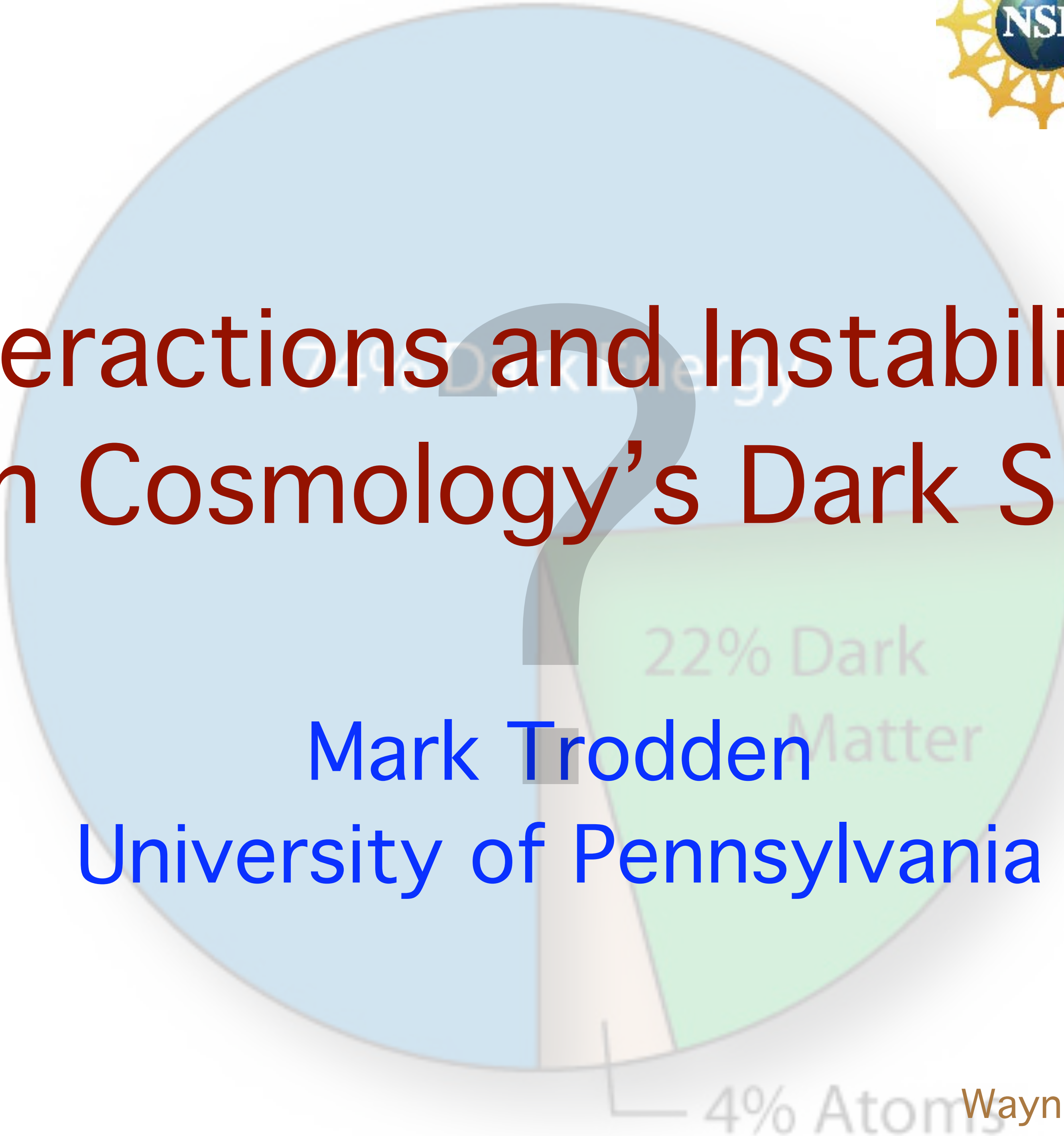
Interactions and Instabilities on Cosmology's Dark Side



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Goals

- The fundamental structure and interactions of the dark sector may be significantly more complex than our simplest models.
- Explore in some generality a subclass of models - those with interactions between dark matter and dark energy, and perhaps with multiple dark components.
- Identify a rather general instability - the adiabatic instability.
- Demonstrate this in a simple model

Coupled Dark Models

There exist many examples (not comprehensive):

O.E. Bjaelde, A.W. Brookfield, C. van de Bruck, S. Hannestad, D.F. Mota, L. Schrempp and D. Tocchini-Valentini, *Neutrino Dark Energy -- Revisiting the Stability Issue*, arXiv:0705.2018 [astro-ph].

