Dalitz Plot Analysis of
$B^- \to D^+\pi^-\pi^-$, $B^+ \to \pi^+\pi^-\pi^+$ and $D_S^+ \to \pi^+\pi^-\pi^+$

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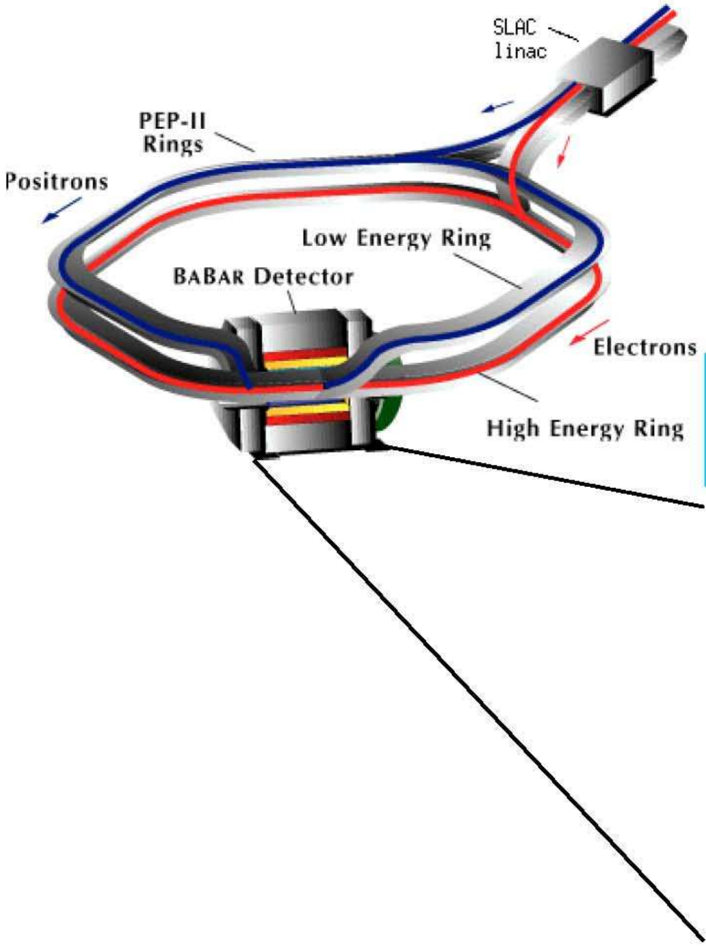
Outline of Talk

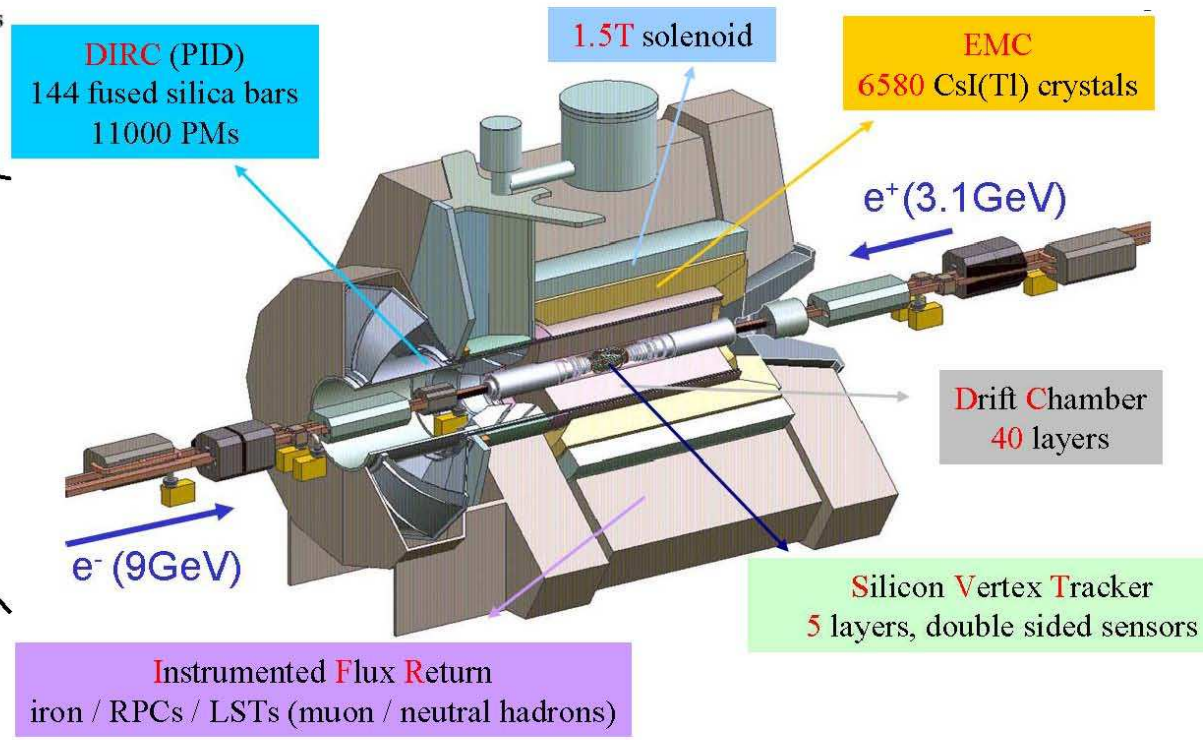
- Analysis Techniques
- Introduction to Dalitz Plot Analysis
- **Dalitz Plot Analysis of $B^- \to D^+\pi^-\pi^-$**
- **Dalitz Plot Analysis of $B^+ \to \pi^+\pi^-\pi^+$**
- **Dalitz Plot Analysis of $D_S^+ \to \pi^+\pi^-\pi^+$**
- Conclusion

DPF 2009

PEP-II and BABAR Detector

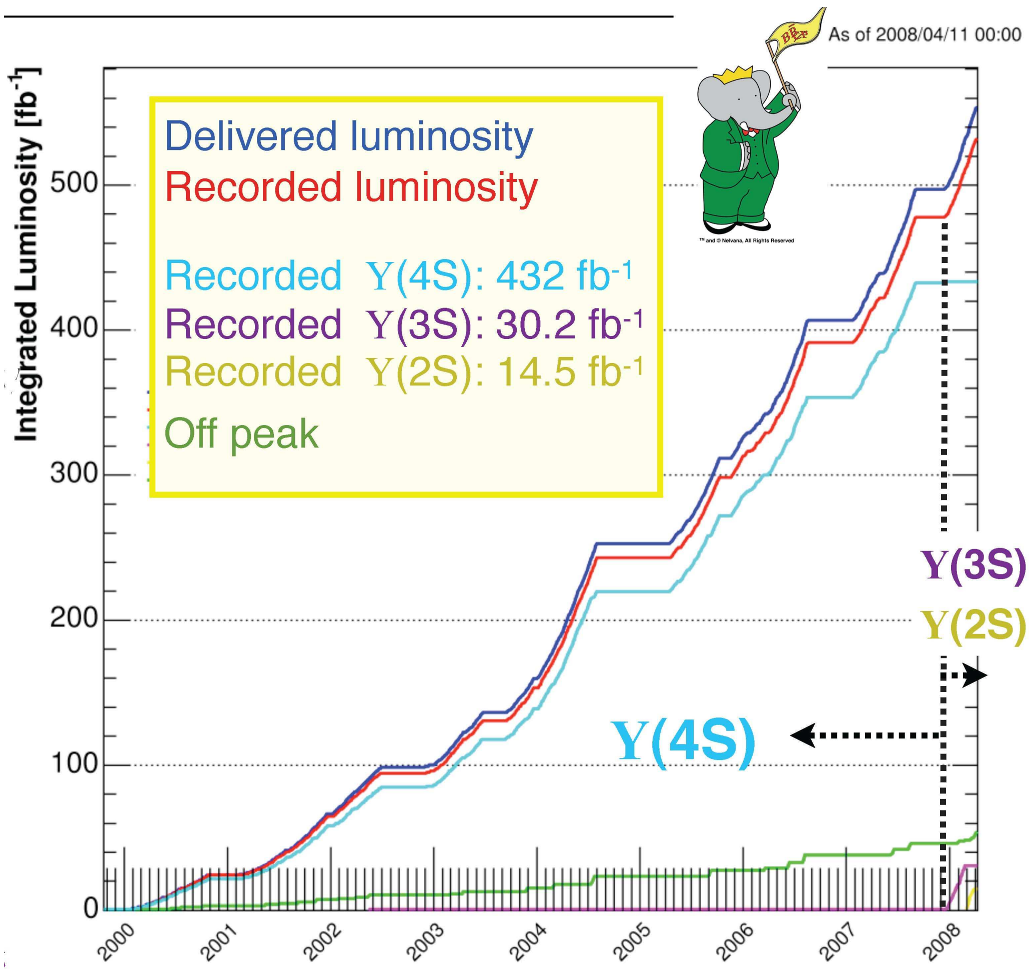


- PEP II/BaBar *B*-Factory located at Stanford Linear Accelerator Center
- Collided beams of electrons and positrons with asymmetric energies



Dataset

Integrated Luminosity



- BaBar data taking ended on 7th April 2008
- BaBar recorded ***470M $B\bar{B}$*** at Υ(4S)

