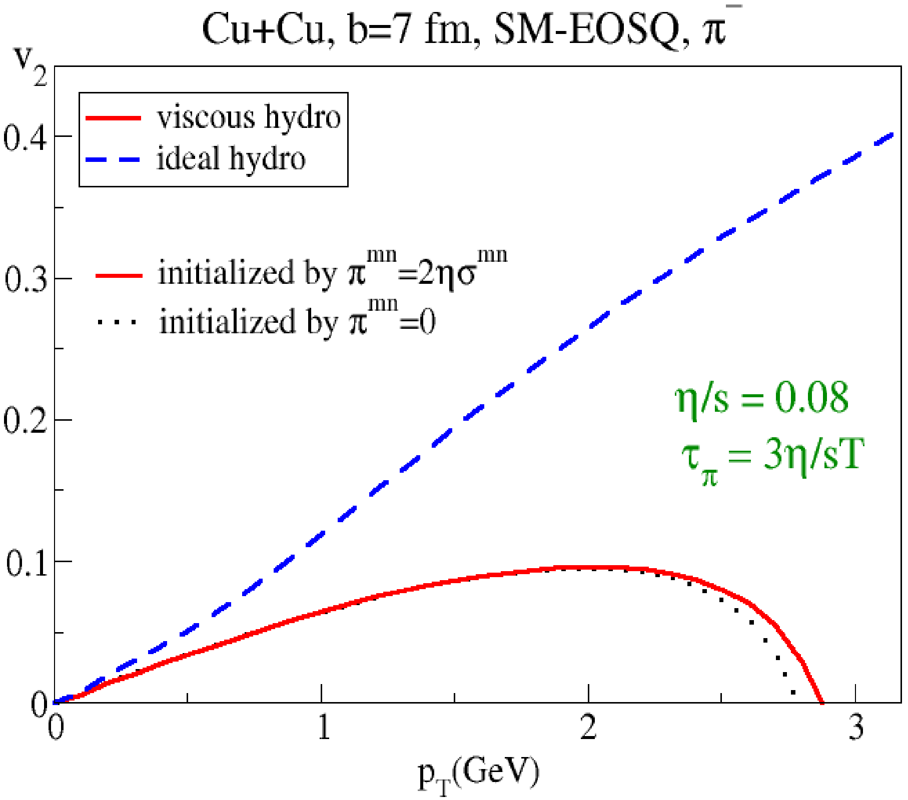
Luzum & Romatschke, PRC 78 (2008)

Effects from initialization of π^{mn}

Song & Heinz, PLB08 & PRC 77(2008)

$$\Sigma = \pi^{xx} + \pi^{yy}$$





$\pi^{mn} = 2\eta\sigma^{mn}$ **vs.** $\pi^{mn} = 0$

$\tau_\pi = 3\eta/sT$

-after ~1fm/c (several relaxation times), viscous pressure loses memory of initial cond.

- v_2 is insensitive to different initializations of π^{mn}

-Effects on entropy production: ~20%