Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

AdS/CFT and RHIC Hydrodynamics

James Alsup

University of Tennessee
Department of Physics and Astronomy
High Energy Theory Group

July 31 2009 / DPF 2009

J. Alsup and G. Siopsis, Phys. Rev. Lett. **101** (2008) arXiv:0712.2164
J. Alsup and G. Siopsis, Phys. Rev. D **79**, 066011 (2009) arXiv:0812.1818

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT

Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence

Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order

Summary

Outline

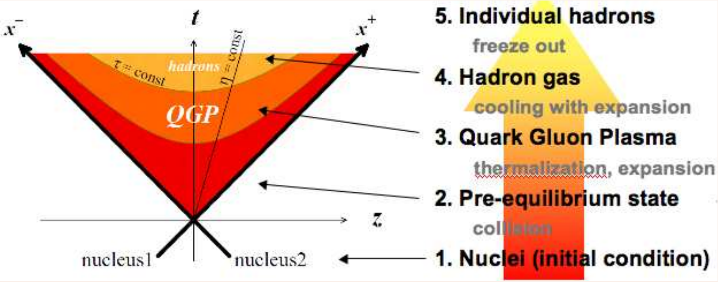
1. Introduction
   - Relativistic Heavy Ion Collisions
   - Bjorken Hydrodynamics
   - AdS/CFT Correspondence
2. Hydrodynamics from AdS/CFT
   - AdS Black Hole
   - Time Dependence
3. Transformation
   - Static Black Hole
   - Dynamic Black Hole
   - CFT Plasma
   - Next-to-leading order
   - Next-to-next-to-leading order

RHIC seen as Hydrodynamics

Experiment seems to suggest QGP is strongly coupled

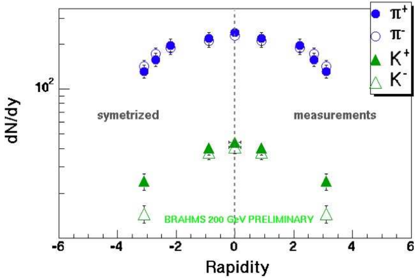
- creating hydrodynamic behavior *[Kolb and Heinz, . . .]*

Hierarchy of times, *[Itakura]*



RHIC seen as Hydrodynamics

Bjorken suggested to study the central rapidity region



[*BRAHMS Collaboration*]

- "plateau" for particle production, $\frac{dN}{dy} = constant$
- all particles share the same proper time, τ, and independent of Lorentz frame

RHIC seen as Hydrodynamics

Initial conditions

	τ_0 (fm/c)	ϵ_0 (GeV/fm^3)	T (GeV)	$\sqrt{s}$ (GeV)
RHIC	0.2	10	0.5	200
LHC	0.1	10	1	5,500

τ: time, ϵ: energy density, T: temprature, $\sqrt{s}$: c.o.m. energy

Hydrodynamic equations

- respect symmetry of initial conditions (boost invariance)
- simple solutions from conservation and conformal invariance

$$\nabla_\mu T^{\mu\nu} = 0\,, \quad T^\mu_\mu = 0\,, \quad T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu + p g_{\mu\nu}$$

$$\Rightarrow \varepsilon = \frac{\varepsilon}{\tau^{4/3}}, \quad T = \frac{T_0}{\tau^{1/3}}, \quad s = \frac{s_0}{\tau}$$

RHIC seen as Hydrodynamics

Initial conditions

	τ_0 (fm/c)	ϵ_0 (GeV/fm^3)	T (GeV)	$\sqrt{s}$ (GeV)
RHIC	0.2	10	0.5	200
LHC	0.1	10	1	5,500

τ: time, ϵ: energy density, T: temprature, $\sqrt{s}$: c.o.m. energy

Hydrodynamic equations

- respect symmetry of initial conditions (boost invariance)
- simple solutions from conservation and conformal invariance

$$\nabla_\mu T^{\mu\nu} = 0\,, \quad T^\mu_\mu = 0\,, \quad T_{\mu\nu} = (\varepsilon + p) u_\mu u_\nu + p g_{\mu\nu}$$

$$\Rightarrow \varepsilon = \frac{\varepsilon}{\tau^{4/3}}, \quad T = \frac{T_0}{\tau^{1/3}}, \quad s = \frac{s_0}{\tau}$$