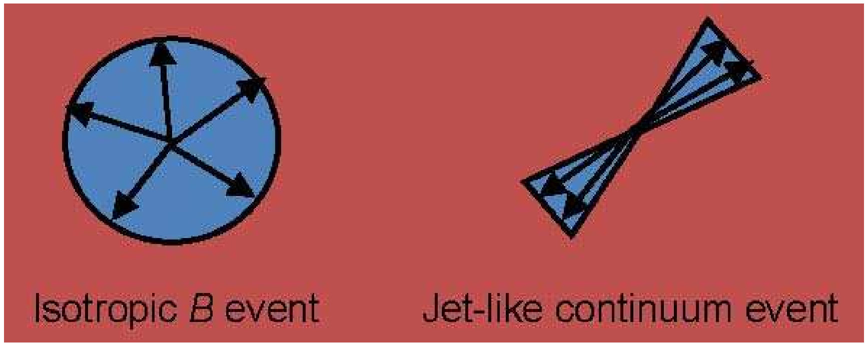
- **BABAR is a B-factory:** $531\ \mathrm{fb}^{-1}$ recorded, $\sim 432\ \mathrm{fb}^{-1}$ at the $\Upsilon(4S)$
- **BABAR is also a charm factory:**
  - $\sigma_{(e^+e^- \to \Upsilon(4S) \to B\bar{B})} = 1.05\mathrm{nb}$,
  - but $\sigma_{(e^+e^- \to c\bar{c})} = 1.3\mathrm{nb}$

  Get ~ 1 charm per B decay:

- Samples used in this talk:
  - $B^- \to D^+\pi^-\pi^-$: $\sim 347\ \mathrm{fb}^{-1}$, ~ 383 million $B\bar{B}$
  - $B^+ \to \pi^+\pi^-\pi^+$: $\sim 424\ \mathrm{fb}^{-1}$, ~ 465 million $B\bar{B}$
  - $D_S^+ \to \pi^+\pi^-\pi^+$: $\sim 384\ \mathrm{fb}^{-1}$, > 500 million $c\bar{c}$

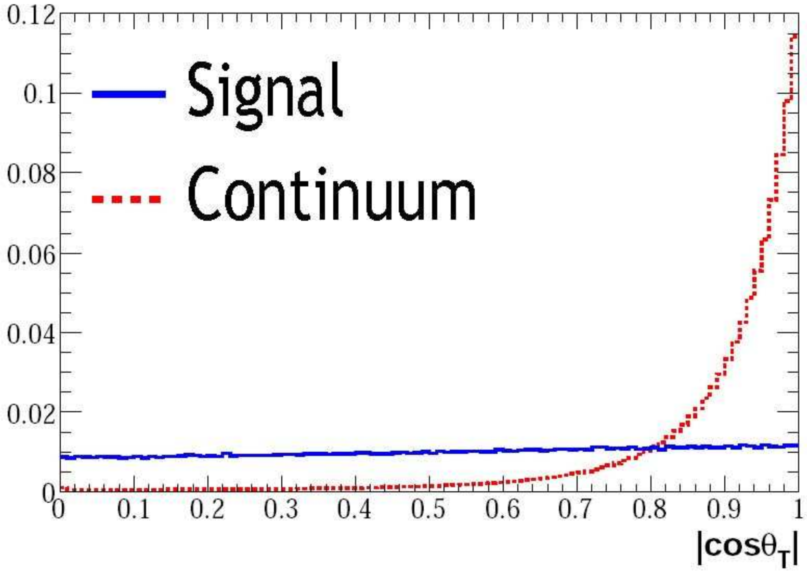
Analysis Variables - Topological

Topological requirements to suppress the continuum background:



- Main background source is continuum $q\bar{q}(q = u, d, s, c)$ production.
- Due to the different masses of light quarks and B mesons, $q\bar{q}$ events have two back-to-back jets; while B decays are more spherical.

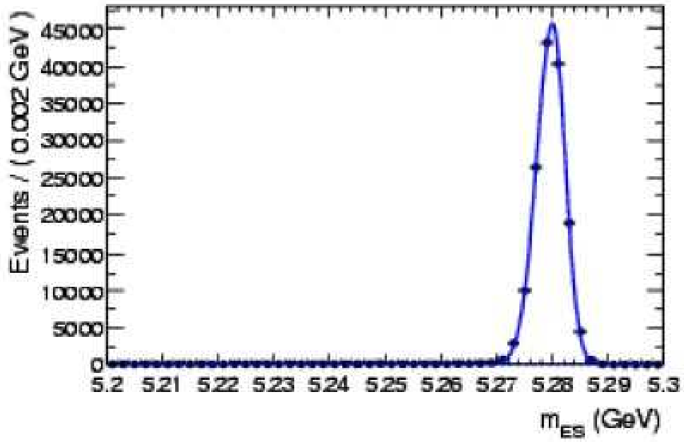
- Use event topology to discriminate.
- For example, $\cos\theta_T$, the cosine between the thrust axis of the B meson and the thrust axis of the rest of the event has great discriminating power.
- Combine topological variables in multi-variate discriminants (Fisher or Neutral network).

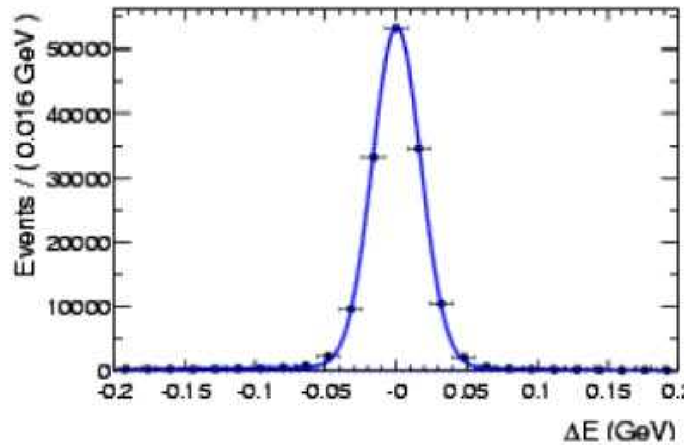


Analysis Variables - Kinematic

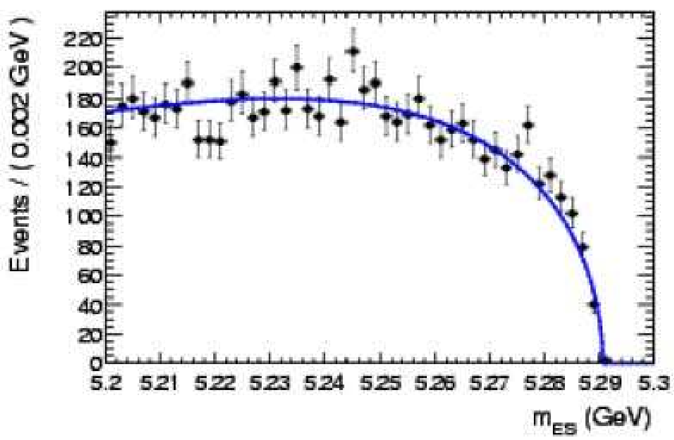
B mesons can be characterized by two kinematic variables:

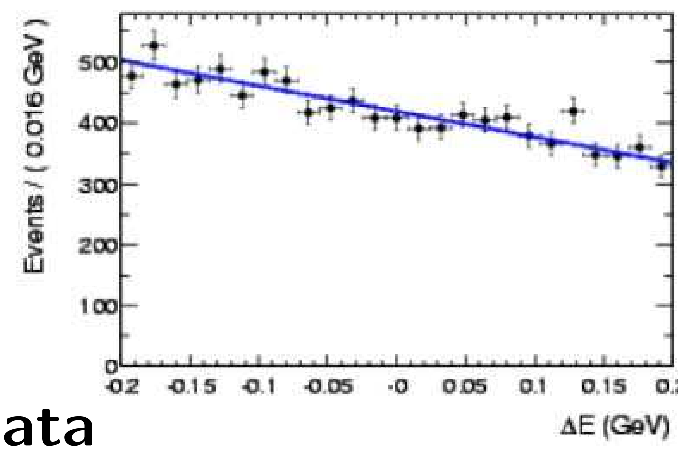
$$m_{ES} = \sqrt{E^{*2}_{beam} - p^{*2}_{B}} \qquad \Delta E = E^*_B - E^*_{beam}$$





Characteristic Signal Distributions





Characteristic Continuum Distributions

Plots show MC data

Make use of precision kinematic information from the beams

Dalitz Plot Analysis

- **Differential decay rate** of a pseudoscalar particle D into three pseudoscalar particles (A,B,C):

$$\frac{d^2\Gamma}{dxdy} \propto |\mathcal{M}|^2 = |\sum_k \rho_k e^{i\phi_k} A_k(x,y) + \rho_n e^{i\phi_n} A_n(x,y)\,|^2$$

  $x = m^2(AB)$ and $y = m^2(BC)$, ρ and ϕ are magnitude and phase, respectively;

  A_n and A_k are the amplitudes for nonresonant and resonant terms.

- Dalitz plot (DP) is a representation of $D \to ABC$ phase space.

- Structure in the DP gives informations on resonance masses, widths and spins, relative phases, interference etc.

- Model each contribution to the Dalitz plot as a separate amplitude with a complex coefficient (isobar model).

- **Dalitz Plot analysis can**

  explore resonances properties, measure corresponding magnitudes and phases, and investigate corresponding CP violation.

