Two-loop soft anomalous dimensions with massive and massless quarks

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- **Soft anomalous dimensions**
- **Two-loop eikonal calculations**
- **Top quark production**

Soft gluon corrections

Increased accuracy in theoretical predictions requires higher-order corrections

One class of corrections are soft-gluon corrections

They arise from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons

Terms of the form $\left[\frac{\ln^k(1-z)}{1-z}\right]_+$, $z \longrightarrow 1$ at threshold

Resum (exponentiate) these soft corrections

At NLL (NNLL) accuracy requires one-loop (two-loop) calculations in the eikonal approximation

Near threshold soft corrections are dominant and can provide good approximations to the complete cross section

Examples: top pair and single top production

jet, direct photon, or W production at high p_T

Resummed cross section

Resummation follows from factorization properties of the cross section

- performed in moment space

Use RGE to evolve function associated with soft-gluon emission

H**: hard-scattering function** S**: soft-gluon function**

$$
\begin{aligned}
\hat{\sigma}^{res}(N) \quad = \quad & \exp\left[\sum_i E_i(N)\right] \; H\left(\alpha_s\right) \\
& \times \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu}\, \Gamma_S^\dagger\left(\alpha_s(\mu)\right)\right] \; S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}}\right)\right) \; \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu}\, \Gamma_S\left(\alpha_s(\mu)\right)\right]
\end{aligned}
$$

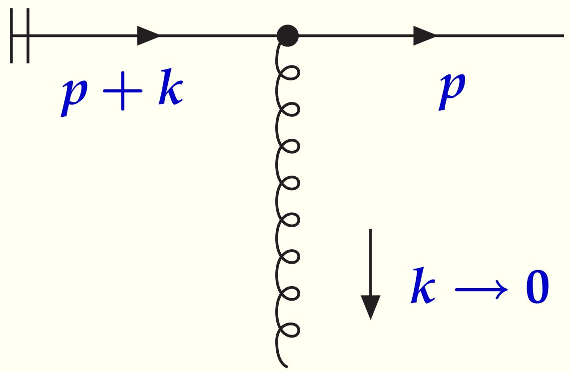
where

Γ_S **is the soft anomalous dimension - a matrix in color space**

and a function of kinematical invariants s, t, u

Calculate Γ_S **in eikonal approximation**

Eikonal approximation



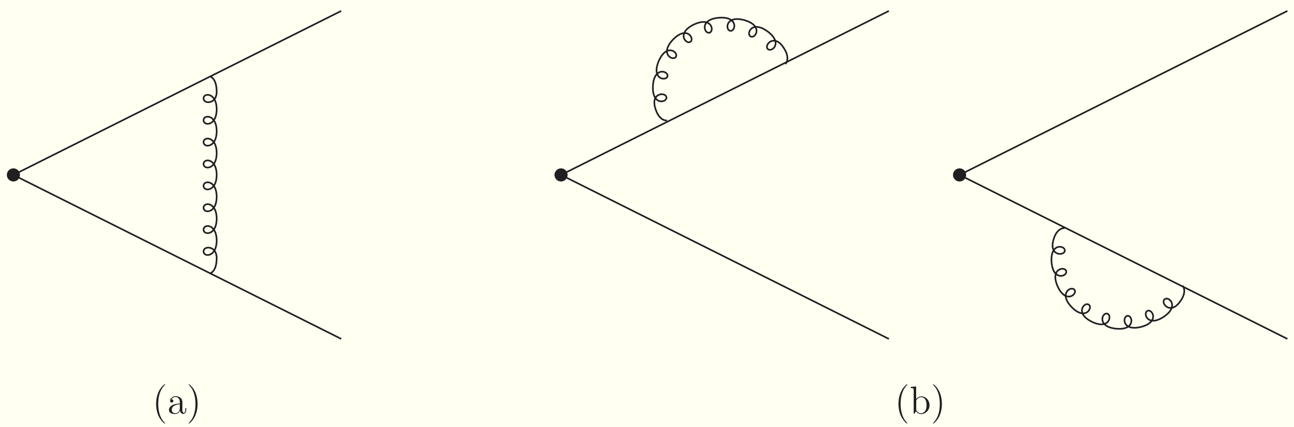
$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2+i\epsilon} \longrightarrow \bar{u}(p)\,g_s T_F^c\,\gamma^\mu \frac{\not{p}+m}{2p\cdot k+i\epsilon} = \bar{u}(p)\,g_s T_F^c\,\frac{v^\mu}{v\cdot k+i\epsilon}$$

