Computation of the string tension in Yang-Mills theory using large N reduction

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Results on

- 3D string tension (T=0) (arXiv:0807.1315)
- 4D string tension (T=0) (Real soon now)
- 3D string tension ($T>T_c$) (arXiv:0906.3015)

Outline

- Quick results
- Introduction
- Details
- Conclusion

Quick results: T=0

3 Dimensions

- 5^3 lattice
- N = 47
- $b = \frac{1}{g^2 N}$ = 0.6 to 0.8
- Wilson loops 1x1 to 7x7
- $\sqrt{\sigma} b = 0.1964 \pm 0.0009$ (continuum extrapolation)

4 Dimensions

- 6^4 lattice
- N = 47 and 59
- $b = \frac{1}{g^2 N}$ = 0.3450 to 0.3500
- Wilson loops 1x1 to 9x9
- $\sigma a^2 = 0.099 \pm 0.016$ at b=0.3480 N=47

Quick result: $T>T_c$

- L^3 lattice L=4, 5, 6, 7
- N = 59
- $b = \frac{1}{g^2 N}$ = 0.9 to 1.75
- Wilson loops 1x1 to 10x10
- $\sigma a^2 \approx \frac{1}{4bL}$ (High T dimensional reduction)
- $\sigma b^2 \approx \frac{T}{4b}$

Introduction

- Large N
- Large N reduction
- Phase structure
- Project description

Large N

- Expansion parameters $\alpha(Q^2)$ or 1/N
- $N \to \infty$ simplifications
  - Planar graphs
  - Factorization
  - Non-interacting mesons
  - OZI rule
- $1/3 \approx 1/\infty$

Large N reduction

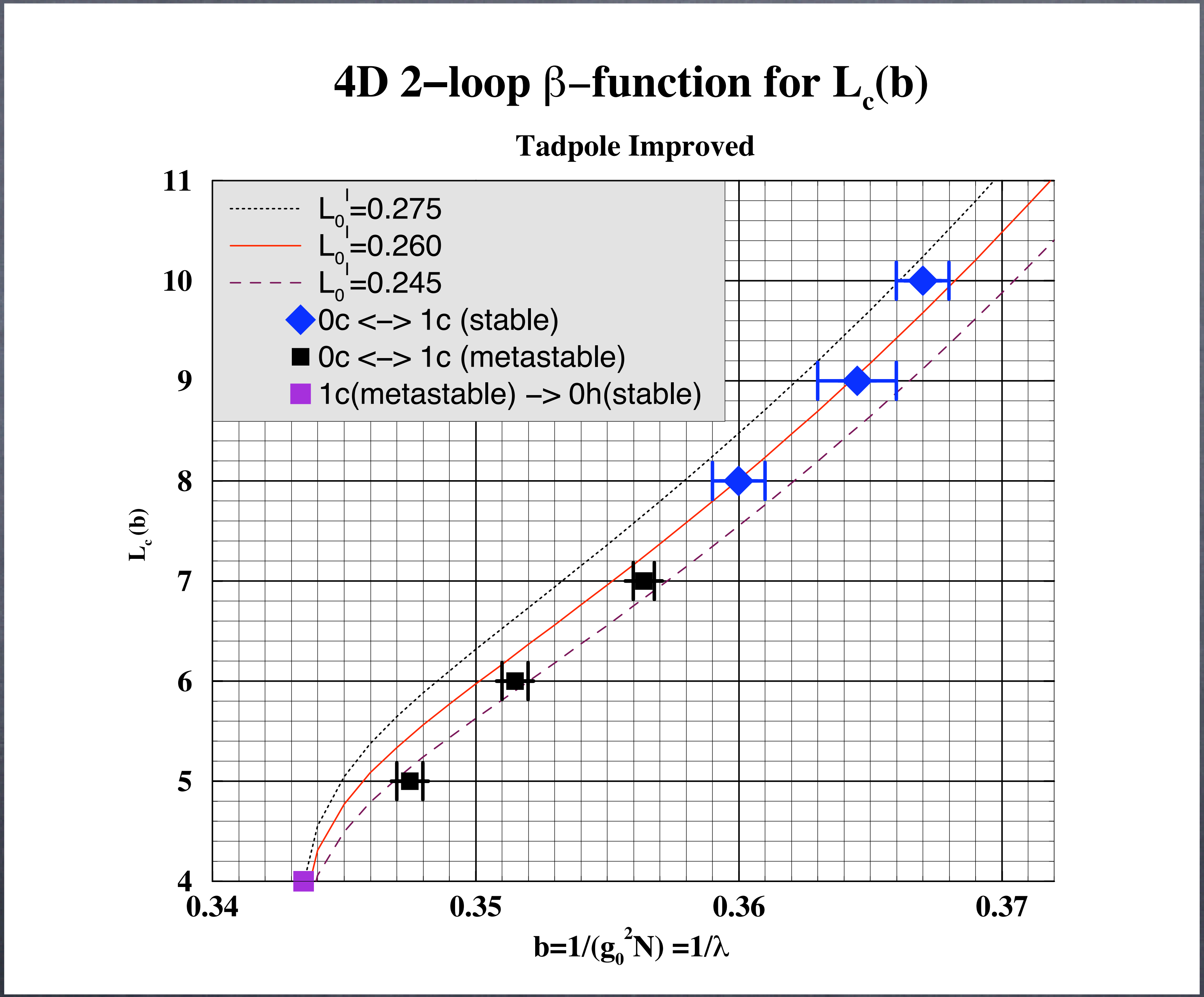
- Reduction to a one point 1^d lattice (Eguchi-Kawai)
- Z_N^d center symmetry
- But broken at weak coupling

Work-arounds

- Quenched E-K
  - But Bringoltz and Sharpe
- Twisted E-K
  - But Teper and Vairinhos
- Continuum or partial reduction
  i.e. reduction to finite physical size
  $l > 1/T_c$

Center symmetry breaking at physical scale

$$Z_N^d \rightarrow Z_N^{d-1} \rightarrow Z_N^{d-2} \rightarrow \ \ldots$$

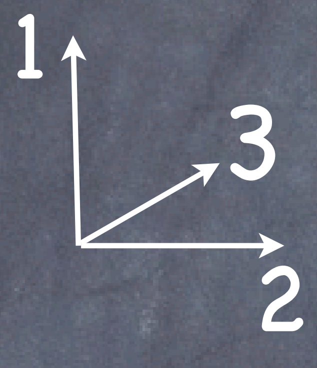


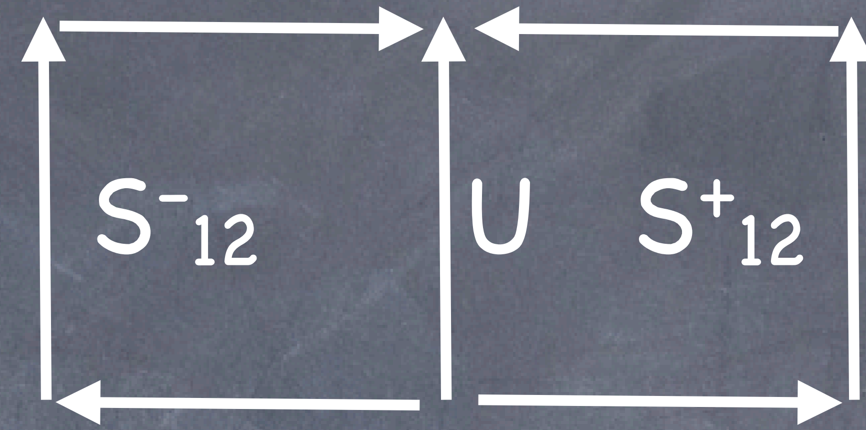
Kiskis, Narayanan, and Neuberger

Details

- Wilson gauge field action with bare coupling g
- Smearing
- $b = \frac{1}{g^2 N}$ Tadpole improved to $b_I = e(b)b$ with e(b) the average plaquette
- 3d: Space-like and time-like separations K, T in lattice units. $k = K/b_I$ $t = T/b_I$