Dalitz Plot Analysis

- **Probability density function** (PDF)

  PDF is the sum of signal and background:

  $$\mathrm{PDF}(x,y) = f_{\mathrm{bg}} \frac{B(x,y)}{\int_{\mathsf{DP}} B(x,y)dxdy} + (1-f_{\mathrm{bg}}) \frac{[S(x,y)\otimes\mathcal{R}]\,\epsilon(x,y)}{\int_{\mathsf{DP}} [S(x,y)\otimes\mathcal{R}]\,\epsilon(x,y)dxdy}$$

  where

  – the integral is performed over the whole Dalitz plot (DP),

  – the $S(x,y)\otimes\mathcal{R}$ is the signal term ($= |\mathcal{M}|^2$) convolved with the signal resolution function $\mathcal{R}$,

  – $B(x,y)$ is the background term, f_{bg} is the fraction of background events,

  – ϵ is the reconstruction efficiency across the Dalitz Plot.

- **Unbinned maximum likelihood fit**
  is performed to maximize the value of

  $$\mathcal{L} = \prod_{i=1}^{N_{\mathrm{event}}} \mathrm{PDF}$$

  – In practice, the negative-log-likelihood (NLL) value $\mathrm{NLL} = -\ln\mathcal{L}$ is minimized in the fit.

Dalitz Plot Analysis

- **Resonant amplitudes:**

$$A_k(x,y) = R_k(m)F_L(D \to AB + C)F_L(AB \to A + B)T_L(\Omega)$$

where

- $R_k(m)$ $(m = \sqrt{x})$ is the k^{th} resonance lineshape:
  - $*$ Most resonances are parametrized with relativistic Breit-Wigner (RBW) function.
  - $*$ $\rho^0(770)$ and $\rho^0(1450)$ can be parametrized with Gounaris-Sakurai (GS) function (PRL 21, 244,1968).
- L is the orbital angular momentum between AB system and C.
- Angular distribution T_L follow the Zemach tensor formalism (Phys. Rev. 140, B97).
- F_L are the Blatt-Weisskopf barrier factors.

- **Fit fraction:**

$$f_k = \frac{\int_{\mathrm{DP}} |\rho_k A_k(x,y)|^2 dxdy}{\int_{\mathrm{DP}} |\mathcal{M}|^2 dxdy}$$

- The uncertainty on f_k is evaluated by propagating the full covariance matrix obtained from the fit.
- The sum of f_k do not necessarily add up to unity because of interference.

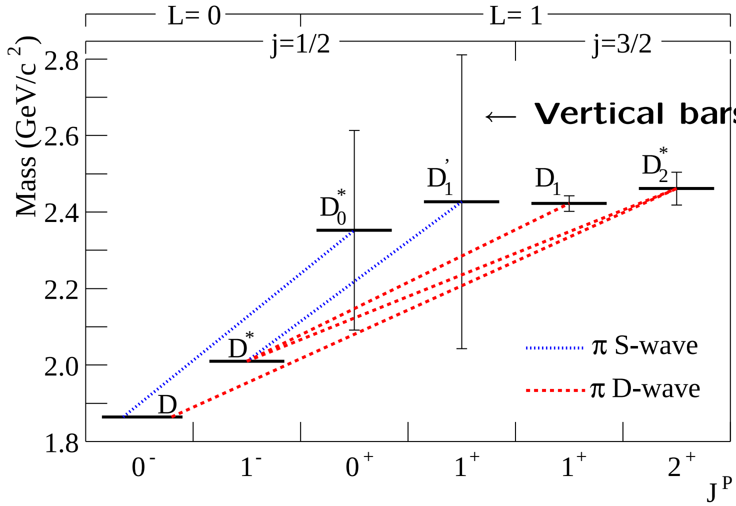
Dalitz plot analysis of $B^- \rightarrow D^+\pi^-\pi^-$

- Based on $\sim 347.2\ \mathrm{fb}^{-1}$ data, corresponding to $\sim 383 \times 10^6\ B\overline{B}$ pairs.
- Published in Phys. Rev. D 79, 112004 (2009).

Analysis of $B^- \to D^+\pi^-\pi^-$

PRD 79, 112004(2009)

- **Motivation**
  - Decay mode of interest: $B^- \to D^+\pi^-\pi^-$ with $D^+ \to K^-\pi^+\pi^+$.
  - Measure the inclusive branching fraction,
  - Study the D^{**0} to test the HQET predictions.

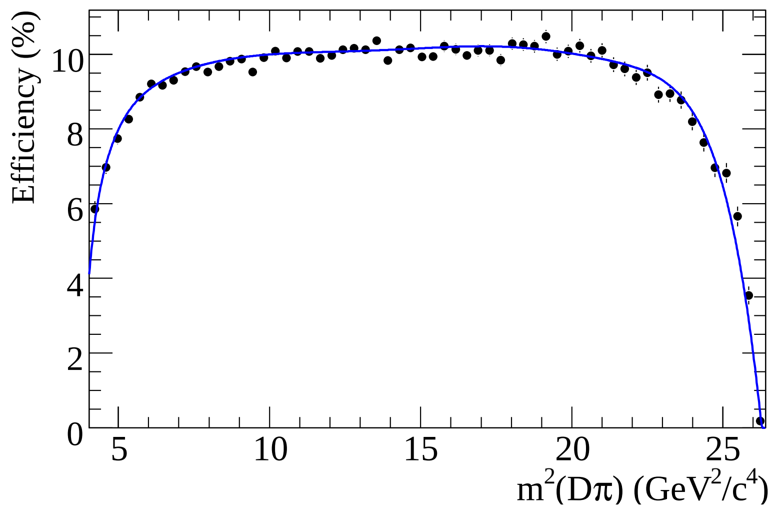


Spectroscopy of D-meson excitations

  - J^P allows D_2^* and D_0^* decay into $D\pi$, not $D_1', D_1 \to D^+\pi^-$.
  - $D^*(2007)^0$ (D_v^{*0}) and B^* (B_v^*) may contribute via virtual process.
  - The J^P of D_0^* not yet confirmed in PDG.

Analysis of $B^- \to D^+\pi^-\pi^-$

- **Analysis Detail:**
  - For narrow D_2^* with width of ~ 40 MeV, signal resolution has been taken into account.
  - Efficiency is determined by fitting the generated & accepted nonresonant $B^- \to D^+\pi^-\pi^-$ Dalitz plot distributions.

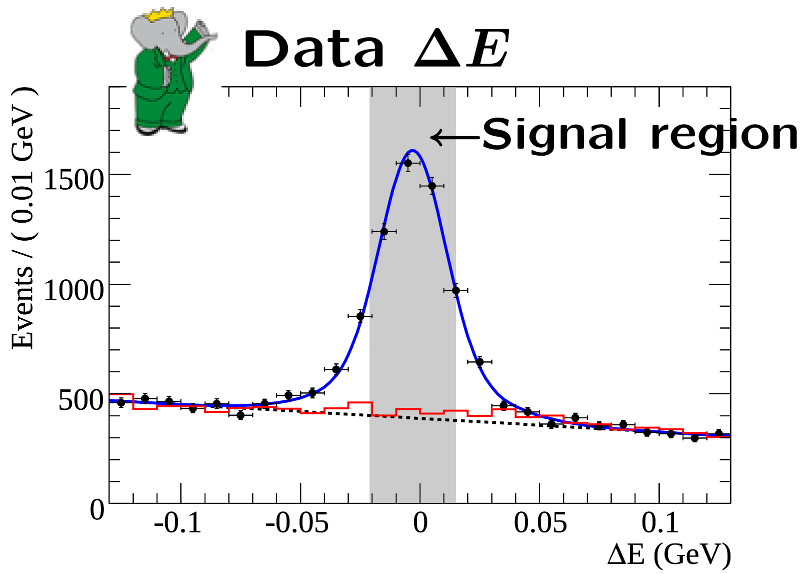


Efficiency projection

  - Background is parametrized using the MC BG events in ΔE signal region.

Analysis of $B^- \to D^+\pi^-\pi^-$

Inclusive $B^- \to D^+\pi^-\pi^-$ branching fraction:



$\mathbf{N_{S_{signal}} = 3414 \pm 85}$

Background $(30.4 \pm 1.1)\%$

Systematic Uncertainties

Number of B^+B^-	1.6%
Tracking efficiencies	2.5%
Particle identification	1.5%
ΔE background shape	1.3%
D^+ branching fraction	2.3%
Fit Models	0.7%
Fit bias	1.0%
Total Systematics	4.4%

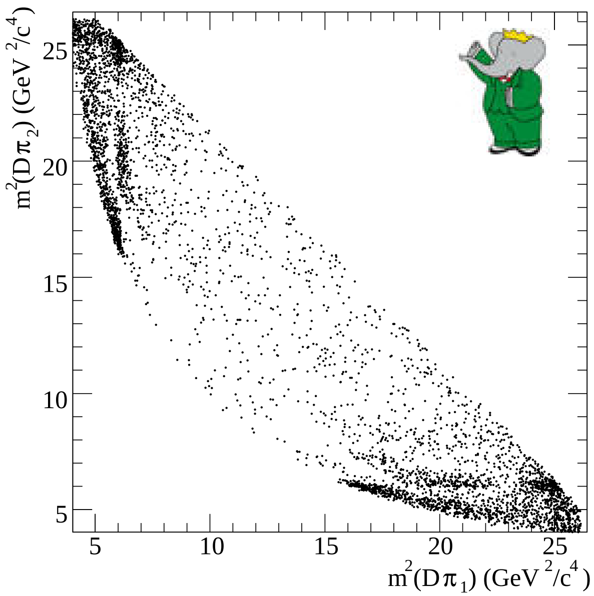
Results

$$\mathcal{B}(B^- \to D^+\pi^-\pi^-) = (1.08 \pm 0.03 \pm 0.05) \times 10^{-3}$$

Belle 60 fb^{-1}: $1.02 \pm 0.04 \pm 0.15$ (PRD 69,112002,2004)

Analysis of $B^- \to D^+\pi^-\pi^-$

- **Data Dalitz plot:**



- **Fitted components:**
  - Nominal Fit: D_2^*, D_0^*, D_v^*, B_v^* and $\mathcal{P}$-wave nonresonant (pnr).
  - $\mathcal{S}$-wave and $\mathcal{D}$-wave nonresonant contributions are not significant.

