Gauge-Invariant Localization of Infinitely Many Gravitational Energies from All Possible Auxiliary Structures

J. Brian Pitts
University of Notre Dame
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Energy Localization Debate from 1910s

- Pseudotensor? 1916 (Einstein, 1923) for *vs.* (Schrödinger, 1918; Bauer, 1918) against.
- Reviews (Fletcher, 1960; Trautman, 1962; Goldberg, 1980; Szabados, 2009).
- $\mathfrak{T}^\mu_\nu \sqrt{-g} =_{def} T^\mu_\nu \sqrt{-g} + t^\mu_\nu \sqrt{-g}$ satisfies $\frac{\partial}{\partial x^\mu}(\mathfrak{T}^\mu_\nu \sqrt{-g}) = 0$ if (and only if) $G^\mu_\nu = T^\mu_\nu$. Here $t^\mu_\nu \sqrt{-g}$ is gravity pseudotensor.
- Problem is not lack of conserved quantities, but *too many* and lack of relationships among them (Anderson, 1967).
- Pseudotensors aren't tensors, not even geometric objects.
- Many (1st derivatives) can vanish at any point or worldline.
- Mix of coordinate artifacts and real physics, it seems.

Tensorial Energy from Additional Background Structure?

- Flat background metric (Rosen, 1963; Cornish, 1964).
- Coordinate change $g_{\sigma\rho} \to e^{\pounds_\xi} g_{\sigma\rho}, u \to e^{\pounds_\xi} u, \eta_{\mu\nu} \to e^{\pounds_\xi} \eta_{\mu\nu}$. u is matter (Grishchuk et al., 1984; Petrov, 2008).
- Problem of coordinate dependence reappears as gauge dependence: $g_{\sigma\rho} \to e^{\pounds_\xi} g_{\sigma\rho}, u \to e^{\pounds_\xi} u, \eta_{\mu\nu} \to \eta_{\mu\nu}$.
- $\eta_{\mu\nu} \to e^{\pounds_\xi} \eta_{\mu\nu}$ affects energy density but no physical meaning.
- Flat connection $\Gamma^\alpha_{\mu\nu} \to e^{\pounds_\xi} \Gamma^\alpha_{\mu\nu}$ (Sorkin, 1991; Fatibene and Francaviglia, 2003) has analogous problem.
- Orthonormal tetrad e^μ_A: extra local $O(3,1)$ (Møller, 1964).
- Just moving lump in carpet, not flattening it out, with *a* flat metric, connection or tetrad: gauge-dependent energy.

Traditional Coping Mechanism

- Blame the question and invoke equivalence principle *ad hoc*: "[a]nybody who looks for a magic formula for 'local gravitational energy-momentum' is looking for the right answer to the wrong question." (Misner et al., 1973, p. 467)
- But Noether's theorems don't care about the equivalence principle. Maybe a different kind of invariance fits what Noether yields?
- Value of messy math as opposed to geometrical shortcuts and picture-thinking (Kiriushcheva et al., 2008; Brown, 2005).
- ∞ many conserved energies (Bergmann, 1958; Komar, 1959).
- Drop *unique* localization, find new kind of co/invariance?

No Need to Assume Energy is Unique

- Universal tacit undefended assumption of just one energy: 10 (or 16) components in one chart should suffice locally.
- (Goldberg, 1980) (Szabados, 2009, section 3.1.3); (Faddeev, 1982): "The energy of the gravitational field is not localized, i.e., a uniquely defined energy density does not exist."
- But *no reason to believe in uniqueness*—just a habit, default.
- ∞ many energies (Bergmann, 1958; Komar, 1959; Anderson, 1967; Regge and Teitelboim, 1974; Fatibene and Francaviglia, 2003). Any vector field gives one! *Why can't they all be real?*
- Uniqueness should have been rejected, localization could have been discovered 50 years ago by Bergmann.

Gauge Independence of Energ**ies** from **All** Background Structure**s**: Case of Flat Metrics

- Don't choose one background; choose them all! $\{(\forall \eta_{\rho\sigma})\ \eta_{\rho\sigma}\}$.
- Gauge invariance: nothing depends on arbitrary choice of $\eta_{\rho\sigma}$.
- Infinite-component invariant energy $\{(\forall \eta_{\rho\sigma})\ t^{\mu\nu}[g_{\alpha\beta}, \eta_{\rho\sigma}]\}$, each conserved: $\partial_{1\mu}(\sqrt{-g}T^{\mu\nu} + \sqrt{-g}t^{\mu\nu}[g, \eta_1]) = 0$, $\partial_{1\mu}\eta_{1\alpha\beta} \equiv 0$, $\partial_{2\mu}(\sqrt{-g}T^{\mu\nu} + \sqrt{-g}t^{\mu\nu}[g, \eta_2]) = 0$,....
- Flat connection is analogous, but angular momentum problems (Chang et al., 2000) (*c.f.*) (Goldberg, 1958).
- $\{(\forall \eta_{\rho\sigma})\ \eta_{\rho\sigma}\}$ a bit like group averaging (Marolf, 2002), but only collect into set, not add, gauge-dependent elements.