with $p \propto v$, T_F^c **generators of SU(3)**

Perform calculation for massive quarks in momentum space and Feynman gauge

Complete two-loop results for $e^+e^- \longrightarrow t\bar{t}$

One-loop diagrams



The one-loop soft anomalous dimension, $\Gamma_S^{(1)}$, can be read off the coefficient of the ultraviolet (UV) pole of the one-loop diagrams

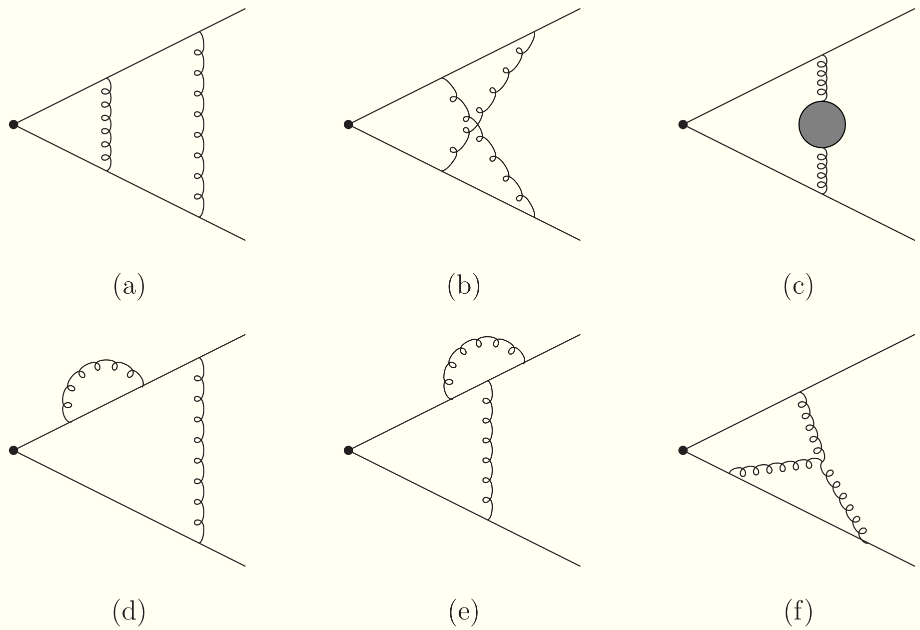
$$\Gamma_S = (\alpha_s/\pi)\Gamma_S^{(1)} + (\alpha_s/\pi)^2\Gamma_S^{(2)} + \cdots$$

$$\Gamma_S^{(1)} = C_F\left[-\frac{(1+\beta^2)}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right) - 1\right]$$

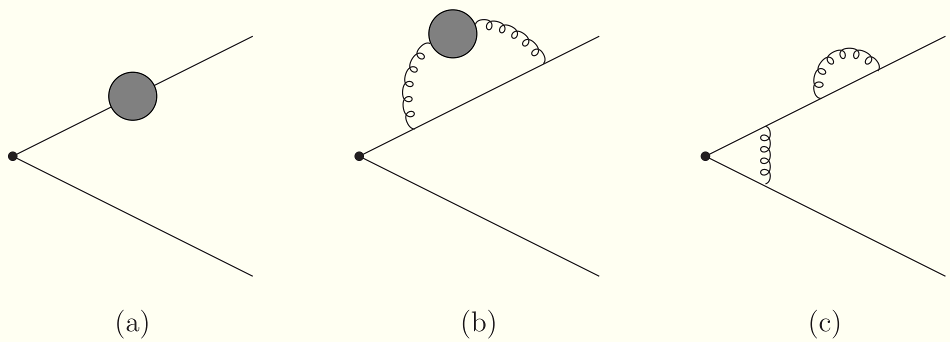
with $\beta = \sqrt{1 - \frac{4m^2}{s}}$