J. Khoury, A. Weltman, *Chameleon fields: Awaiting surprises for tests of gravity in space, Phys. Rev. Lett.* **93**, 171190 (2004).

J. Khoury and A. Weltman, *Chameleon cosmology, Phys. Rev.* **D69**, 044026 (2004).

D.F. Mota and D.J. Shaw, *Strongly coupled chameleon fields: New horizons in scalar field theory, Phys. Rev. Lett.* **97**, 151102 (2006)

R. Fardon, A.E. Nelson and N. Weiner, *Dark energy from mass varying neutrinos, JCAP* **0410**, 005 (2004)

R. Fardon, A.E. Nelson and N. Weiner, *Supersymmetric theories of neutrino dark energy, JHEP* **0603**, 042 (2006)

D.B. Kaplan, A.E. Nelson and N. Weiner, *Neutrino oscillations as a probe of dark energy, Phys. Rev. Lett.* **93**, 091801 (2004)

S.M. Carroll, V. Duvvuri, M. Trodden and M.S. Turner, *Is cosmic speed-up due to new gravitational physics?, Phys. Rev.* **D70**, 043528 (2004)

S.M. Carroll, I. Sawicki, A. Silvestri and M. Trodden, *Modified-Source Gravity and Cosmological Structure Formation, New J. Phys.* **8**, 323 (2006)

G.R. Farrar and P.J.E. Peebles, *Interacting dark matter and dark energy, Astrophys. J.* **604**, 1 (2004)

N. Afshordi, M. Zaldarriaga and K. Kohri, *On the stability of dark energy with mass-varying neutrinos, Phys. Rev.* **D72**, 065024 (2005)

R. Bean, E. Flanagan and M.T., [arxiv:0709.1124]
R. Bean, E. Flanagan and M.T., [arxiv:0709.1128])
R. Bean, E. Flanagan, I. Laszlo and M.T., [...]

Modeling Dark Couplings

Consider writing a general action as

$$S = S[g_{ab}, \phi, \Psi_j] = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right] + \sum_j S_j [e^{2\alpha_j(\phi)} g_{ab}, \Psi_j]$$

We'll focus just on DM/DE couplings here $\alpha_j(\phi) = \alpha(\phi)$

Model matter as a perfect fluid. EOMs become

$$M_{pl}^2 G_{ab} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} (\nabla\phi)^2 - V(\phi) g_{ab} + e^{4\alpha(\phi)} [(\bar{\rho} + \bar{p}) u_a u_b + \bar{p} g_{ab}]$$

$$\nabla_a \nabla^a \phi - V'(\phi) = \alpha'(\phi) e^{4\alpha(\phi)} (\bar{\rho} - 3\bar{p})$$

Coupled DM/DE

For DM, need $\bar{p} = 0$

Define $\rho \equiv e^{3\alpha(\phi)}\bar{\rho}$ EOMs become

$$M_{pl}^2 G_{ab} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}(\nabla\phi)^2 - V(\phi)g_{ab} + e^{\alpha(\phi)}\rho u_a u_b$$

$$\nabla_a\nabla^a\phi - V'_{eff}(\phi) = 0 \qquad V_{eff}(\phi) = V(\phi) + e^{\alpha(\phi)}\rho$$

$$\nabla_a(\rho u^a) = 0 \qquad u^b\nabla_b u^a = -(g^{ab} + u^a u^b)\nabla_b\alpha$$

