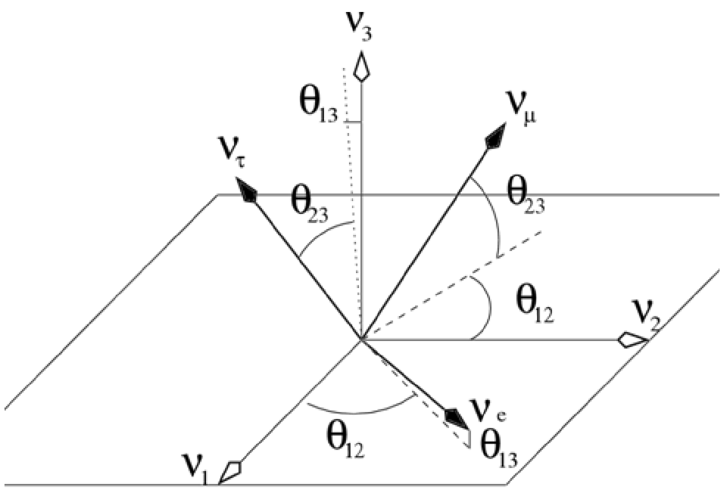
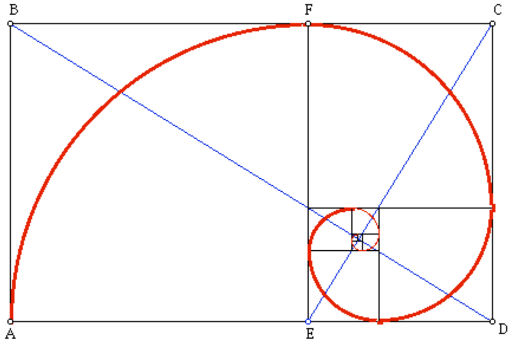
Understanding Lepton Flavor Mixing





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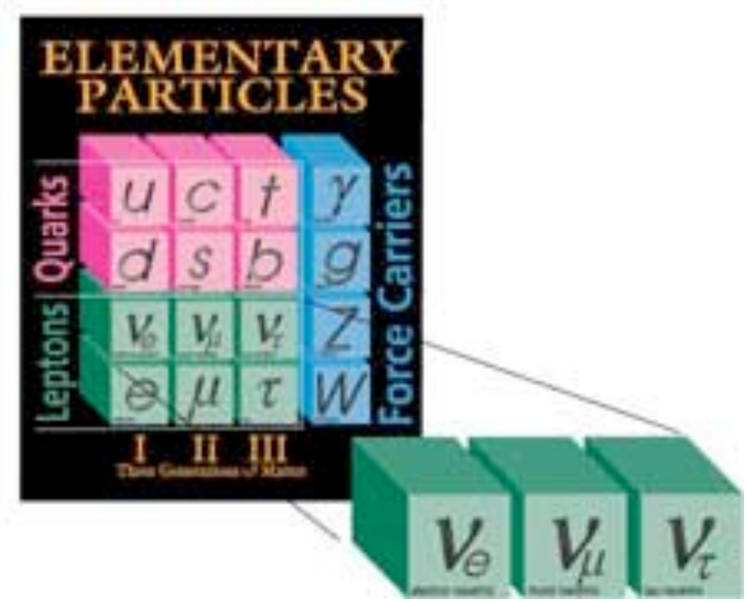
Introduction/Motivation

Observation of Neutrino Oscillations:

$$\mathcal{P}_{\nu_\alpha \to \nu_\beta}(L) = \sum_{ij} \mathcal{U}_{i\alpha}\mathcal{U}^*_{i\beta}\mathcal{U}^*_{j\alpha}\mathcal{U}_{j\beta} e^{-\frac{i\Delta m^2_{ij} L}{2E}}$$

- Neutrinos are massive
- Lepton mixing is observable



Standard Model ⟶ " ν Standard Model "

(νSM)

The ν Flavor Puzzle

- SM flavor puzzle:
  origin of charged fermion masses, quark mixings
  Dirac mass terms, parametrized by Yukawas:

$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj}$$

- νSM flavor puzzle:
  origin of neutrino masses (Dirac or Majorana), lepton mixings,...