- **Fitted results:**

Parameter	Fitted result
$m_{D_2^*}$ (MeV)	2460.4 ± 1.2
$\Gamma_{D_2^*}$ (MeV)	41.8 ± 2.5
$m_{D_0^*}$ (MeV)	2297 ± 8
$\Gamma_{D_0^*}$ (MeV)	273 ± 12
$f_{D_2^*}$ (%)	32.2 ± 1.3

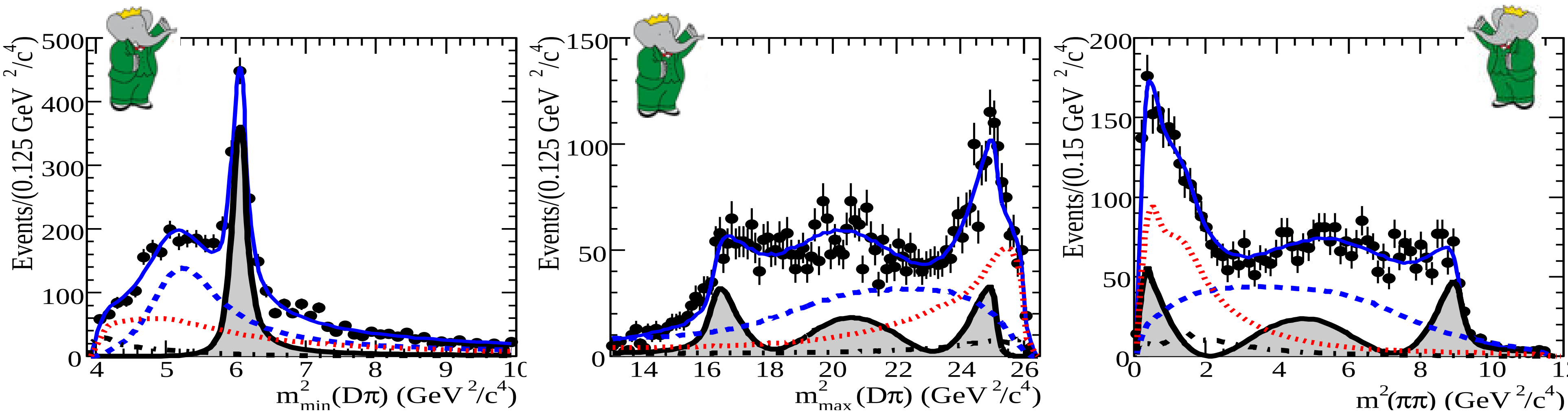
Parameter	Fitted result
$f_{D_0^*}$ (%)	62.8 ± 2.5
$\phi_{D_0^*}$ (rad)	-2.07 ± 0.06
$f_{D_v^*}$ (%)	10.1 ± 1.4
$\phi_{D_v^*}$ (rad)	3.00 ± 0.12
$f_{B_v^*}$ (%)	4.6 ± 2.6

Parameter	Fitted result
$\phi_{B_v^*}$ (rad)	2.80 ± 0.21
f_{pnr} (%)	5.4 ± 2.4
ϕ_{pnr} (rad)	-0.89 ± 0.18

Analysis of $B^- \to D^+\pi^-\pi^-$

Fitting the data

Projections on $m^2_{\min}(D\pi)$, $m^2_{\max}(D\pi)$ and $m^2(\pi\pi)$:

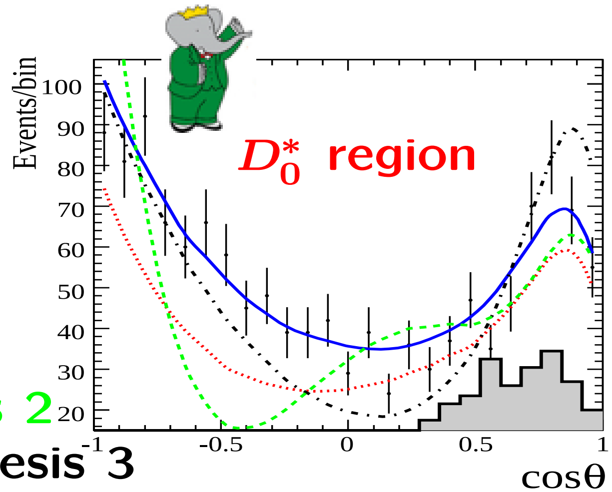


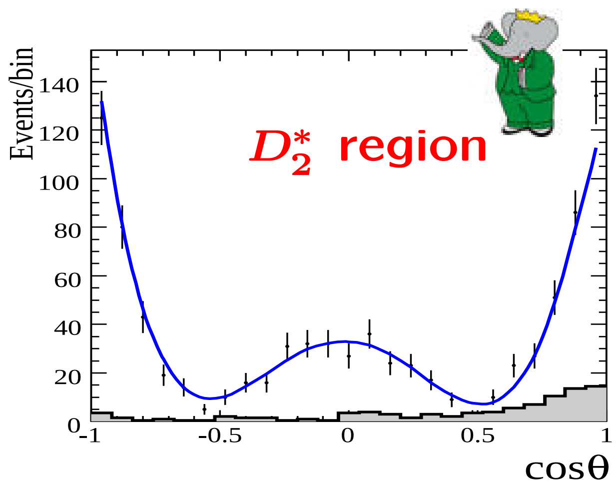
$$\chi^2/dof = 220/153$$

- Dots with error bars are data, blue curves represent the nominal fit,
- Shaded areas show the D_2^{*0} contribution,
- Blue dashed curves show the D_0^{*0} signal, black dash-dotted curves show the D_v^* and B_v^* signals,
- Red dotted curves show the background.

J^P **assignment test:** Fits with or without the broad resonance D_0^*

Hypothesis	Model	$-2\ln\mathcal{L}$
	Nominal fit	22970
1	D_2^*, D^{*0}, B^*, P-NR	23761
2	D_2^*, D^{*0}, B^*, P-NR, (2^+)	23699
3	D_2^*, D^{*0}, B^*, P-NR, (1^-)	23427
4	D_2^*, D^{*0}, B^*, P-NR, S-NR	23339





full: Nominal fit
dash: hypothesis 2
dashdot: hypothesis 3
dot: hypothesis 4

- In all cases, the $-2\ln\mathcal{L}$ values are significantly worse than that of nominal fit.
- Angular distribution in D_2^* region consistent with the expected $\mathcal{D}$-wave.
- Conclusion: the broad scalar D_0^* is required in the fit to the data.

Systematic Uncertainties

	$\Delta m_{D_2^*}$ (MeV)	$\Delta\Gamma_{D_2^*}$ (MeV)	$\Delta m_{D_0^*}$ (MeV)	$\Delta\Gamma_{D_0^*}$ (MeV)	$\Delta f_{D_2^*}$ (%)	$\Delta f_{D_0^*}$ (%)	$\Delta\phi_{D_0^*}$ (rad)
BG parameterization	1.0	1.1	3	5	1.2	0.0	0.04
BG fraction	0.1	0.4	2	1	0.4	0.4	0.00
Event selection	0.6	1.6	1	14	0.3	0.8	0.08
Fit bias	0.3	0.7	4	8	0.7	1.4	0.02
PID efficiency	0.0	0.1	0	0	0.0	0.1	0.01
Total systematic error	1.2	2.1	5	17	1.5	1.7	0.09
Fits models	1.3	0.7	12	40	1.5	17.2	0.07
r constant	1.4	1.9	12	21	3.8	7.8	0.17
Total model-dependent error	1.9	2.0	19	45	4.1	18.9	0.18

- **BG parameterization:** using the fit with BG parameters allowed to float.
- **BG fraction:** changing the BG fraction within their statistical error.
- **Event selection:** using looser cuts.
- **Fit bias:** using MC & the fit with bad χ^2 cells removed.
- **r constant:** changing Blatt-Weisskopf barrier factor r from 0 to 5 GeV^{-1}.