Critique of Komar's Vector, Møller's Orthonormal Tetrad

- Komar's conserved quantities tensorial, depend on vector field.
- Problem: factor of 2 wrong answers for Komar (Katz, 1985; Katz and Ori, 1990; Iyer and Wald, 1994; Petrov and Katz, 2002; Fatibene and Francaviglia, 2003).
- Møller's orthonormal tetrad: accept them all?
- Worry: local Lorentz $O(3,1)$ group is gratuitous, so gauge invariance from *all tetrads* is Pickwickian.
- Flat metric $\eta_{\rho\sigma}$, flat connection $\Gamma^{\alpha}_{\rho\sigma}$ give invariant localization.
- $\eta_{\rho\sigma}$ or $\Gamma^{\alpha}_{\rho\sigma}$ needed in action S (Faddeev, 1982; Hawking and Horowitz, 1996; Fatibene and Francaviglia, 2003).
- Gauge-invariant multi-action $S[g_{\mu\nu}, \{(\forall \eta_{\rho\sigma})\ \eta_{\rho\sigma}\}]$.

Spinorial Almost Geometric Objects, So No Exception

- Spinors in coordinates (Ogievetskiĭ and Polubarinov, 1965; Bilyalov, 2002; Gates et al., 1983): $\langle r_{\mu\nu}, \psi \rangle$, where $r_{[\mu\alpha]} = 0$, $g_{\mu\nu} = \sum_{\alpha=A} \sum_{\beta=B} r_{\mu\alpha} \eta^{AB} r_{\beta\nu}$: symmetric square root.
- $\langle r_{\mu\nu}, \psi \rangle$ nonlinear spinor representation of coordinate transformations (up to sign), linear for conformal subgroup.
- No tetrad, no local Lorentz group; $r_{\mu\alpha}$ locally like tetrad in symmetric gauge.
- Any coordinates, but swap to get time first (Bilyalov, 1992).
- $\langle g_{\mu\nu}, \psi \rangle$ equivalent to $\langle r_{\mu\nu}, \psi \rangle$. Lie, covariant derivatives defined (Ogievetskiĭ and Polubarinov, 1965; Szybiak, 1966).
- So energy localization proposed here suits spinors also.

Pseudotensor in All Coordinates as Invariant Localization

- Tensors $t^{\mu\nu}$ from $\eta_{\rho\sigma}$ or $\Gamma^{\alpha}_{\rho\sigma}$: clear transformation properties.
- Ignoring global issues, can fix coordinates: each flat metric in Cartesian coordinates: $\eta_{RS} = diag(-1, 1, 1, 1), \Gamma^{A}_{RS} = 0$.
- Yields pseudotensor in $g_{\mu\nu}$ in every gauge/coordinate system.
- Invariant: for every chart U, $\{\forall U \; t^{\mu}_{\nu}[g_{\mu\nu}, \eta_{MN}]\}$.
- A *pseudotensor* is *okay*; just take it in *all* coordinate systems.
- Not ∞ many faces of same entity as with tensor, but ∞ many distinct entities, each in own adapted coordinate system.
- Not just metric, but natural bases enter definition of energy.
- Good pseudotensor maybe same for all solutions (Katz et al., 1997), maybe not (Nester, 2004).

Objections to Pseudotensors Assume Unique Energy

- Worry: t^{μ}_{ν} vanishing at point or worldline in some coordinates.
- Reply: some but not all energies vanish there.
- Worry: Minkowski in unimodular spherical coordinates has nonzero Einstein pseudotensor (Bauer, 1918; Pauli, 1921).
- Reply: different coordinate system gives different energies.
- Worry: total energy in these coordinates diverges.
- Reply: spherical coordinate singularities, strongly curved basis.
- Worry: Einstein pseudotensor 0 [for $r > 2M$] in Schwarzschild in $\approx$ Cartesian $\sqrt{-g} = 1$ coordinates (Schrödinger, 1918).
- Reply: Many energies exist; some vanish outside horizon, some don't (Petrov, 2005; Petrov, 2008).

Noether Operator Is Invariant in New Sense

- Non-GR fields: $\nabla_\mu (T^{\mu\nu} \sqrt{-g} \xi_\nu) = (T^{\mu\nu} \sqrt{-g} \xi_\nu),_\mu = 0$: conserved vector density $T^{\mu\nu} \sqrt{-g} \xi_\nu$ is algebraic in ξ^ν.
- For GR, Noether's theorem gives nontensorial differential operator in ξ^ν (Schutz and Sorkin, 1977; Sorkin, 1977; Thirring and Wallner, 1978; Szabados, 1992).
- Feeding it natural basis yields pseudotensor components.
- Feeding it *all* natural bases yields pseudotensor in all coordinates: invariant in sense proposed here.
- Not unique due to possibility of adding curls.
- But even scalar fields in flat space-time have a bit of that problem (Callan et al., 1970): "improved" stress tensor.