Ultimate Goal: satisfactory, credible, complete flavor theory

very difficult!

but first, let's look at the data...

fits: Schwetz, Tortola, Valle

The Data: Neutrino Masses

Homestake, Kam, SuperK, KamLAND, SNO, SuperK, MINOS, miniBOONE,...

$$\Delta m^2_{ij} \equiv m_i^2 - m_j^2$$

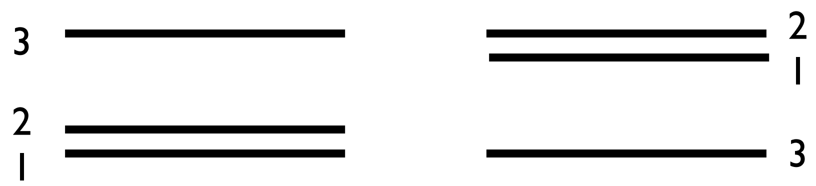
assume: 3 neutrino mixing

Solar: $$\Delta m^2_\odot = |\Delta m^2_{12}| = 7.65^{+0.23}_{-0.20} \times 10^{-5}\,\mathrm{eV}^2$$

(best fit $\pm 1\sigma$)

Atmospheric: $$\Delta m^2_{31} = \pm 2.4^{+0.12}_{-0.11} \times 10^{-3}\,\mathrm{eV}^2$$

Normal Hierarchy | Inverted Hierarchy



Cosmology (WMAP): $$\sum_i m_i < 0.7\,\mathrm{eV}$$

fit: Schwetz, Tortola, Valle '08

The Data: Lepton Mixing

Homestake, Kam, SuperK, KamLAND, SNO, SuperK, Palo Verde, CHOOZ, MINOS..

$$\mathcal{U}_{\mathrm{MNSP}} = \mathcal{R}_1(\theta_\oplus)\mathcal{R}_2(\theta_{13}, \delta_{\mathrm{MNSP}})\mathcal{R}_3(\theta_\odot)\mathcal{P}$$

Maki, Nakagawa, Sakata
Pontecorvo

$$|\mathcal{U}_{\mathrm{MNSP}}| \simeq \begin{pmatrix} \cos\theta_\odot & \sin\theta_\odot & \epsilon \\ -\cos\theta_\oplus \sin\theta_\odot & \cos\theta_\oplus \cos\theta_\odot & \sin\theta_\oplus \\ \sin\theta_\oplus \sin\theta_\odot & -\sin\theta_\oplus \cos\theta_\odot & \cos\theta_\oplus \end{pmatrix}$$

Solar: $\theta_\odot = \theta_{12} = 33.4^\circ \pm 1.4^\circ$

Atmospheric: $\theta_\oplus = \theta_{23} = 45.0^\circ\,{}^{+4.0}_{-3.4}$ (best fit $\pm 1\sigma$)

Reactor: $\epsilon = \sin\theta_{13}, \quad \theta_{13} = 5.7^\circ\,{}^{+3.5}_{-5.7}$

2 large angles, 1 small angle! (no constraints on CP violation)

Compare: Quark Mixing

Cabibbo; Kobayashi, Maskawa

$$\mathcal{U}_{\rm CKM} = \mathcal{R}_1(\theta_{23}^{\rm CKM})\mathcal{R}_2(\theta_{13}^{\rm CKM}, \delta_{\rm CKM})\mathcal{R}_3(\theta_{12}^{\rm CKM})$$

Mixing Angles: $\theta_{12}^{\rm CKM} = 13.0^\circ \pm 0.1^\circ$ $\longleftrightarrow$ Cabibbo angle θ_c

$$\theta_{23}^{\rm CKM} = 2.4^\circ \pm 0.1^\circ$$

$$\theta_{13}^{\rm CKM} = 0.2^\circ \pm 0.1^\circ$$

CP violation: $J \equiv {\rm Im}(\mathcal{U}_{\alpha i}\mathcal{U}_{\beta j}\mathcal{U}^*_{\beta i}\mathcal{U}^*_{\alpha j})$

Jarlskog
Dunietz, Greenberg, Wu

$$J_{\rm CP}^{(\rm CKM)} \simeq \sin 2\theta_{12}^{\rm CKM} \sin 2\theta_{23}^{\rm CKM} \sin 2\theta_{13}^{\rm CKM} \sin \delta_{\rm CKM}$$