Smearing





$$U' = P_{SU(N)}[(1-f)U + \frac{f}{4}S^{+}_{12} + \frac{f}{4}S^{-}_{12} + \frac{f}{4}S^{+}_{13} + \frac{f}{4}S^{-}_{13}]$$

Iterate n times

$$\tau = fn$$

4d: $f = 0.45 \quad n = 5 \quad \tau = 2.25 \quad \sqrt{\tau} = 1.5$

3d (4->2): $f = 0.1 \quad n = 25 \quad \tau = 2.5 \quad \sqrt{\tau} = 1.6$

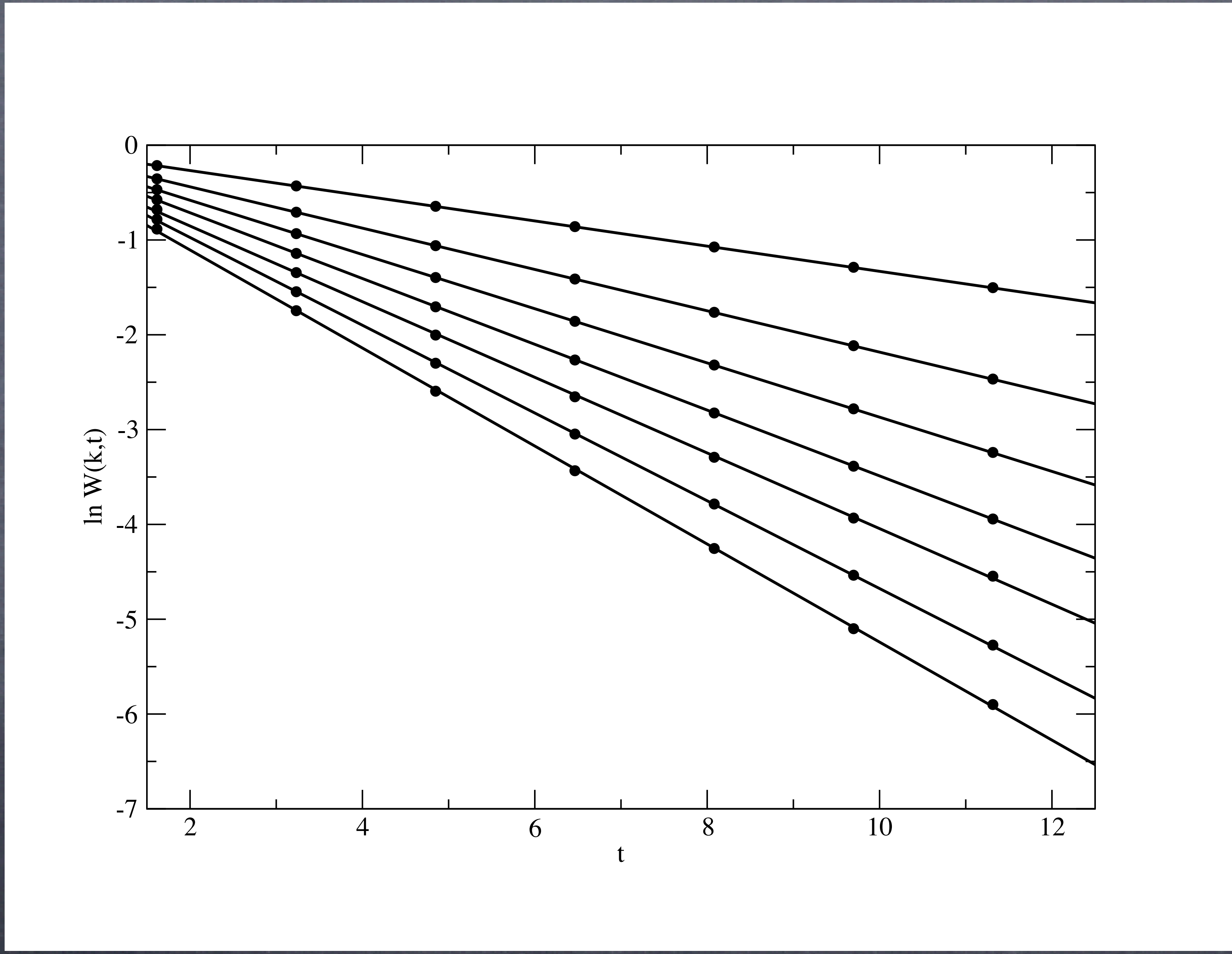
- 3-dimensions, T=0
  - 5^3 lattice
  - N = 47
  - b = 0.6 to 0.8
  - Smear space-like links with staples in the same time slice
  - Wilson loops 1x1 to 7x7 (folded loops)
  - Fit to get quark-antiquark potential and string tension

- 3-dimensions, $T>T_c$
  - L^3 lattice L=4, 5, 6, 7
  - N = 59
  - b = 0.9 to 1.75
  - Smear space-like links with staples in the same time slice
  - Wilson loops 1x1 to 10x10 (folded loops)
  - Fit to get quark-antiquark potential and string tension

- 4-dimensions, T=0
  - 6^4 lattice
  - N = 37, **47**, 59
  - b = 0.3450, **0.3480**, 0.3500
  - Smear space-like links with staples in the same time slice
  - Wilson loops 1x1 to 9x9 (folded loops)
  - Fit to get quark-antiquark potential and string tension

