Top Mass Reconstruction from Jets

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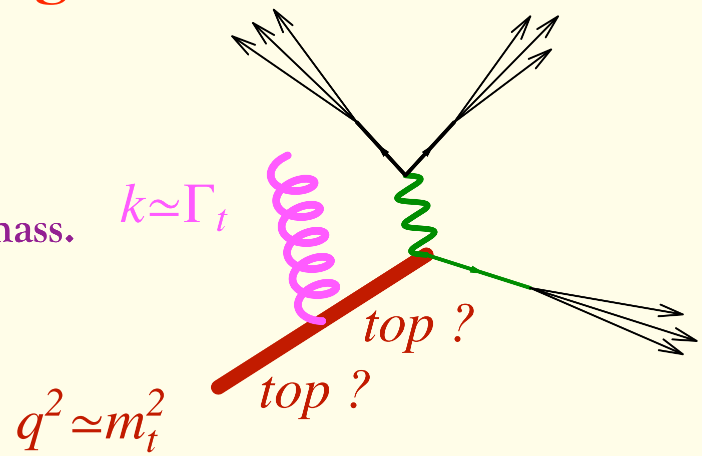
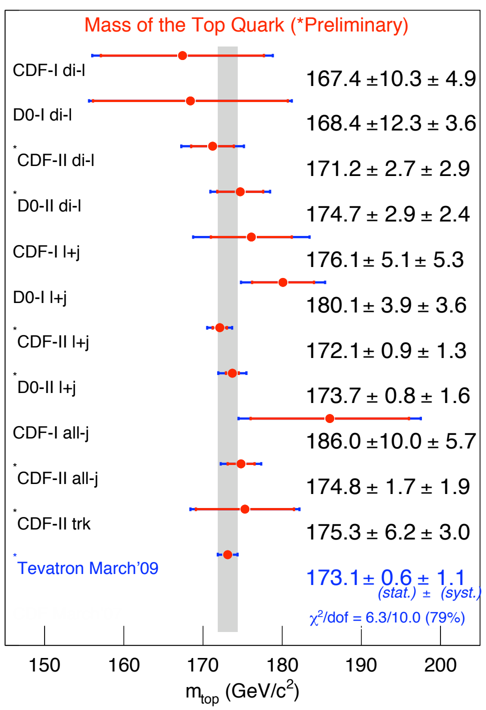
What are we Measuring?

- What is the top mass?
  - Top is a colored parton. Cannot define physical on-shell mass.
  - Top mass is a parameter of the Lagrangian.
  - Top mass parameter is scheme dependent.





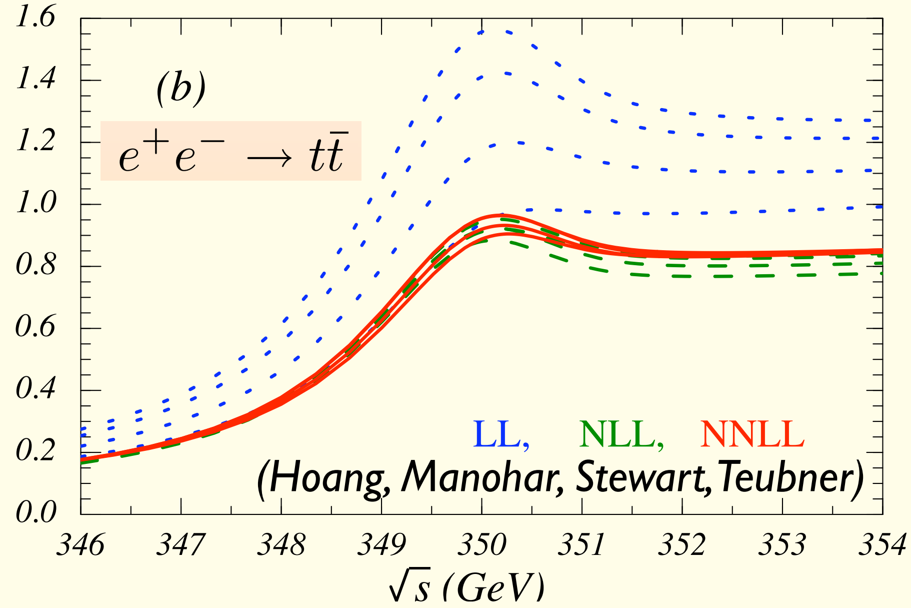
$$M_t = 173.1 \pm 1.3 \text{ GeV}/c^2$$

- Which top mass?
  - Which mass are the experimentalists measuring?
  - Pole mass? : $\delta m \sim \Lambda_{QCD}$ renormalon ambiguity, poor perturbative behavior.
  - For better precision we need a short distance top mass.
  - How can we extract a short distance mass? Which mass?