RHIC seen as Hydrodynamics

Initial conditions

	τ_0 (fm/c)	ϵ_0 (GeV/fm^3)	T (GeV)	$\sqrt{s}$ (GeV)
RHIC	0.2	10	0.5	200
LHC	0.1	10	1	5,500

τ: time, ϵ: energy density, T: temprature, $\sqrt{s}$: c.o.m. energy

Hydrodynamic equations

- respect symmetry of initial conditions (boost invariance)
- simple solutions from conservation and conformal invariance

$$\nabla_\mu T^{\mu\nu} = 0\,, \quad T^\mu_\mu = 0\,, \quad T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu + p g_{\mu\nu}$$

$$\Rightarrow \varepsilon = \frac{\varepsilon}{\tau^{4/3}}, \quad T = \frac{T_0}{\tau^{1/3}}, \quad s = \frac{s_0}{\tau}$$

Dissipative Effects

Viscosity is essential to understanding QGP

RHIC - $\eta/s = (.1 - .2)\frac{\hbar}{k_B}$ *[Teaney]*

- Smallest known value
- Inclusion can account for elliptic flow

- Boost invariant viscous flow

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon(\tau) & 0 & 0 & 0 \\ 0 & \frac{p(\tau)}{\tau^2} - \frac{4}{3}\frac{\eta(\tau)}{\tau^3} & 0 & 0 \\ 0 & 0 & p(\tau) + \frac{2}{3}\frac{\eta(\tau)}{\tau} & 0 \\ 0 & 0 & 0 & p(\tau) + \frac{2}{3}\frac{\eta(\tau)}{\tau} \end{pmatrix}$$

Dissipative Effects

Viscosity is essential to understanding QGP

RHIC - $\eta/s = (.1 - .2)\frac{\hbar}{k_B}$ *[Teaney]*

- Smallest known value
- Inclusion can account for elliptic flow

- Boost invariant viscous flow

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon(\tau) & 0 & 0 & 0 \\ 0 & \frac{p(\tau)}{\tau^2} - \frac{4}{3}\frac{\eta(\tau)}{\tau^3} & 0 & 0 \\ 0 & 0 & p(\tau) + \frac{2}{3}\frac{\eta(\tau)}{\tau} & 0 \\ 0 & 0 & 0 & p(\tau) + \frac{2}{3}\frac{\eta(\tau)}{\tau} \end{pmatrix}$$

Dissipative Effects

- From conservation equations

$$
\begin{aligned}
\varepsilon &= 3p = \frac{\varepsilon_0}{\tau^{4/3}} - \frac{2\eta_0}{\tau^2} + \dots \\
T &= T_0\left(\frac{1}{\tau^{1/3}} - \frac{\eta_0}{2\varepsilon_0\tau} + \dots\right) \\
s &= \frac{dp}{dT} = s_0\left(\frac{1}{\tau} - \frac{3\eta_0}{2\varepsilon_0}\frac{1}{\tau^{5/3}} + \dots\right)
\end{aligned}
$$

- where

$$\frac{\eta}{s} = \frac{\eta_0}{s_0} = \frac{3\eta_0}{4\varepsilon_0}T_0$$

$\Rightarrow$ constant cannot be determined from hydrodynamics!

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

Dissipative Effects

- From conservation equations

$$\begin{aligned} \varepsilon &= 3p = \frac{\varepsilon_0}{\tau^{4/3}} - \frac{2\eta_0}{\tau^2} + \dots \\ T &= T_0 \left(\frac{1}{\tau^{1/3}} - \frac{\eta_0}{2\varepsilon_0 \tau} + \dots \right) \\ s &= \frac{dp}{dT} = s_0 \left(\frac{1}{\tau} - \frac{3\eta_0}{2\varepsilon_0} \frac{1}{\tau^{5/3}} + \dots \right) \end{aligned}$$

- where

$$\frac{\eta}{s} = \frac{\eta_0}{s_0} = \frac{3\eta_0}{4\varepsilon_0} T_0$$

$\Rightarrow$ constant cannot be determined from hydrodynamics!