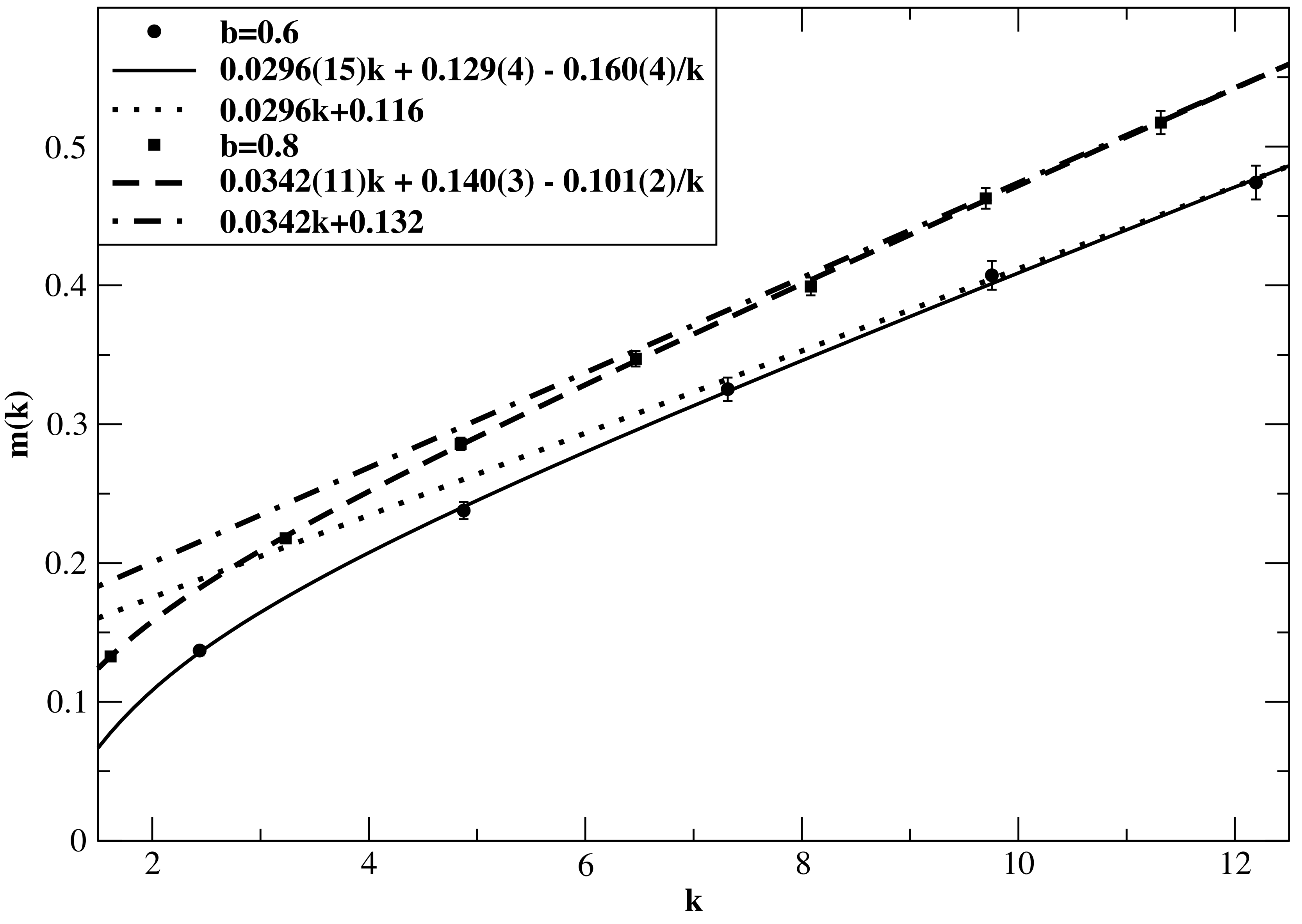
Compute all Wilson loops 1x1 to 7x7

Fit to $W(k,t) = e^{-a-m(k)t}$



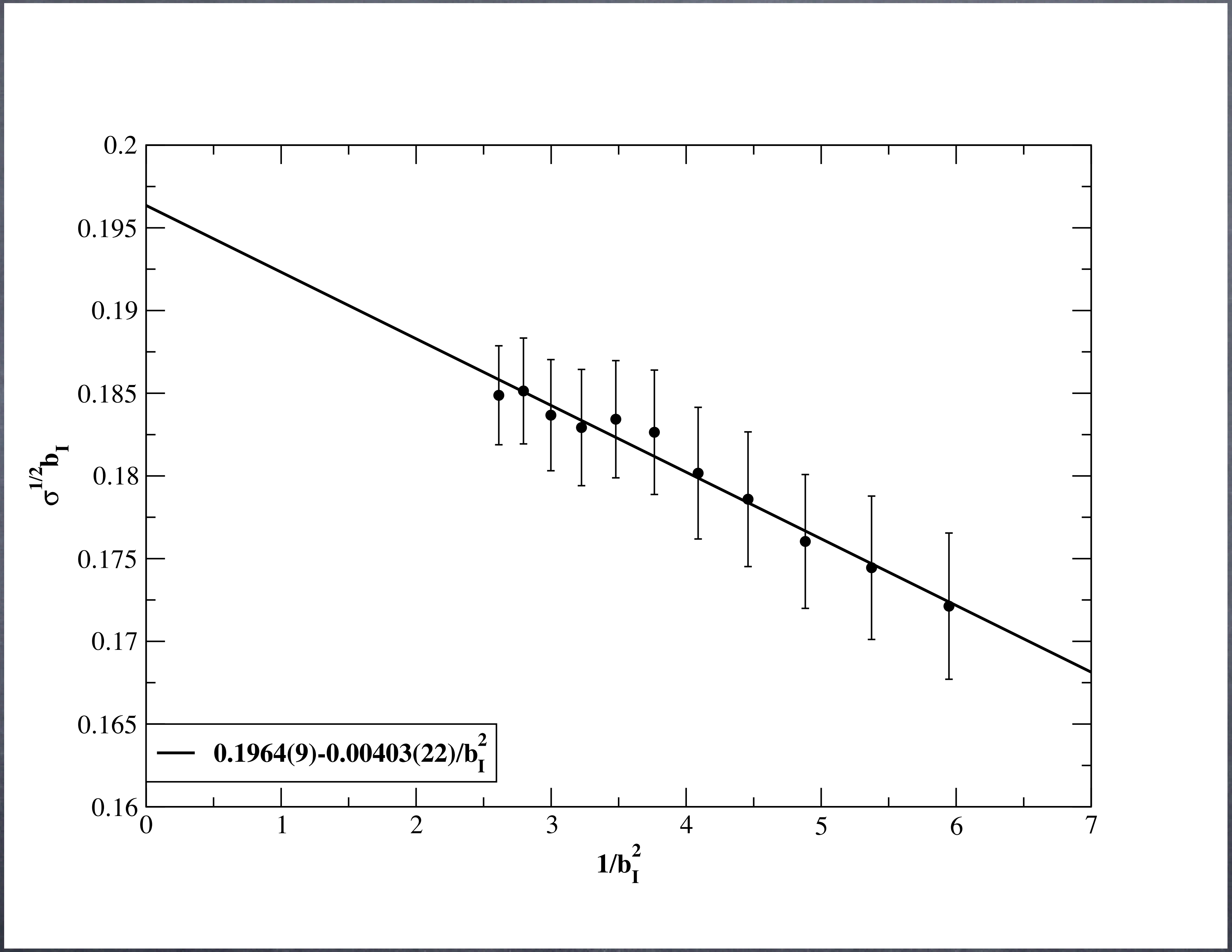
3d s

Fit m(k) to $m(k) = \sigma b_I^2 k + c_0 b_I + \frac{c_1}{k}$



