# Top Mass Reconstruction from Jets

# Sonny Mantry, University of Wisconsin at Madison. DPF 2009, Wayne State University

Collaborators: Sean Fleming, Andre Hoang, Iain Stewart

**arXiv:0711.2079, Phys.Rev.D77:114003,2008 hep-ph/0703207,Phys.Rev.D77:074010,2008**

#### What are we Measuring?  $f(x)$  scales  $\epsilon$

 $\mathcal{L}_{\mathcal{A}}$  , and the set of th

**•What is the top mass?**  $| \cdot \mathbf{w} |$  $\sim$  vv nat is the top mass.  $\mid \cdot$  Wha

(1 − s)2 , s)2

s – 3s)<br>s – 3s) – 3s) – 3s)

1 **+ s + 2** 

 $1 - 2$ 

1 − 2<br>1 − 2<br>2 *s − 2* 

<sup>5</sup> <sup>−</sup> <sup>3</sup>s<sup>2</sup>

(1 <sup>−</sup> <sup>s</sup>)<sup>2</sup> <sup>−</sup> <sup>λ</sup><sup>2</sup>

 $\overline{\phantom{a}}$ 

- Top is a colored parton. Cannot define physical on-shell mass. Top is a colored parton. Cannot define physical on-shell mass.  $k \approx 1$
- **• Top mass is a parameter of the Lagrangian.**<br>
 **Cop mass parameter is scheme dependent**

**Reconstruction at LHC and ILC**

• Top mass parameter is scheme dependent. corresponding values for  $\mathbf{r}$  and  $\mathbf{r}$  are  $\mathbf{r}$  and  $\mathbf{r}$  are  $\mathbf{r}$  are  $\mathbf{r}$  and The combined value for the top-quark mass is:



There is s.t. there is s.t. there is s.t. there is s.t. the understand here  $\alpha$ 

, (ii) and (iii)  $\alpha$ 



Figure 1: A summary of the input measurements and resulting world average mass of the top

$$
M_{\rm t} \;\; = \;\; 173.1 \pm 1.3 \,\, \mathrm{GeV}/c^2
$$

*<sup>m</sup>*pole <sup>−</sup> *<sup>m</sup>*MS(*m*) <sup>∼</sup> <sup>8</sup> GeV

### In the decay rates we use

- 
- LAISIS Measuring:<br>Emalon ambiquity, noor perturbative behavior **• Pole mass? :**  $\delta m \sim \Lambda_{\text{QCD}}$  renormalon ambiguity, poor perturbative behavior.  $\widetilde{R}m \sim \Lambda_{\rm QCD}$  renormalon ambiguity, noor r  $\left| \begin{matrix} \frac{1}{200\cdot 11.4j} & \frac{1}{200\cdot 11.4j} \\ \frac{1}{200\cdot 11.4j} & \frac{1}{200\cdot 11.4j} \end{matrix} \right|$   $\bullet$  Pole mass':  $\delta m \sim \Lambda_{\text{QCD}}$  renormalon ambiguity, poor perturbative behavior.

*q2*

- For better precision we need a short distance top mass.
- 

# **Threshold Scan**

√s (GeV) 347 350 353

ν  $=$  0.1255  $\pm$  0.1255  $\pm$  0.1255  $\pm$  0.1255  $\pm$  0.1355  $\pm$  0.1355  $\pm$  0.1355  $\pm$  0.1355  $\pm$  0.1355  $\pm$ 

*(Fadin & Khoze; Peskin & Strassler; Hoang, Manohar, Stewart,Teubner,...)* δ*m<sup>t</sup>* ∼ 1 GeV

*pp* <sup>→</sup> *ttX*¯

**Reconstruction at LHC and ILC**

#### **•Top pair production in the threshold region**  *1.4 1.6 (a)*

- **•Shape of total cross-section sensitive to top mass.** *1.0*
- $\bullet$  Top width provides IR cutoff.
- **•Non-perturbative effects are small.** *0.8 Q*

#### **•Physics well understood**  *\_ 0.0*

- NRQCD is the appropriate EFT.
- $\bullet$  Well defined relation to short distance mass. eg. 1S mass  $\frac{1}{2}$ t mass for the fixed order and resummed expansions. The fixed order and resummed expansions. The fixed order and  $\alpha$
- NNLL results known.
- **Theoretical uncertainty: •** Theoretical uncertainty:

# $\delta m_t^{th} \sim 100MeV$



# *•• Pet Reconstruction •* Review existing attempts to explain this puzzle

# • Jet reconstruction methods not so well understood

*•* Review existing attempts to explain this puzzle

- Suitable observables with a well defined relation to a short distance mass SUITED READING
	- **•Summation of large logarithms**

*•* Explain how situation is more complicated at twist-4: mixing between many ops.

- **•Final state radiation**
- **•Initial state radiation** 
	- **•PDFs**
	- **•Jet Energy scale**
	- **•Beam remnants**
	- **•...**



*•* Explain how situation is more complicated at twist-4: mixing between many ops.



**•We study high energy top pair production at the linear collider** *econstructed top mass [GeV]*<br> **hereonstructed top mass [GeV]**<br> **hereonstructed top mass [GeV]** 

 $e^+e^- \to t\bar{t}X$ ,  $Q \gg m \gg \Gamma > \Lambda_{QCD}$ 

*•* Explain how situation is simple at twist-2(only two types of ops).

*•* Explain how situation is simple at twist-2(only two types of ops).

#### As an observable sensitive to the top mass, we considered in Ref. [2] the double differential invariant mass distribution in the peak region in the peak region  $\mathbf{L}$  region around the top resonance: Incer Collidat Observable  $\frac{1}{\sqrt{2}}$ At leading order in the expansion in the expansion in the expansion in many show that the double differential o **Linear Collider Observable**

t

<sup>t</sup> and M<sup>2</sup>

 $\tau$  scales  $\tau$  and  $\tau$  and  $\tau$  and  $\tau$  are between  $\tau$  and  $\tau$  are between  $\tau$  are between  $\tau$  are between  $\tau$ 

 $\Gamma$  that acts as an imaginary residual mass term  $\Gamma$  is usually understood in understood is usually understood in

as being formulated close to the rest frame of the rest frame of the heavy  $\alpha$  the soft cross-talk without t

−<br>− m2<br>− m2

 $\tau$  scales  $Q, \mu$  and  $\mu$  and  $\mu$  are between  $Q$  and  $\mu$  are between  $Q$  are between  $Q$ 

<sup>t</sup> described below.