Two-loop diagrams

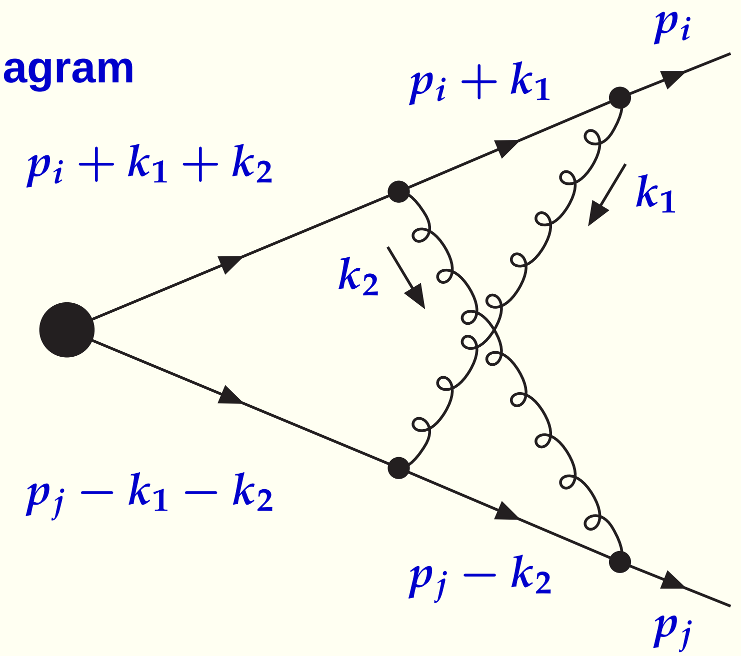
Vertex correction graphs



Heavy-quark self-energy graphs



Example: two-loop crossed diagram



$$I_{2b} = g_s^4 \int \frac{d^n k_1}{(2\pi)^n} \frac{d^n k_2}{(2\pi)^n} \frac{(-i)g^{\mu\nu}}{k_1^2} \frac{(-i)g^{\rho\sigma}}{k_2^2} \frac{v_i^\mu}{v_i \cdot k_1} \frac{v_i^\rho}{v_i \cdot (k_1+k_2)} \frac{(-v_j^\nu)}{-v_j \cdot (k_1+k_2)} \frac{(-v_j^\sigma)}{-v_j \cdot k_2}$$

Perform k_2 integral first

$$I_{2b} = -i\,\frac{\alpha_s^2}{\pi^2}\, 2^{-4+\epsilon}\, \pi^{-2+3\epsilon/2}\, \Gamma\left(1-\frac{\epsilon}{2}\right) \Gamma(1+\epsilon)(1+\beta^2)^2 \int_0^1 dz \int_0^1 \frac{dy\,(1-y)^{-\epsilon}}{\left[2\beta^2(1-y)^2 z^2 - 2\beta^2(1-y)z - \frac{(1-\beta^2)}{2}\right]^{1-\epsilon/2}}$$

$$\times \int \frac{d^n k_1}{k_1^2\, v_i \cdot k_1\, [\left((v_i - v_j)z + v_j\right) \cdot k_1]^{1+\epsilon}}$$

Then proceed with the k_1 integral, and isolate UV and IR poles. After many steps

$$I_{2b}^{UV} = \frac{\alpha_s^2}{\pi^2} \frac{(1+\beta^2)^2}{8\beta^2} \frac{1}{\epsilon} \left\{ -\ln\left(\frac{1-\beta}{1+\beta}\right) \left[\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) + \zeta_2 \right] - \frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right) + \mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) - \zeta_3 \right\}$$

Include counterterms for all graphs and multiply with corresponding color factors

