Equation of State and the finite temperature transition in hot QCD

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The HotQCD collaboration

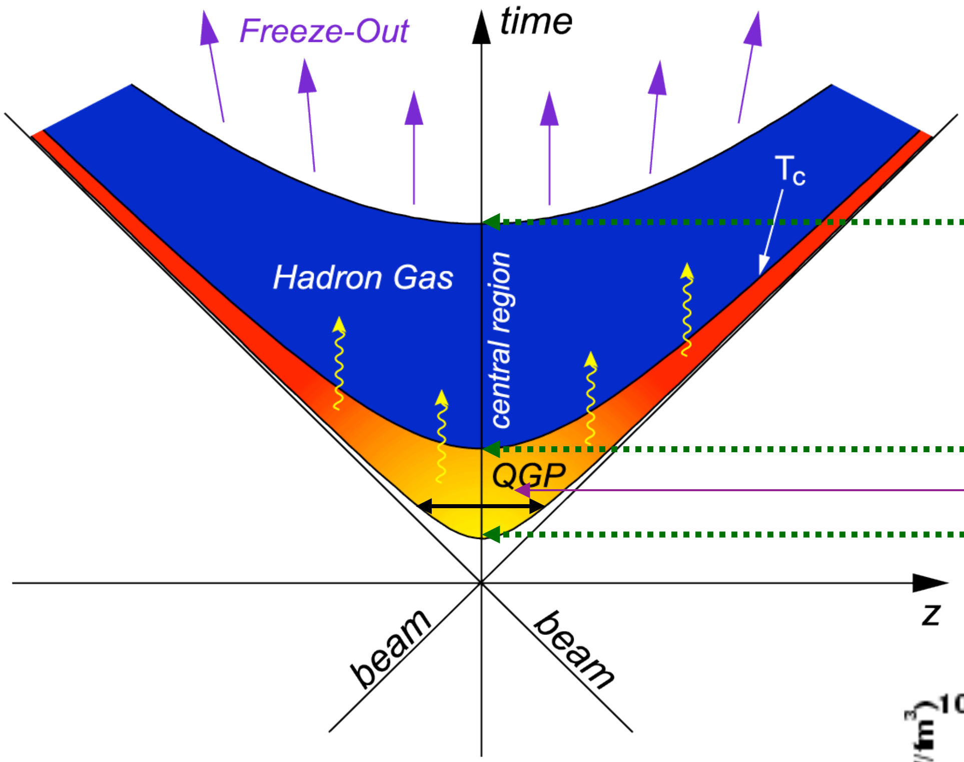
hep-lat/arXiv:0903.4379 to appear in PRD

HotQCD Collaboration

A US wide collaboration studying QCD at finite temperature by simulating Lattice QCD on the BlueGene L at LLNL, NYBlue, …

- A. Bazavov (Arizona)
- T. Bhattacharya (LANL)
- M. Cheng (Columbia)
- N. Christ (Columbia)
- C. DeTar (Utah)
- S. Ejiri (BNL)
- S. Gottlieb (Indiana)
- R. Gupta (LANL)
- U. Heller (APS)
- K. Huebner (BNL)
- C. Jung (BNL)
- F. Karsch (BNL/Bielefeld)
- E. Laermann (Bielefeld)
- L. Levkova (Utah)
- C. Miao (LLNL)
- R. Mawhinney (Columbia)
- P. Petreczky (BNL)
- D. Renfrew (Columbia)
- C. Schmidt (BNL)
- R. Soltz (LLNL)
- W. Soeldner (BNL)
- R. Sugar (UCSB)
- D. Toussaint (Arizona)
- P. Vranas (LLNL)

Probe QGP at RHIC and LHC



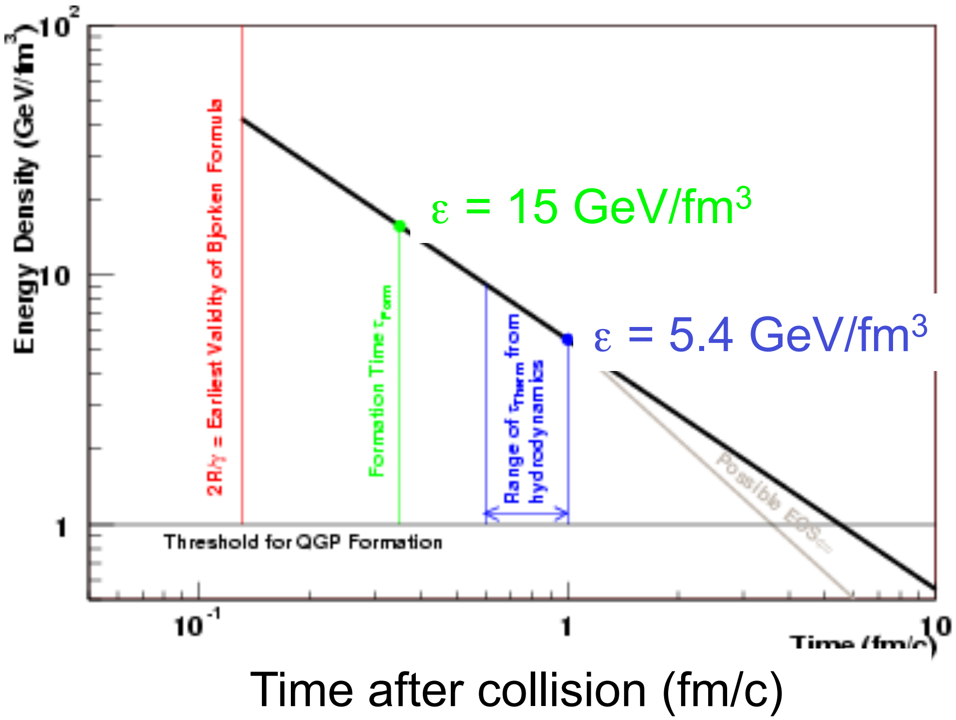
Thermal freeze-out

Hadronization

EoS, viscosity

Equilibration

- RHIC probes QGP between 150-250 MeV
- LHC will probe the range 150-700 MeV



HotQCD Collaboration: Goals

- **Nature of the transition**
  - **Deconfinement and χS restoration?**
- **Crossover temperature Tc**
- **Equation of State (EOS) at (μ=0, $\mu \neq$0)**
- Spectral Functions
- Spatial and temporal correlators versus T
- Transport coefficients of the quark gluon plasma

HotQCD Collaboration: Precision LQCD

(Controlling all systematic errors)

- Two improved staggered formulations (asqtad, p4) (different $O(a^2)$ errors)
- Continuum limit ($N_\tau = 4, 6, 8, \ldots$)
- Chiral extrapolation ($m_l/m_s = 0.2, 0.1, 0.05$)
- Dense sampling of the transition region
- High statistics
- $N_{space} \geq 4\ N_\tau$
- Testing domain wall fermions

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Taking the continuum limit along a line of constant physics (LCP)