ل<br>م**ان**سه on of top jets:  $\overline{\text{e}}$  jets: **• Hemisphere invariant mass distribution of top jets:** 

variables M<sup>2</sup>

t − m2<br>t = m2<br><del>= m2</del>

namely where  $\frac{1}{2}$  M2  $\frac{1}{2}$  M2  $\frac{1}{2}$  M2  $\frac{1}{2}$  M2  $\frac{1}{2}$  M2  $\frac{1}{2}$ 

because it is only the invariant mass distribution close to the peak that we wish to peak that we wish to pred<br>It is only the peak that we wish to peak that we wish to predict the peak that we wish to peak the peak that w

 $H_{\rm eff}$  is setting a lower bound on the width on the invariant mass distribution on the invariant mass distribution on the invariant mass distribution of the invariant mass distribution on the invariant mass distribution

namely where m<sup>2</sup> \$ M<sup>2</sup>

to the top mass, so that M<sup>2</sup>



− m2 and either M2 and either M2 and either M2 and

st,t ¯

We do not consider the region where  $\alpha$  is the region where  $\alpha$ 

invariant masses starting just past the peak where the cross-section begins to fall off rapidly,

t

−<br>− m2<br>− m2

− m2 and either M2<br>2 and either M2 and either M2 and either M2

M<sup>2</sup> t,t

t − m2 ∞ m2 ∞ m2 ∞ m2 ∞ m2

¯ <sup>−</sup> <sup>m</sup><sup>2</sup> <sup>&</sup>gt;

invariant massesstarting just past the peak where the cross-section begins to fall off rapidly,

 $\sim$   $\sim$ 

¯ % m. However, for

 $\sim$   $\sim$   $\sim$ 

, we have an ultra-tail region where the cross-section is very small. The cross-section is very small.

¯ − m<sup>2</sup> ∼ mΓ. It is convenient to introduce

− − m2<br>− m2 + m2 + m2<br>− m2 + m2 + m2

− m2 m<br>2 marther

¯ − m<sup>2</sup> \$ m Γ. Farther

 $\mathcal{F} = \mathcal{F} \setminus \{ \mathcal{F} \mid \mathcal{F} \}$  . The tail region is defined by

, map  $\sim$ 

, (1) and (1) and (1) and (1)  $\sim$ 

, #

1

thrust

### **Relevant Energy Scales**

**• Center of mass energy** 

*Q* ∼ 1TeV

**• Top quark mass** *<sup>m</sup>* <sup>∼</sup> 174GeV

**• Top quark width** <sup>Γ</sup> <sup>∼</sup> 2GeV

**• Confinement Scale** <sup>Λ</sup> <sup>∼</sup> <sup>500</sup>*MeV*

### Disparate energy scales **– Effective Field Theory!**

# **Effective Field Theories**

### **Kinematics for Top Jets: I**  $W$ ing the scale of the scale of the invariant mass distribution  $V$  is setting the invariant mass distribution and  $V$

**• High Energy Condition: Top quark pairs are produced with a center of mass energy much larger than the top mass** ∼ Γ. The are the are than the top quark pairs are produced with a center of mass<br>are the system of the system, and the system, and the system, and the system of the system, and the system, and



**•** In this limit one can treat top quarks as collinear degrees of freedom in the **Soft Collinear Effective Theory (SCET)** *(Bauer, Fleming, Luke, Pirjol, Stewart)***.**   $t_{\text{out}}$  back-to-bac

the energetic jets coming from the top quark decay and coming from the top  $\alpha$  and collinear radiation. Frequently in

this work we refer to this work we refer to the jets coming from the top and antitop  $\alpha$  and antitop  $\alpha$ 

and antitop jet, respectively, respectively, respectively, but we stress that we do not require the jets from <br>The from the jets from the jets from the jets from the top and the top and the jets from the top and the top a

and dilepton channels (not shown).

the shifted variable s

### **Kinematics for Top Jets: II** definition of the invariant mass of the invariant masses. The invariant mass of each  $\mathbf{K}$  invariant mass of  $\mathbf{K}$  invariant mass of  $\mathbf{K}$  invariant mass of  $\mathbf{K}$  in  $\mathbf{K}$  in  $\mathbf{K}$  in  $\mathbf{K}$  in  $\mathbf{K}$

 $\overline{\phantom{a}}$  . In this section we can recommon to any carry out manipulations that are common to any common to any common to any

 $\frac{1}{\sqrt{2}}$ 

<sup>X</sup>n¯ + K<sup>+</sup>

 $\frac{1}{\sqrt{2}}$ 

**• Invariant Mass Condition: We characterize on shell production by the de literature in the Invariant I**<br> **Prequirement:** 

$$
M_{t,\bar t}^2 - m^2 \lesssim m\Gamma
$$

**• This condition looks like the invariant mass constraint on a heavy quark in Heavy Quark Effective Theory (HQET)** *(Isgur,Wise,...)***.** From here on the sense of the sense of power constraint on a heavy quark in a very constraint on a heavy quark in the sense of power counting ∴ Counting ∴ We are all the sense of power counting and the sense of power cou the collinear and soft momenta into  $\frac{1}{2}$  ( $\frac{1}{2}$ )  $\frac{1}{2}$  ( $\frac{1}{2}$ )  $\frac{1}{2}$ 

**• HQET has been generalized to unstable particles***(Beneke, Chapovsky, Signer, Zanderighi).* port = producer extensive = producer = produ<br>P <u>+ kn</u><br>P <u>+ kn</u>  $X \in \mathbb{R}^n$  ,  $X \in \mathbb{R}^n$ 

<sup>X</sup><sup>n</sup> , P <sup>+</sup>

= P

+ K−<br>+ K− K−

 $\frac{1}{\sqrt{2}}$ 

hemisphere is close to the top mass by including in the restrictions,

, on the states the

### **Group Photo of Effective Field Theories**





\*BEGIN\*

• The sum over final states  $X$  is restricted to contain a top jet and an anti-top iet with invariant masses close to the top mass. **let with invariant masses close to the top mass.** *ii* et with invariant masses close to the top mass.  $\int$ ugu $\int$ 

γ

= γ*<sup>µ</sup>* and Γ*<sup>µ</sup>*

*<sup>Z</sup>* =

*y* quand the indicates are produced by photon and *Z* exemange. **• The top quark currents are produced by photon and Z exchange: a** The top quark currents are produced by photon and *Z*<sup>*,*</sup> corresponding to contributions from photon and *Z*<sup>*,*</sup> tion from this draft) and of the observable to be treated (only the final one from the f

$$
J_i^{\mu}(x) = \bar{\psi}(x)\Gamma_i^{\mu}\psi(x), \quad \Gamma_{\gamma}^{\mu} = \gamma^{\mu}, \Gamma_{Z}^{\mu} = g^V\gamma^{\mu} + g^A\gamma^{\mu}\gamma_5
$$

summation and α*<sup>s</sup>* corrections are of similar size. This is a hybrid LL-NLO.