Dissipative Effects

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

- From conservation equations

$$\varepsilon = 3p = \frac{\varepsilon_0}{\tau^{4/3}} - \frac{2\eta_0}{\tau^2} + \dots$$

$$T = T_0\left(\frac{1}{\tau^{1/3}} - \frac{\eta_0}{2\varepsilon_0\tau} + \dots\right)$$

$$s = \frac{dp}{dT} = s_0\left(\frac{1}{\tau} - \frac{3\eta_0}{2\varepsilon_0}\frac{1}{\tau^{5/3}} + \dots\right)$$

- where

$$\frac{\eta}{s} = \frac{\eta_0}{s_0} = \frac{3\eta_0}{4\varepsilon_0}T_0$$

$\Rightarrow$ constant cannot be determined from hydrodynamics!

AdS_5 Black Hole

Duality

At $T \sim T_C$ $\mathcal{N} = 4$ SYM and QGP become more similar

- For Bjorken hydrodynamics dual description
  $\rightarrow$ introduce time dependence and same symmetries into bulk metric

- AdS_5 Schwarzschild black hole approximate solution for large longitudinal proper time, τ *[Janik and Peschanski]*

$$ds^2 = \frac{1}{\tilde{z}^2}\left(-(1 - \frac{2\mu\tilde{z}^4}{\tau^{4/3}})d\tau^2 + \tau^2 dy^2 + (d\tilde{x}^\perp)^2 + \frac{d\tilde{z}^2}{1 - \frac{2\mu\tilde{z}^4}{\tau^{4/3}}}\right)$$

▶ Dual CFT stress-energy tensor follows Bjorken!

- Holographic Renormalization $\langle T_{\mu\nu}\rangle \sim g^{(4)}_{\mu\nu}$ *[Haro, Skenderis, and Solodukhin]*

AdS_5 Black Hole

Duality

At $T \sim T_C$ $\mathcal{N} = 4$ SYM and QGP become more similar

- For Bjorken hydrodynamics dual description $\rightarrow$ introduce time dependence and same symmetries into bulk metric

- AdS_5 Schwarzschild black hole approximate solution for large longitudinal proper time, τ *[Janik and Peschanski]*

$$ds^2 = \frac{1}{\tilde{z}^2}\left(-(1 - \frac{2\mu\tilde{z}^4}{\tau^{4/3}})d\tau^2 + \tau^2 dy^2 + (d\tilde{x}^\perp)^2 + \frac{d\tilde{z}^2}{1 - \frac{2\mu\tilde{z}^4}{\tau^{4/3}}}\right)$$

▶ Dual CFT stress-energy tensor follows Bjorken!

- Holographic Renormalization $\langle T_{\mu\nu}\rangle \sim g^{(4)}_{\mu\nu}$ *[Haro, Skenderis, and Solodukhin]*

Time Dependence

Thermodynamics

- Temperature and entropy are well understood for a static black hole
- Concepts become murky with time dependence

Several approximate solutions have been found *[Heller, Janik, Sin, Nakamura, Kim, Buchel . . .]*

- higher orders in τ
  - $\eta/s = 1/4\pi$, relaxation time
  - break down at 3rd order
- boosted black branes
  - redefinition of expansion parameter τ
  - in principle good to all orders

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

Time Dependence

Thermodynamics

- Temperature and entropy are well understood for a static black hole
- Concepts become murky with time dependence

Several approximate solutions have been found *[Heller, Janik, Sin, Nakamura, Kim, Buchel . . .]*

- higher orders in τ
  - $\eta/s = 1/4\pi$, relaxation time
  - break down at 3rd order
- boosted black branes
  - redefinition of expansion parameter τ
  - in principle good to all orders

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

Time Dependence

Thermodynamics

- Temperature and entropy are well understood for a static black hole
- Concepts become murky with time dependence

Several approximate solutions have been found *[Heller, Janik, Sin, Nakamura, Kim, Buchel . . .]*

- higher orders in τ
  - $\eta/s = 1/4\pi$, relaxation time
  - break down at 3rd order
- boosted black branes
  - redefinition of expansion parameter τ
  - in principle good to all orders

Static to Dynamic solutions

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

Exact Solution

Time dependent metric is known exactly in 3 dimensions

- equivalent to static Schwarzschild black hole

  [Kajantie, Louko, Tahkokallio]

Gives rise to 2-dim Bjorken hydrodynamics

Temperature, entropy are better understood

$\Rightarrow$ but 3-dim gravity is special

Can this be done in other than 3 dimensions? 5-D?