Use simulations at T=0 to fix $m_{\{u,d\}}$ and m_s

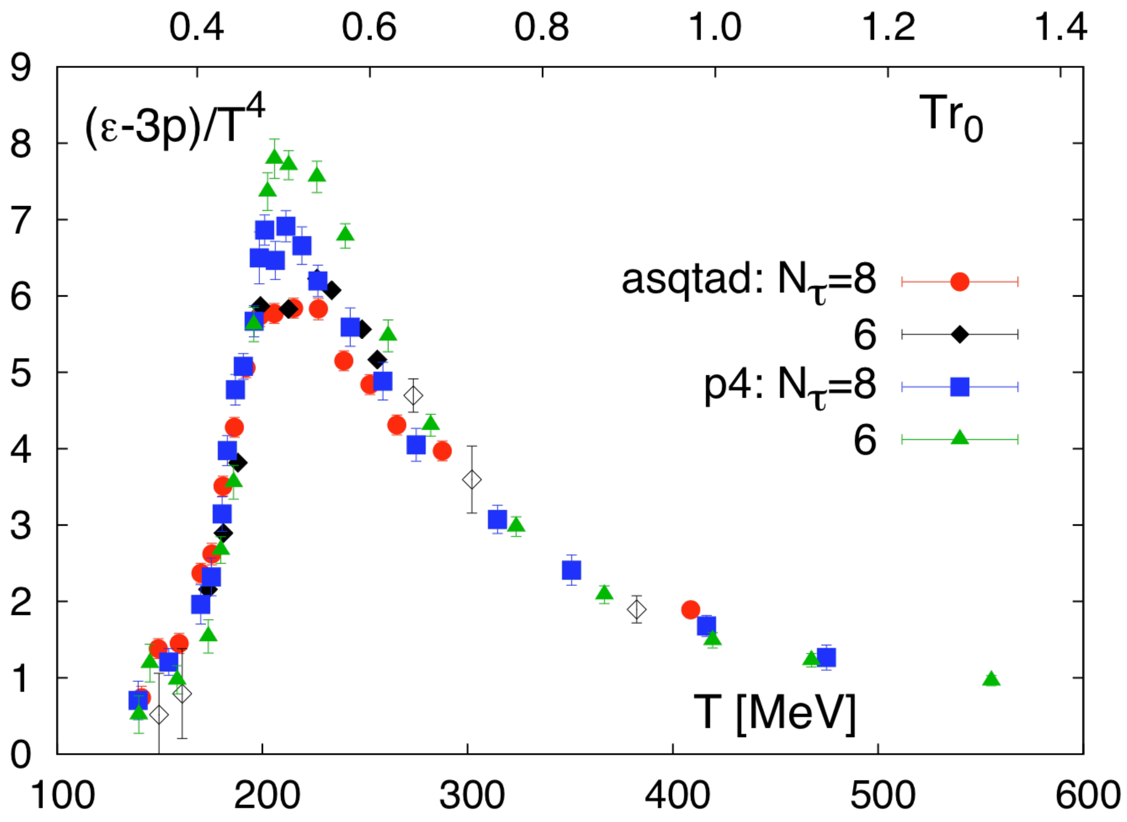
- Lattice scale *a* set using static $q\bar{q}$ potential (r_0 and r_1) with $r_0 = 0.469(7)$ from $\Upsilon(2S-1S)$
- **$M_{ss}\, r_0 = 1.58$ ($M_\pi\, r_0 \approx 0.52 \rightarrow M_\pi \approx 220$ MeV)**
- Quark mass $m_l/m_s = 0.1$ (real world ~0.04)
- Take $a \rightarrow 0$ along this LCP varying just the gauge coupling β

Equation of State

Experiments at the LHC will probe the QGP over 700-200 MeV.

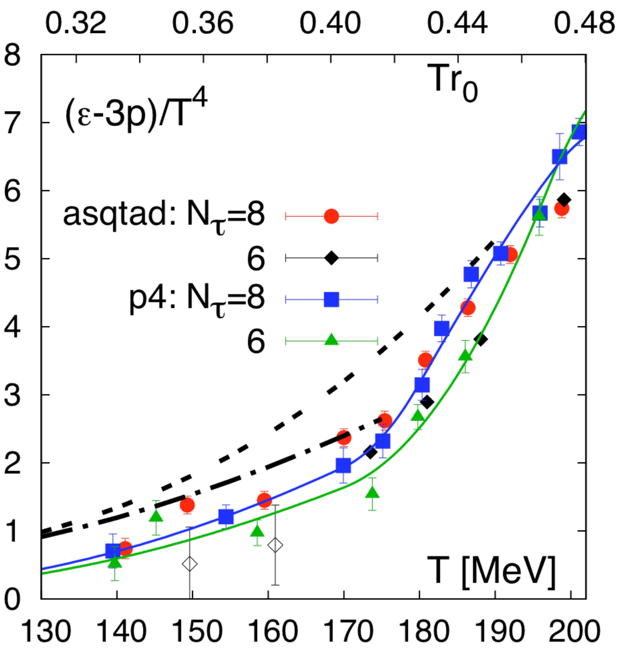
Lattice QCD provides EoS from first principles calculation as input into hydrodynamical models used to describe the evolution of the QGP.

Trace Anomaly $(\varepsilon\text{-}3p)/T^4$

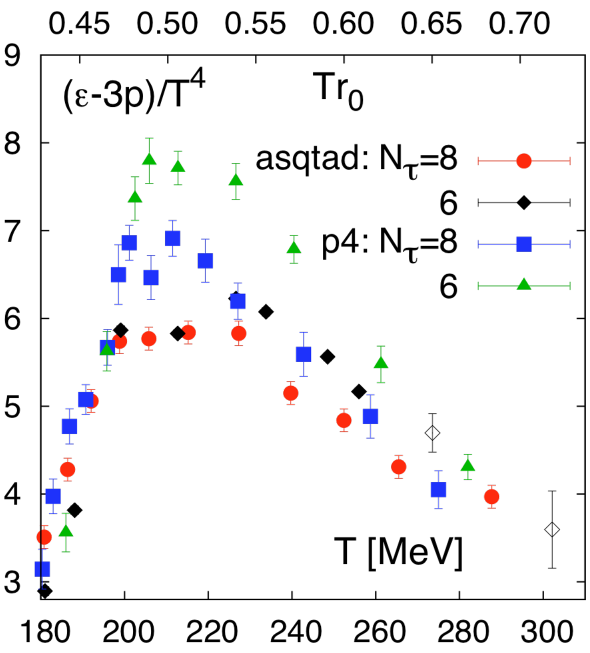


This is the single basic quantity we calculate

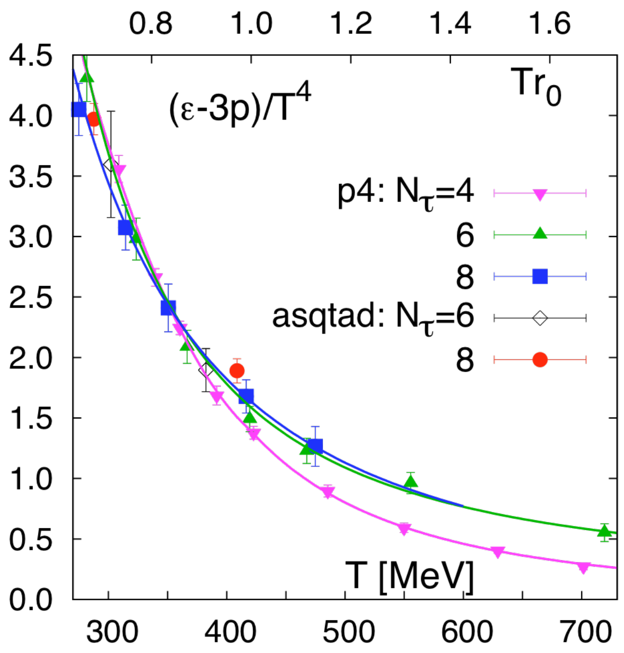
Trace Anomaly -- details



- Shift to smaller T by ~5 MeV with $N_\tau=6\rightarrow8$.
- Data lies below hadron resonance gas (dashed curve)