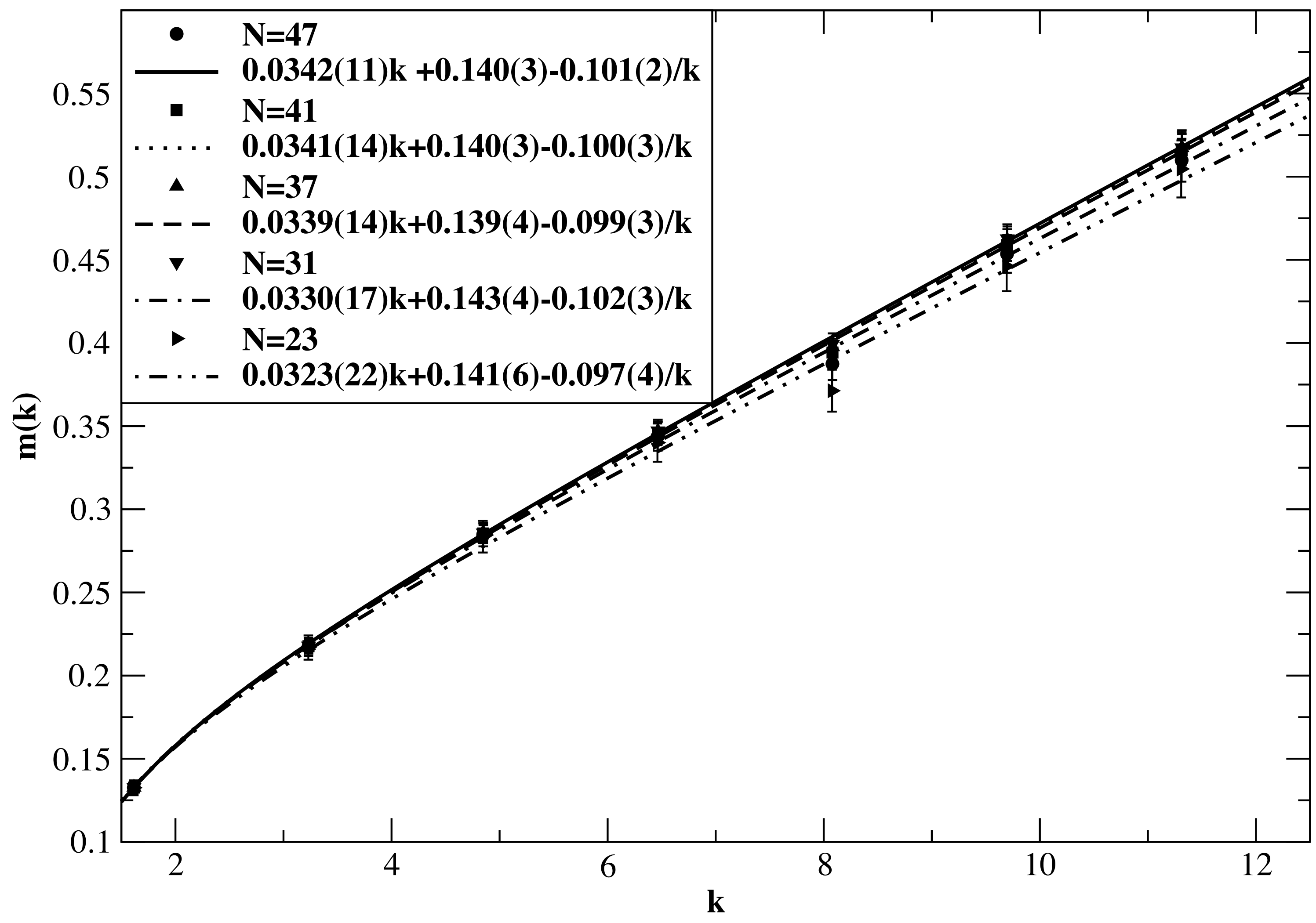
3d s

Extrapolate: $b_I \to \infty \quad \sqrt{\sigma} b_I \to 0.1964 \pm 0.0009$

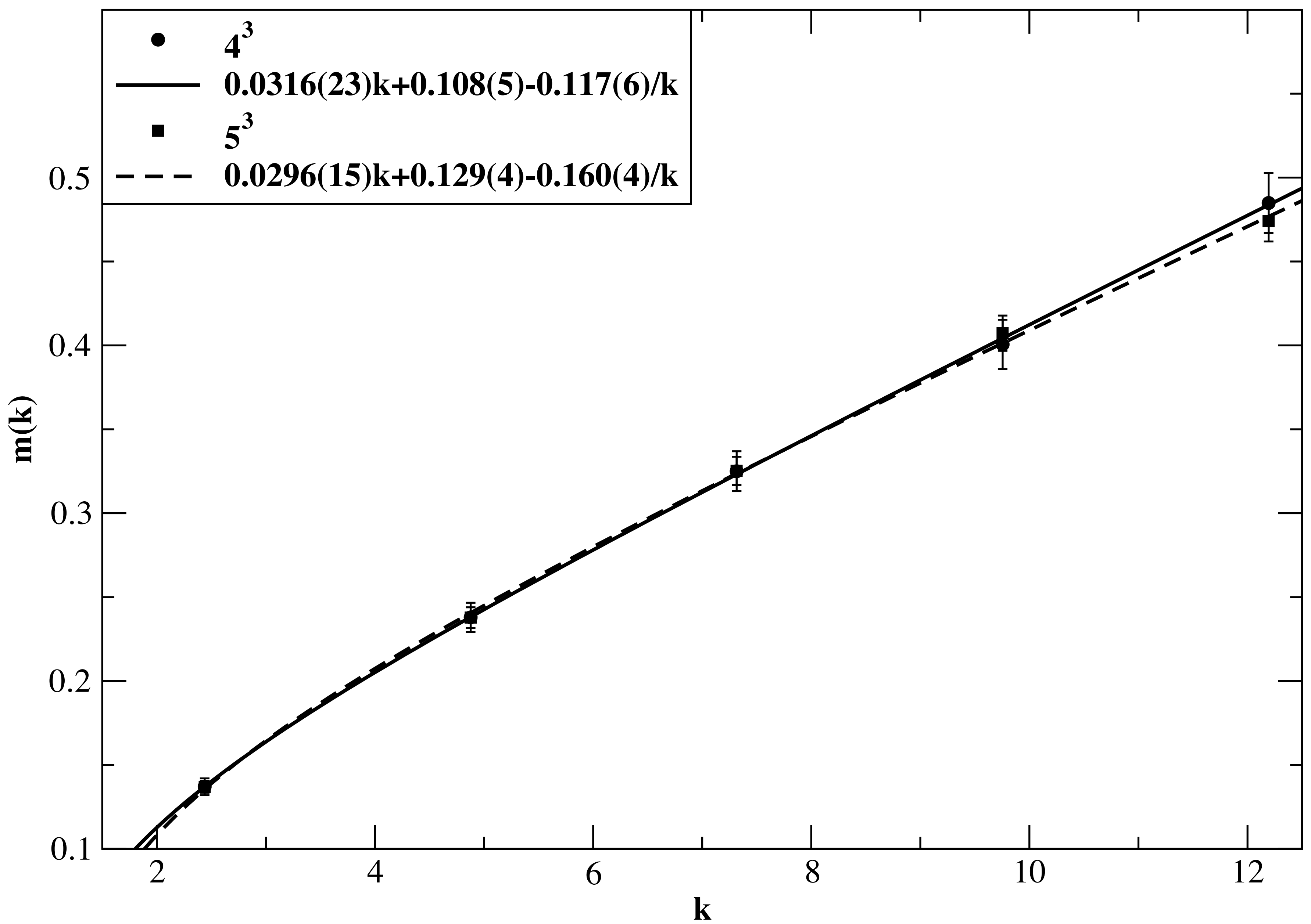


3d

Are N and L large enough?