*•* Section II. Recap of the factorization theorem from the other paper (remove the deriva-

*•* Section III. SCET computations, matching from QCD. Computation of the running.

to 1/! in the anom.dim. with tree level matching), iii) one-loop LL, as in ii) but also including the one-loop matching results in the boundary conditions in case the log

 $A$  important outstanding theoretical issue is the formulation of a consistent framework framework framework framework which incorporates finite width effects in the production of massive unstable particles such as  $\mu$ as the top quark or the *W* boson. The issue is a pressing one in the era of the large hadron

boson exchange respectively. The corresponding Dirac structures are Γ*<sup>µ</sup>*

than the pole mass scheme.

and the indices *i, j* run over *{*γ*,Z}* corresponding to contributions from photon and *Z*

 $*^*\mathbb{R}^n$  . The contribution of the contribution  $\mathbb{R}^n$  ,  $\mathbb{R}^n$  ,

*<sup>i</sup>* ψ(*x*)*,* (6) QCDcurrents

= γ*<sup>µ</sup>* and Γ*<sup>µ</sup>*

restriction on the sum over final states *X*. The final states are restricted to contain top and

anti-top jets with invariant masses close to the top  $\alpha$  top  $\alpha$  the top  $\alpha$  the explicit form of these top  $\alpha$ 

restrictions will depend on the specific jet algorithms and invariant mass definitions used.



*Lee, Manohar, Wise* ) Entertainment of the United States of the United Sta ¯) = δ(*s<sup>t</sup>* δ("<sup>+</sup>)δ("−), and integrating Eq. (81) over *s<sup>t</sup>* and *s<sup>t</sup>* **• The same soft function appears in massless dijets***(Korchemsky & Sterman; Bauer,*  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ , in the anomaly dimensional matching  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **Summar** ( Lee, Manohar, Wise )

In the factorization theorem in Eq. (81) the jet-functions *J<sup>n</sup>* and *Jn*¯ describe the dynamics

of the top and antitop and antitop jets. In the next see that the next see that the next see that the next see

computed in perturbation theory and at the tree level are just Breit-Wigner distributions.

*<sup>n</sup> <sup>Y</sup>n*¯(0)*|Xs*#"*Xs|Y*˜ *†*

σ0. This provides a check for the normalization of Eq. (81).

*•* Section IV. bHET computations, matching and running. Results in schemes other

An important outstanding theoretical issue is the formulation of a consistent framework which incorporates finite width effects in the production of massive unstable particles such as  $\mu$ as the top quark or the *W* boson. The issue is a pressing one in the era of the large hadron collider (LHC) with expectations of a wealth of data where QCD backgrounds involving top quarks and *W* bosons must be understood at a precision level in order to tease out

than the pole mass scheme.

 $\mathsf{I}^{\mathsf{I}}$ 

measurements of exotic new physics. For example *tt*

*,* "−) =

 $\mathcal{L}_{\mathcal{A}}$  the total tree-level  $\mathcal{A}_{\mathcal{A}}$  total tree-level  $\mathcal{A}_{\mathcal{A}}$  tree-level  $\mathcal{A}_{\mathcal{A}}$ 



top quarks and *W* bosons must be understood at a precision level in order to tease out

measurements of exotic new physics. For example *tt*

<sup>−</sup>*, <sup>µ</sup>*) <sup>=</sup> <sup>1</sup>

&

*Xs*

*N<sup>c</sup>*

*<sup>n</sup>*¯(0)*|*0\$*.* (91)

#### **Matching QCD onto SCET at One Loop**  $\overline{ }$ *m* <sup>δ</sup>(2*v*<sup>+</sup> *· <sup>k</sup>*) <sup>=</sup> <sup>1</sup> *m* δ(*s*ˆ) = δ(*s*)*,* (89)  $m$  $\alpha$  is the corresponding  $\alpha$ **The ∪CD onto SCET at One Loop** and found for the fo **d**<sup>2</sup>

*v*<sup>+</sup> *· k* + *i*0

essentially corresponds to running the bHQET top pair production current of Eq. (33), and the bHQET top pair p<br>In the bHQET top pair production current of Eq. (33), and the bHQET top pair production current of Eq. (33), a

*ds*ˆ# *U<sup>B</sup>*<sup>+</sup> (*s*ˆ− *s*ˆ#

To evolve the Wilson coefficient to lower scales we need to solve the RG equation in

*, µm, µ*) *B*+(*s*ˆ#

*m*

"

 $= 10$ 

Imerican de la Constantina<br>Imperience

"

<sup>4</sup>π*Ncm*(−*Nc*) Disc! *<sup>i</sup>*

$$
\left(\frac{d^2\sigma}{dM_t^2 dM_t^2}\right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^- B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_t - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
$$
\n
$$
\sum_{\text{B}} \left(\frac{d^2\sigma}{dM_t^2 dM_t^2}\right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^- B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_t - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
$$
\n
$$
\sum_{\text{B}} \left(\frac{d^2\sigma}{dM_t^2 dM_t^2}\right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^- B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_t - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
$$

ching QCD production current onto SCET at the hard scale:  $p \propto \Omega_{\rm C} \Omega_{\rm max}$  and the matrices with any infrared regulator, and the pick and to pi  $\arg \chi$ UD production ch momenta to regulate the IR divergences, letting *p*<sup>2</sup> − *m*<sup>2</sup> = *p*¯<sup>2</sup> − *m*<sup>2</sup> = ∆<sup>2</sup> "= 0. Since  $t$  current current of the  $\mathbf{S}$  current physics of the interaction physics of the  $\mathbf{S}$ perform the matching with any external states with any external states we like, and to pick any infrared regula Latching OCD production current onto SCFT at the hard s run the Wilson coefficient *H<sup>m</sup>* of we run the individual functions *B<sup>±</sup>* and *S*. The first option we will call this method *"top-down"*. The relation *H<sup>m</sup>* ! *m, Q <sup>m</sup>, <sup>µ</sup>m, <sup>µ</sup>* " = *Hm*(*m, µm*)*U<sup>H</sup>m*(*µm, µ*) (92)  $\sigma$  OCD meaduction outgoint onto  $\text{SCET}$  of the hand explaint  $\sigma \sim$  the phase space space space space is  $\sigma$  and no directions is carried the n and no directions is carried in the number of  $\sigma$ **•Matching QCD production current onto SCET at the hard scale:** 