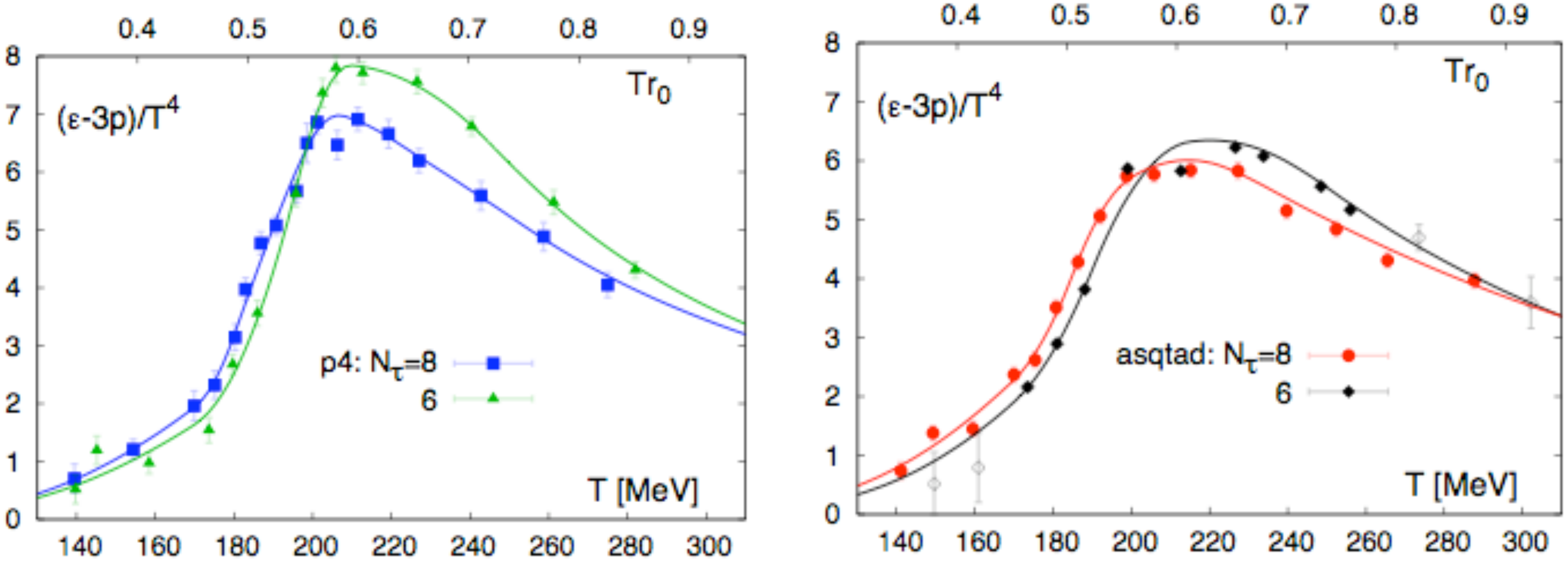


- Peak above 200 MeV.
- 15-20% change with $N_\tau=6\rightarrow8$



- Need to estimate finite volume uncertainty for T>400 MeV (point at 400 MeV with 8×64^3).
- Need more points to fix the shape of the curve
- Fit: $c_0 + c_2/T^2 + c_4/T^4$
  Asymptotic g^4 behavior not evident in c_0

Lattice artifacts ($N_\tau=6\rightarrow 8$)



A shift by ~5 MeV of $N_\tau=6$ data (estimate of discretization errors in lattice scale a between $N_\tau=6$ and 8) takes care of most of the difference.

Extracting *ε* and *p* from the trace anomaly

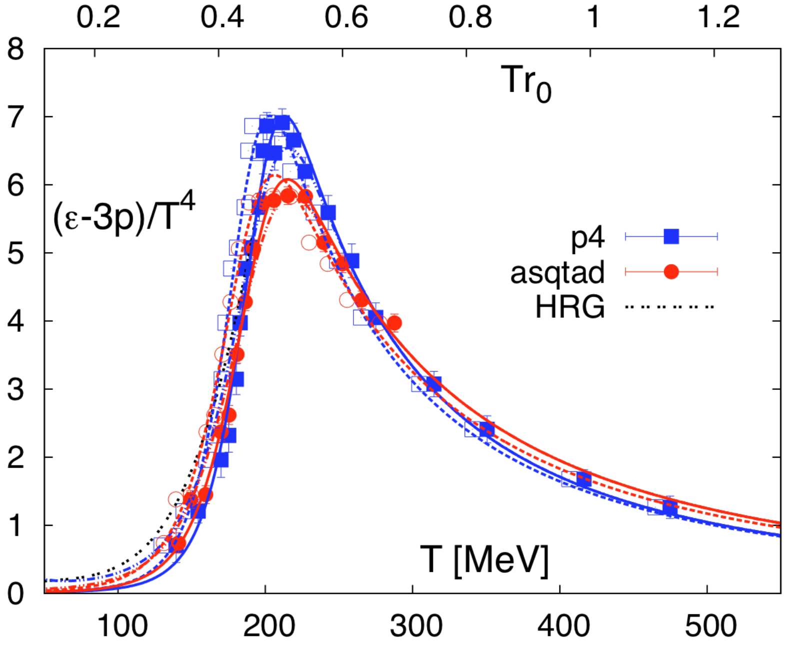
$$\frac{\Theta^{\mu\mu}(T)}{T^4} \equiv \frac{\varepsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4)$$

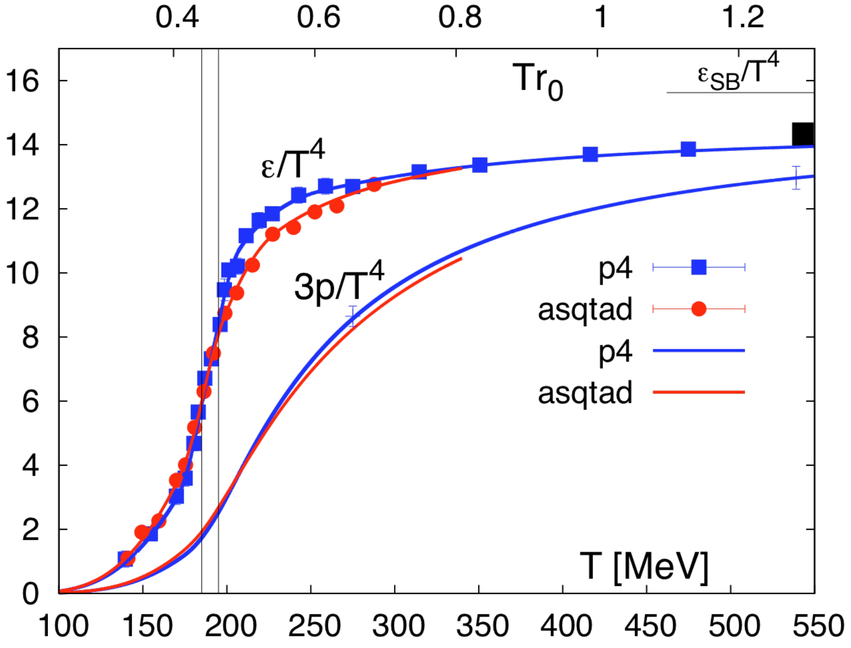
⇒

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dt\,\frac{\Theta^{\mu\mu}(t)}{t^5}$$

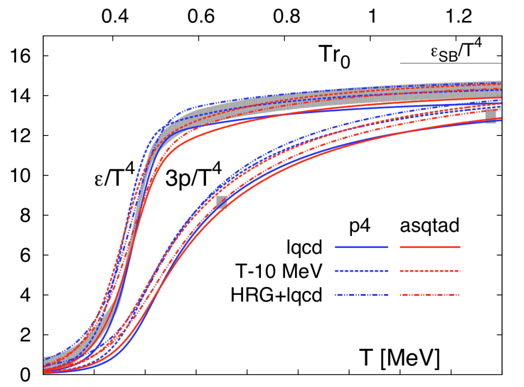
- $(\varepsilon\text{-}3p)/T^4$ calculated at a discrete set of points
- To integrate with respect to T we need to
  - Fit the data with a smooth interpolating function
  - Choose T_0
  - Choose the value of $(\varepsilon\text{-}3p)/T^4$ at T_0