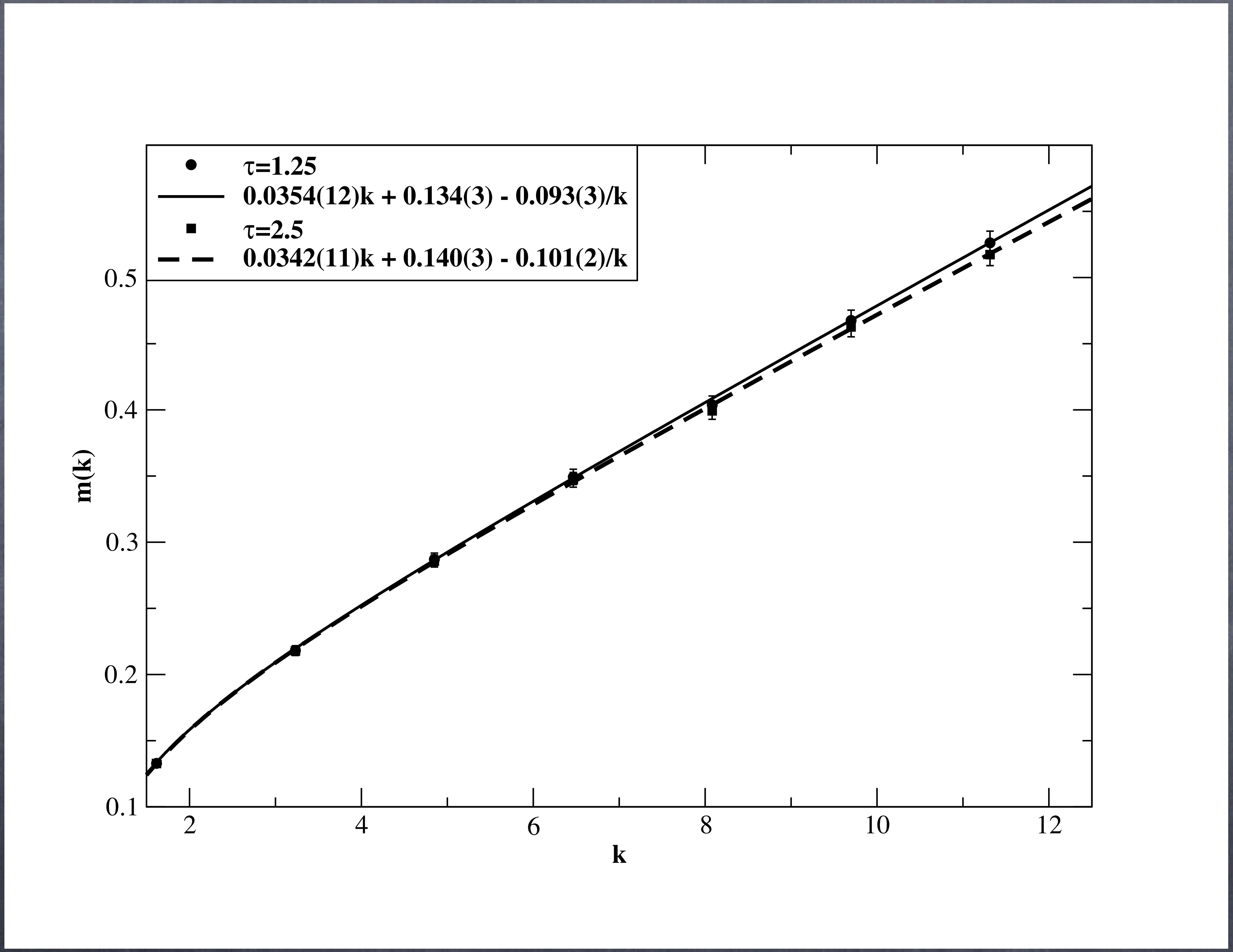
3d s



4^3
0.0316(23)k+0.108(5)-0.117(6)/k
5^3
0.0296(15)k+0.129(4)-0.160(4)/k
m(k)
k
0.1
0.2
0.3
0.4
0.5
2
4
6
8
10
12

3d s

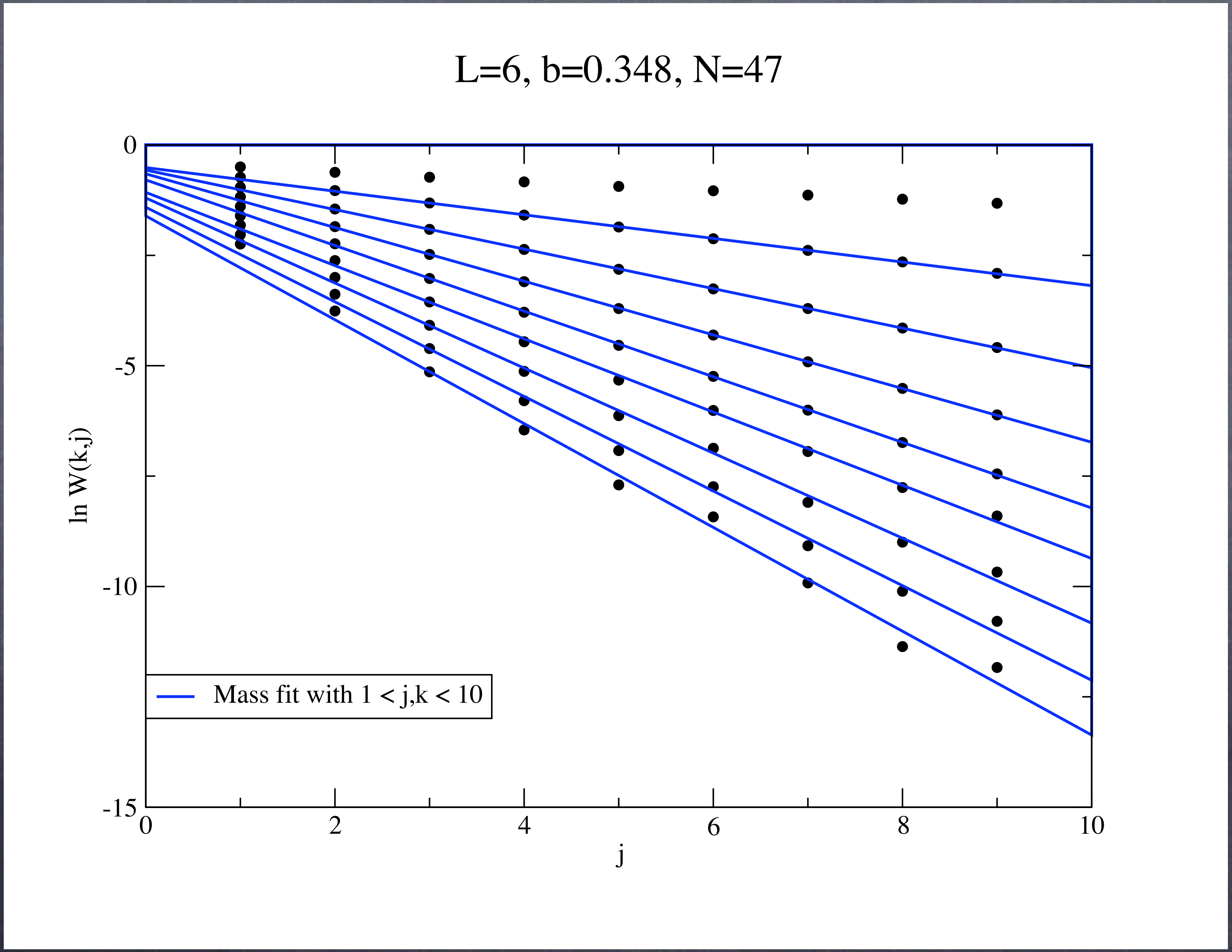
Are the results sensitive to smearing?