*v*<sup>+</sup> *· k* + *i*0

*B*<sup>Γ</sup>=0

<sup>+</sup> (*s*ˆ) <sup>=</sup> <sup>−</sup><sup>1</sup>

*m*

elements, which we will simply call simply call amplitudes. The  $\mathcal{L}^{\text{max}}$  are given in Fig. 3 and 3 an

where  $\alpha$ <sup>÷</sup> and *p* and *p*<sup>om</sup> are defined. We use dimension of the UV divergences and offshell dimensional offshell dimensional dimensional dimensional dimensional dimensional dimensional dimensional dimensional dimens

demonstrates that massive SCET has the same IR structure as in QCD. Evaluatingthe

<sup>∆</sup>¯ <sup>=</sup> <sup>∆</sup> *<sup>&</sup>gt;* <sup>0</sup> and again taking <sup>∆</sup><sup>2</sup> # *<sup>m</sup>*<sup>2</sup> # *<sup>Q</sup>*<sup>2</sup> this gives

*dµB*+(*s,* <sup>ˆ</sup> *<sup>µ</sup>*) <sup>=</sup>

*µ*

shell particles, but also the production of events with multiple particles, but also the production of events <br>The production of events with multiple particles, with multiple particles, with multiple particles, with multip

# ln # *<sup>µ</sup>*<sup>2</sup> −*Q*<sup>2</sup>

zero <sup>∆</sup><sup>2</sup> <sup>=</sup> *<sup>p</sup>*<sup>2</sup> <sup>−</sup> *<sup>m</sup>*<sup>2</sup> and <sup>∆</sup>¯ <sup>2</sup> <sup>=</sup> *<sup>p</sup>*¯<sup>2</sup> <sup>−</sup> *<sup>m</sup>*<sup>2</sup>. The sum of collinear and soft vertex graphs,

*ds*ˆ# γ*<sup>B</sup>*<sup>+</sup> (*s*ˆ− *s*ˆ#

evolution factors *U<sup>B</sup><sup>±</sup>* (*µ, µm*) and *US*(*µ, µm*) respectively, as is also illustrated in Fig. 4. This

#<sup>2</sup> <sup>+</sup> .<br>...

4π

$$
C(\mu) = 1 + \frac{\alpha_s C_F}{4\pi} \Big[ 3 \ln \frac{-Q^2}{\mu^2} - \ln^2 \frac{-Q^2}{\mu^2} - 8 + \frac{\pi^2}{6} \Big]
$$

$$
H_Q(Q, \mu_Q) = |C(Q, \mu_Q)|^2 = 1 + \frac{\alpha_s C_F}{4\pi} \Big[ -2 \ln^2 \Big( \frac{Q^2}{\mu_Q^2} \Big) + 6 \ln \Big( \frac{Q^2}{\mu_Q^2} \Big) - 16 + \frac{7\pi^2}{3} \Big]
$$

−∆<sup>2</sup>

*, µ*)*,*

*B*+(*s,* ˆ *µm*) =

) *B*+(*s*ˆ#

the pole mass scheme

*, µ*)*,* (93) *{*Brun*}*

(90) *{*bHQETcross-hem*}*

Matching SCET onto BHQET at One Loop											
\n $\left(\frac{d^2\sigma}{dM_t^2 dM_t^2}\right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m(m, \frac{Q}{m}, \mu_m, \mu) \times \int_{-\infty}^{\infty} dt^* dt^{-} B_+(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu) B_-(\hat{s}_t - \frac{Q\ell^-}{m}, \Gamma, \mu) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$ \n											
\n $\text{SCET}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots\n\end{bmatrix}$ \n	\n $\begin{bmatrix}\n\vdots$

#### **•Matching SCET jet functions onto bHQET jet functions:**

 $T_{\pm}(\mu, m) = 1 +$  $\alpha_s C_F$  $4\pi$  $\left(\ln^2 \frac{m^2}{\mu^2} - \ln \frac{m^2}{\mu^2} + 4 + \right)$  $\pi^2$ 6  $J_n(m\hat{s}, \Gamma, \mu_m) = T_+(m, \mu_m) B_+(\hat{s}, \Gamma, \mu_m)$ ,  $T_{\pm}(\mu, m) = 1 + \frac{\alpha_s C_F}{4} \left( \ln^2 \frac{m^2}{r^2} - \ln \frac{m^2}{r^2} + 4 + \frac{\pi^2}{c} \right)$ .  $J_{\bar{n}}(m\hat{s},\Gamma,\mu_m)=T_{-}(m,\mu_m) B_{-}(\hat{s},\Gamma,\mu_m)$ 

$$
H_m\left(m,\mu_m\right) = T_+(m,\mu_m)T_-(m,\mu_m) = 1 + \frac{\alpha_s C_F}{2\pi} \left(\ln^2\frac{\mu_m^2}{m^2} + \ln\frac{\mu_m^2}{m^2} + 4 + \frac{\pi^2}{6}\right)
$$



 $\sim$  m2  $\sim$  m2 defines the peak region, which is the peak region, which is the peak region, which is the region mos

**•Running between the different scales mostly affects only the normalization!**  $\overline{P}$ 

$$
\frac{d\sigma}{dM_t^2 dM_{\bar{t}}^2} = \sigma_0 H_Q(Q, \mu_m) H_m(m_J, \frac{Q}{m_J}, \mu_m, \mu)
$$
  
\$\times \int d\ell^+ d\ell^- B\_+\left(\hat{s}\_t - \frac{Q\ell^+}{m\_J}, \Gamma\_t, \mu\right) B\_-\left(\hat{s}\_{\bar{t}} - \frac{Q\ell^-}{m\_J}, \Gamma\_t, \mu\right) S(\ell^+, \ell^-, \mu)\$

Monday, July 27, 2009 16

 $\Gamma$   $\sim$  The restriction  $\Gamma$ 

# **Short Distance Mass for Jets**

#### $\Omega$ *x*  $\sim$   $\Omega$ *x*  $\sim$   $\Omega$ <sup>*x*</sup> $\sim$   $\Omega$ <sup>*x*</sup> $\sim$   $\Omega$ **E** Observable to a Short Distance Mass Scheme *v*<sup>+</sup> *· k* + *i*0 *v*<sup>+</sup> *· k* + *i*0  $\mathbf{u}$  $\overline{\phantom{a}}$  $\overline{\smash{\big)}\text{D}\text{S}\text{C}\text{I}}\text{val}\ \overline{\text{V}}$  $\overline{e}$ **(b)** a SHOLL DIStance Mass Scheme  $\Gamma$  Connecting the Observable to a Short Distance Mass Scheme **Connecting the Observable to a Short Distance Mass Scheme**

a review) and, as a consequence, to artificially large perturbative corrections. This behavior  $\alpha$ 

is particularly important for observables that have a strong dependence on the heavy quark