ε/T^4 and p/T^4



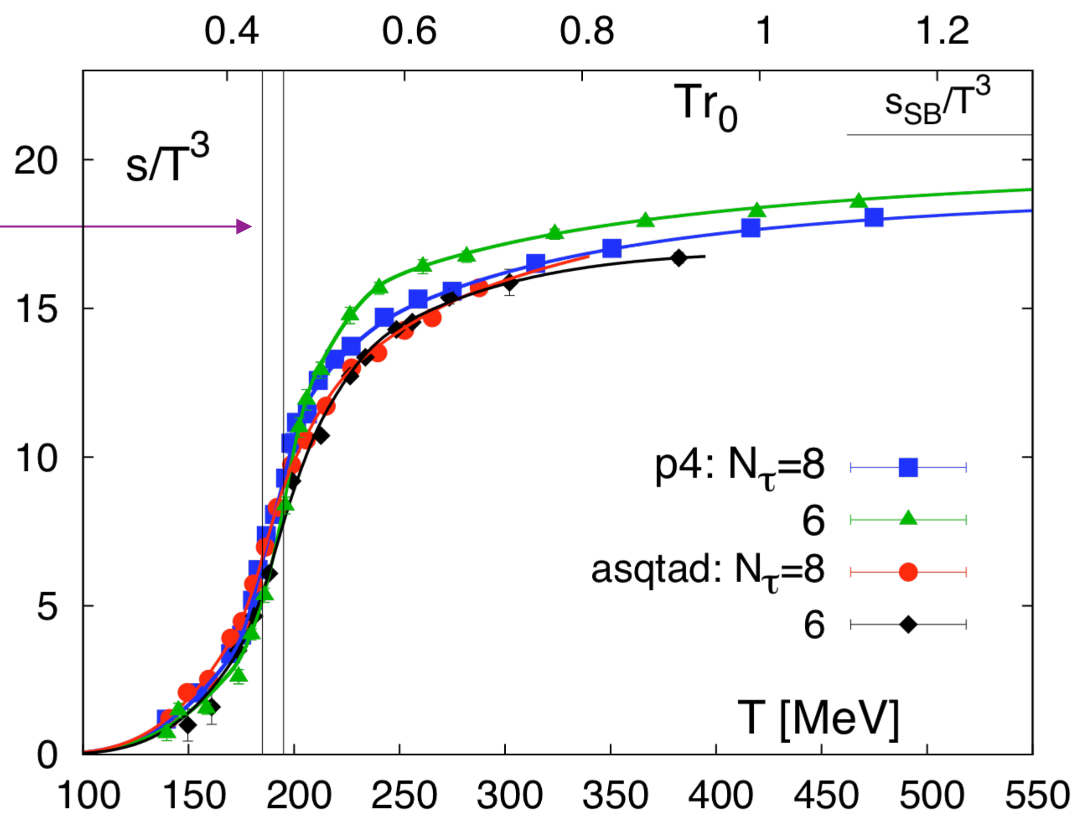


- Explored sensitivity to
  - Interpolation function
  - Choice of T_0
  - Choice of the value of $(\varepsilon-3p)/T^4$ at T_0



Parameterized fit for hydro

Entropy density $s/T^3=(\varepsilon+p)/T^4$

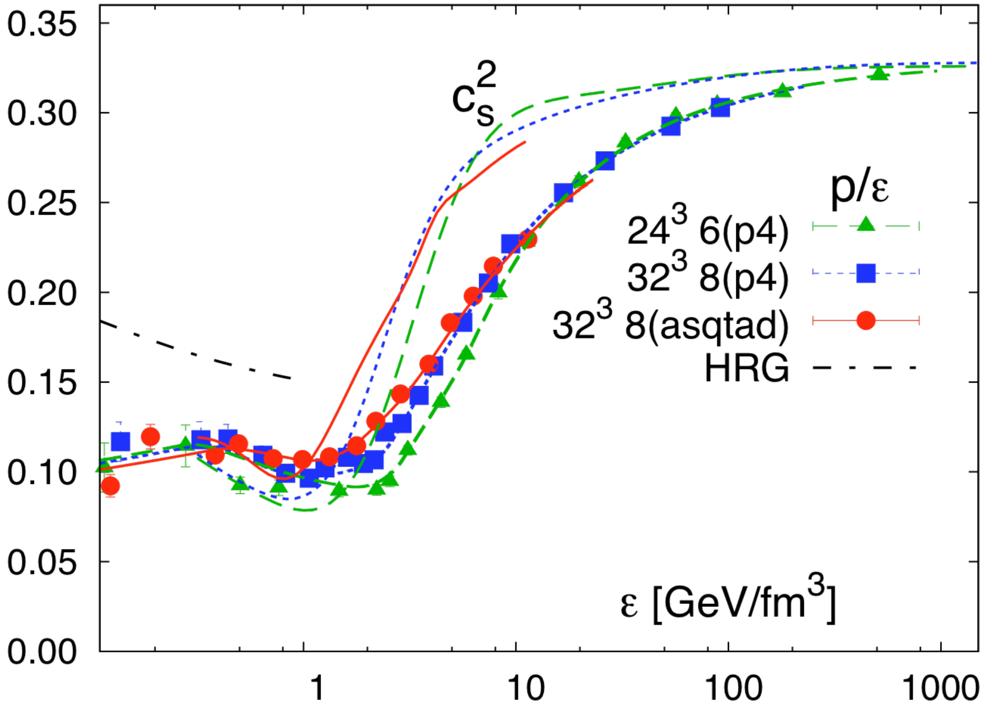


Band in T: 185-195 MeV

The degrees of freedom in the initial state at LHC (~700 MeV) have a much cleaner interpretation as quarks and gluons.

Crossover T in (ε and s) is what is relevant to HI experiments!

Speed of sound c_s^2



$$c_s^2 = \frac{dp}{d\varepsilon} = \varepsilon\frac{d(p/\varepsilon)}{d\varepsilon} + \frac{p}{\varepsilon}$$

Lines without points are obtained using interpolating functions for $(\varepsilon\text{-}3p)/T^4$ *and* p/T^4

- Rapid saturation to free field value of 1/3
- Comparing N_τ=6 with N_τ=8
  - The dip is shallower
  - The rise after the dip is less steep

Transition Temperature

- Genuine phase transition only at $m_l < m_{phy}$ (maybe only at $m_l=0$)
- To estimate crossover T at m_{phy} we study
  - Polyakov line and its susceptibility
  - Chiral condensate $\langle \bar{\psi}\psi \rangle$ and its susceptibility
  - Quark number $\langle \bar{\psi}\gamma_0\psi \rangle$ and its susceptibility