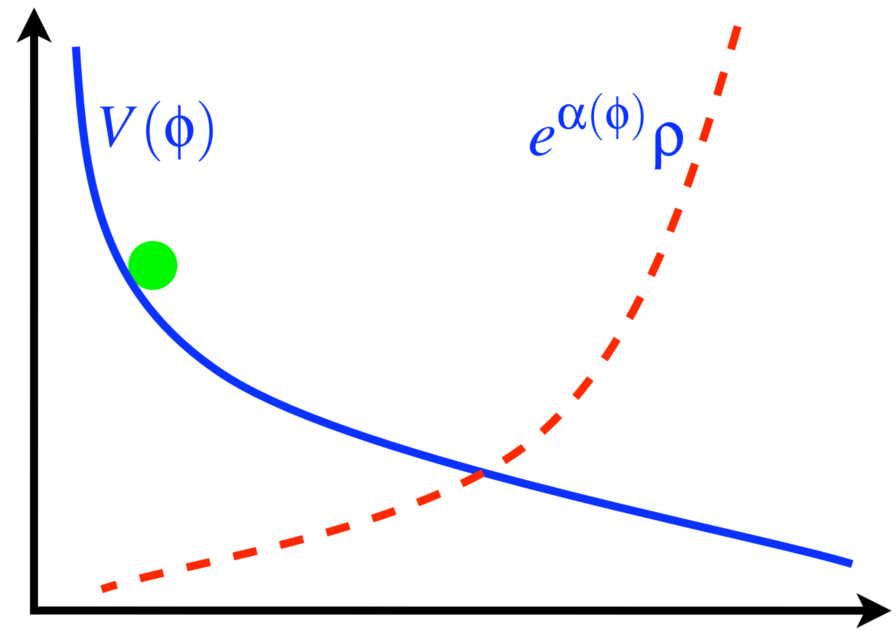
The Adiabatic Regime

Interested in a particular regime:

DE field adiabatically tracks minimum of effective potential

Define background solution $\phi_m(\rho)$ by

$$V'_{eff}(\phi) = V'(\phi) + \alpha'(\phi)e^{\alpha(\phi)}\rho = 0$$



So effective equations of motion are now

$$p_{eff}(\rho) = -V[\phi_m(\rho)]$$

$$M_{pl}^2 G_{ab} = [(\rho_{eff} + p_{eff})u_a u_b + p_{eff} g_{ab}]\, \rho_{eff}(\rho) = e^{\alpha[\phi_m(\rho)]}\rho + V[\phi_m(\rho)]$$

Observed Value of w

$$w_{obs}(a) = -1 - \frac{1}{3}\frac{d\ln\rho_{DE}}{d\ln a}$$

[This is an important point in general when it comes to interpreting cosmic acceleration]

$$w_{obs} = \frac{-1}{1 - \frac{d\ln V}{d\alpha}\left(1 - e^{\alpha_0 - \alpha}\right)}$$

Note: w<-1 today, *for all models*!

Also, can expand about a=1

$$\frac{1}{w_{obs}} \simeq -1 + \ln(V/V_0)$$

So w<-1 in the recent past, *for all models*!

The Adiabatic Instability

(Bean, Flanagan and M.T., [arxiv:0709.1124], [arxiv:0709.1128])

There exist several ways to look at the instability that may occur in the adiabatic regime. We'll just briefly touch on them.

The Hydrodynamic Viewpoint

Recall that $$\rho_{eff} = V + e^{\alpha}\rho = V - \frac{dV/d\phi}{d\alpha/d\phi} = V - \frac{dV}{d\alpha}$$

The adiabatic sound speed is then given by

$$\frac{1}{c_a^2} = \frac{d\rho_{eff}}{dp_{eff}} = \frac{d\rho_{eff}/d\alpha}{dp_{eff}/d\alpha} = \frac{\frac{d}{d\alpha}\left[V - \frac{dV}{d\alpha}\right]}{\frac{d}{d\alpha}\left[-V\right]} = -1 + \frac{\frac{d^2V}{d\alpha^2}}{\frac{dV}{d\alpha}}$$

In adiabatic regime $c_s^2(k,a) \equiv \dfrac{\delta P(k,a)}{\delta\rho(k,a)} \rightarrow c_a^2$

Instability if $c_s^2 < 0$ occurs if $c_a^2 < 0$

Jeans Viewpoint

In Einstein frame instability is nothing to do with gravity - all about the scalar and the coupling.

But in Jordan frame, instability involves gravity - mediated partly by a tensor interaction and partly by a scalar interaction.

Effective Newton's constant for self interaction of dark matter

$$G_{cc} = G\left[1 + \frac{2^2\alpha'(\phi)^2}{1 + \frac{m_{eff}^2}{k^2}}\right]$$

Tensor Scalar

At long scales $G_{cc} \approx G$ and at short ones $G_{cc} \approx G[1 + 2^2(\alpha')^2]$

But for $M_p|\alpha'| \gg 1$ have an intermediate range of length-scales

[Important condition, missed in lots of other treatments]

$$\frac{m_{eff}}{|\alpha'|} \ll k \ll m_{eff}$$

where

$$G_{cc} \approx G\frac{2^2(\alpha')^2}{m_{eff}^2}k^2$$

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{cc}e^{\alpha}\rho = 0$$

Hubble damping is ineffective - Jeans instability causes approximate exponential growth rather than power law growth.