Summary

- Inclusive branching fraction:

  $\mathcal{B}(B^- \to D^+\pi^-\pi^-) = (1.08 \pm 0.03 \pm 0.05) \times 10^{-3}$

- Mass, width and product branching fraction of D_2^*:

  $m_{D_2^*} = (2460.4 \pm 1.2 \pm 1.2 \pm 1.9)\text{MeV/c}^2$; $\Gamma_{D_2^*} = (41.8 \pm 2.5 \pm 2.5 \pm 2.0)\text{MeV}$

  $\mathcal{B}(B^- \to D_2^*\pi^-) \times \mathcal{B}(D_2^* \to D^+\pi^-) = (3.5 \pm 0.2 \pm 0.2 \pm 0.4) \times 10^{-4}$

- Confirm the D_0^*, mass, width and product branching fraction of D_0^*:

  $m_{D_0^*} = (2297 \pm 8 \pm 5 \pm 19)\text{MeV/c}^2$; $\Gamma_{D_0^*} = (273 \pm 12 \pm 17 \pm 45)$ MeV

  $\mathcal{B}(B^- \to D_0^*\pi^-) \times \mathcal{B}(D_0^* \to D^+\pi^-) = (6.8 \pm 0.3 \pm 0.4 \pm 2.0) \times 10^{-4}$

- Relative phase of the scalar and tensor amplitude:

  $\phi_{D_0^*} = -2.07 \pm 0.06 \pm 0.09 \pm 0.18$

Our results are consistent with but more precise than Belle measurements (based on 60fb^{-1} data, PRD 69,112002,2004).

Dalitz plot analysis of $B^+ \to \pi^+\pi^-\pi^+$

- Based on $\sim 426\ \text{fb}^{-1}$ data, corresponding to $\sim 465 \times 10^6$ $B\bar{B}$ pairs.
- Published in Phys. Rev. D 79, 072006 (2009).

Analysis of $B^+ \to \pi^+\pi^-\pi^+$

PRD 79, 072006(2009)

- **Motivation**
  - This mode can extract CKM angle γ from the interference between $B^+ \to \chi_{c0}\pi^+$ and modes such as $B^+ \to \rho^0\pi^+$.
  - can also provide important information to improve the DP model in $B^0 \to \pi^+\pi^-\pi^0$, which is used to measure the CKM angle α.
  - Branching fraction and A_{CP} measurements can be used to test factorisation and other effective theories.
  - Light meson spectroscopy aided by information from as many different final states as possible.

- **Analysis Detail:**

  – We fit the B^- and B^+ samples independently:

  $$A = \sum_j c_j F_j(m^2_{\rm max}, m^2_{\rm min}), \quad \bar{A} = \sum_j \bar{c}_j F_j(m^2_{\rm max}, m^2_{\rm min})$$

  – We use the following parametrization for amplitude coefficients:

  $$c_j = (x_j + \Delta x_j) + i(y_j + \Delta y_j), \quad \bar{c}_j = (x_j - \Delta x_j) + i(y_j - \Delta y_j)$$

  x_j and y_j (Δx_j and Δy_j) are the CP-conserving (-violating) components.

  – The CP asymmetry is determined by:

  $$\mathcal{A}_{CP,\,j} = (|\bar{c}_j|^2 - |c_j|^2)/(|\bar{c}_j|^2 + |c_j|^2)$$

  – Nonresonant component amplitude:

  $$A_{\rm nr} = c_{\rm nr}(e^{-\alpha_{\rm nr} m^2_{\rm max}} + e^{-\alpha_{\rm nr} m^2_{\rm min}})$$

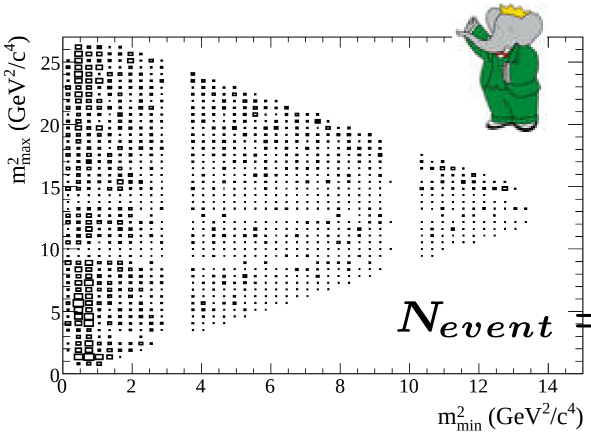
  which has been found to accurately describe nonresonant contributions in other charmless three-body B decay.

  – Dalitz Plot Vetoes: against K^0_S, D^0, J/ψ and $\psi(2S)$ regions.

  – We fit the $m_{\rm ES}$, ΔE and Dalitz Plot simultaneously.

- **Data Dalitz plot:**

$(B^+ \ \& \ B^-)$ combined
(background-subtracted)
(Dalitz Plot Vetoes applied)



$N_{event} = 4335$

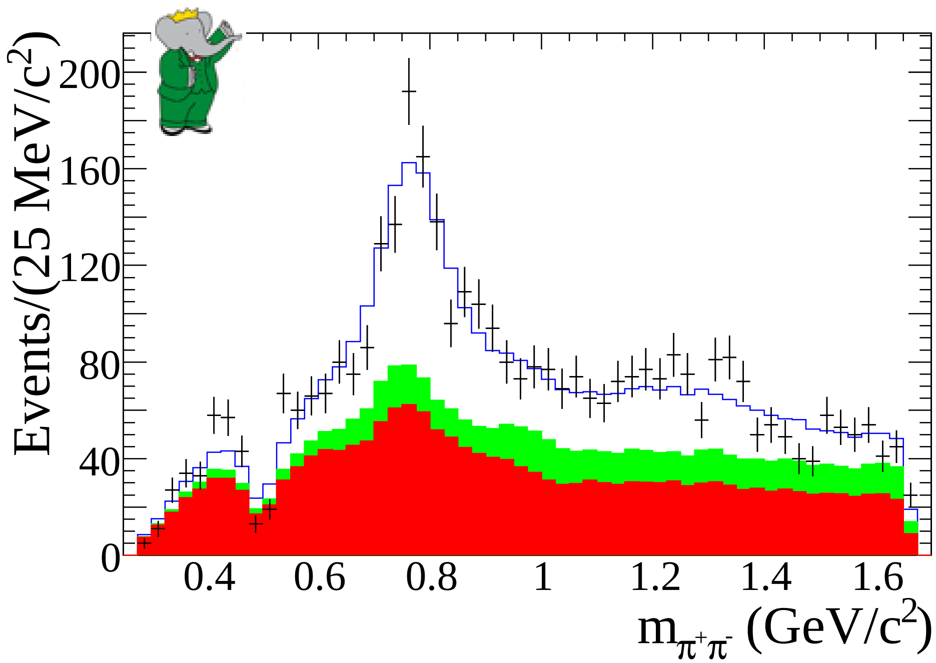
- **Fitted components:**
  - Nominal Fit: $\rho^0(770)$, $\rho^0(1450)$, $f_2(1270)$, $f_0(1370)$ and nonresonant.
  - No significant signal from $f_0(980)$.
  - $\chi_{c0}\pi^\pm$ and $\chi_{c2}\pi^\pm$ contributions are not significant.
  - All CP asymmetries are consistent with zero.

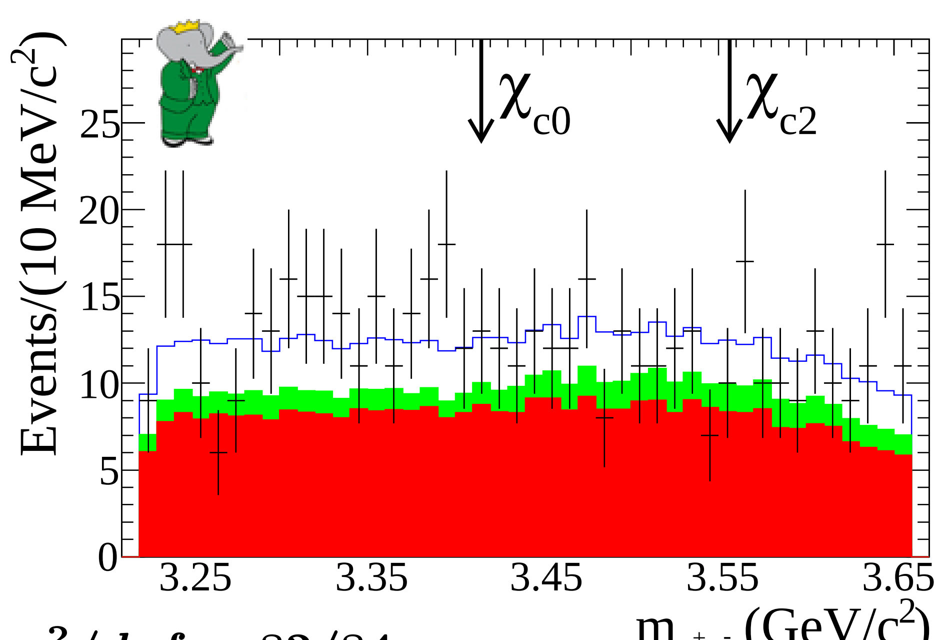
- **Fitted results:**

Resonance	Lineshape	Mass (MeV/c^2)	Width (MeV)	significant
$\rho^0(770)$	GS	775.49 ± 0.34	149.4 ± 1.0	
$\rho^0(1450)$	GS	1465 ± 25	400 ± 60	4.6σ
$f_2(1270)$	RBW	1275.1 ± 1.2	$185.0^{+2.9}_{-2.4}$	6.1σ
$f_0(1370)$	RBW	1400 ± 40	300 ± 80	3.9σ
nonresonant	$\alpha_{nr} = 0.28 \pm 0.06_{sta}$ GeV$^{-2}c^4$			

Fitting the data:

Projections on $m(\pi^+\pi^-)$ in the regions of $\rho^0(770)$ and χ_{c0}, χ_{c2}:





$\chi^2/dof = 82/84$

- Points with error bars are data, blue histograms represent the nominal fit.
- Red histograms are the $q\bar{q}$, Green histograms are the $B\bar{B}$ background.
- Lack of signal in χ_c modes precludes measurement of CKM angle γ in this decay mode with present statistics.
- Dip ~ 0.5 GeV$/c^2$ is due to the rejection of events containing K_S^0.