Polyakov loop

Quark number susceptibility

} **Deconfinement**

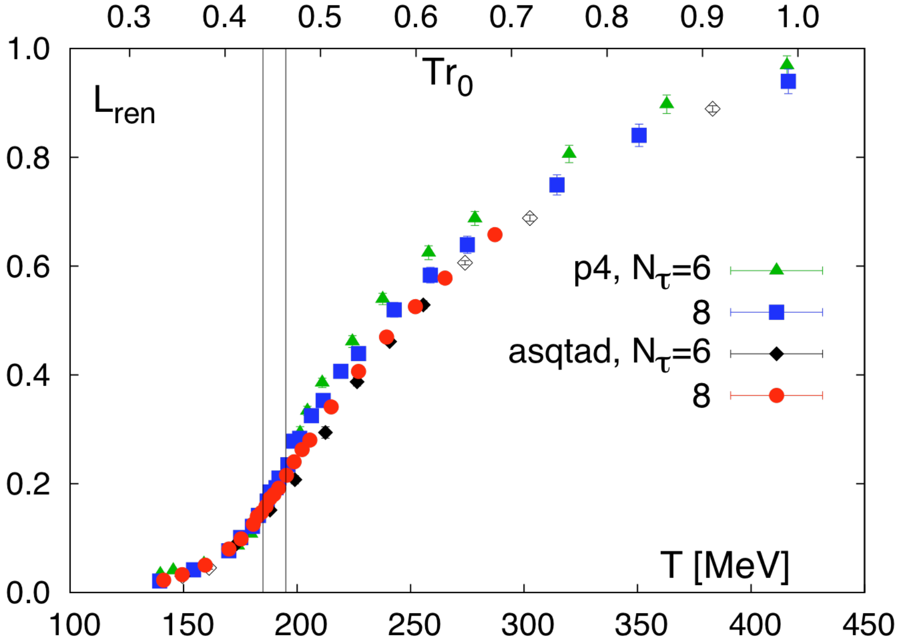
Chiral condensate

Chiral susceptibility

} **χS Restoration**

Renormalized Polyakov Loop

$$\langle L_{ren} \rangle = Z(\beta)^{N_\tau} \langle L_{Bare} \rangle$$



The band indicates the range T=185-195 MeV

- Difference between asqtad and p4 for Nτ = 6, 8 ≤ 10%
- No clear inflection point (peak in the susceptibility)
- <L> does not probe the singular part of Z

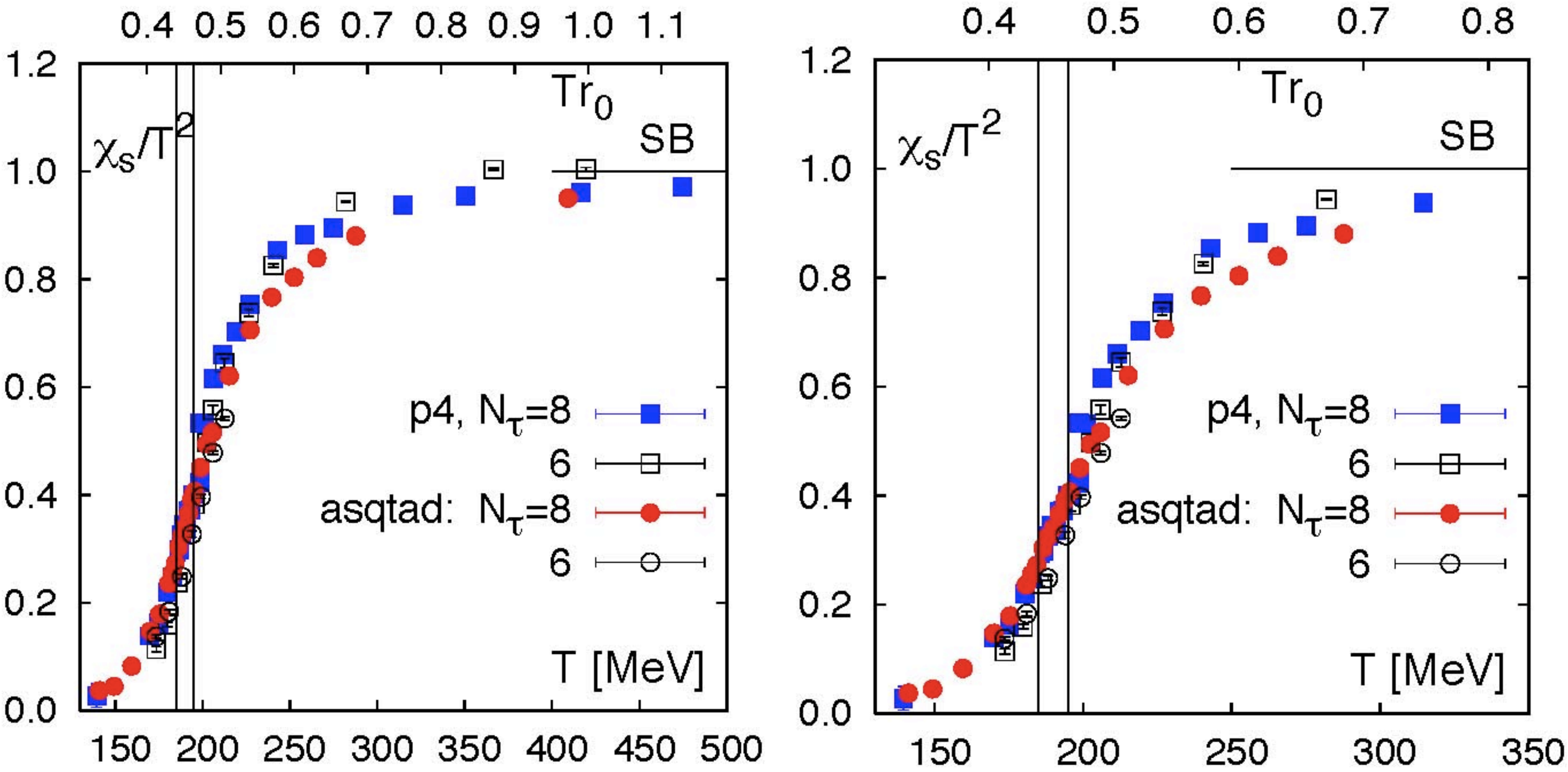
Quark Number Susceptibility

$$\frac{\chi_{l,s}}{T^2} = \frac{1}{TV}\frac{\partial^2 \log Z}{\partial \mu_{l,s}^2}$$

χ_l Probes confinement

- Basic operator is $\langle\bar{\psi}\gamma_0\psi\rangle$ (→ baryon number/strangeness)
- States carrying quantum numbers given by $\langle\bar{\psi}\gamma_0\psi\rangle$ are heavy (mesons & baryons) at low T and light (quarks) at high T.
- χ_l Does not require renormalization
- Peak is in fourth derivative → computationally hard!