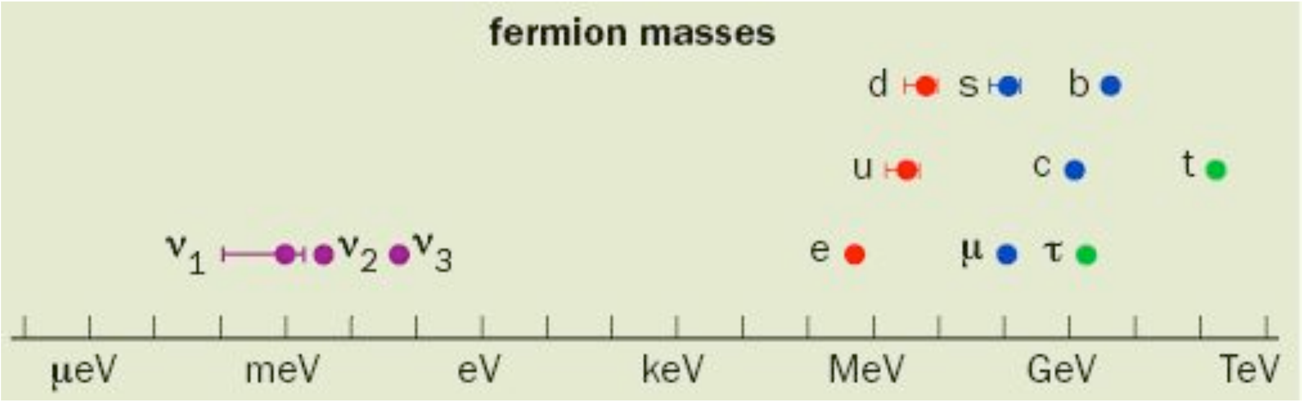
$$J \sim 10^{-5} \qquad \delta_{\rm CKM} = 60^\circ \pm 14^\circ$$

3 small angles, 1 large phase!

A paradigm shift

Strikingly different flavor patterns for quarks and leptons!

- Mass scales, hierarchies of neutral and charged fermions:



- Mixing Angles: quarks small, leptons 2 large, 1 small

step 1 for theory: suppressing neutrino mass scale

Beyond physics of Yukawa couplings!

$$-\mathcal{L}_\nu = Y_{\nu ij}\bar{L}_{Li}H\nu_{Rj} + \frac{\lambda_{ij}}{\Lambda}(L_{Li}H)(L_{Lj}H) + \frac{1}{2}(M_{ij}\bar{\nu}_{Ri}(\nu_{Rj})^c + h.c.)$$

Prototype: Type I neutrino seesaw

Minkowski; Yanagida;
Gell-Mann, Ramond, Slansky

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$m \sim \mathcal{O}(100\,\mathrm{GeV})$

$M \gg m$

$m_1 \sim \frac{m^2}{M} \qquad m_2 \sim M \gg m_1$

$\nu_{1,2} \sim \nu_{L,R} + \frac{m}{M}\nu_{R,L}$

but also many other possibilities...

Majorana (Type II, III seesaws, double seesaw...), suppressed Dirac masses (most mechanisms exploit SM singlet nature of ν_R)

This talk: implications of large lepton mixings (step 2)

Flavor Model Building in the νSM (I)

Standard paradigm: spontaneously broken flavor symmetry

$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj} \longrightarrow \left(\frac{\varphi}{M}\right)^{n_{ij}} H \cdot \bar{\psi}_{Li} \psi_{Rj}$$

Froggatt, Nielsen

Recall for quarks:

- hierarchical masses, small mixings: continuous family symmetries
- CKM matrix: small angles and/or alignment

$$\mathcal{U}_{\mathrm{CKM}} = \mathcal{U}_u \mathcal{U}_d^\dagger \sim 1 + \mathcal{O}(\lambda)$$

$$\lambda \sim \frac{\varphi}{M}$$

Wolfenstein parametrization: $\lambda \equiv \sin\theta_c = 0.22$

suggests Cabibbo angle may be a useful flavor expansion parameter

Flavor Model Building in the νSM (II)

- Main issue: what is $\mathcal{U}_{\rm MNSP}$ in limit of exact symmetry?

for the leptons, large angles suggest

$$\mathcal{U}_{\rm MNSP} = \mathcal{U}_e \mathcal{U}_\nu^\dagger \sim \mathcal{W} + \mathcal{O}(\lambda')$$

"bare" mixing angles ($\mathcal{W}$): $(\theta^0_{12}, \theta^0_{13}, \theta^0_{23})$

flavor expansion parameter (λ')

- useful, and motivated in unified/string scenarios, to take

$$\lambda' = \lambda \equiv \sin\theta_c$$

ideas of "Cabibbo haze" and quark-lepton complementarity

(Datta, L.E., Ramond) (more shortly...)