3d s

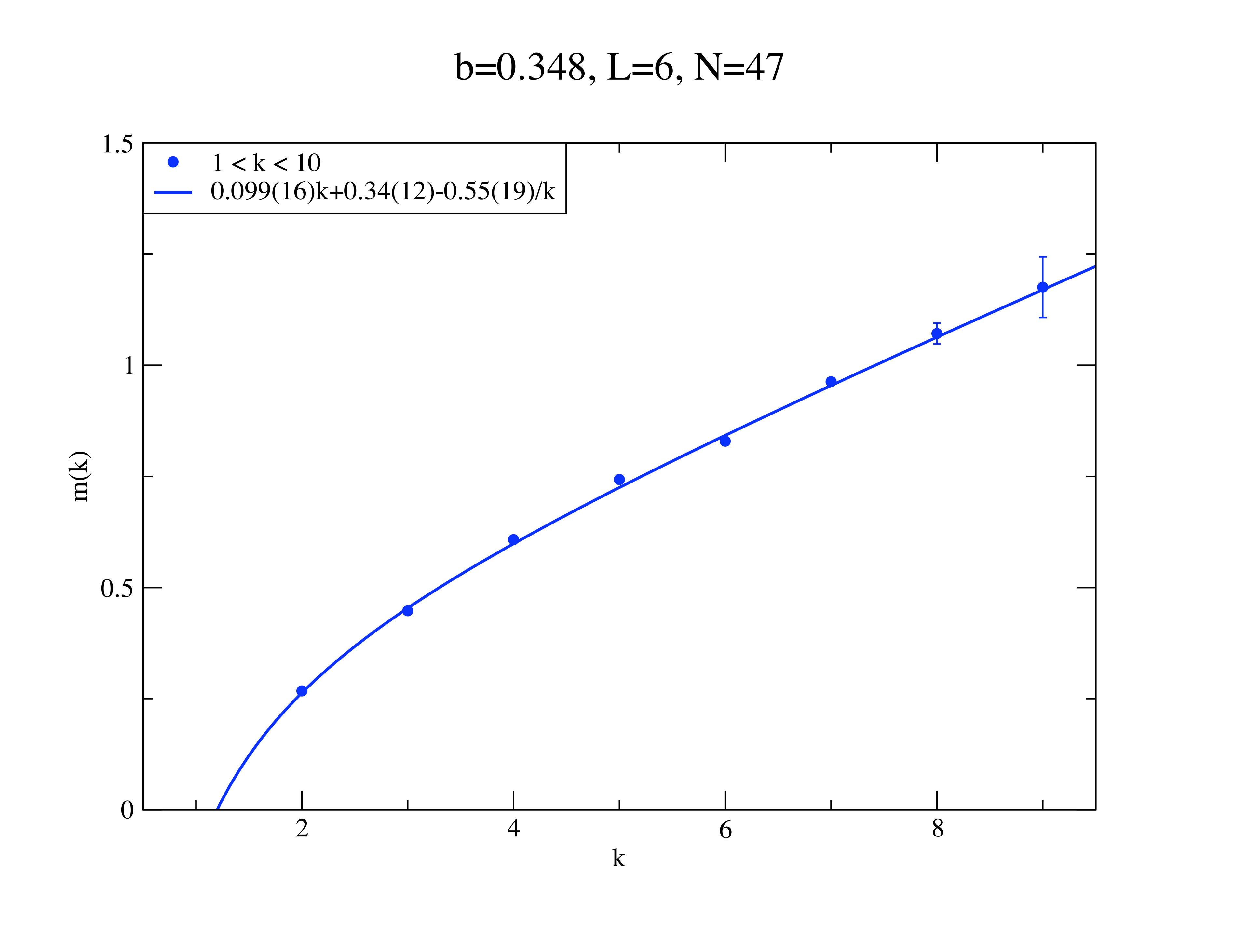
Compute all Wilson loops 1x1 to 9x9

Fit 2x2 through 9x9 to $W(k,t) = e^{-a-m(k)t}$

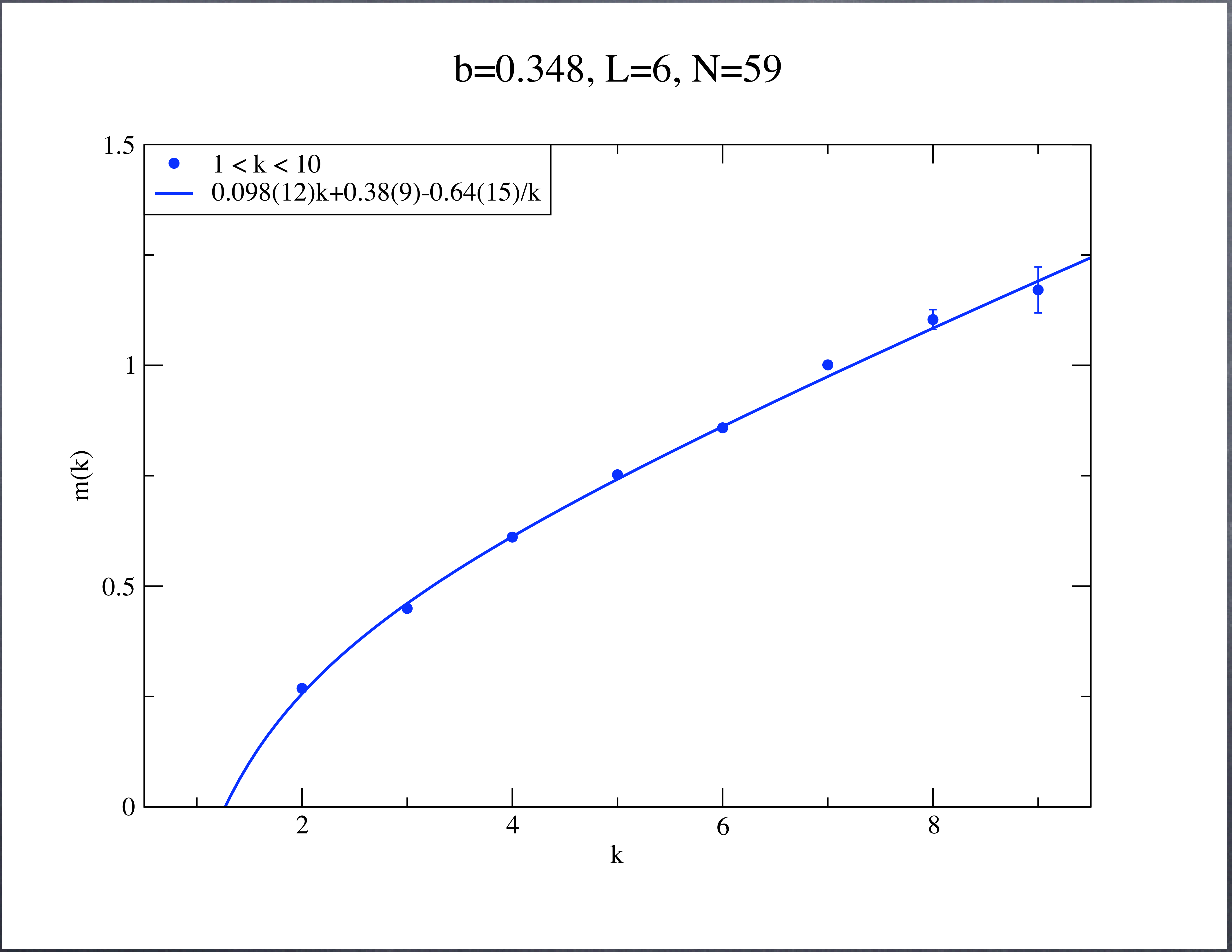


4d

Fit m(k) to $$m(k) = \sigma a^2 k + c_0 + \frac{c_1}{k}$$