Threshold Scan

(Fadin & Khoze; Peskin & Strassler; Hoang, Manohar, Stewart, Teubner,...)

- **Top pair production in the threshold region**
  - Shape of total cross-section sensitive to top mass.
  - Top width provides IR cutoff.
  - Non-perturbative effects are small.

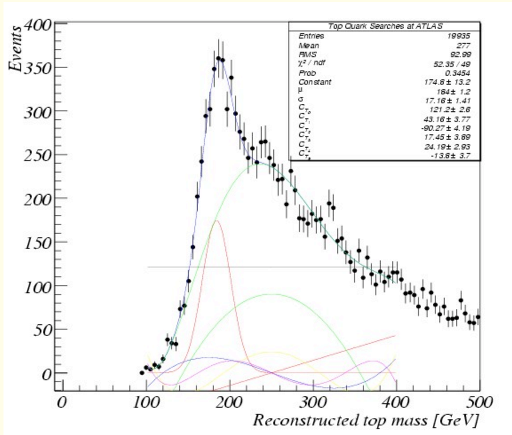


- **Physics well understood**
  - NRQCD is the appropriate EFT.
  - Well defined relation to short distance mass. eg. 1S mass
  - NNLL results known.
  - Theoretical uncertainty:

$$\delta m_t^{th} \sim 100 MeV$$

Jet Reconstruction

- **Jet reconstruction methods not so well understood**
  - Suitable observables with a well defined relation to a short distance mass ★
  - Summation of large logarithms ★
  - Final state radiation ★
  - Initial state radiation
  - PDFs
  - Jet Energy scale ★
  - Beam remnants
  - ...



★ Issues common to the ILC & LHC

- **We study high energy top pair production at the linear collider**

$$e^+e^- \to t\bar{t}X, \quad Q \gg m \gg \Gamma > \Lambda_{QCD}$$

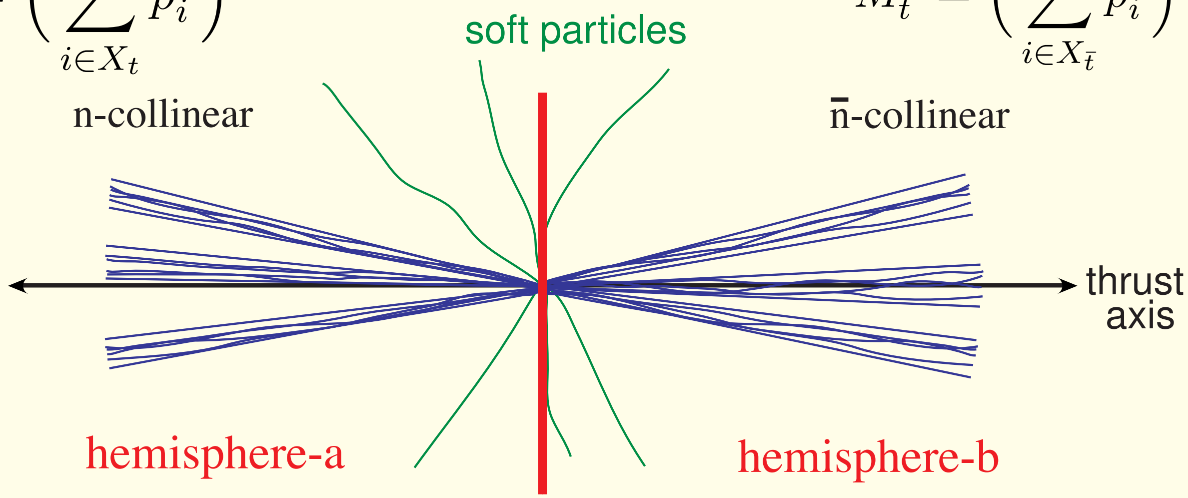
Linear Collider Observable

- **Hemisphere invariant mass distribution of top jets:**

$$\left(\frac{d\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\text{hemi}}$$

(S. Chekanov, V. Morgunov)

$$M_t^2 = \Big(\sum_{i\in X_t} p_i^\mu\Big)^2 \qquad M_{\bar t}^2 = \Big(\sum_{i\in X_{\bar t}} p_i^\mu\Big)^2$$