"

=

<sup>=</sup> <sup>1</sup>

Imerican<br>Imerican

Imerican<br>Imerican

"

• Top mass sensitivity comes from the bHQET jet functions intrinsic ambiguity in the heavy quark pole mass parameter of order the hadronization scale *C* TOP mass sensitivity comes from the DTQCT Jet functions **•Top mass sensitivity comes from the bHQET jet functions**

<sup>4</sup>π*Ncm*(−*Nc*) Disc! *<sup>i</sup>*

*B*<sup>Γ</sup>=0

*B*<sup>Γ</sup>=0

the pole mass scheme mass scheme  $\mathcal{L}$ 

<sup>+</sup> (*s*ˆ) <sup>=</sup> <sup>−</sup><sup>1</sup>

<sup>+</sup> (*s*ˆ) <sup>=</sup> <sup>−</sup><sup>1</sup>

where we still have *HQ*(*Q, µ*) = *|C*(*Q, µ*)*|*

$$
\left(\frac{d^2\sigma}{dM_t^2 dM_t^2}\right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)
$$

$$
\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
$$

*T*<sup>+</sup> = *T*<sup>−</sup> = 1. Plugging Eq. (87) into Eq. (81), the final form for differential cross section is

**<sub>P</sub>** have an analytic fo *N<sup>c</sup>* We have an analytic formula for the double differential jet invariant mass distribution in terms of the pole mass. Note that Eq. (90) depends on two renormalization scales, *µ<sup>m</sup>* and *µ*. The matching scale *N<sup>c</sup>*  $\overline{\phantom{a}}$  $\bullet$  We have an analytic formula for the double differential jet invariant mass

• We can now switch to a short distance mass scheme in bHQET. matching at *m* we get the dependence on *µ<sup>m</sup>* in *Hm*, and from running below *m* we get

$$
m_{\rm pole} \, = \, m + \delta m
$$

Note that Eq. (90) depends on two renormalization scales, *µ<sup>m</sup>* and *µ*. The matching scale

So to sum the remaining large logarithms we have in principle two choices. We can either

where  $\alpha$  or higher, and must be strictly expanded perturbatively to the strictly expanded perturbatively to the strictly expanded perturbative limit be strictly expanded perturbatively to the strictly expanded perturbat

functions and the soft function.

**Switching Mass Schemes in bHQET**  
\n
$$
\left(\frac{d^2\sigma}{dM_t^2 dM_t^2}\right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m(m, \frac{Q}{m}, \mu_m, \mu) \times \int_{-\infty}^{\infty} dt^+ dt^- B_+(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu) B_-(\hat{s}_t - \frac{Q\ell^-}{m}, \Gamma, \mu) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
$$
\n**Anti-Top HQET**  
\n
$$
\mathcal{L}_+ = \bar{h}_{v_+}(iv_+ \cdot D_+ - \delta m + \frac{i}{2}\Gamma) h_{v_+}, \qquad \mathcal{L}_- = \bar{h}_{v_-}(iv_- \cdot D_- - \delta m + \frac{i}{2}\Gamma) h_{v_-}
$$
\n**Top mass scheme**

where  $\delta$  or higher, and must be strictly expanded perturbatively to the strictly expanded perturbatively to the strictly expanded perturbative limits be strictly expanded perturbative limits of  $\delta$ 

4π*m*

<sup>δ</sup>(2*v*<sup>+</sup> *· <sup>k</sup>*) <sup>=</sup> <sup>1</sup>

 $\alpha$  times logs that are summed up by renormalization group improved perturbation group improved perturbation theory.

<sup>=</sup> <sup>1</sup> *m*

*v*<sup>+</sup> *· k* + *i*0

same order as other O(αs) corrections. (This strict expansion does not apply to powers of

¯ ∼ Γ ! m. For our top-quark analysis, the center of mass frame is

which is identical to the result for the result for the corresponding  $S_{\rm C}$  jet  $\sigma$ 

δ(*s*ˆ) = δ(*s*)*,* (89)

*v*<sup>+</sup> *· k* + *i*0

*m*

• Power counting in bHQET requires *■* Power counting in bHOET requires matching at *m* we get the dependence on *µ<sup>m</sup>* in *Hm*, and from running below *m* we get  $S_{\infty}$  to sum the remaining large logarithms we have in principal two choices. We can either  $\mathcal{L}$ 

Γ

#

initially by setting  $\delta$  in Eq. (31). In this case the mass-dependence appears in Eq. (31). In this case the mass-dependence appears in Eq. (31). In this case of the mass-dependence appears in Eq. (31). In this case of th

<sup>4</sup>π*Ncm*(−*Nc*) Disc! *<sup>i</sup>*

 $\delta m \sim \hat{s}_t \sim \hat{s}_{\bar{t}} \sim \Gamma$  $\bar{t} \sim \Gamma$  $\delta m \propto \hat{s}$ .  $\sim \hat{s}$  the  $\Gamma$ distance mass scheme which violates  $\zeta$  $\sigma_{l}$  and  $\sigma_{l}$  and  $\sigma_{l}$  in essentially corresponds to  $s_t \sim s_{\bar t} \sim 1$ 

Eq. (94) Final restricts us to a suitable counting preaks down in the mass scheme: the notion of a top-quark Breit  $\frac{1}{2}$  top-quark Breit  $\frac{1}{2}$  invariant becomes invalid. The most prominent prominent becomes invariant becomes invariant becomes invariant becomes invariant becomes invariant becomes example for an excluding shows so an excluding shows scheme in the MS mass scheme. • Note that this power counting breaks down in the  $\overline{\text{MS}}$  scheme: run the Wilson coefficient *H<sup>m</sup>* of we run the individual functions *B<sup>±</sup>* and *S*. The first option we will call this method *"top-down"*. The relation *H<sup>m</sup>*  $\mathbf{a}$ *m,*  $V1$ *<sup>m</sup>, <sup>µ</sup>m, <sup>µ</sup>*  $\mathbf{r}$ = *Hm*(*m, µm*)*U<sup>H</sup>m*(*µm, µ*) (92)

and converting to the MS scheme with the MS scheme with the MS scheme with the O( $\alpha$ 