Logical Equivalence of All Conservation Laws to Einstein's Equations

- Fields: conservation because every field has Euler-Lagrange equations or (generalized) Killing vectors (Trautman, 1966).
- GR (without $\eta_{\rho\sigma}$ or $\Gamma^{\alpha}_{\rho\sigma}$): every field has E-L equations.
- GR: conservation from Einstein's equations without using matter equations (Anderson, 1967; Wald, 1984).
- Coordinate form $\frac{\partial}{\partial x^{\mu}}(\mathfrak{T}^{\mu}_{\nu}\sqrt{-g}) = 0$ in all coordinates.
- Sheds light on relation of GR to first law of thermodynamics: GR obviously entails it.

Logical Equivalence of All Conservation Laws to Einstein's Equations

- Reverse entailment also holds (Anderson, 1967): pseudotensor law in all coordinates entails Einstein's equations!
- Illuminates spin-2 derivations of Einstein's equations (Einstein and Grossmann, 1996; Deser, 1970; Pitts and Schieve, 2001).
- Clearly, nothing logically equivalent to Einstein's equations depends viciously on coordinates.
- Thus pseudotensor laws don't depend viciously on coordinates.
- Pseudotensor laws give invariant localization of energy.
- Another way to see that pseudotensor localization is real.

Angular Momentum Localization

- Generalization straightforward: invariant ∞-component angular momentum. Depends on x^μ explicitly.
- Matrix $diag(-1,1,1,1)$ helps with angular momentum (Chang et al., 2000) (*c.f.*) (Goldberg, 1958).
- Symmetric total energy-momentum $\sqrt{-g}\mathfrak{T}^{\mu\nu}$ gives angular momentum complex $\mathfrak{M}^{\mu\nu\alpha} =_{def} \sqrt{-g}\mathfrak{T}^{\mu\nu}x^\alpha - \sqrt{-g}\mathfrak{T}^{\mu\alpha}x^\nu$.
- $\frac{\partial}{\partial x^\mu}\mathfrak{M}^{\mu\nu\alpha} = 0$ due to $\frac{\partial}{\partial x^\mu}(\sqrt{-g}\mathfrak{T}^{\mu\nu}) = 0$ and $\mathfrak{T}^{[\mu\nu]} = 0$.
- Choice of pseudotensor (Katz et al., 1997; Nester, 2004) affects distributions of angular momenta.
- Using $\{(\forall\eta_{\rho\sigma})\ \eta_{\rho\sigma}\}$, position 4-vectors replace x^μ.

Conceptual Benefits of Local Energy Conservation

Lack of local energy conservation in GR in general or Big Bang cosmology has been invoked for unwarranted conclusions, such as:

1. GR is false (Logunov and Folomeshkin, 1977; Logunov et al., 1986)—addressed in (Faddeev, 1982; Zel'dovich and Grishchuk, 1988; Grishchuk, 1990);
2. Big Bang cosmology is false (by Robert Gentry)—addressed in (Pitts, 2004a; Pitts, 2004b);
3. Big Bang cosmology is plausibly true and yet violates a principle so fundamental as to transcend physics into metaphysics (Bunge, 2000);

4. Big Bang cosmology is a heat sink for anomalous terrestrial heat (by D. Russell Humphreys)—addressed in (Pitts, 2009);
5. GR makes it easier than other theories for souls to affect bodies (Collins, 2008)—addressed in (Pitts, 2010);
6. Universes with zero total energy can pop into being without violating energy conservation (Tryon, 1973; Thirring, 2003).

- Concerning **6**, *all* energy densities needed for gauge invariance must vanish; impossible except maybe in boring cases.
- Detailed analysis undermines these 6 conclusions, if one tries. But new claims keep arising.
- Play with matches and keep fire extinguishers handy?
- Such claims would be unthinkable in ordinary field theories due to well-known gauge-invariant local conservation laws.

Conclusions

- Ironic that Einstein took energy conservation as criterion for GR equations, yet GR is widely held to have no such law.
- Irony resolved by ∞-component gauge-invariant localization.
- Natural bases, not just metric, determine energies.
- GR logically equivalent to conservation of ∞ many energies, hence *more* conserving of energy than other theories.
- Gauge (in)dependence largely orthogonal to question of 'right answers' for the conserved quantities.
- Non-uniqueness of relocalizing (Anderson, 1967), as in other field theories. Maybe case-by-case (Nester, 2004).
- Best functional form technically, make covariant in above way.

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