Results of the fit:

Resonance	x	y	Δx	Δy
$\rho^0(770)\pi^\pm$	1.0 (fixed)	0.0 (fixed)	$-0.092\pm0.036\pm0.027\,^{+0.071}_{-0.012}$	0.0 (fixed)
$\rho^0(1450)\pi^\pm$	$-0.292\pm0.071\pm0.065\,^{+0.182}_{-0.054}$	$0.175\pm0.078\pm0.048\,^{+0.133}_{-0.042}$	$0.109\pm0.080\pm0.059\,^{+0.038}_{-0.116}$	$0.211\pm0.073\pm0.038\,^{+0.032}_{-0.146}$
$f_2(1270)\pi^\pm$	$0.136\pm0.064\pm0.040\,^{+0.178}_{-0.029}$	$0.149\pm0.052\pm0.030\,^{+0.022}_{-0.077}$	$0.101\pm0.063\pm0.016\,^{+0.031}_{-0.183}$	$-0.248\pm0.052\pm0.024\,^{+0.024}_{-0.026}$
$f_0(1370)\pi^\pm$	$0.397\pm0.067\pm0.058\,^{+0.047}_{-0.050}$	$-0.151\pm0.081\pm0.052\,^{+0.057}_{-0.187}$	$-0.387\pm0.064\pm0.029\,^{+0.072}_{-0.082}$	$-0.168\pm0.086\pm0.055\,^{+0.160}_{-0.046}$
Nonresonant	$-0.200\pm0.091\pm0.029\,^{+0.239}_{-0.045}$	$-0.682\pm0.070\pm0.038\,^{+0.032}_{-0.082}$	$-0.392\pm0.089\pm0.055\,^{+0.037}_{-0.128}$	$0.046\pm0.069\pm0.055\,^{+0.101}_{-0.124}$

Measurements of branching fractions:

Mode	Fit Fraction (%)	$\mathcal{B}(B^\pm \to \text{Mode})(10^{-6})$	$\mathcal{A}_{CP}$ (%)
$\pi^\pm\pi^\pm\pi^\mp$ Total		$15.2 \pm 0.6 \pm 1.2\,^{+0.4}_{-0.3}$	$+3.2 \pm 4.4 \pm 3.1\,^{+2.5}_{-2.0}$
$\rho^0(770)\pi^\pm$; $\rho^0(770) \to \pi^+\pi^-$	$53.2 \pm 3.7 \pm 2.5\,^{+1.5}_{-7.4}$	$8.1 \pm 0.7 \pm 1.2\,^{+0.4}_{-1.1}$	$+18 \pm 7 \pm 5\,^{+2}_{-14}$
$\rho^0(1450)\pi^\pm$; $\rho^0(1450) \to \pi^+\pi^-$	$9.1 \pm 2.3 \pm 2.4\,^{+1.9}_{-4.5}$	$1.4 \pm 0.4 \pm 0.4\,^{+0.3}_{-0.7}$	$-6 \pm 28 \pm 20\,^{+12}_{-35}$
$f_2(1270)\pi^\pm$; $f_2(1270) \to \pi^+\pi^-$	$5.9 \pm 1.6 \pm 0.4\,^{+2.0}_{-0.7}$	$0.9 \pm 0.2 \pm 0.1\,^{+0.3}_{-0.1}$	$+41 \pm 25 \pm 13\,^{+12}_{-8}$
$f_0(1370)\pi^\pm$; $f_0(1370) \to \pi^+\pi^-$	$18.9 \pm 3.3 \pm 2.6\,^{+4.3}_{-3.5}$	$2.9 \pm 0.5 \pm 0.5\,^{+0.7}_{-0.5}$ (< 4.0)	$+72 \pm 15 \pm 14\,^{+7}_{-8}$
$\pi^\pm\pi^\pm\pi^\mp$ nonresonant	$34.9 \pm 4.2 \pm 2.9\,^{+7.5}_{-3.4}$	$5.3 \pm 0.7 \pm 0.6\,^{+1.1}_{-0.5}$	$-14 \pm 14 \pm 7\,^{+17}_{-3}$
$f_0(980)\pi^\pm$; $f_0(980) \to \pi^+\pi^-$	-	< 1.5	-
$\chi_{c0}\pi^\pm$; $\chi_{c0} \to \pi^+\pi^-$	-	< 0.1	-
$\chi_{c2}\pi^\pm$; $\chi_{c2} \to \pi^+\pi^-$	-	< 0.1	-

Systematic Uncertainties

Source	x	y	Δx	Δy	Fit fraction	$\mathcal{A}_{CP}$	Signal yield	Signal asymmetry
$B\bar{B}$ yields	0.02	0.04	0.02	...	0.02	0.08	1.4	0.01
$B\bar{B}$ PDF	...	...	...	...	...	0.01	3.3	...
Signal PDF	0.02	0.02	0.01	0.02	0.01	0.08	48.3	0.01
$q\bar{q}$ Dalitz plot	0.06	0.03	0.05	0.05	0.03	0.14	47.7	...
$B\bar{B}$ Dalitz plot	0.03	0.03	0.02	0.02	0.02	0.10	31.6	0.02
Efficiency Dalitz plot	...	0.01	...	...	...	0.03	0.6	...
Fit bias	0.01	0.02	0.02	0.01	...	0.05	2.8	...

- **$B\bar{B}$ yields:** floating the yield and CP asymmetries of the $B\bar{B}$ background.
- **$B\bar{B}$ PDF:** fluctuating the histogram bin contents in accordance with their errors.
- **Signal PDF:** using the control sample $B^+ \to \bar{D}^0\pi^+, \bar{D}^0 \to K^+\pi^-$.
- **$q\bar{q}$ Dalitz plot:** using MC background events in signal region instead of sideband.
- **$B\bar{B}$ Dalitz plot:** varying the components parameters of signal model and composition of the signal model.
- **Efficiency Dalitz plot:** fluctuating the efficiency parameters in accordance with the covariance matrix.
- **Fit bias:** using MC sample.

Summary

- Total $\mathcal{B}(B^\pm \to \pi^\pm\pi^\mp\pi^\pm) = (15.2 \pm 0.6 \pm 1.2 \pm 0.4) \times 10^{-6}$.
- Dominated by the $\rho^0(770)$, $\mathcal{B}(B^\pm \to \rho^0(770)\pi^\pm) = (8.1 \pm 0.7 \pm 1.2^{+0.4}_{-1.1}) \times 10^{-6}$.

  BF consistent with theoretical predictions (PRD 73,114027,2006 & J. High Energy Phys. 03,055,2009).
- Also a large nonresonant component:

  $\mathcal{B}(B^\pm \to \pi^\pm\pi^\mp\pi^\pm \text{ nonresonant}) = (5.3 \pm 0.7 \pm 0.6^{+1.1}_{-0.5}) \times 10^{-6}$,

  and $\alpha_{\rm nr} = 0.28 \pm 0.06_{sta}\ \mathrm{GeV}^{-2}c^4$.
- The mass and width of $f_0(1370)$ are determined to be

  $m_{f_0(1370)} = 1400 \pm 40_{sta}\ \mathrm{MeV}/c^2$ and $\Gamma_{f_0(1370)} = 300 \pm 80_{sta}$ MeV.
- First observation of the decay $B^\pm \to f_2(1270)\pi^\pm$ with significance of 6.1σ: $\mathcal{B}(B^\pm \to f_2(1270)\pi^\pm) = (1.57 \pm 0.42 \pm 0.16\,^{+0.53}_{-0.19}) \times 10^{-6}$.
- We do not find $\chi_{c0}\pi^\pm$, $\chi_{c2}\pi^\pm$ and $f_0(980)\pi^\pm$ contributions.
- We do not find any significant CP asymmetries for the components in the Dalitz Plot model.

Dalitz plot analysis of $D_S^+ \to \pi^+\pi^-\pi^+$

- Based on $\sim 384\ \mathrm{fb}^{-1}$ data, corresponding to $> 500 \times 10^6\ c\bar{c}$ pairs.
- Published in Phys. Rev. D 79, 032003 (2009).

Analysis of $D_S^+ \to \pi^+\pi^-\pi^+$

PRD 79, 032003(2009)

- **Motivation**
  - Study the light meson spectroscopy:
    - $\sigma/f_0(600)$ need experimental confirmation.
    - $f_0(980)$ and $f_0(1370)$ have parameters not well measured.
    - $f_0(980)$ property: may be 4-quark states ?
    - Too many candidates to fit the I=0 $J^{PC} = 0^{++}$ nonet: $\sigma/f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ ....