"(M<sup>2</sup>

means running the jet functions *B<sup>±</sup>* and the soft function *S*hemi independently with the

evolution factors *U<sup>B</sup><sup>±</sup>* (*µ, µm*) and *US*(*µ, µm*) respectively, as is also illustrated in Fig. 4. This

 $\frac{1}{\sqrt{2}}$ 

*µ*

$$
\delta \overline{m} \sim \alpha_s \overline{m} \gg \Gamma
$$

mpole − m = δm. Here δm \$ 8 GeV % Γ, or parameterizally δm ₹ 8 GeV % Γ, or parameterizally δm ₹ αsm % Γ. Using E<br>Eq. (92)

*dµB*+(*s,* <sup>ˆ</sup> *<sup>µ</sup>*) <sup>=</sup>

<sup>m</sup><sup>2</sup> + Γ<sup>2</sup>

#2

#

*<u>x</u>* 

evolution factors *U<sup>B</sup><sup>±</sup>* (*µ, µm*) and *US*(*µ, µm*) respectively, as is also illustrated in Fig. 4. This

*ds*ˆ# γ*<sup>B</sup>*<sup>+</sup> (*s*ˆ− *s*ˆ#

<sup>m</sup><sup>2</sup> + Γ<sup>2</sup>

) *B*+(*s*ˆ#

*, µ*)*,*

 $\bullet$  We need a short-distance mass that respects the nower counting of bHOET. d a short distance mass that re " (M<sup>2</sup> <sup>t</sup> −m2)<sup>2</sup> he power count <u>t</u> • We need a short distance mass that respects the power counting of bHQET.  $\mathbf{di}$  $\frac{1}{2}$ defines the corresponding evolution factor *U<sup>H</sup><sup>m</sup>* that is shown in Fig. 4. The second option *d* %

<sup>m</sup><sup>2</sup> + Γ<sup>2</sup>

!<br>!<br>!

functions and the soft function.

|
|-<br>|-

*, µ*)*,* (93) *{*Brun*}*

#### **Pole in B+ and which is identical to the SCET of Top Resonance Mass Schemes** *m T*<sup>+</sup> = *T*<sup>−</sup> = 1. Plugging Eq. (87) into Eq. (81), the final form for differential cross section is *T*<sup>+</sup> = *T*<sup>−</sup> = 1. Plugging Eq. (87) into Eq. (81), the final form for differential cross section is **p Resot**  $\frac{1}{2}$ **.** *m, Q <sup>m</sup>, <sup>µ</sup>m, <sup>µ</sup>*  $\mathbf{e}$

<sup>=</sup> <sup>1</sup>

*v*<sup>+</sup> *· k* + *i*0

 $\tilde{D}_{\rm eff}$  can calculate B+(s $\tilde{D}_{\rm eff}$  ) in two equivalent ways. In two equivalent ways. In the pole-mass scheme schem

4π*m*

<sup>δ</sup>(2*v*<sup>+</sup> *· <sup>k</sup>*) <sup>=</sup> <sup>1</sup>

Im a construction of the construction of the construction of the construction of

*v*<sup>+</sup> *· k* + *i*0

which is identical to the result for the result for the corresponding  $S_{\rm C}$  jet  $\sigma$ 

= 10 minutes and 10 minutes and 10 minutes

initially by setting  $\delta$  in Eq. (31). In this case the mass-dependence appears in Eq. (31). In this case the mass-dependence appears in Eq. (31). In this case of  $\delta$ 

$$
\left(\frac{d^2\sigma}{dM_t^2\,dM_t^2}\right)_{\text{hemi}} = \sigma_0\,H_Q(Q,\mu_m)H_m\left(m,\frac{Q}{m},\mu_m,\mu\right) \times \int_{-\infty}^{\infty} \!\!d\ell^+d\ell^- \,B_+\left(\hat{s}_t-\frac{Q\ell^+}{m},\Gamma,\mu\right)B_-\left(\hat{s}_{\bar{t}}-\frac{Q\ell^-}{m},\Gamma,\mu\right)S_{\text{hemi}}(\ell^+,\ell^-,\mu)
$$

**•Top resonance mass schemes are compatible with measurements relying on an underlying Breit-Wigner which incorporates the top width:** De resonance mass senemer In this section we explore the resonance mass-schemes for m. With the notation for  $\frac{1}{\sqrt{2}}$  $\mathcal{A}$ . Potential Jet-Mass Definitions and Anomalous Dimensions and Anomalous Dimensions  $\mathcal{A}$  $A$ . Potential  $\alpha$  and  $\alpha$  and  $\alpha$  and  $A$ In the meeting we explore the top matter.  $\mathbf{n}$ **nass sch** *s*<br>*s***1** ∴ 1 *r cot* .<br>~ *B*<sup>−</sup>  $\ddot{\mathbf{t}}$ ∂*ible* wit *,* Γ*, µ*  $\overline{a}$ <mark>asurements</mark> 1 | diluctlying Dicit *w* igher which hieorporates the  $\frac{1}{2}$  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$ ates  $\sim$   $\sim$  $\Phi$  top width: *<sup>n</sup>*¯(0)*|*0\$*.* (91)

$$
\delta m \sim \hat{s}_t \sim \hat{s}_{\bar{t}} \sim \Gamma
$$

**• Peak Mass**

sˆpole = (M<sup>2</sup> − m<sup>2</sup>

**BΓ**=00 **BΓ**=00 **BΓ** 

+ (*s*<sup>2</sup>) = −1 (s<sup>2</sup>) = −1 (s<sup>2)</sup> = −1 (s<sup>2)</sup> = −1 (s<sup>2)</sup> = −1 (s<sup>2)</sup> = −1 (s2) =

*m*

- **Moment Mass**
- **•** Position Mass (Jain, Scimemi, Stewart)  $\int ds \hat{s} B(\hat{s},\delta m^{\text{in}})$  $f(x) = \frac{1}{2} \cos(x) \sin(x) \sin(x)$