- **Peak Region**

$$\hat{s}_{t,\bar t} \equiv \frac{s_{t,\bar t}}{m} \equiv \frac{M_{t,\bar t}^2 - m^2}{m} \sim \Gamma \ll m$$

Relevant Energy Scales

- Center of mass energy $Q \sim 1\text{TeV}$
- Top quark mass $m \sim 174\text{GeV}$
- Top quark width $\Gamma \sim 2\text{GeV}$
- Confinement Scale $\Lambda \sim 500 MeV$

Disparate energy scales ⟶ Effective Field Theory!

Effective Field Theories

Kinematics for Top Jets: I

- **High Energy Condition**: Top quark pairs are produced with a center of mass energy much larger than the top mass

$$Q \gg m$$

- In this limit one can treat top quarks as collinear degrees of freedom in the Soft Collinear Effective Theory (SCET) *(Bauer, Fleming, Luke, Pirjol, Stewart)*.

Kinematics for Top Jets: II

- Invariant Mass Condition: We characterize on shell production by the requirement:

$$M_{t,\bar{t}}^2 - m^2 \lesssim m\Gamma$$

- This condition looks like the invariant mass constraint on a heavy quark in Heavy Quark Effective Theory (HQET) *(Isgur, Wise,...)*.

- HQET has been generalized to unstable particles*(Beneke, Chapovsky, Signer, Zanderighi)*.