Aside: Lepton Mixing Angles are "non-generic"

Classify scenarios by the form of $\mathcal{U}_{\rm MNSP}$ in symmetry limit

note: lepton mixing angle pattern has the most challenges (w/3 families)

- 3 small angles $\longrightarrow$ $\sim$ diagonal $\mathcal{M}_\nu$
- 1 large, 2 small $\longrightarrow$ $\sim {\rm Rank}\mathcal{M}_\nu < 3$
- 3 large angles $\longrightarrow$ "anarchical" $\mathcal{M}_\nu$
- 2 large, 1 small $\longrightarrow$ fine-tuning, non-Abelian

Issues: size of θ_{13}, origin of non-maximal θ_{12}

large angles also suggest discrete non-Abelian family symmetries!

Flavor Model Building in the νSM (III)

$$\mathcal{U}_{\rm MNSP} = \mathcal{U}_e \mathcal{U}_\nu^\dagger \sim \mathcal{W} + \mathcal{O}(\lambda')$$

Classify models by form of $\mathcal{W}(\theta^0_{12}, \theta^0_{13}, \theta^0_{23})$:

- In general: $\theta^0_{23} = 45^\circ$ $\theta^0_{13} = 0^\circ$ (reasonable)
- More variety in choice of bare solar angle θ^0_{12}:
  - "bimaximal" mixing (quark-lepton complementarity)
  - "tri-bimaximal" mixing Harrison, Perkins, Scott (HPS)
  - "golden ratio" mixing $\phi = (1 + \sqrt{5})/2$

Scenario 1. Bimaximal Mixing

peak popularity: ~2004-2006

"bare" solar angle $\theta_{12}^0 = 45^\circ$ $\tan\theta_{12}^0 = 1$

$$\mathcal{U}_{\mathrm{MNSP}}^{(\mathrm{BM})} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Requires large perturbations:

$$\theta_{12} = \theta_{12}^0 + \mathcal{O}(\lambda) \sim \frac{\pi}{4} - \theta_c$$

"quark-lepton complementarity"

Raidal; Minakata, Smirnov; Frampton, Mohapatra; Xing; Ferrandis, Pakvasa; King; L.E., Ramond; Plentinger, Lindner; Dighe, Rodejohann, many others (>100 papers)...

Bimaximal mixing scenarios:

useful framework for exploring Cabibbo effects in quark+lepton sectors

$$\frac{m_u}{m_t} \sim \lambda^8 \qquad \frac{m_d}{m_b} \sim \lambda^4 \qquad \frac{m_e}{m_\tau} \sim \lambda^5$$

$$\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_\oplus}} \sim \lambda$$

$$\frac{m_c}{m_t} \sim \lambda^4 \qquad \frac{m_s}{m_b} \sim \lambda^2 \qquad \frac{m_\mu}{m_\tau} \sim \lambda^2$$

$$\theta_{13} \leq \mathcal{O}(\lambda)$$

$$\frac{m_b}{m_\tau} \sim 1 \qquad \frac{m_b}{m_t} \sim \lambda^3$$

(GUT scale)

$$\Delta\theta_{23} < \mathcal{O}(\lambda)$$

$$\theta_{12}^{\rm CKM} \sim \lambda \qquad \theta_{23}^{\rm CKM} \sim \lambda^2 \qquad \theta_{13}^{\rm CKM} \sim \lambda^3$$

but implementation in full grand unified theories: very challenging

recent resurgence in context of discrete non-Abelian family symms

Altarelli, Feruglio, and Merlo, '09,...

Scenario II. Tri-bimaximal (HPS) Mixing

peak popularity: ~2006-now

“bare” solar angle $\tan\theta_{12}^0 = \frac{1}{\sqrt{2}}$ $\theta_{12}^0 = 35.26^\circ$

Harrison, Perkins, Scott ’02

$$\mathcal{U}_{\rm MNSP}^{\rm (HPS)} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Does not require large perturbations! $\theta_{12} = \theta_{12}^0 + \mathcal{O}(\lambda^2)$

amusing note: MNSP looks like Clebsch-Gordan coeffs Meshkov; Zee,...