Determine two-loop soft anomalous dimension from UV poles of the sum of the graphs

$$\Gamma_S^{(2)} = \left\{\frac{K}{2} + \frac{C_A}{2}\left[-\frac{1}{3}\ln^2\left(\frac{1-\beta}{1+\beta}\right) + \ln\left(\frac{1-\beta}{1+\beta}\right) - \zeta_2\right]\right.$$
$$\left. + \frac{(1+\beta^2)}{4\beta} C_A \left[\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) + \frac{1}{3}\ln^2\left(\frac{1-\beta}{1+\beta}\right) + \zeta_2\right]\right\} \Gamma_S^{(1)}$$
$$+ C_F C_A \left\{\frac{1}{2} + \frac{1}{2}\ln\left(\frac{1-\beta}{1+\beta}\right) + \frac{1}{3}\ln^2\left(\frac{1-\beta}{1+\beta}\right) - \frac{(1+\beta^2)^2}{8\beta^2}\left[-\mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) + \zeta_3\right]\right.$$
$$\left. - \frac{(1+\beta^2)}{2\beta}\left[\ln\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{(1+\beta)^2}{4\beta}\right) - \frac{1}{6}\ln^2\left(\frac{1-\beta}{1+\beta}\right) - \mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]\right\}$$

where $K = C_A(67/18 - \zeta_2) - 5n_f/9$

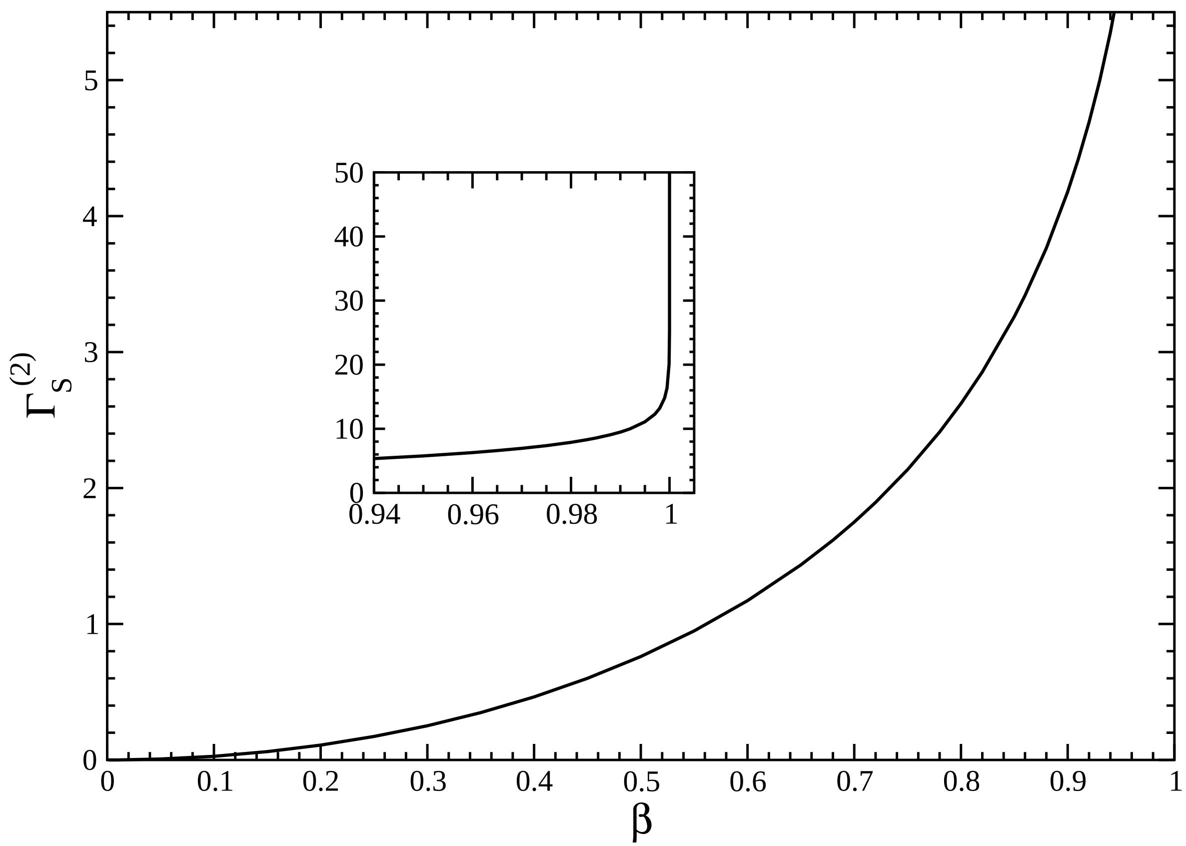
The color structure of $\Gamma_S^{(2)}$ involves only the factors $C_F C_A$ and $C_F n_f$