Group Photo of Effective Field Theories

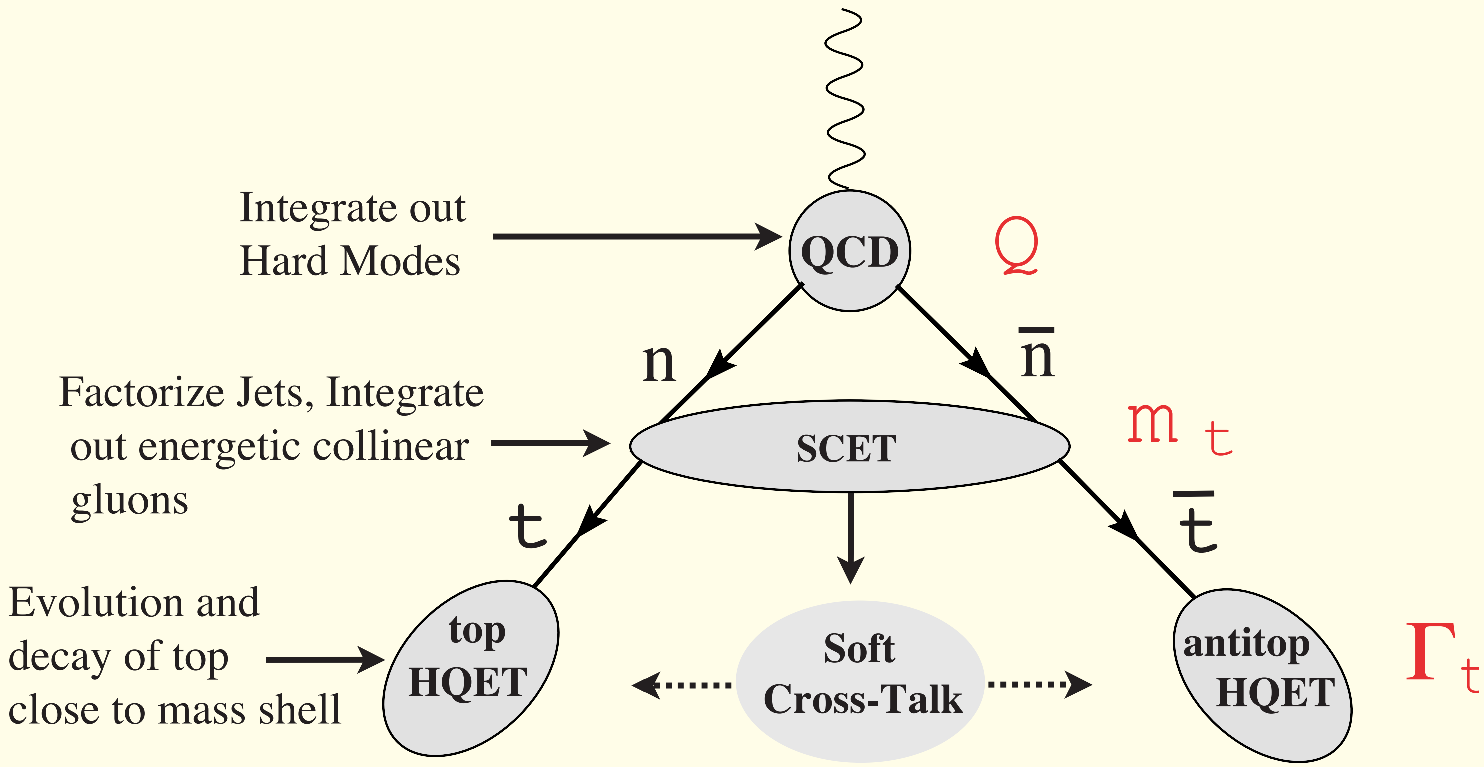
QCD Q

↓

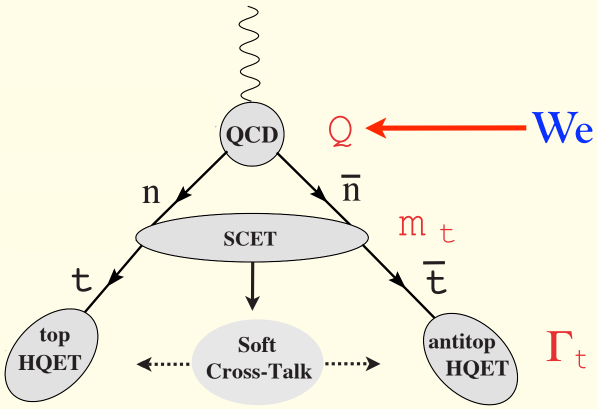
SCET m

↓

HQET Γ



The QCD Cross-Section



- The cross-section in QCD has the general form :

$$\sigma \;=\; \sum_{X}^{res.} (2\pi)^4\, \delta^4(p_e + p_{\bar{e}} - p_X) \sum_{ij} L_{\mu\nu}^{(ij)} \;\langle 0|J_i^{\mu}(0)|X\rangle\langle X|J_j^{\dagger\nu}(0)|0\rangle$$

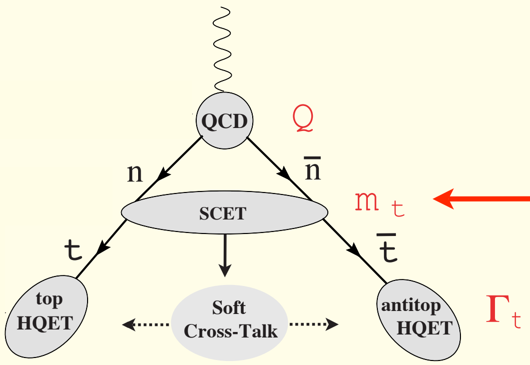
- The sum over final states X is restricted to contain a top jet and an anti-top jet with invariant masses close to the top mass.

- The top quark currents are produced by photon and Z exchange:

$$J_i^{\mu}(x) = \bar{\psi}(x)\Gamma_i^{\mu}\psi(x)\,, \quad \Gamma_{\gamma}^{\mu} \;=\; \gamma^{\mu},\, \Gamma_Z^{\mu} = g^V\gamma^{\mu} + g^A\gamma^{\mu}\gamma_5$$

SCET Cross-section

(Fleming, Hoang, Mantry, Stewart)



We are here.

- The SCET cross section takes the form:

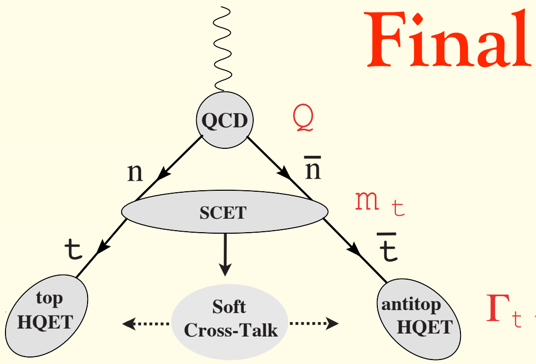
$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2} = \sigma_0\, H_Q(Q,\mu)\int_{-\infty}^{\infty} d\ell^+ d\ell^-\; J_n(s_t - Q\ell^+,\mu)\, J_{\bar n}(s_{\bar t} - Q\ell^-,\mu)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

Hard Wilson Coefficient — Top Jet Function — Anti-Top Jet Function — Soft Cross Talk Function

- The same soft function appears in massless dijets *(Korchemsky & Sterman; Bauer, Lee, Manohar, Wise)*

Final Form of Factorized Cross-Section

(Fleming, Hoang, Mantry, Stewart)



We are here.