e.g. 2-component DM Models

Two components. One not coupled, one coupled, exponential potential, constant coupling

$$3^2H^2 = V + e^{\alpha}\rho_{co} + \rho_c$$

Fractional amount in coupled must be small in large coupling limit

Evolution equations for fractional density perturbations in the adiabatic limit on subhorizon scales given by

$$\ddot{\delta}_j + 2H\dot{\delta}_j - 4\pi\sum_k G_{jk}\rho_k e^{\alpha_k}\delta_k = 0$$

$$\ddot{\delta}_c + 2H\dot{\delta}_c = \frac{1}{2^2}\rho_c\delta_c + \frac{1}{2^2}e^{\alpha}\rho_{co}\delta_{co}$$

$$\ddot{\delta}_{co} + 2H\dot{\delta}_{co} = \frac{1}{2M_p^2}\rho_c\delta_c + \frac{1}{2M_p^2}\left[1 + \frac{2\beta^2C^2}{1+\frac{m_{eff}^2a^2}{k^2}}\right]e^{\alpha}\rho_{co}$$

Effective equation of state (black full line)

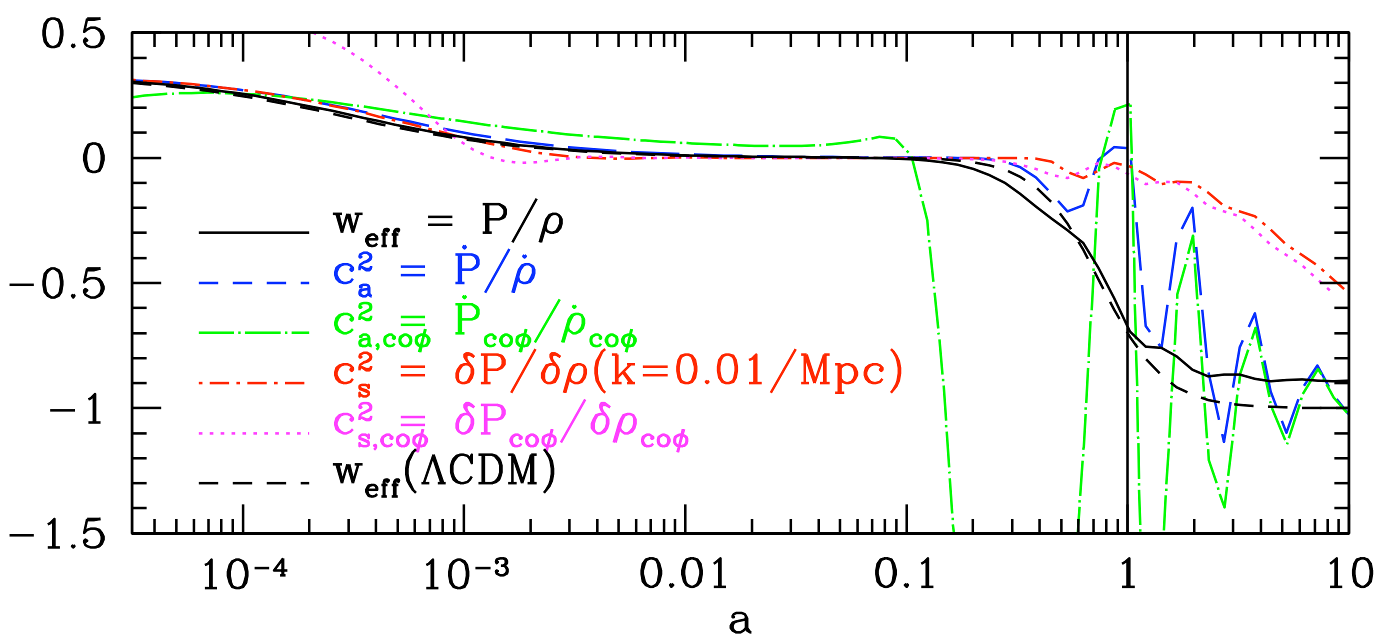
Adiabatic speed of sound, for all components (blue long dashed line)