In terms of the cusp angle (Korchemsky& Radyushkin) $\gamma = \ln[(1+\beta)/(1-\beta)]$ we get

$\Gamma_S^{(1)} = C_F(\gamma \coth\gamma - 1)$ and

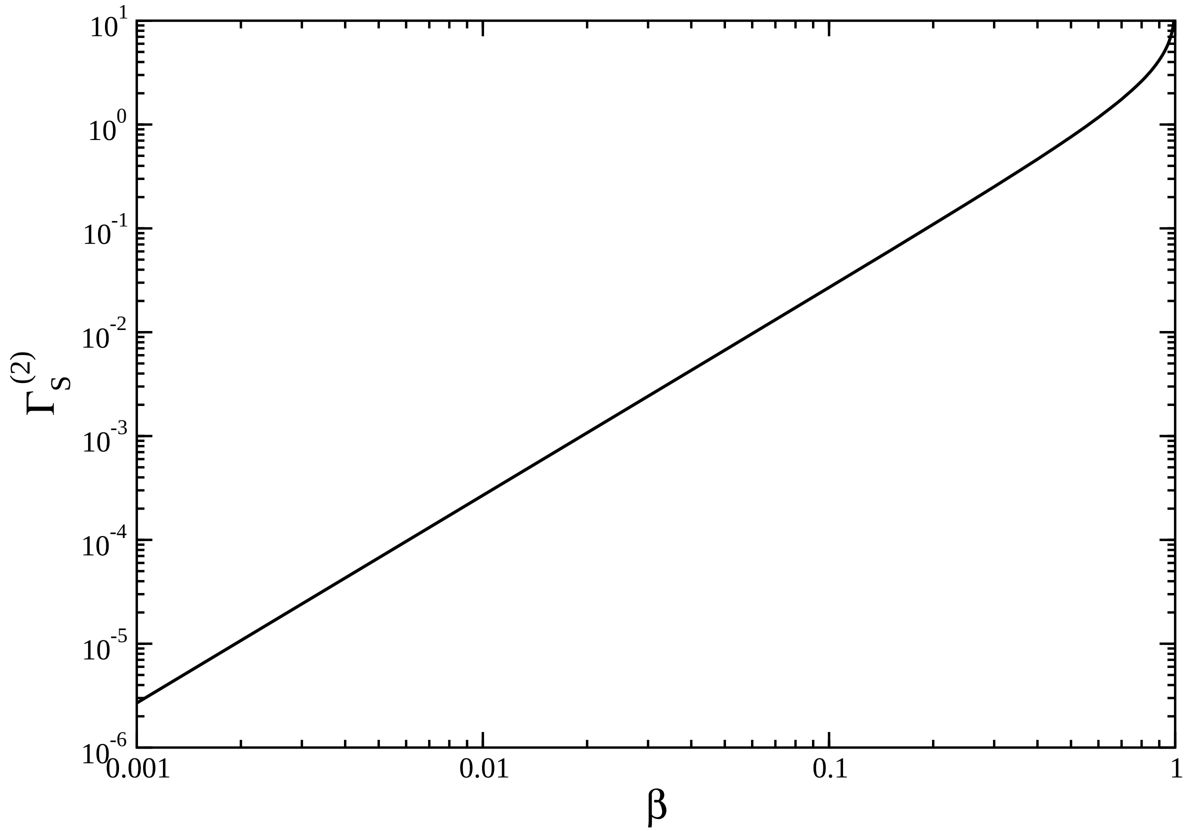
$$\Gamma_S^{(2)} = \frac{K}{2}\Gamma_S^{(1)} + C_F C_A \left\{\frac{1}{2} + \frac{\zeta_2}{2} + \frac{\gamma^2}{2} - \frac{1}{2}\coth^2\gamma\left[\zeta_3 - \zeta_2\gamma - \frac{\gamma^3}{3} - \gamma\,\mathrm{Li}_2\left(e^{-2\gamma}\right) - \mathrm{Li}_3\left(e^{-2\gamma}\right)\right]\right.$$
$$\left. - \frac{1}{2}\coth\gamma\left[\zeta_2 + \zeta_2\gamma + \gamma^2 + \frac{\gamma^3}{3} + 2\gamma\ln\left(1-e^{-2\gamma}\right) - \mathrm{Li}_2\left(e^{-2\gamma}\right)\right]\right\}$$