JA and GS, PRL

Static to Dynamic solutions

Exact Solution

Time dependent metric is known exactly in 3 dimensions

- equivalent to static Schwarzschild black hole

  [Kajantie, Louko, Tahkokallio]

Gives rise to 2-dim Bjorken hydrodynamics

Temperature, entropy are better understood

$\Rightarrow$ but 3-dim gravity is special

Can this be done in other than 3 dimensions? 5-D?

JA and GS, PRL

AdS_5 static black hole

Schwarzschild black hole

- Large AdS_5 Schwarzschild black hole exact solution

$$R_{\mu\nu} - \left(\frac{1}{2}R + \Lambda_5\right) g_{\mu\nu} = 0, \quad \Lambda_5 = -6$$

$$ds^2 = \frac{1}{z^2}\left(-(1-2\mu z^4)dt^2 + d\vec{x}^2 + \frac{dz^2}{1-2\mu z^4}\right)$$

- the horizon occurs at

$$z_H = (2\mu)^{-1/4}, \quad \vec{x} \in \mathcal{R}^3$$

- with temperature

$$T_H = \frac{1}{\pi z_+}$$

AdS_5 static black hole

Schwarzschild black hole

- Large AdS_5 Schwarzschild black hole exact solution

$$R_{\mu\nu} - \left(\frac{1}{2}R + \Lambda_5\right) g_{\mu\nu} = 0, \quad \Lambda_5 = -6$$

$$ds^2 = \frac{1}{z^2}\left(-(1-2\mu z^4)dt^2 + d\vec{x}^2 + \frac{dz^2}{1-2\mu z^4}\right)$$

- the horizon occurs at

$$z_H = (2\mu)^{-1/4}, \quad \vec{x} \in \mathcal{R}^3$$

- with temperature

$$T_H = \frac{1}{\pi z_+}$$

AdS_5 dynamic black hole

AdS_5 boundaries

- Two types of boundaries

$$ds^2_{\text{b.h.}} \rightarrow \frac{1}{z^2}\left(-dt^2 + d\vec{x}^{\,2} + dz^2\right)$$

$$ds^2_{\text{Bjorken}} \rightarrow \frac{1}{\tilde{z}^2}\left(-d\tau^2 + \tau^2 dy^2 + (d\tilde{x}^{\perp})^2 + d\tilde{z}^2\right)$$

- Instead of $z =$ const. hypersurfaces at the boundary, slice with $\tilde{z} =$ const.
  $\Rightarrow$ gives rise to flowing hydrodynamics

AdS_5 dynamic black hole

- For $\tau \to \infty$ with $\tilde{x}^\perp$, τy, and $\frac{\tilde{z}}{\tau^{1/3}}$ fixed

$$t = \frac{3}{2}\tau^{2/3}, \quad x^1 = \tau^{2/3} y, \quad x^\perp = \frac{\tilde{x}^\perp}{\tau^{1/3}}, \quad z = \frac{\tilde{z}}{\tau^{1/3}}$$

Transformed Metric

$$ds^2_{\mathrm{b.h.}} = \frac{1}{\tilde{z}^2}\left[-\left(1 - \frac{2\mu\tilde{z}^4}{\tau^{4/3}}\right)d\tau^2 + \frac{d\tilde{z}^2}{1 - \frac{2\mu\tilde{z}^4}{\tau^{4/3}}} + \tau^2 dy^2 + (d\tilde{x}^\perp)^2\right] + \mathcal{O}(\tau^{-4/3})$$