<sup>4</sup>π*Ncm*(−*Nc*) Disc! *<sup>i</sup>*

<sup>δ</sup>(2*v*<sup>+</sup> *· <sup>k</sup>*) <sup>=</sup> <sup>1</sup>

\n- \n Some mass schemes in this context are\n 
$$
\frac{d}{d\hat{s}} B(\hat{s}, \delta m^{\text{peak}}, \Gamma_t, \mu) \Big|_{\hat{s}=0} = 0
$$
\n
\n- \n Peak Mass\n
	\n- Month Mass (Jain, Scimemi, Stewart)
	\n- \n $\int_{-\infty}^{R} d\hat{s} \hat{s} B(\hat{s}, \delta m^{\text{mom}}, \mu) = 0$ \n
	\n- \n $\delta m_J = \frac{-i}{2 \tilde{B}(y, \mu)} \frac{d}{dy} \tilde{B}(y, \mu) \Big|_{y=-ie^{-\gamma_E}/R}$ \n
	\n\n
\n

δ(*s*ˆ) = δ(*s*)*,* (89)

 $\frac{1}{2}$  +  $\frac{1$ 

*, µ*)*,*

) *B*+(*s*ˆ#

#2

 $\mathcal{L}(\mathcal{G})$ **•These top resonance mass schemes can be related to the more familiar mass schemes***.*  $W$  refers to a), c), c), c), c), c), as the peak-mass, and position-mass respectively. The peak-mass definition-mass definition-mass definition-mass definition-mass definition-mass definition-mass definition uses definit ass schemes can be refated to the more familiar mass the jet function with a non-zero width and satisfies the  $\alpha$  Ft power counting criteria  $\alpha$  $W$  refers to a), c), c), c), c), c), as the peak-mass, and position-mass respectively. The peak-mass definition-mass definition-mass definition-mass definition-mass definition-mass definition-mass definition uses definit the jet function with a non-zero width and satisfies the  $\alpha$  Ft power counting criteria  $\alpha$  function  $\alpha$ **• These top resonance mass schemes can be related to the more familiar mass** evolution factors *U<sup>B</sup><sup>±</sup>* (*µ, µm*) and *US*(*µ, µm*) respectively, as is also illustrated in Fig. 4. This

*µ*

 $\frac{1}{2}$  +  $\frac{1$ 

*dµB*+(*s,* <sup>ˆ</sup> *<sup>µ</sup>*) <sup>=</sup>

#

"<br>"(M2)<br>"(M2)

*ds*ˆ# γ*<sup>B</sup>*<sup>+</sup> (*s*ˆ− *s*ˆ#

*ds*ˆ# *U<sup>B</sup>*<sup>+</sup> (*s*ˆ− *s*ˆ#

" (M2)<br>" (M2)<br>" (M2)

πm

defines the corresponding evolution factor *U<sup>H</sup><sup>m</sup>* that is shown in Fig. 4. The second option

means running the jet functions *B<sup>±</sup>* and the soft function *S*hemi independently with the

the jet function with a non-zero width and satisfies the ower counting criteria [12]. In b) and c) the schemes<br>In b) and c) the schemes counting criteria [12]. In b) and c) the schemes counting counting counting counting

depend on a parameter R, and we must take R ∼ Γ<sup>t</sup> in order to satisfy the power counting criteria. Different choices for R specify different schemes, and are analogous to the difference between the MS and MS mass-schemes. All three

schemes in Eq. (57) are free from leading renormalon ambiguities [65]. In the following we will argue that only the

 $\mathbb{R}^2$ 

(90) *{*bHQETcross-hem*}*

depend on a parameter R, and we must take R ∼ Γt in order to satisfy the power counting criteria. Different ch<br>The power counting criteria. Different choices counting criteria. Different choices counting counting counting

depend on a parameter R, and we must take R ∞ Γt in order to satisfy the power counting criteria. Different ch<br>The power counting criteria. Different choices counting criteria. Different choices counting criteria. Differe

for R specify different schemes, and are analogous to the difference between the MS and MS mass-schemes. All three

*, µ*)*,* (93) *{*Brun*}*

for R specify different schemes, and are analogous to the difference between the MS and MS mass-schemes. All three



! !

%

 $\sim$  0  $\sim$ 

define resummed jet masses where one applies the condition of Eq.(104) to the LL, NLL,

in the pole mass scheme it remains stable in the jet mass scheme. As a result, experimentally

one will be sensitive to the jet mass is extracted from experiment it can be sensitive to the jet  $\alpha$ 

*.* (98)

be related to the more familiar pole mass via Eq.(102) or any other mass scheme such as

 $T$  simplify the notation we will use the notation  $\mathcal{L}$   $\mathcal{L}$   $\mathcal{L}$  $\sim$  At iNLO the jet-mass is related to the At NLO the jet mass is related to the pole mass scheme as follows:

where  $m$  is the pole-mass jet function to  $\mathcal{A}+\mathcal{A}$  is the pole-mass jet function to  $\mathcal{A}+\mathcal{A}$ 

*<sup>m</sup><sup>J</sup>* (*µ*) <sup>=</sup> *<sup>m</sup>*pole <sup>−</sup> <sup>Γ</sup>α*s*(*µ*)

corrections that are power suppressed by Γ/m. The one-loop relation between the pole and

determination that relies on the reconstruction of the peak position of an invariant mass

*dB*+(*s,* ˆ *µ,* δ*m<sup>J</sup>* )

$$
m_J(\mu) = m_{\text{pole}} - \Gamma \frac{\alpha_s(\mu)}{3} \Big[ \ln \left( \frac{\mu}{\Gamma} \right) + \frac{3}{2} \Big]
$$

 $F_{\rm eff}$   $=$   $F_{\rm eff}$   $=$   $F_{\rm eff}$   $=$   $F_{\rm eff}$   $=$   $F_{\rm eff}$  one-loop pole mass is  $\sim$ 

 $E_{\rm eff}$  also shows that the jet-mass is substantially different from the short-distantial different from the short-distance  $\sigma$ 

¯-threshold analyses [12], where <sup>δ</sup><sup>m</sup> <sup>∼</sup> <sup>α</sup><sup>2</sup>

ln '*<sup>µ</sup>*

improve the perturbative stability of the peak position.

+

corrections that are power suppressed by  $\Gamma$  m. The one-loop relation by  $\Gamma$  m. The one-loop relation between the pole and pole an

jet-mass is [60]

jet-mass is [? ]

#### **IR Group Flow of the Top Mass**  $\overline{C}$  and  $\overline{C}$  and  $\overline{C}$  and  $\overline{C}$  and  $\overline{C}$  and  $\overline{C}$  and  $\overline{C}$   $\overline{C}$  mpole and  $\overline{C}$  an

series for the normalization of the u = 1/2 singularity in

*•* explain what we are going to do: a complete analysis of twist-4 *n* = 2 perturbative effects.