- The HQET cross section takes the form:

Hard Production modes integrated out

"Hard" collinear gluons integrated out

Evolution and decay of top quark close to mass shell

Non-perturbative Cross talk

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big)$$

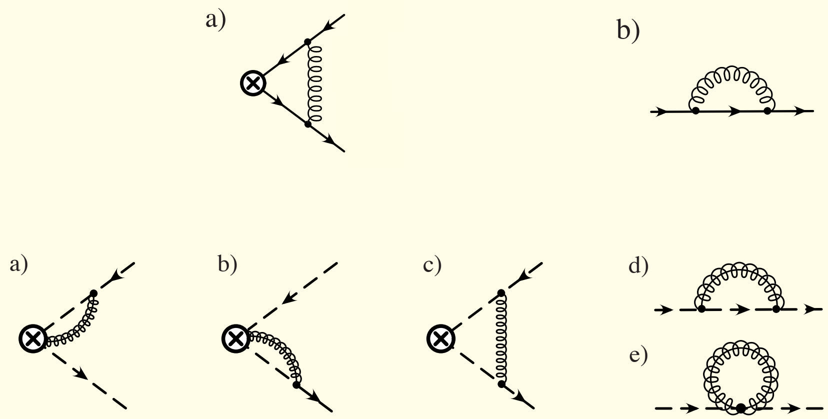
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Matching QCD onto SCET at One Loop

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

QCD

SCET



- **Matching QCD production current onto SCET at the hard scale:**

$$C(\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left[3 \ln\frac{-Q^2}{\mu^2} - \ln^2\frac{-Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

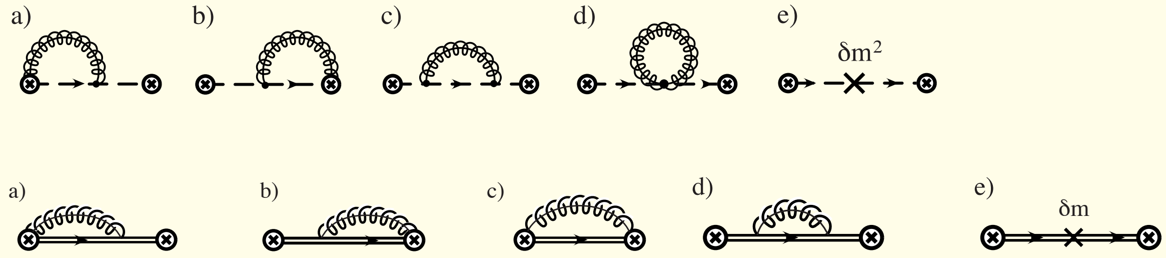
$$H_Q(Q,\mu_Q) = |C(Q,\mu_Q)|^2 = 1 + \frac{\alpha_s C_F}{4\pi}\left[-2\ln^2\left(\frac{Q^2}{\mu_Q^2}\right) + 6\ln\left(\frac{Q^2}{\mu_Q^2}\right) - 16 + \frac{7\pi^2}{3}\right]$$