Quark Number Susceptibility



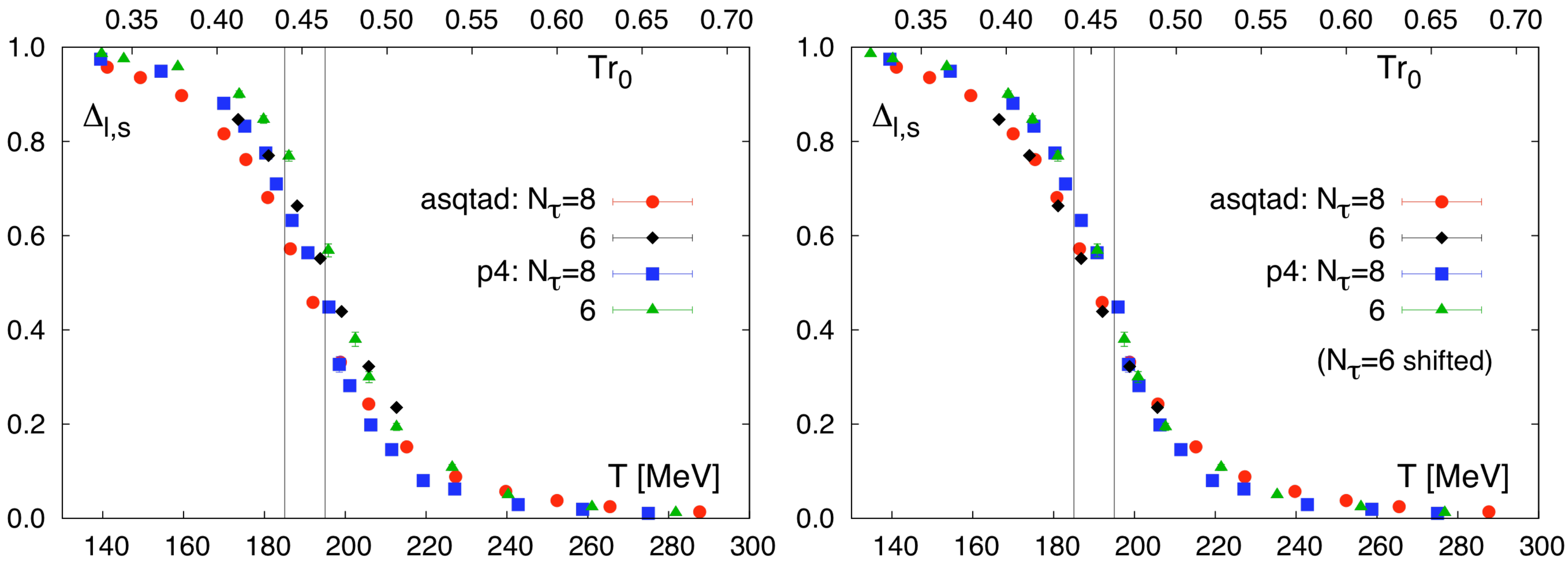
Difference between asqtad and p4 most pronounced between 200-300 MeV

Chiral Condensate: χS restoration

$$\Delta_{l,s} = \frac{\left\langle \bar{l} l \right\rangle_T - \frac{m_l}{m_s} \left\langle \bar{s} s \right\rangle_T}{\left\langle \bar{l} l \right\rangle_{T=0} - \frac{m_l}{m_s} \left\langle \bar{s} s \right\rangle_{T=0}}$$

- Subtract $< \bar{s}s >$ to eliminate additive renormalization
- Divide by T=0 value to cancel multiplicative renormalization
- Sharp decrease in Δ reflects χ Symmetry restoration

N_τ=6, 8 data for p4 and asqtad

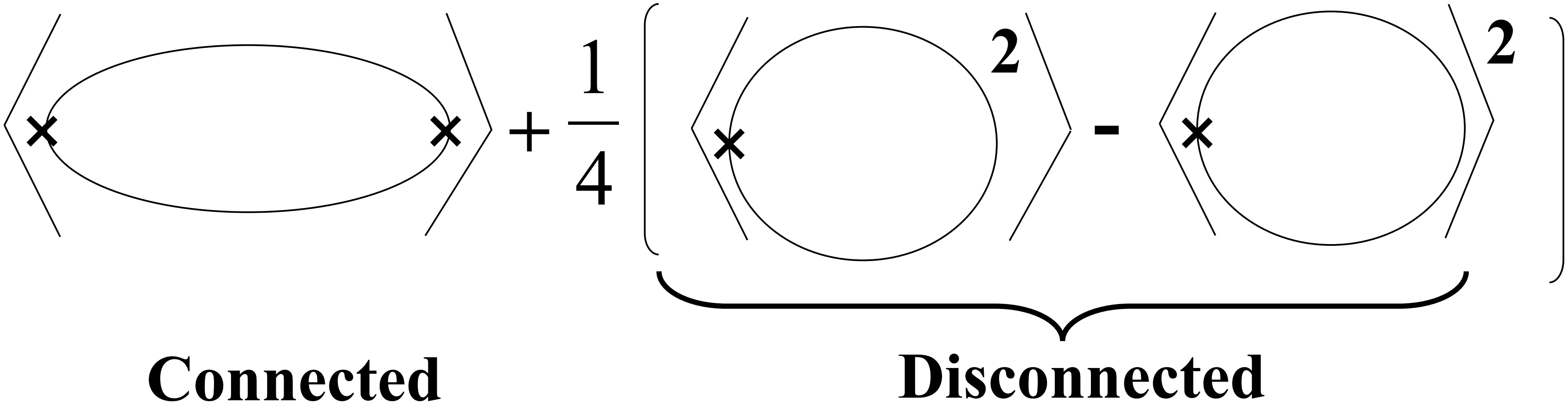


- Inflection point estimated to be in the range 185-195 MeV

Chiral Susceptibility: N_τ = 4, 6, 8

One flavor susceptibility for light (*l* = u,d) and strange (s) quarks:

$$\frac{\chi_{l,s}}{T^2} = \frac{T}{V}\frac{\partial^2 \log Z}{\partial^2 m_{l,s}^2} = \left\langle Tr(M_{l,s}^{-2})\right\rangle + \frac{1}{4}\left(\left\langle (TrM_{l,s}^{-1})^2\right\rangle - \left\langle TrM_{l,s}^{-1}\right\rangle^2\right)$$