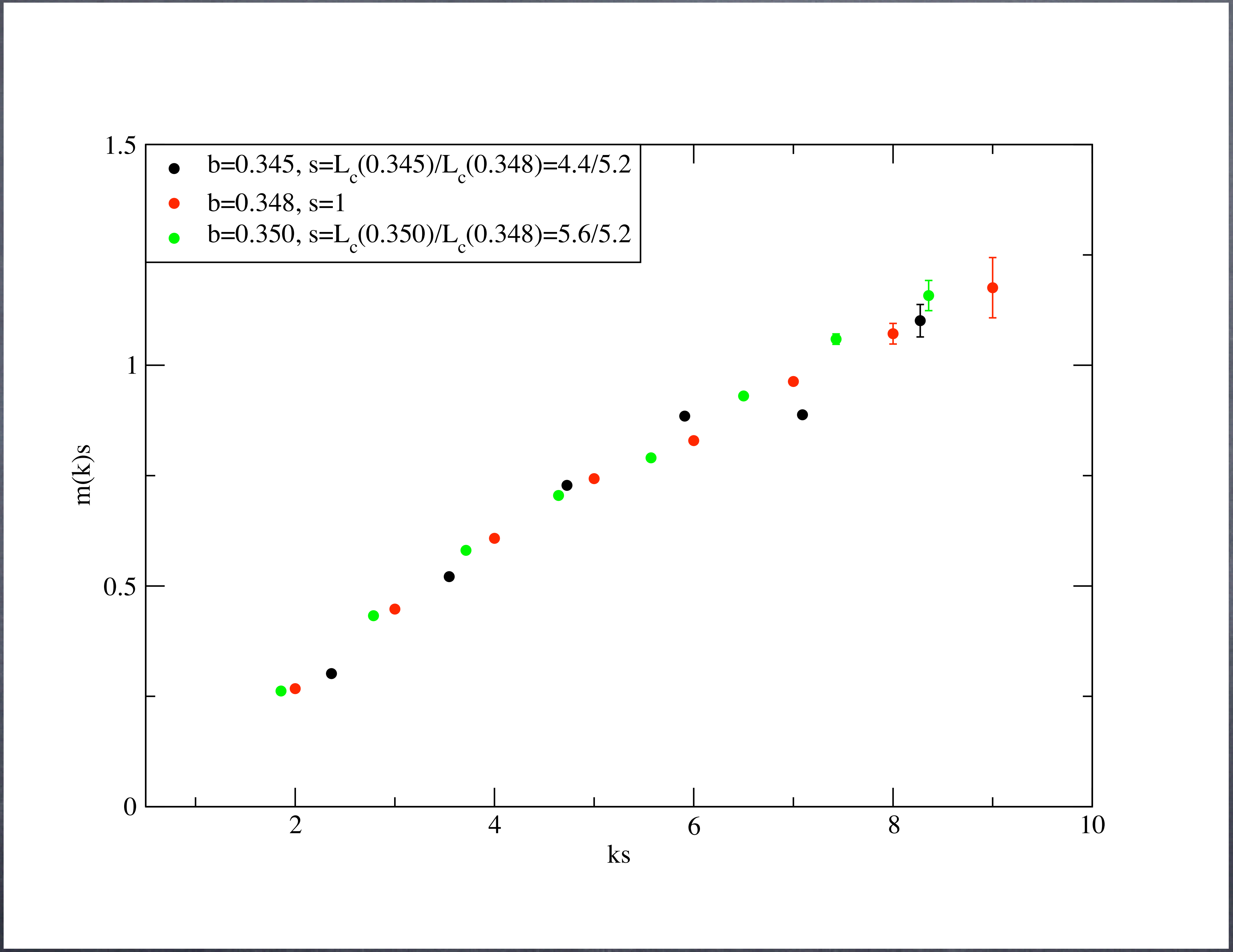


4d



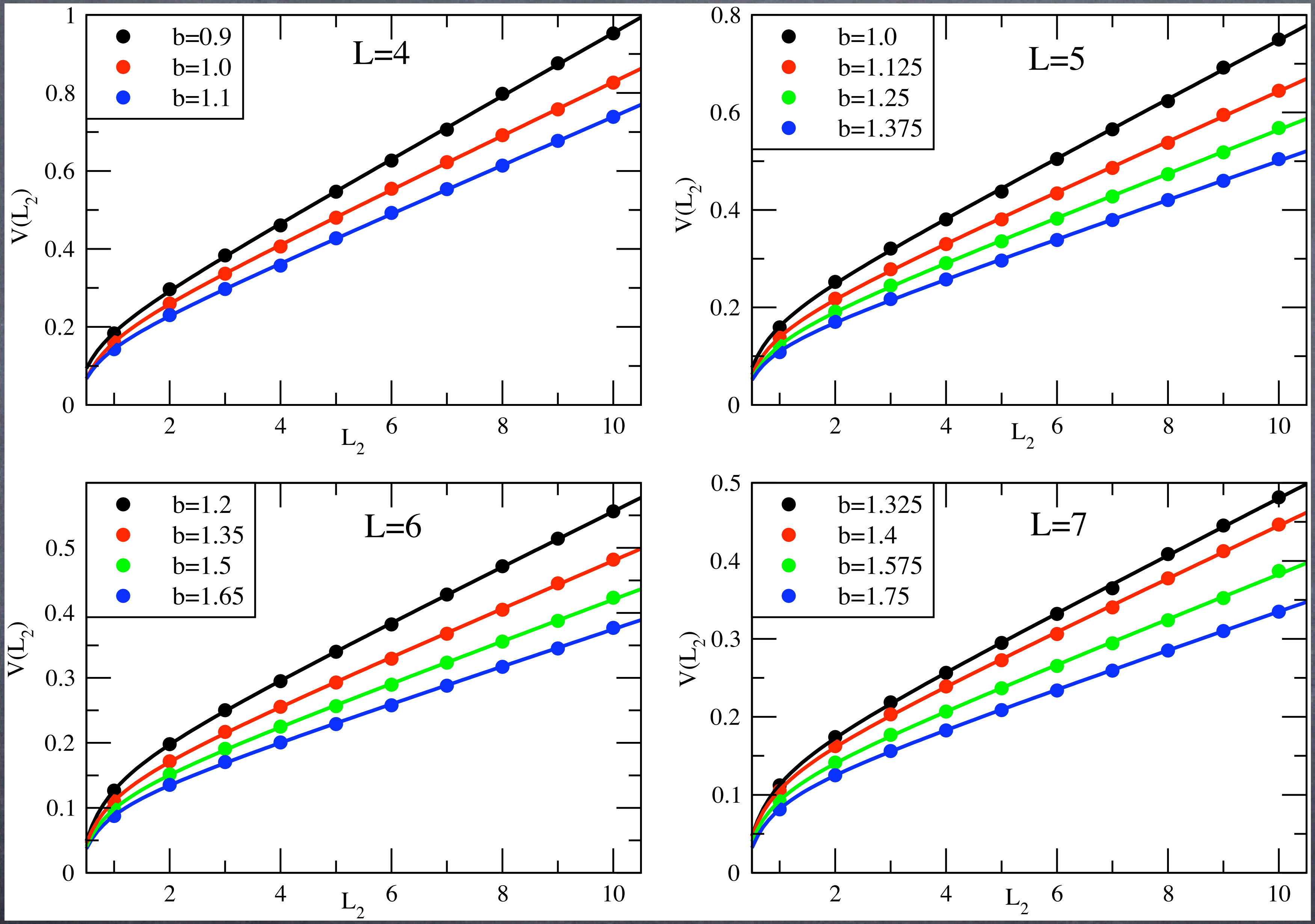
4d

Scaling



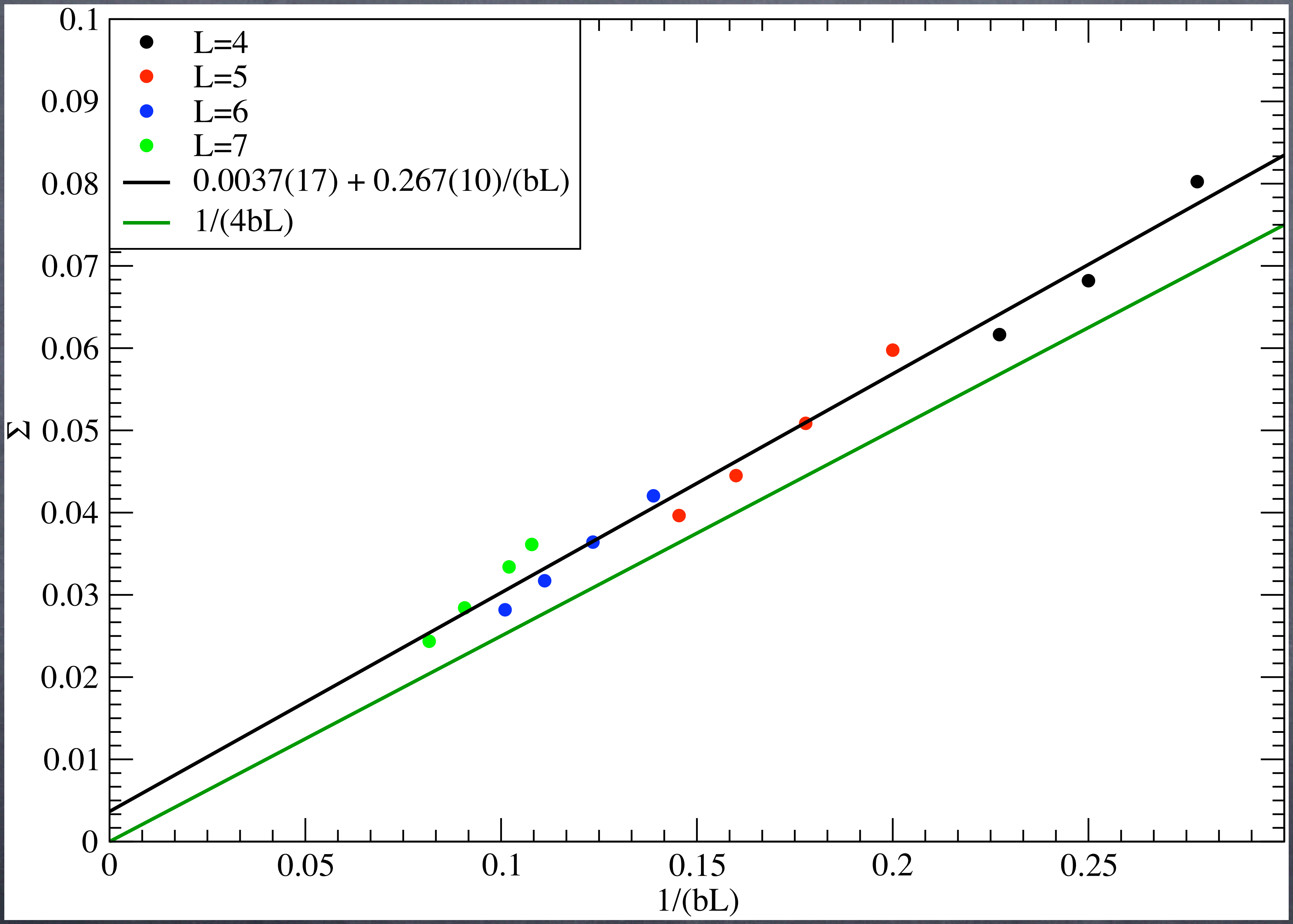
b=0.345, s=L_c(0.345)/L_c(0.348)=4.4/5.2
b=0.348, s=1
b=0.350, s=L_c(0.350)/L_c(0.348)=5.6/5.2
m(k)s
ks
1.5
1
0.5
0
2
4
6
8
10