$\Rightarrow$ JP Metric

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

AdS_5 dynamic black hole

- For $\tau \to \infty$ with $\tilde{x}^\perp$, τy, and $\frac{\tilde{z}}{\tau^{1/3}}$ fixed

$$t = \frac{3}{2}\tau^{2/3}, \quad x^1 = \tau^{2/3} y, \quad x^\perp = \frac{\tilde{x}^\perp}{\tau^{1/3}}, \quad z = \frac{\tilde{z}}{\tau^{1/3}}$$

Transformed Metric

$$ds^2_{\text{b.h.}} = \frac{1}{\tilde{z}^2}\left[-\left(1 - \frac{2\mu\tilde{z}^4}{\tau^{4/3}}\right)d\tau^2 + \frac{d\tilde{z}^2}{1 - \frac{2\mu\tilde{z}^4}{\tau^{4/3}}} + \tau^2 dy^2 + (d\tilde{x}^\perp)^2\right] + \mathcal{O}(\tau^{-4/3})$$

$\Rightarrow$ JP Metric

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT

Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence

Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order

Summary

Boundary Theory

Thermodynamics

- Temperature may be calculated at boundaries

$$ds^2_{z\to 0} = \tau^{-2/3}\left[ds^2_{\tilde{z}\to 0} + \mathcal{O}\left(\frac{\tilde{x}^i\tilde{x}^j}{\tau^2}\right)\right]$$

- Temperature is proportional to conformal factor$^{1/2}$

$$T = \frac{T_H}{\tau^{1/3}}$$

- the entropy density is found by

$$s = \frac{dp}{dT} = \frac{s_0}{\tau}$$

▶ Bjorken Flow from exact solution of Einstein equations!

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

Boundary Theory

Thermodynamics

- Temperature may be calculated at boundaries

$$ds^2_{z\to 0} = \tau^{-2/3}\left[ds^2_{\tilde{z}\to 0} + \mathcal{O}\left(\frac{\tilde{x}^i\tilde{x}^j}{\tau^2}\right)\right]$$

- Temperature is proportional to conformal factor$^{1/2}$

$$T = \frac{T_H}{\tau^{1/3}}$$

- the entropy density is found by

$$s = \frac{dp}{dT} = \frac{s_0}{\tau}$$

▶ Bjorken Flow from exact solution of Einstein equations!

Next-to-leading order

Can viscosity be understood from a Schwarzschild BH?

JA and GS, to be published

Higher orders

- Respect boost and transverse coordinates invariance
- Introduce terms $\mathcal{O}(1/\tau)$
  - systematically done with Mathematica

$$t = \frac{3}{2}\tau^{2/3} - \mathcal{C}_1 \ln\tau + \frac{f_1(v)}{\tau^{2/3}}\,, \quad z = \tilde{z}\left(\frac{1}{\tau^{1/3}} - \frac{\mathcal{C}_1}{\tau}\right)$$

$$x^1 = \tau y\left(\frac{1}{\tau^{1/3}} - \frac{\mathcal{C}_1 + b_1(v)}{\tau}\right), x^\perp = \tilde{x}^\perp\left(\frac{1}{\tau^{1/3}} - \frac{\mathcal{C}_1 + c_1(v)}{\tau}\right)$$

- with $v = \tilde{z}/\tau^{1/3}$ kept fixed and $\mathcal{C}_1,\ b_1(v),\ c_1(v)$ to be determined

Next-to-leading order

Dual conformal field theory

- $b_1(v),\ c_1(v)$ found with next-to-leading order Einstein equations
- Stress-energy tensor and thermo can then be calculated

▶ viscous Bjorken hydrodynamics with

$$\eta_0 = 2\mathcal{C}_1\varepsilon_0 \longrightarrow \eta/s = \frac{3\mathcal{C}_1}{2\pi}(2\mu)^{1/3}$$

No constraint on η/s

$\Rightarrow$ go to next-to-next-to-leading order

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

Next-to-next-to-leading order

Transformation

Alter the transformation to include next-to-next-to-leading order, $\mathcal{O}(\tau^{4/3})$