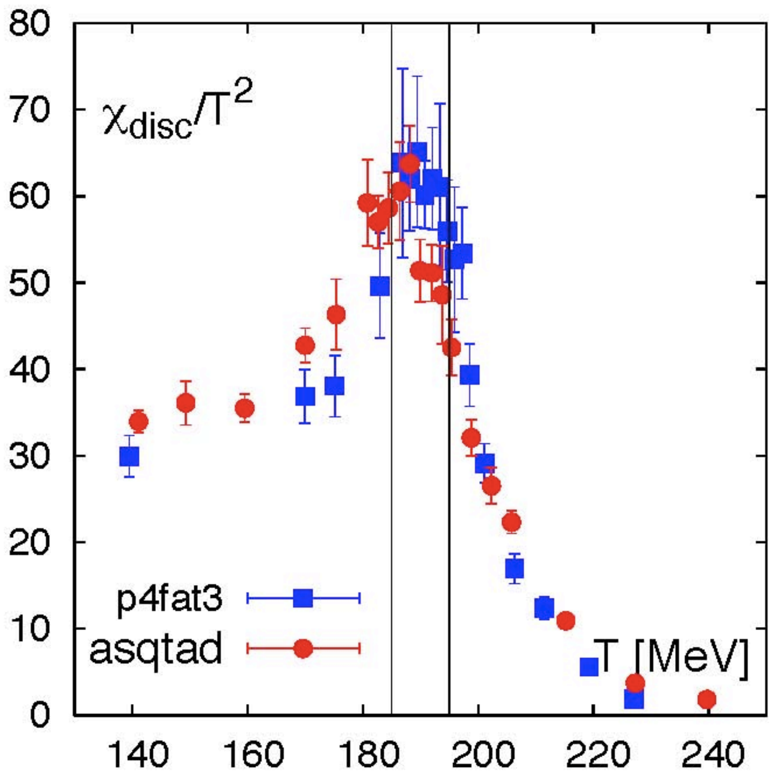


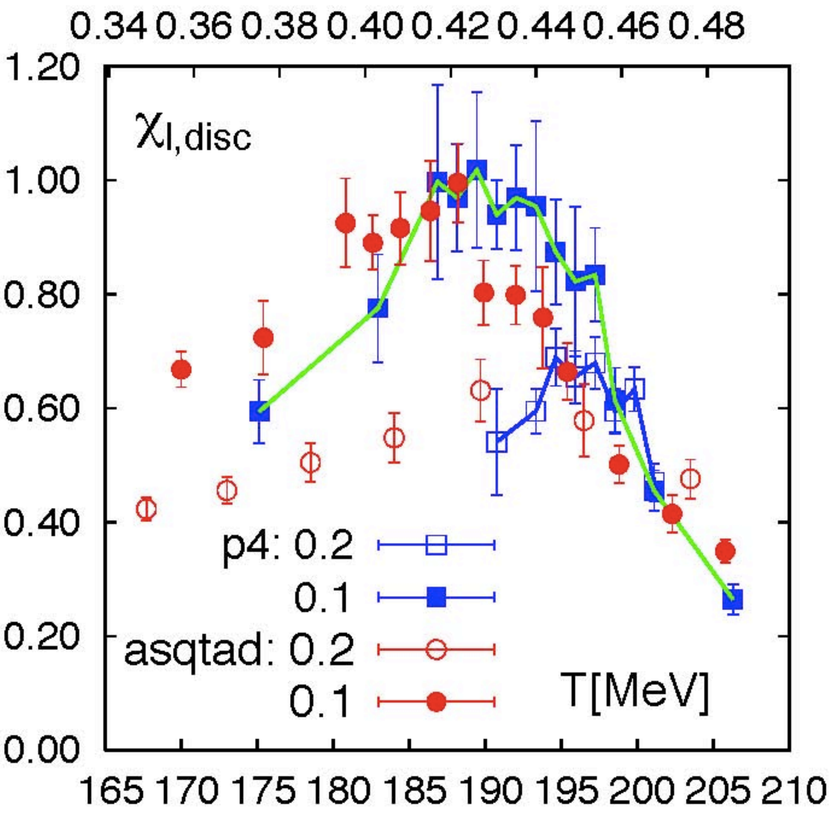
Note: Isosinglet $\chi = \chi_{\text{disconnected}} + 2\chi_{\text{connected}}$

One flavor $\chi = \chi_{\text{disconnected}} + 4\chi_{\text{connected}}$

Disconnected Chiral Susceptibility: $N_\tau = 8$

Preliminary



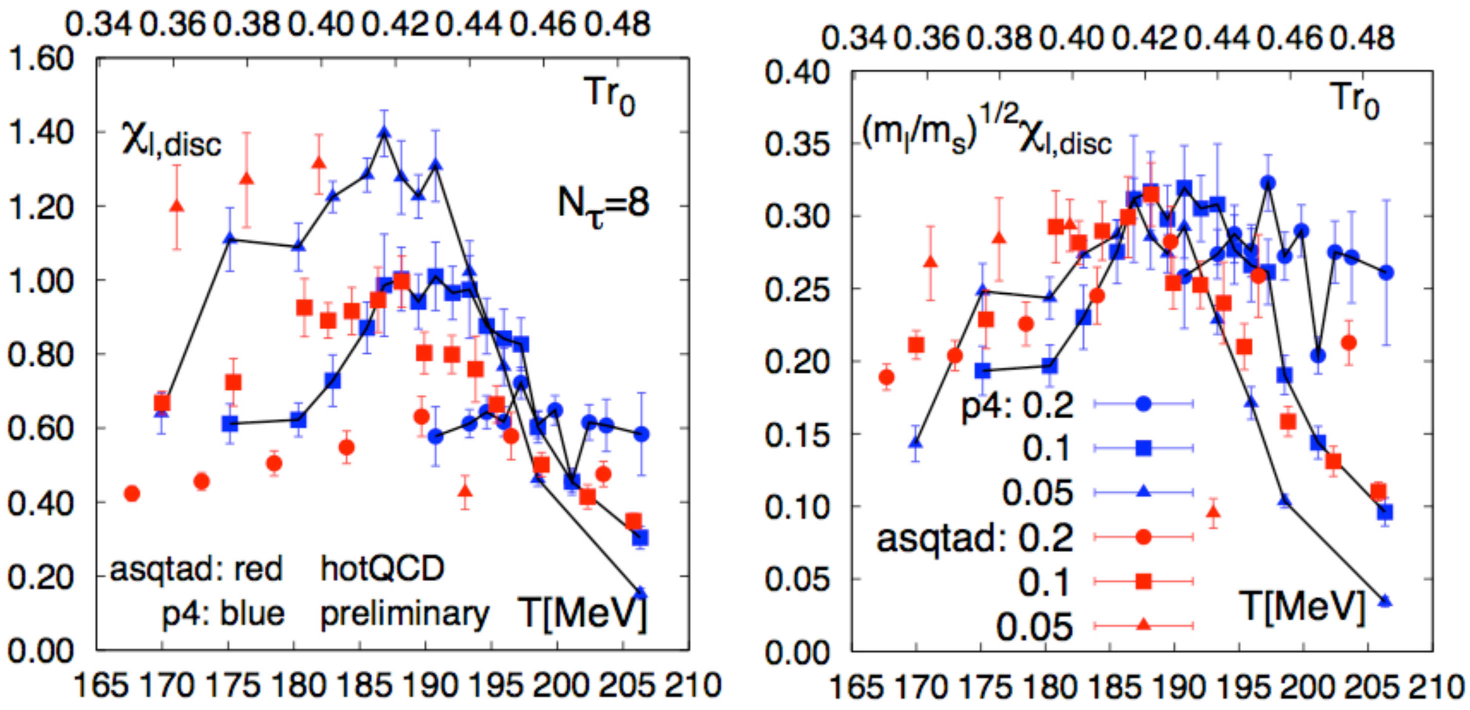


The peak becomes broader as mq → 0!

What T value should be designated Tc?

Disconnected Chiral Susceptibility: $N_\tau = 8$

Preliminary



For m_q=0 the pions are massless in the χS broken phase and χ_l should diverge for all T < Tc. At finite *m* we expect a broad peak for T < Tc.

➡ Choose right edge as the point at which χS is recovered = T_c

HotQCD Conclusions

- QCD exhibits a crossover over 160-220 MeV
- asqtad and p4 data are consistent
  - Differences at $O(a^2)$ level
  - Maximum differences seen between 200-300 MeV
- Estimate of EoS over the range 150-500 MeV ready for input into hydrodynamics models
- Need ($m_l \rightarrow 0$) limit to define transition temperature.
- Estimates from ($N_\tau = 8$, $m_l/m_s = 0.1$) ~185-195 MeV
- Extend $N_\tau = 6, 8$ ($m_l/m_s = 0.2, 0.1$) runs to
  - $m_l/m_s = 0.05$
  - $N_\tau = 12$

Resolving Differences Between HotQCD (arXiv:0903:4379v1) and Wuppertal-Budapest (Aoki et al)