Matching SCET onto BHQET at One Loop

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

SCET

BHQET



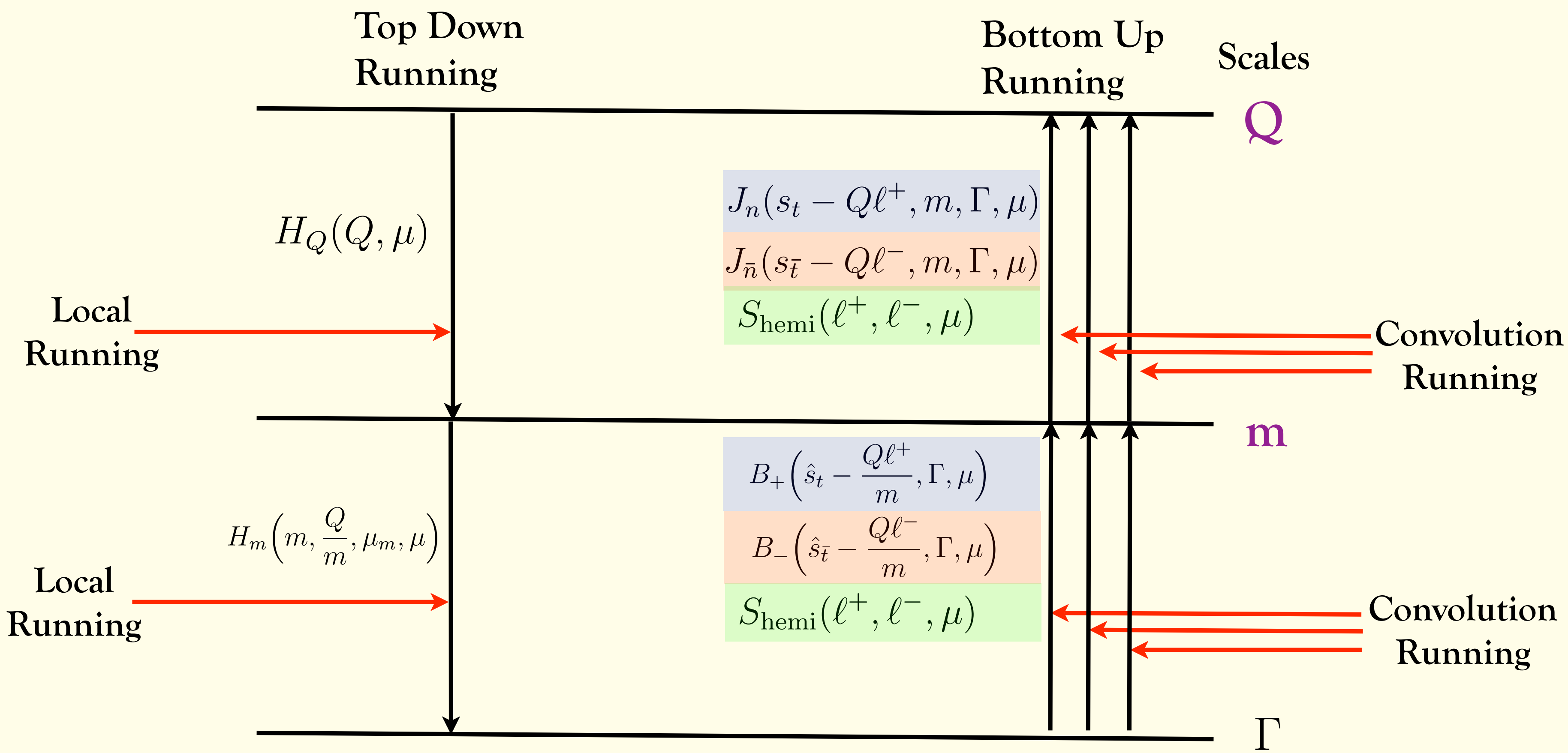
- **Matching SCET jet functions onto bHQET jet functions:**

$$J_n(m\hat s, \Gamma, \mu_m) = T_+(m,\mu_m)\, B_+(\hat s, \Gamma, \mu_m)\,,$$
$$J_{\bar n}(m\hat s, \Gamma, \mu_m) = T_-(m,\mu_m)\, B_-(\hat s, \Gamma, \mu_m)$$

$$T_\pm(\mu, m) = 1 + \frac{\alpha_s C_F}{4\pi}\Big(\ln^2\frac{m^2}{\mu^2} - \ln\frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6}\Big)\,.$$

$$H_m\Big(m, \mu_m\Big) = T_+(m,\mu_m) T_-(m,\mu_m) = 1 + \frac{\alpha_s C_F}{2\pi}\Big(\ln^2\frac{\mu_m^2}{m^2} + \ln\frac{\mu_m^2}{m^2} + 4 + \frac{\pi^2}{6}\Big)$$

Equivalence of Top-Down vs. Bottom Up



- Running between the different scales mostly affects only the normalization!

$$\frac{d\sigma}{dM_t^2\, dM_{\bar{t}}^2} = \sigma_0\, H_Q(Q, \mu_m) H_m\left(m_J, \frac{Q}{m_J}, \mu_m, \mu\right)$$

$$\times \int d\ell^+ d\ell^- B_+\left(\hat{s}_t - \frac{Q\ell^+}{m_J}, \Gamma_t, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m_J}, \Gamma_t, \mu\right) S(\ell^+, \ell^-, \mu)$$

Short Distance Mass for Jets

Connecting the Observable to a Short Distance Mass Scheme

- Top mass sensitivity comes from the bHQET jet functions

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

- We have an analytic formula for the double differential jet invariant mass distribution in terms of the pole mass.