- introduce $a_2(v), b_2(v), c_2(v), f_2(v)$ and $\mathcal{C}_2$

y and $x^\perp$ dependence is unavoidable

$\Rightarrow$ Metric must be perturbed to produce Bjorken flow

$$ds^2_{\text{perturbed}} = ds^2_{\text{b.h.}} - \frac{1}{\tilde{z}^2}\left[\frac{v^2\mathcal{A}(v)}{\tau^{4/3}}d\tilde{z}^2 + 2\mathcal{A}_\mu d\tilde{x}^\mu d\tilde{z}\right]$$

$\mathcal{A}(v)$ is a gauge freedom and $\mathcal{A}_\mu$ kills $y, x^\perp$ dependence

Next-to-next-to-leading order

Transformation

Alter the transformation to include next-to-next-to-leading order, $\mathcal{O}(\tau^{4/3})$

- introduce $a_2(v), b_2(v), c_2(v), f_2(v)$ and $\mathcal{C}_2$

y and $x^\perp$ dependence is unavoidable
$\Rightarrow$ Metric must be perturbed to produce Bjorken flow

$$ds^2_{\text{perturbed}} = ds^2_{\text{b.h.}} - \frac{1}{\tilde{z}^2}\left[\frac{v^2\mathcal{A}(v)}{\tau^{4/3}}d\tilde{z}^2 + 2\mathcal{A}_\mu d\tilde{x}^\mu d\tilde{z}\right]$$

$\mathcal{A}(v)$ is a gauge freedom and $\mathcal{A}_\mu$ kills $y, x^\perp$ dependence

Next-to-next-to-leading order

Solution

- The Einstein equations allow for the solutions to $b_2(v), c_2(v), f_2(v)$
  - $a_2(v)$ remains arbitrary due to $\mathcal{A}(v)$
- The solution has a divergent curvature invariant $\mathcal{R}^2 = R_{ABCD}R^{ABCD}$ at the horizon

$\Rightarrow$ Constraint on $\mathcal{C}_1$

Nonsingular for only

$$\mathcal{C}_1 = \frac{1}{6(2\mu)^{1/4}} \quad \Rightarrow \quad \frac{\eta}{s} = \frac{1}{4\pi}$$

- Equivalent to AdS perturbations and subleading approximate solutions

Next-to-next-to-leading order

Solution

- The Einstein equations allow for the solutions to $b_2(v), c_2(v), f_2(v)$
  - $a_2(v)$ remains arbitrary due to $\mathcal{A}(v)$
- The solution has a divergent curvature invariant $\mathcal{R}^2 = R_{ABCD}R^{ABCD}$ at the horizon

$\Rightarrow$ Constraint on $\mathcal{C}_1$

Nonsingular for only

$$\mathcal{C}_1 = \frac{1}{6(2\mu)^{1/4}} \quad \Rightarrow \quad \frac{\eta}{s} = \frac{1}{4\pi}$$

- Equivalent to AdS perturbations and subleading approximate solutions

Next-to-next-to-leading order

Solution

- The Einstein equations allow for the solutions to $b_2(v), c_2(v), f_2(v)$
  - $a_2(v)$ remains arbitrary due to $\mathcal{A}(v)$
- The solution has a divergent curvature invariant $\mathcal{R}^2 = R_{ABCD}R^{ABCD}$ at the horizon

$\Rightarrow$ Constraint on $\mathcal{C}_1$

Nonsingular for only

$$\mathcal{C}_1 = \frac{1}{6(2\mu)^{1/4}} \qquad \Rightarrow \qquad \frac{\eta}{s} = \frac{1}{4\pi}$$

- Equivalent to AdS perturbations and subleading approximate solutions

Introduction
RHIC
Bjorken Hydrodynamics
AdS/CFT
Hydrodynamics from AdS/CFT
AdS Black Hole
Time Dependence
Transformation
Static Black Hole
Dynamic Black Hole
CFT Plasma
Next-to-leading order
Next-to-next-to-leading order
Summary

Summary

RHIC

- Connections are being made for string theory and the experimental results of RHIC

- Viscous Bjorken hydrodynamics found by slicing near the boundary of a large AdS_d Schwarzschild black hole
  - Exact solution to be used to study the plasma
  - Has been generalized to d-dim at NLO

- Future
  - Go beyond perturbative analysis for more complex RHIC phenomenon
  - Account for deviations from boost invariance