Two-loop soft anomalous dimension $\Gamma_S^{(2)}$ for $e^+e^- \rightarrow t\bar{t}$



$\Gamma_S^{(2)}$ vanishes at $\beta = 0$, the threshold limit, and diverges at $\beta = 1$, the massless limit.

Logarithmic plot of $\Gamma_S^{(2)}$ for $e^+e^- \to t\bar{t}$



Small and large β behavior of $\Gamma_S^{(2)}$

Small β behavior - expand around $\beta = 0$

$$\Gamma_{S\,\mathrm{exp}}^{(2)} = -\frac{2}{27}\beta^2\left[C_F C_A(18\zeta_2 - 47) + 5C_F n_f\right] + \mathcal{O}(\beta^4)$$

$\Gamma_S^{(2)}$ is an even function of β

Large β behavior: as $\beta \to 1$, $\Gamma_S^{(2)} \to \frac{K}{2}\Gamma_S^{(1)} + C_F C_A \frac{(1-\zeta_3)}{2} \to \frac{K}{2}\Gamma_S^{(1)}$

[In massless case $\Gamma_S^{(2)} = \frac{K}{2}\Gamma_S^{(1)}$ (Aybat, Dixon, Sterman)

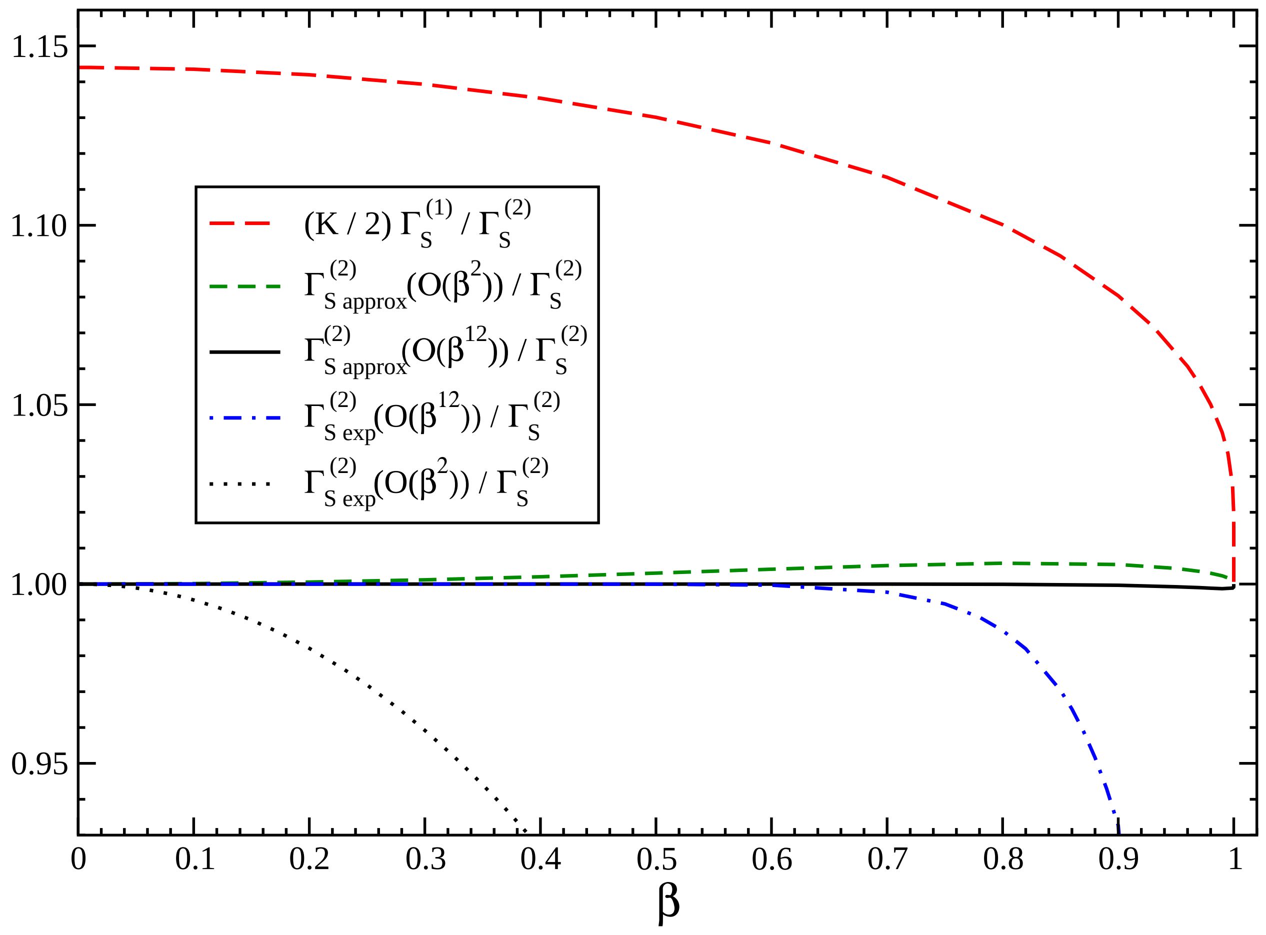
In massive-massless case $\Gamma_S^{(2)} = \frac{K}{2}\Gamma_S^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$]