*•• explain, Scimemi, Stewart)* and the structure function moments  $(\mathsf{Hoang}, \mathsf{Join}, \mathsf{Scimemi}, \mathsf{Stewart})$  $\alpha$ ny other scheme metal scheme metal scheme metal scheme metal scheme metal scheme metal specifies to specifying ad-

 $\blacksquare$   $\blacksquare$  Mass s • Mass schemes can be parameterized by 'R'.

$$
m_{\text{pole}} = m(R, \mu) + \delta m(R, \mu),
$$

$$
\delta m(R, \mu) = R \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk} \left[ \frac{\alpha_s(\mu)}{4\pi} \right]^n \ln^k \left( \frac{\mu}{R} \right)
$$

1.<br>|-<br>| Single as (1990) Here are numbers, and as in the MS-scheme with the MS-scheme with the MS-scheme with the MS-scheme with the MS-**•MSbar and top resonance schemes satisfy**   $\overline{L}$  running, mixing, etc...

<sup>∆</sup> *· <sup>Q</sup>*1(*k,*!)

 $\frac{1}{\sqrt{2}}$  =  $\frac{1}{\sqrt{2}}$ 

<sup>∆</sup> *· <sup>Q</sup>*2(*k,*!) *n*

dαs(µ)  $MS: R' = \bar{m}(\mu) \gg \Gamma_t$ , Top Resonance scheme :  $R \sim \Gamma_t \ll m$ 

**Conversion betwe** is not generic. For instance, the 1S-mass is defined as  $=$   $\frac{1}{2}$ αs( $\frac{1}{2}$ αs(<u>ntro</u> βn **• Conversion between such schemes can introduce large logs of** 

= *g*ψ¯*R*∆*/ d*

γR

= *g*ψ¯*R*τ*a*∆*/ d*

$$
\log \frac{R'}{R}
$$

←! *d* →*k*

<u>|</u><br>|-<br>| αs(α)| αs(α)| αs(α)|

'<sup>α</sup>s(R)

• These logs are summed by an IR group flow(more details in 0803.4214, HJSS). <sup>f</sup> , on integrals  $R\frac{d}{dR}m(R) = -R\,\gamma_R[\alpha_s(R)]$  $\frac{1}{1000}$   $\frac{1}{1000}$   $\frac{1}{100}$ m(R, µ) = 0. In these schemes a<sup>11</sup> = 0, and ank with k ≥ 1 are determined by the a<sup>n</sup> ≡ an<sup>0</sup> s<br>Se logs are summed by an IR grou These mass schemes come in two categories. In µ-independent schemes (such as RGI, kinetic, 1S):  $m_{\rm H}$  = 0. In these schemes at  $1$ drm(R) = − drm(R) = d<br>Cristian = drm(R) =

*d*

(n+1

ψ*<sup>R</sup>* ψ¯*R*∆*/ d*

γR

ψ*<sup>R</sup>* ψ¯*R*∆*/ d*

→*n*−2−*k*−!

(<sup>n</sup>+1

'<sup>α</sup>s(R)

. <u>(6) 1961 - Johann Amerikaanse kon</u><br>1960 - Johann Amerikaanse koning<br>1960 - Johann Amerikaanse koning

ψ*R,*

τ*a*ψ*R,*

. <u>(6) 1972 - Johann John Stein, deutscher Karl</u>

's specific the specific the scheme. Masses in the scheme.

d ln µ

II. OPERATOR BASIS

asymptotically for large n. For the heavy quark masses

 $(\alpha,\beta)$  that we study, the study, that we study, the study, this behavior is re-

ferred to as the pole-mass O(ΛQCD) renormalon prob-

lem [1], where the Borel transform of the series has a

singularity at u = 1/2. Schemes without this infrared

problem are known as short-distance masses, and always

distance quark mass definition, mR(µ). Examples are

 $\mathbb{R}$ 

one-half the mass of the heavy quarkonium <sup>3</sup>S<sup>1</sup> state in

perturbation theory, and its R is of order the inverse Bohr

ank with k ≥ 1 are determined by the a<sup>n</sup> ≡ an<sup>0</sup>

other category have a µ-anomalous dimension (like MS),

 $M$  : m(n),  $R$  : m(n),  $R$ 

|
|
|

other category have a µ-anomalous dimension (like MS),

m(R, µ) =

 $\overline{ }$ 

's specify the scheme. Masses in the

and using d/d ln µ mpole = 0 one finds <sup>d</sup>

#### Which is identical to the Short Distance Top Mass which is identical to the result for the corresponding  $\mathbf{S}^{\text{c}}$  function, so at tree level  $\mathbf{S}^{\text{c}}$ **Extraction of the Short Distance Top Mass**

*v*<sup>+</sup> *· k* + *i*0

 $= 100$ 

4π*m*

Imerican de la Constantina de la Const<br>La Constantina de la Constantina de la

*v*<sup>+</sup> *· k* + *i*0

*Q* " *m* " Γ *>* Λ*QCD* (3)



*ds*ˆ# γ*<sup>B</sup>*<sup>+</sup> (*s*ˆ− *s*ˆ#

*ds*ˆ# γ*<sup>B</sup>*<sup>+</sup> (*s*ˆ− *s*ˆ#

 $\ddot{\phantom{0}}$ 

) *B*+(*s*ˆ#

) *B*+(*s*ˆ#

*, µ*)*,*

*, µ*)*,*

*B*<sup>Γ</sup>=0

<sup>+</sup> (*s*ˆ) <sup>=</sup> <sup>−</sup><sup>1</sup>

*µ*

*n*

*µ d*

*dµB*+(*s,* <sup>ˆ</sup> *<sup>µ</sup>*) <sup>=</sup>

*dµB*+(*s,* <sup>ˆ</sup> *<sup>µ</sup>*) <sup>=</sup>

*m*

<sup>4</sup>π*Ncm*(−*Nc*) Disc! *<sup>i</sup>*

<sup>4</sup>π*Ncm*(−*Nc*) Disc! *<sup>i</sup>*

*m*

<sup>δ</sup>(2*v*<sup>+</sup> *· <sup>k</sup>*) <sup>=</sup> <sup>1</sup>

FIG. 15: F(Mt, M<sup>t</sup>

is shown at NLL order.

# **Conclusions**

**• An analytic framework in effective field theory now exists for high energy pair production of tops at a linear collider:**

- **•Factorization for high energy top pair production at a linear collider.**
- **•Large logarithms summed using RG equations in effective field theory; NLL resummation**
- **•Short distance mass schemes suitable for reconstruction from jets; Top resonance schemes**
- **•Measured peak position can be related to the short distance mass.**