- We can now switch to a short distance mass scheme in bHQET.

$$m_{\rm pole} = m + \delta m$$

Switching Mass Schemes in bHQET

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \; B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\; B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\; S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Top HQET

$$\mathcal{L}_+ = \bar h_{v_+}\big(i v_+ \cdot D_+ - \delta m + \frac{i}{2}\Gamma\big) h_{v_+},$$

Anti-Top HQET

$$\mathcal{L}_- = \bar h_{v_-}\big(i v_- \cdot D_- - \delta m + \frac{i}{2}\Gamma\big) h_{v_-}$$

Top mass scheme

- Power counting in bHQET requires

$$\delta m \sim \hat s_t \sim \hat s_{\bar t} \sim \Gamma$$

- Note that this power counting breaks down in the $\overline{\text{MS}}$ scheme:

$$\delta \overline{m} \sim \alpha_s \overline{m} \gg \Gamma$$

- We need a short distance mass that respects the power counting of bHQET.

Top Resonance Mass Schemes

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

- Top resonance mass schemes are compatible with measurements relying on an underlying Breit-Wigner which incorporates the top width:

$$\delta m \sim \hat s_t \sim \hat s_{\bar t} \sim \Gamma$$

- Some mass schemes in this context are
  - Peak Mass
  - Moment Mass
  - Position Mass *(Jain, Scimemi, Stewart)*

$$\frac{d}{d\hat s} B(\hat s, \delta m^{\rm peak}, \Gamma_t, \mu)\Big|_{\hat s = 0} = 0$$

$$\int_{-\infty}^{R} d\hat s\ \hat s\ B(\hat s, \delta m^{\rm mom}, \mu) = 0$$

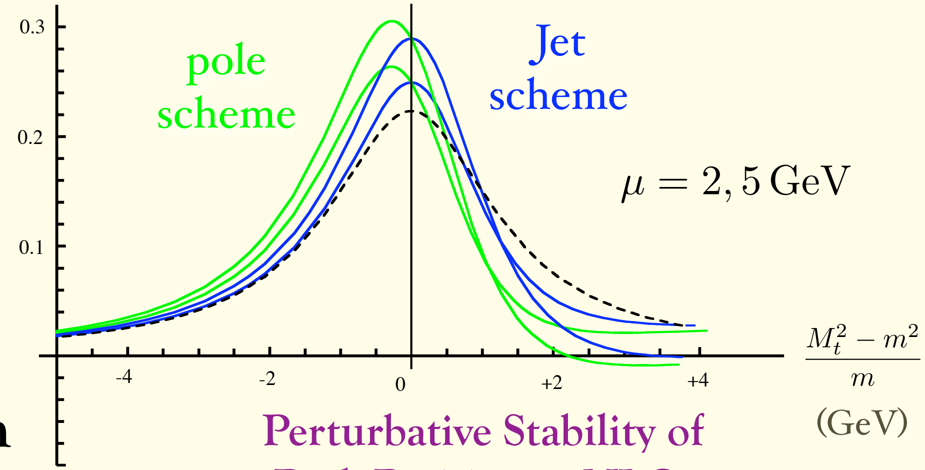
$$\delta m_J = \frac{-i}{2\, \tilde B(y,\mu)} \frac{d}{dy} \tilde B(y,\mu)\Big|_{y = -ie^{-\gamma_E}/R}$$

- These top resonance mass schemes can be related to the more familiar mass schemes.

Peak Mass

- **Define peak mass as:**

$$\left.\frac{dB_+(\hat{s},\mu,\delta m_J)}{d\hat{s}}\right|_{\hat{s}=0} = 0$$



Perturbative Stability of Peak Position at NLO.