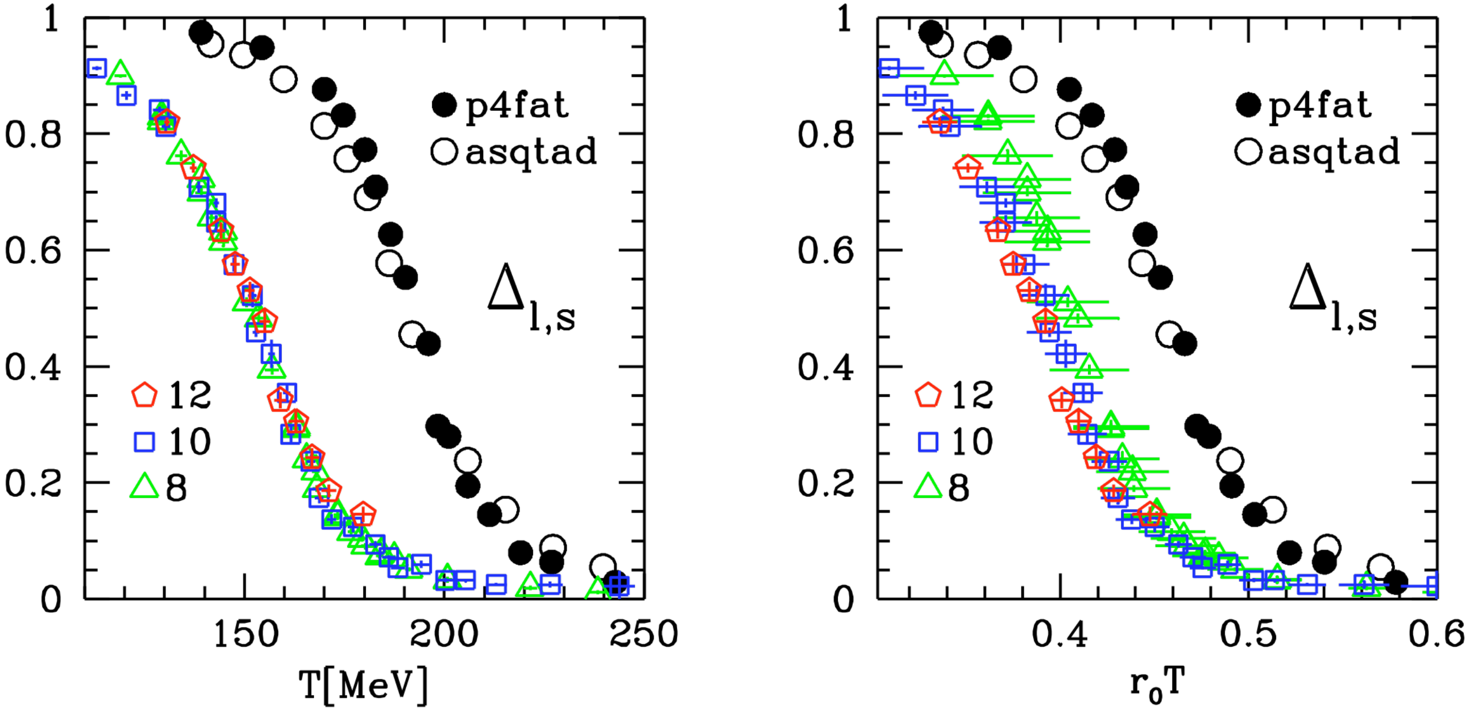
(JHEP 0906:088,2009 / arXiv:0903.4155v1)

Collaboration results

Estimates of crossover T

- Wuppertal-Budapest (Aoki et al)
  - Tc = 146(2)(3)-157(3) Chiral Symmetry restoration
  - Tc = 170(4) Deconfinement
- HotQCD ($N_\tau = 8$; $m_l/m_s = 0.1$)
  - All observables exhibit crossover between 185-195 MeV
  - Polyakov Loop and Strange Quark Number Susceptibility do not probe singular structure of the theory and, thus, are not good observables to determine Tc
    - No clear inflection point in Polyakov loop or peak in its susceptibility for $N_\tau > 4$
  - Need to resolve the ~30 MeV difference in the light quark chiral susceptibility

Data as presented by Aoki el al: Light Quark Chiral Condensate



The shape of the curves is similar:

→ the simplest possibility is difference in setting the lattice scale in the two calculations i.e. we can explain the differenced by shifting T by ~30MeV?

→ The answer is most likely a combination of effects.

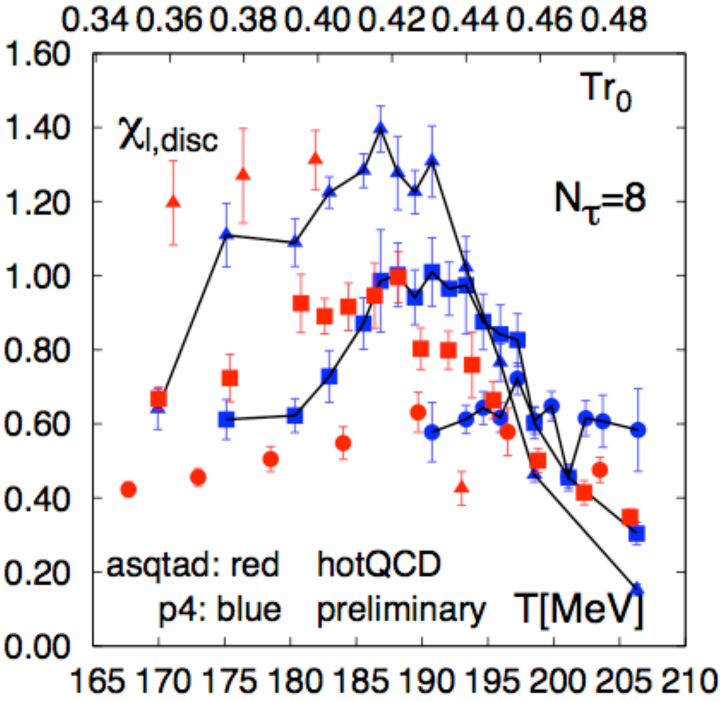
We consider this issue unresolved and are studying it further.

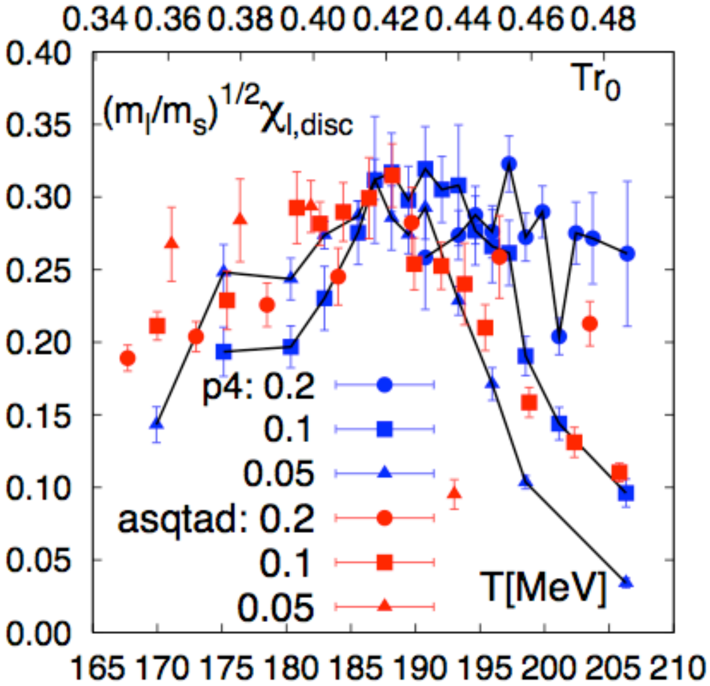
Differences in calculations

	Staggered Action	Quark masses	Lattice Scale *a*
Wuppertal-Budapest (WB)	Unimproved Stout (N_τ = 8,10,12)	m_l/m_s = 1/27	M_Ω, M_ϕ(1020), M_{K^*}(892), f_K, r_0
HotQCD	asqtad & p4 Both actions O(a^2) improved (N_τ = 6,8)	m_l/m_s = 0.2, 0.1, 0.05	r_0
Data Show (all three estimates need to be refined)	Remaining shift in Tc to lower T with a→0 is ~5 MeV based on above N_τ	Remaining shift in Tc to lower T with $m_q \rightarrow m_{physical}$ is ~5 MeV in HotQCD data	WB result: All 5 quantities give consistent lattice scale in a=0 limit

Understanding the peak in Chiral Susceptibility: $N_\tau = 8$

Preliminary





For $m_q=0$ the pions are massless in the χS broken phase and χ_l should diverge as $m_q^{1/2}$ for all T < Tc. Data are roughly consistent with this expectation.

⇨ Should choose right edge for T_c!

Future

The two collaborations (Hot QCD and WB) have started new calculations on the IBM BG/P at Juelich to resolve this difference