Construct approximation for all β

$$\Gamma_{S\,\mathrm{approx}}^{(2)} = \Gamma_{S\,\mathrm{exp}}^{(2)} + \frac{K}{2}\Gamma_S^{(1)} - \frac{K}{2}\Gamma_{S\,\mathrm{exp}}^{(1)}$$

$$= \frac{K}{2}\Gamma_S^{(1)} + C_F C_A\left(1 - \frac{2}{3}\zeta_2\right)\beta^2 + \mathcal{O}\left(\beta^4\right)$$

Expansions and approximations to $\Gamma_S^{(2)}$ for $e^+e^- \longrightarrow t\bar{t}$



$\Gamma_{S\,\mathrm{approx}}^{(2)}$ is a remarkably good approximation to complete $\Gamma_S^{(2)}$

In general $\Gamma_S^{(2)} \neq \frac{K}{2} \Gamma_S^{(1)}$, i.e. more complicated than massless case, for all heavy quark processes ($e^+e^- \longrightarrow t\bar{t}$ and heavy quark hadroproduction)

For $e^+e^- \longrightarrow t\bar{t}$ soft logarithms of the form $\frac{\ln^{n-1}(\beta^2)}{\beta^2}$

NLO soft-gluon corrections

$$\sigma^{(1)} = \sigma^B \frac{\alpha_s}{\pi} 2\,\Gamma_S^{(1)} \frac{1}{\beta^2}$$

with σ^B the Born cross section

Second-order soft-gluon corrections

$$\sigma^{(2)} = \sigma^B \frac{\alpha_s^2}{\pi^2} \left\{ \left[4(\Gamma_S^{(1)})^2 - \beta_0\, \Gamma_S^{(1)}\right] \frac{\ln(\beta^2)}{\beta^2} + \left[2\, T_1\, \Gamma_S^{(1)} + 2\, \Gamma_S^{(2)}\right] \frac{1}{\beta^2} \right\}$$

with T_1 the NLO virtual corrections

At the Tevatron and the LHC the $t\bar{t}$ cross section receives most contributions in the region around $0.3 < \beta < 0.8$ which peak roughly around $\beta \sim 0.6$.

NNLO approximate cross sections include soft-gluon contributions

Summary

- **Soft-gluon corrections and resummation**
- **Two-loop calculations in eikonal approximation**
- **Massive quarks involve further complications**
- $\Gamma_S^{(2)}$ **calculated for** $e^+e^- \rightarrow t\bar{t}$
- **Extensions to single top and top pair hadroproduction**