    It is important to have precise information on the structure of $\pi\pi$ $\mathcal{S}$-wave.
  - Confirmation of the existence of all the scalar mesons and the measurement of their parameters have a fundamental role for understanding the QCD at the low energy limit.
  - D mesons very useful:

    Large coupling to scalar mesons & well defined initial state ($J^P = 0^-$).
  - We also measure the $\frac{B(D_S^+\to\pi^+\pi^-\pi^+)}{B(D_S^+\to K^+K^-\pi^+)}$ to improve the precision.

Analysis of $D_S^+ \to \pi^+\pi^-\pi^+$

- **Analysis Detail:**
  - $\pi\pi$ $\mathcal{S}$-wave amplitude:

    Because of

    * Scalar resonances have large uncertainties,
    * Modeling the $\mathcal{S}$-wave with relativistic BW violates unitarity when broad resonances overlap.

    We parameterize $\pi\pi$ $\mathcal{S}$-wave amplitude in a Model-Independent way:

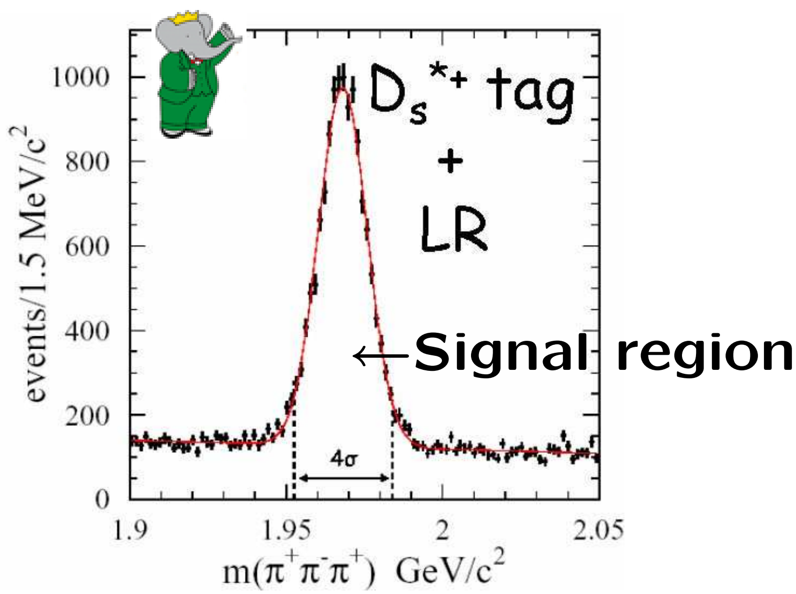
    $$A_{S-wave}(m_{\pi\pi}) = Interp(c_k(m_{\pi\pi}) \exp^{i\phi_k(m_{\pi\pi})})_{k=1,\ldots,30}$$

    * Divide the $\pi^+\pi^-$ mass spectrum into 29 slices, each slice has $\sim$ same number of events.
    * Interpolate between 30 endpoints in the complex plane.
    * Fit for the amplitude and phase of each endpoint (30 c_k's and ϕ_k's).

  - The background shape is obtained by fitting the D_S^+ sideband.

Analysis of $D_S^+ \to \pi^+\pi^-\pi^+$

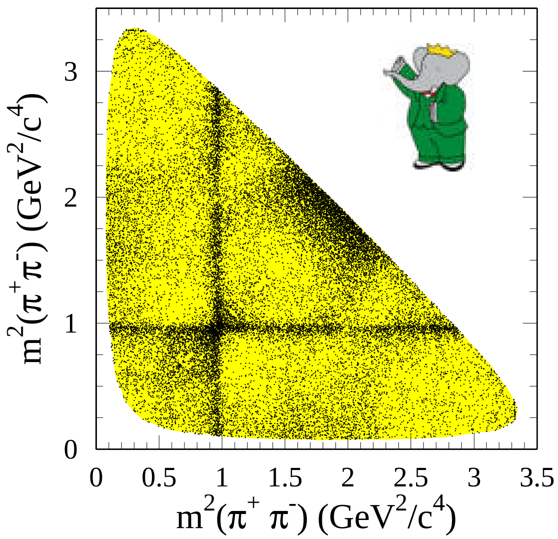
- D_S^+ **Selection:**
  - Use $D_S^*(2112)^+ \to D_S^+\gamma$ tag.
  - Use a likelihood ratio (LR) to improve signal over background ratio.
    - Use $D_S^+ \to K^+K^-\pi^+$ to build signal PDFs:
      - Decay distance transverse to beam plane
      - vertex information
      - center of mass momentum of D_S^*
  - Eliminate reflections from $D^{*+} \to D^0\pi^+ \to (K^-\pi^+/\pi^-\pi^+)\pi^+$.



Signal region contains:
13179 events
with 80% purity

Analysis of $D_S^+ \to \pi^+\pi^-\pi^+$

- **Data Dalitz plot:**



- **Fitted components:**
  - Dominated by $(\pi\pi)_{\mathcal{S}-wave}\pi^+$.
  - $\mathcal{S}$-wave shows clear signal of $f_0(980)$.
  - $\mathcal{S}$-wave shows further activity in the regions of $f_0(1370)$ and $f_0(1500)$.
  - $\mathcal{S}$-wave is small in the $f_0(600)$ region.
  - contributions of $f_2(1270)\pi^+$ is $\sim 10\%$.
  - $\omega(780)$, $f_2'(1525)$ do not improve fit.

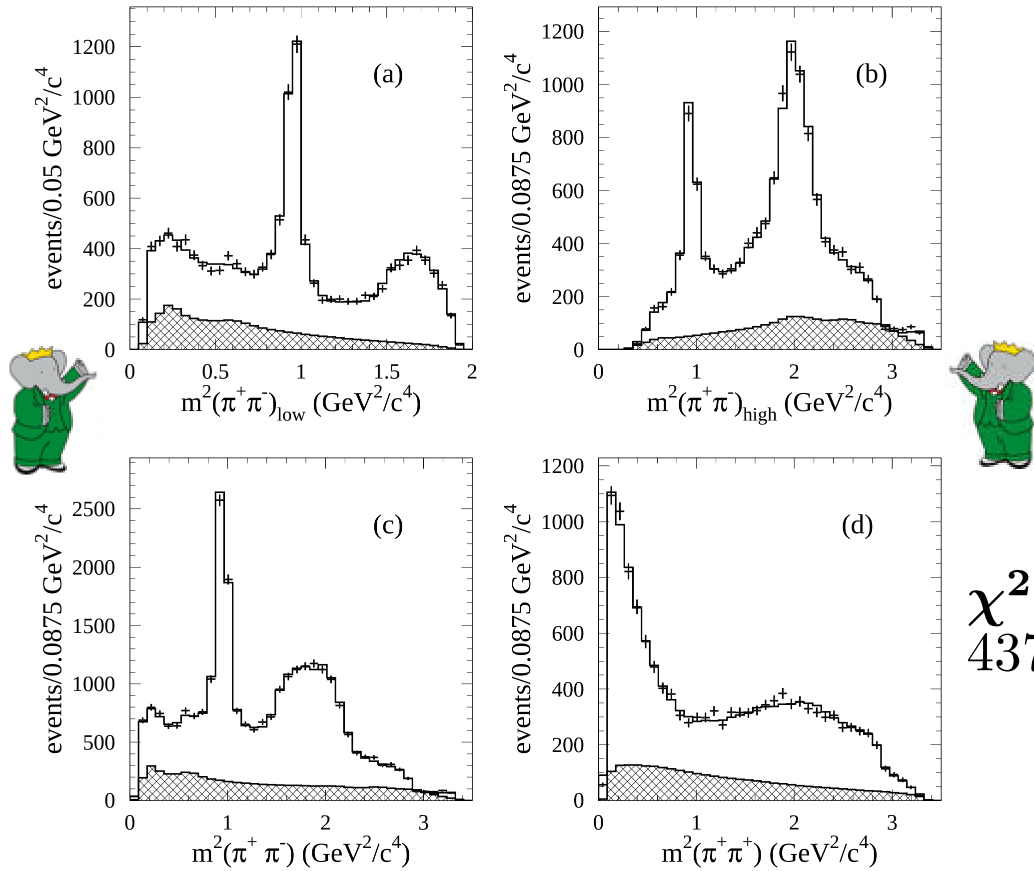
- **Fitted results:**

Decay Mode	Decay fraction(%)	Amplitude	Phase(rad)
$f_2(1270)\pi^+$	$10.1 \pm 1.5 \pm 1.1$	1.(Fixed)	0.(Fixed)
$\rho(770)\pi^+$	$1.8 \pm 0.5 \pm 1.0$	$0.19 \pm 0.02 \pm 0.12$	$1.1 \pm 0.1 \pm 0.2$
$\rho(1450)\pi^+$	$2.3 \pm 0.8 \pm 1.7$	$1.2 \pm 0.3 \pm 1.0$	$4.1 \pm 0.2 \pm 0.5$
$\mathcal{S}$-wave	$83.0 \pm 0.9 \pm 1.9$		
Total	$97.2 \pm 3.7 \pm 3.8$		

Analysis of $D_S^+ \to \pi^+\pi^-\pi^+$

Fitting the data:

Projections on (a) $m^2(\pi^+\pi^-)_{low}$, (b) $m^2(\pi^+\pi^-)_{high}$, (c) total $m^2(\pi^+\pi^-)$, (d) $m^2(\pi^+\pi^+)$:

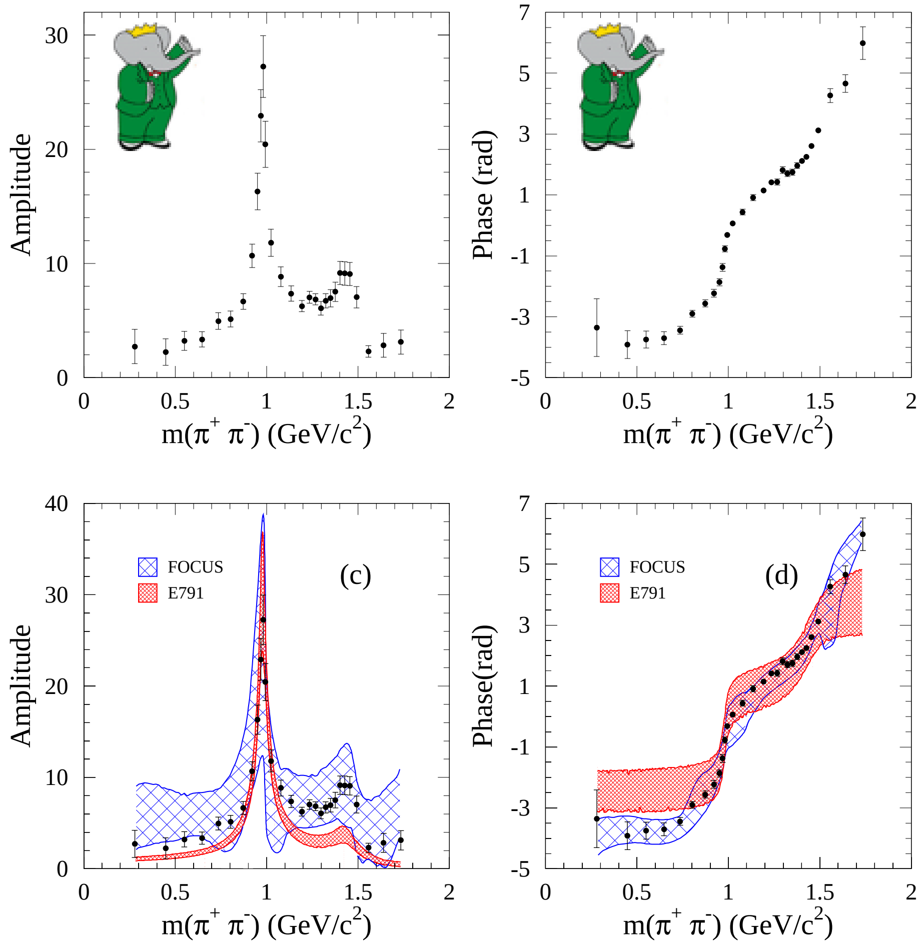


$\chi^2/dof =$
$437/(422 - 64) = 1.2$

- Hatched shaded histograms show the background contribution

Analysis of $D_S^+ \to \pi^+\pi^-\pi^+$

$\mathcal{S}$-wave amplitude and phase extracted from the best fit:



← BABAR: isobar with model-independent $\pi\pi$ $\mathcal{S}$-wave

FOCUS: K-matrix

E791: isobar model

- Our $\mathcal{S}$-wave amplitude and phase agree better with results from FOCUS than those from E791.

Systematic Uncertainties

- **BG parameterization:** using information from the lower/higher sideband only or both
- **Event selection:** estimated by relaxing likelihood cut but narrowing the mass cut on the $\pi^+\pi^-\pi^+$ in order to obtain a similar purity.
- **r constant:** changing Blatt-Weisskopf barrier factor r from 0 to 3 GeV^{-1}.
- **$\rho(770)$ and $\rho(1450)$ parametrization:** using the Gounaris-Sakurai model.
- **Fit model:** using the models which giving equivalent Dalitz plot descriptions and similar sums of fractions.
- **$\mathcal{S}$-wave:** varying the number of steps used to describe the $\mathcal{S}$-wave by ± 2.

Analysis of $D_S^+ \to \pi^+\pi^-\pi^+$

Measurement of $\frac{B(D_S^+ \to \pi^+\pi^-\pi^+)}{B(D_S^+ \to K^+K^-\pi^+)}$ ratio:

- We select events from the two D_S^+ decay modes using similar selection criteria.
- The ratios of branching fractions is evaluated as:

$$Ratio = \frac{\sum_{x,y} N_1(x,y)/\epsilon_1(x,y)}{\sum_{x,y} N_0(x,y)/\epsilon_0(x,y)}$$

where $N_i(x,y)$ represents the number of events measured for channel i $\epsilon_i(x,y)$ is the corresponding efficiency in a given Dalitz Plot cell (x,y).

Systematic Uncertainties

MC statistic	0.9%
Δm cut	1.5%
Likelihood cut	2.6%
Particle identification	3.0%
Total Systematics	4.3%

Results

$$\frac{B(D_S^+ \to \pi^+\pi^-\pi^+)}{B(D_S^+ \to K^+K^-\pi^+)} = 0.199 \pm 0.004 \pm 0.009$$

CLEO: $0.202 \pm 0.011 \pm 0.009$ (PRL100,161804,2008)

PDG: $0.265 \pm 0.041 \pm 0.031$

Summary

- The amplitude and phase of the $\pi^+\pi^-$ $\mathcal{S}$-wave is extracted in a model-independent way for the first time.
- The decay is dominated by $\pi^+\pi^-$ $\mathcal{S}$-wave.
  - $\pi^+\pi^-$ $\mathcal{S}$-wave has amplitude & phase expected for $f_0(980)$.
  - $\pi^+\pi^-$ $\mathcal{S}$-wave has contributions from $f_0(1370)$ and $f_0(1500)$.
  - $\pi^+\pi^-$ $\mathcal{S}$-wave is small contribution in $f_0(600)$ region, small coupling to $s\bar{s}$.
- Important contribution from $f_2(1270)$:
  amplitude 2X smaller than E791, comparable to FOCUS results.
- We measure with high precision the ratio:
  $\frac{B(D_S^+ \to \pi^+\pi^-\pi^+)}{B(D_S^+ \to K^+K^-\pi^+)} = 0.199 \pm 0.004 \pm 0.009.$
  - consistent with CLEO: $0.202 \pm 0.011 \pm 0.009$ (PRL 100, 161804, 2008),
  - consistent with PDG within 1σ: $0.265 \pm 0.041 \pm 0.031$.

Conclusions

- $\boldsymbol{B^- \to D^+\pi^-\pi^-}$:
  - Total $\mathcal{B}(B^- \to D^+\pi^-\pi^-) = (1.08 \pm 0.03(\text{stat}) \pm 0.05(\text{syst})) \times 10^{-3}$
  - $m_{D_2^*} = (2460.4 \pm 1.2 \pm 1.2 \pm 1.9)\text{MeV/c}^2$; $\Gamma_{D_2^*} = (41.8 \pm 2.5 \pm 2.5 \pm 2.0)\text{MeV}$
  - $m_{D_0^*} = (2297 \pm 8 \pm 5 \pm 19)\text{MeV/c}^2$, $\Gamma_{D_0^*} = (273 \pm 12 \pm 17 \pm 45)$ MeV
  - $\mathcal{B}(B^- \to D_2^*\pi^-) \times \mathcal{B}(D_2^* \to D^+\pi^-) = (3.5 \pm 0.2 \pm 0.3 \pm 0.4) \times 10^{-4}$
  - $\mathcal{B}(B^- \to D_0^*\pi^-) \times \mathcal{B}(D_0^* \to D^+\pi^-) = (6.8 \pm 0.3 \pm 0.4 \pm 2.0) \times 10^{-4}$
- $\boldsymbol{B^+ \to \pi^+\pi^-\pi^+}$
  - Total $\mathcal{B}(B^\pm \to \pi^\pm\pi^\mp\pi^\pm) = (15.2 \pm 0.6 \pm 1.2 \pm 0.4) \times 10^{-6}$
  - $\mathcal{B}(B^\pm \to \pi^\pm\pi^\mp\pi^\pm \text{ nonresonant}) = (5.3 \pm 0.7 \pm 0.6^{+1.1}_{-0.5}) \times 10^{-6}$
  - $\mathcal{B}(B^\pm \to \rho^0(770)\pi^\pm) = (8.1 \pm 0.7 \pm 1.2^{+0.4}_{-1.1}) \times 10^{-6}$
  - $m_{f_0(1370)} = 1400 \pm 40\,\text{MeV}/c^2$ and $\Gamma_{f_0(1370)} = 300 \pm 80_{sta}$ MeV
  - $\mathcal{B}(B^\pm \to f_2(1270)\pi^\pm) = (1.57 \pm 0.42 \pm 0.16\,^{+0.53}_{-0.19}) \times 10^{-6}$
- $\boldsymbol{D_S^+ \to \pi^+\pi^-\pi^+}$
  - $B(D_S^+ \to \pi^+\pi^-\pi^+)/B(D_S^+ \to K^+K^-\pi^+) = 0.199 \pm 0.004 \pm 0.009$
  - Amplitude and phase of the $\pi^+\pi^-$ $\mathcal{S}$-wave is extracted in a model-independent way for the first time