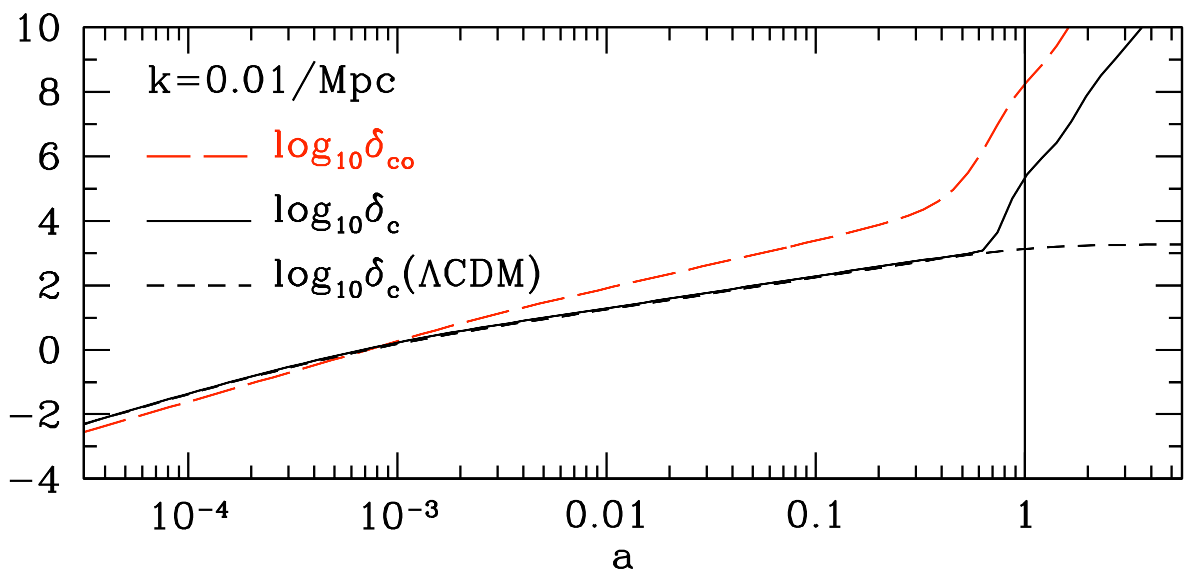
For the coupled components only (green dot long dashed line)

Effective speed of sound for all components (red dot-dashed line)

For the coupled components alone (magenta dotted line).

Effective equation of state for LCDM with $\Omega_c = 0.25$, $\Omega_b = 0.05$, and $\Omega_\Lambda = 0.05$ (black dashed line)





Fractional over-density for coupled CDM component (red long dashed line)
For uncoupled component (black full line)
For LCDM (black dashed line).

At late times adiabatic behavior triggers a dramatic increase in the rate of growth of both uncoupled and coupled components, leading to structure predictions inconsistent with observations.

2-component DM instability

These models are example of a class of theories for which

- the background cosmology is compatible with observations,
- but which are ruled out by the adiabatic instability of the perturbations.

Summary

It may be that there are new energy components that are driving acceleration.

Or, perhaps the response of spacetime to existing matter/energy components differs from GR at long distances - modified gravity.

In general many kinds of new instabilities can exist

In many particle physics motivated proposals, there may well be nontrivial couplings, either between different energy components, or between them and new gravitational degrees of freedom.
For many such couplings, there exist environments for which the coupled system evolves adiabatically...

... and this can lead to an unacceptable instability - the adiabatic instability

Thank You!

Validity of the Adiabatic Regime

Local Adiabatic Condition: consider a perturbation with timescale or lengthscale L and density ρ

Rough necessary condition for LAC is

$$L \gg m_{eff}^{-1}(\rho) \equiv \left(\frac{\partial^2 V_{eff}}{\partial \phi^2}(\phi, \rho)\Big|_{\phi=\phi_m(\rho)} \right)^{-1/2}$$

More precisely, LAC holds if

$$\frac{d \ln V}{d \ln \rho} \left(\frac{1}{m_{eff}^2 L^2} \right) \ll 1$$

If holds everywhere, adiabatic approx. holds. If not, can fail nonlocally

Can then show are in adiabatic regime and adiabatically unstable in two regions of parameter space

$$\frac{dV}{d\alpha} < \frac{d^2V}{d\alpha^2} < 0 \qquad \text{or} \qquad \frac{dV}{d\alpha} < 0 < \frac{d^2V}{d\alpha^2}$$

Instability operates in a range of scales. Must be long enough and must grow on a time-scale less than the Hubble one

$$m_{eff}^{-1} \ll L \ll \frac{\sqrt{|c_s^2|}}{H}$$

Generally, for a fluid, instability time-scale must be shorter than the gravitational dynamical time. Can put all this together to show

$\frac{L_{max}}{L_{min}} \leq |\alpha'[\phi_m(\rho)]|$ Nonempty only if $|\alpha'| \gg 1$