- **In the jet mass scheme the NLO jet function is modified as:**

$$\tilde{B}_\pm(\hat{s},\mu) \;=\; B_\pm(\hat{s},\mu) + \frac{1}{\pi m_J}\,\frac{(4\,\hat{s}\,\Gamma)\,\delta m_J}{(\hat{s}^2+\Gamma^2)^2}$$

- **At NLO the jet mass is related to the pole mass scheme as follows:**

$$m_J(\mu) = m_{\rm pole} - \Gamma\frac{\alpha_s(\mu)}{3}\left[\ln\left(\frac{\mu}{\Gamma}\right) + \frac{3}{2}\right]$$

IR Group Flow of the Top Mass

(Hoang, Jain, Scimemi, Stewart)

- Mass schemes can be parameterized by 'R'.

$$m_{\rm pole} = m(R,\mu) + \delta m(R,\mu)\,,$$

$$\delta m(R,\mu) = R\sum_{n=1}^{\infty}\sum_{k=0}^{n} a_{nk}\left[\frac{\alpha_s(\mu)}{4\pi}\right]^n \ln^k\left(\frac{\mu}{R}\right)$$

- MSbar and top resonance schemes satisfy

$$\overline{\rm MS} : R' = \bar{m}(\mu) \gg \Gamma_t, \qquad \text{Top Resonance scheme} : R \sim \Gamma_t \ll m$$

- Conversion between such schemes can introduce large logs of

$$\log\frac{R'}{R}$$

- These logs are summed by an IR group flow(more details in 0803.4214, HJSS).

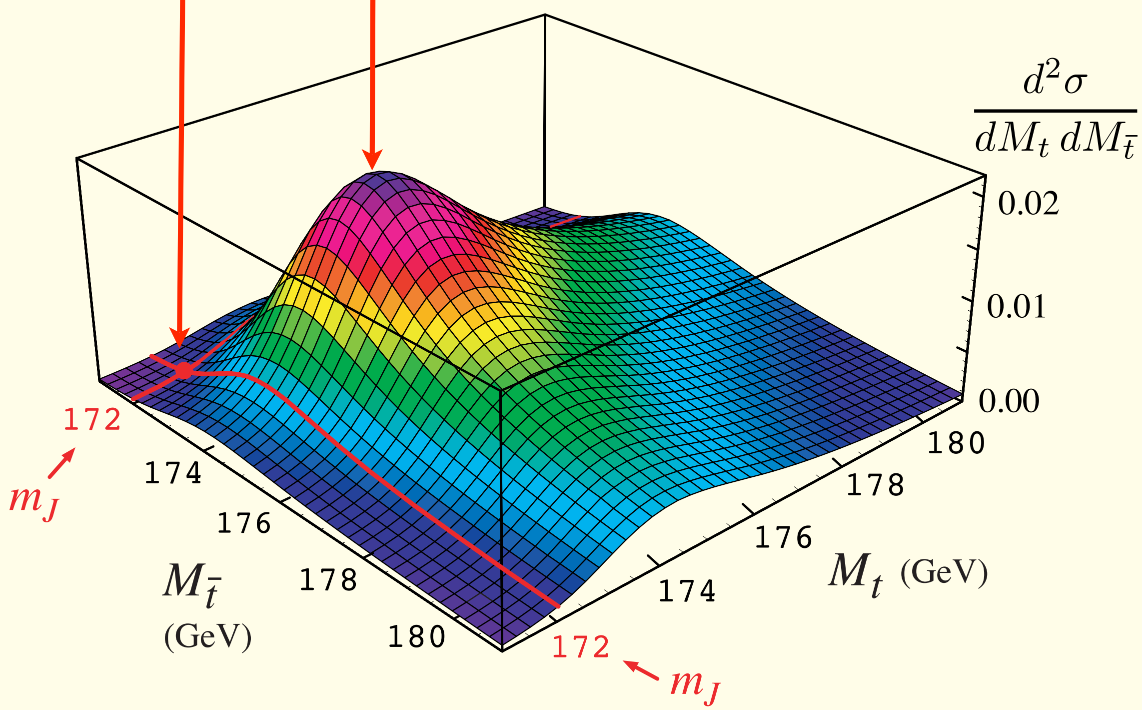
$$R\frac{d}{dR}m(R) = -R\,\gamma_R[\alpha_s(R)]$$

Extraction of the Short Distance Top Mass

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Extract top mass

$$m = M_{\text{peak}} - \Gamma(\alpha_s + \alpha_s^2 + \cdots) - \frac{Q\Lambda_{QCD}}{m}$$



- **Final result with NLL resummation.**

(Fleming, Hoang, Mantry, Stewart)

Conclusions

- An analytic framework in effective field theory now exists for high energy pair production of tops at a linear collider:

    - Factorization for high energy top pair production at a linear collider.
    - Large logarithms summed using RG equations in effective field theory; NLL resummation
    - Short distance mass schemes suitable for reconstruction from jets; Top resonance schemes
    - Measured peak position can be related to the short distance mass.