Naturally obtained from discrete non-Abelian symmetries (subgroups of $SO(3)$, $SU(3)$)

A Few Examples: $\mathcal{A}_4$ (tetrahedron) — Ma and collaborators (earliest in '01), Altarelli, Feruglio, Babu and He, Valle, Hirsch et al., King et al., many, many others...

$\mathcal{S}_4$ (cube) — Ma; Hagedorn, Lindner, Mohapatra; Cai, Yu; Zhang,...

$\mathcal{T}'$ — Aranda, Carone, Lebed; Chen, Mahanthappa,...

$\Delta(3n^2)$ — Luhn, Nasri, Ramond; Ma; King, Ross,...

$\mathcal{A}_5$ (icosahedron) — L.E., Stuart (in progress)

Most popular scenario! many models, elegant results

issue of incorporating quarks: much recent progress

Scenario III. Golden Ratio Mixing

peak popularity: hopefully soon!!

Idea: solar angle related to "golden ratio"

$$\phi = (1+\sqrt{5})/2$$

- Two proposed scenarios:
  - $\tan\theta_{12} = \frac{1}{\phi}$ $\theta_{12} = 31.72^\circ$

    L.E., Stuart '08, in progress

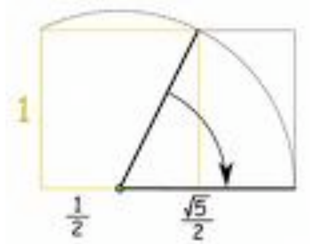
    Implementation: icosahedral flavor symmetry $\mathcal{I}\ (\mathcal{A}_5)$

  - $\cos\theta_{12} = \frac{\phi}{2}$ $\theta_{12} = 36^\circ$

    Adulpravitchai, Blum, Rodejohann '09

    Implementation: dihedral flavor symmetry $\mathcal{D}_{10}$

Scenario III: GR1



Idea: Ramond et al., hep-ph/0306002 (footnote)

Kajiyama, Raidal, Strumia 0705.4559 [hep-ph] $\mathcal{Z}_2 \times \mathcal{Z}_2$

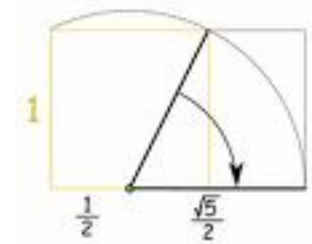
Everett and Stuart, 0812.1057 [hep-ph],... $\mathcal{A}_5$

$$\mathcal{U}_{\rm MNSP}^{\rm (GR1)} = \begin{pmatrix} \sqrt{\frac{\phi}{\sqrt{5}}} & -\sqrt{\frac{1}{\sqrt{5}\phi}} & 0 \\ \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\sqrt{5}\phi}} & \frac{1}{\sqrt{2}}\sqrt{\frac{\phi}{\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\sqrt{5}\phi}} & \frac{1}{\sqrt{2}}\sqrt{\frac{\phi}{\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\mathcal{A}_5$ rich and virtually unexplored model building territory!

A. Stuart's talk

Scenario III: GR2



Idea: Rodejohann, 0810. 5239 [hep-ph] (phenomenology)

Adulpravitchai, Blum, and Rodejohann, 0903.0531 [hep-ph] $\mathcal{D}_{10}$

$$\mathcal{U}_{\rm MNSP}^{(\rm GR2)} = \begin{pmatrix} \frac{\phi}{2} & -\frac{1}{2}\sqrt{\frac{\sqrt{5}}{\phi}} & 0 \\ \frac{1}{2}\sqrt{\frac{5}{2\phi}} & \frac{\phi}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2}\sqrt{\frac{5}{2\phi}} & \frac{\phi}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

complete flavor theory based on dihedral symmetry!

(solar angle prediction based on exterior angle of decagon)

Conclusions

- Lepton data has given us a νSM flavor puzzle!
- Theoretically favored mixing angle patterns:
  - Bimaximal mixing and Cabibbo-sized effects
  - Tri-bimaximal mixing: tetrahedral (+others)
  - "golden ratio" mixings: icosahedral/dihedral
- themes: discrete non-Abelian family symmetries, embedding quarks together with leptons...
- lots of interesting work to do! Data will be crucial!