4d



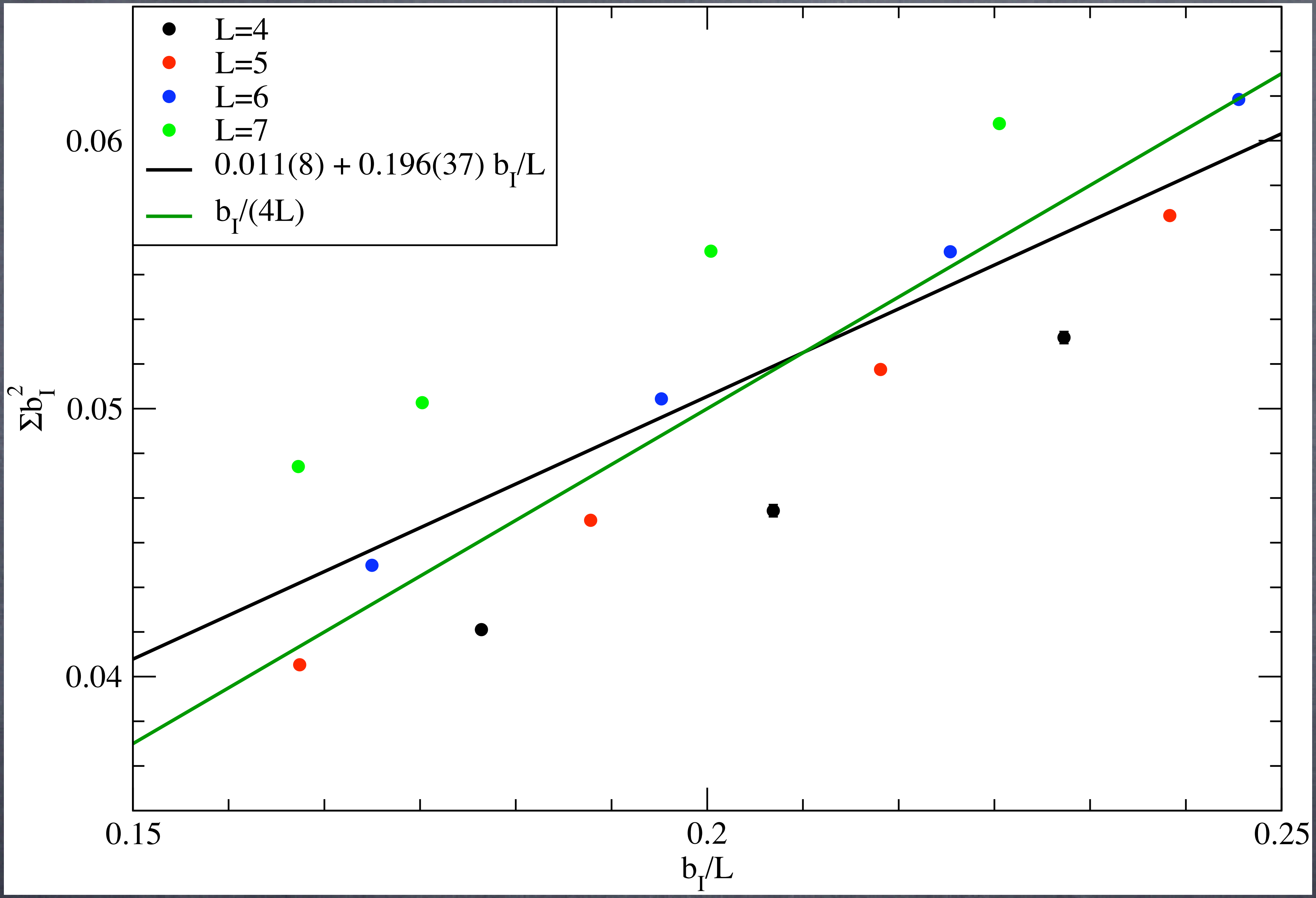
b=0.9
b=1.0
b=1.1
L=4
V(L2)
L2
b=1.0
b=1.125
b=1.25
b=1.375
L=5
b=1.2
b=1.35
b=1.5
b=1.65
L=6
b=1.325
b=1.4
b=1.575
b=1.75
L=7

3d T



L=4
L=5
L=6
L=7
0.0037(17) + 0.267(10)/(bL)
1/(4bL)
Σ
1/(bL)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0
0.05
0.1
0.15
0.2
0.25

3d T


L=4
L=5
L=6
L=7
0.011(8) + 0.196(37) b_I/L
$b_I/(4L)$
0.06
0.05
0.04
Σb_I^2
0.15
0.2
0.25
b_I/L

3d T

Comparison with large L results

3-dimensions, T=0

- Karabali, Kim, and Nair $\sqrt{\sigma} b = \frac{1}{\sqrt{8\pi}} \approx 0.1995$
- Bringoltz and Teper $\sqrt{\sigma} b = 0.1975 \pm 0.0002 - 0.0005$
- J.K. & R.N. arXiv:0807.1315 $\sqrt{\sigma} b = 0.1964 \pm 0.0009$

Comparison with large L results

4-dimensions, T=0

- Lucini, Teper, and Wenger $\frac{T_c}{\sqrt{\sigma}} = 0.597 \pm 0.004$
- J.K. & R.N. $\frac{T_c}{\sqrt{\sigma}} = 0.61 \pm 0.05$
- At the same b_I=0.182
  Lucini, Teper, and Wenger N=8 $\sigma a^2 = 0.116 \pm 0.001$
  J.K. & R.N. $\sigma a^2 = 0.099 \pm 0.016$

Conclusion

- In both 3 and 4 dimensions, continuum reduction gives good results for quantities based on the space-time dependence of large Wilson loops, e.g. the heavy quark potential and the string tension.