A_5 Family Symmetry and the Golden Ratio Prediction for Solar Neutrino Mixing

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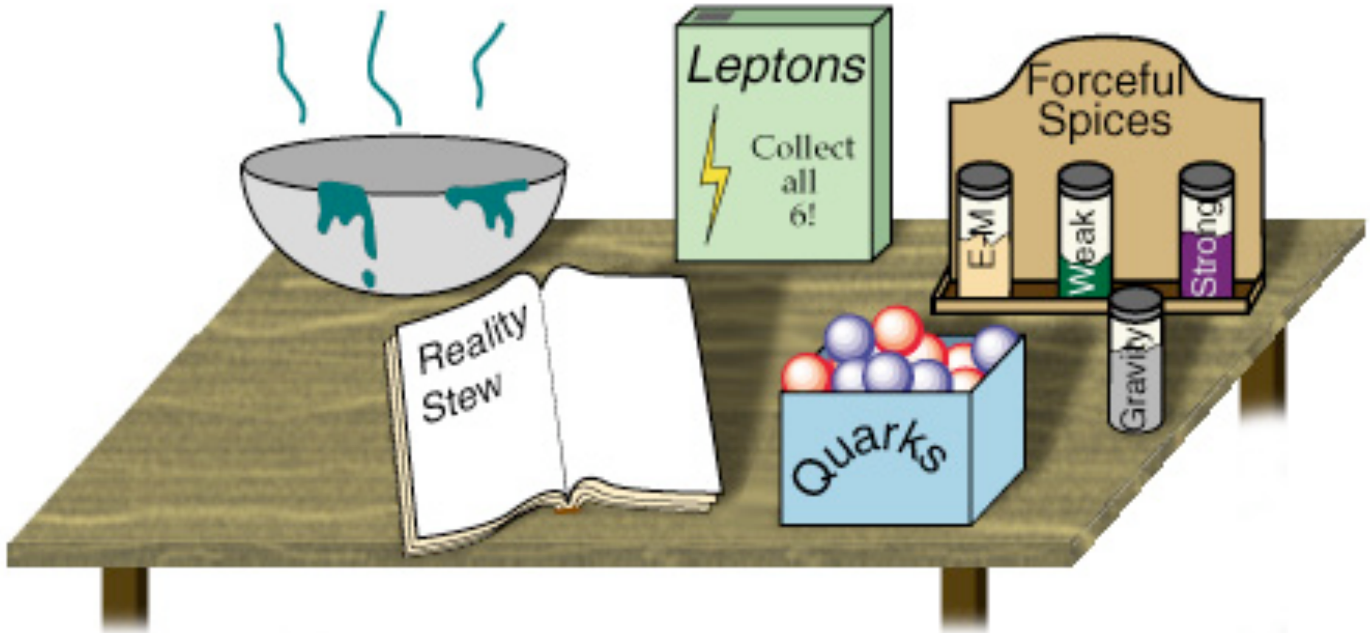
Based on: L. Everett and A. Stuart, 0812.1057 [hep-ph], to appear in PRD

The Standard Model

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

•Triumph of modern science, but incomplete. Predicts massless neutrinos.

•How can we mix neutrinos into our recipe?



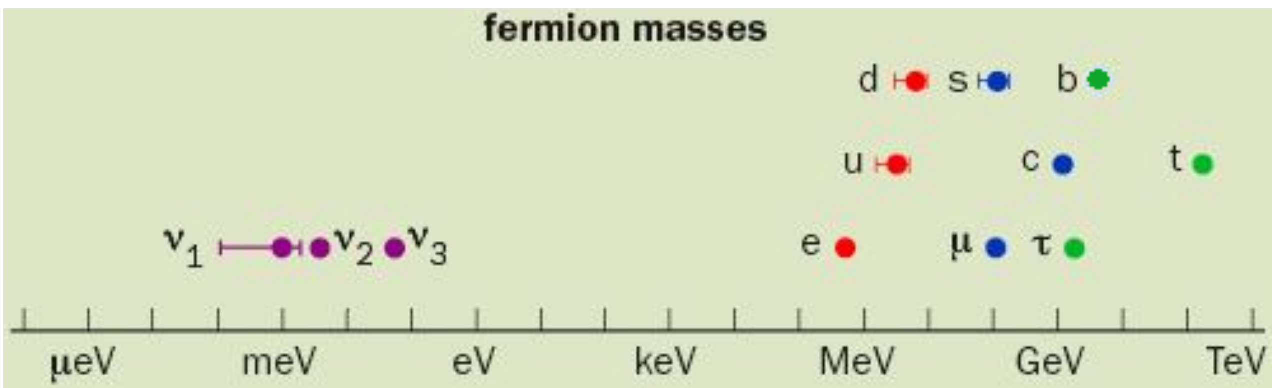
Figuring Out the Ingredients

$$\theta_{12} = 34.5^{+4.8}_{-4.0}, \quad \theta_{23} = 42.3^{+11.3}_{-7.7}, \quad \theta_{13} = 0.0^{+12.9}_{-0.0}$$

$$\Delta m^2_{ij} \equiv m^2_i - m^2_j$$

$$\Delta m^2_{12} = 7.67^{+0.67}_{-0.61} \times 10^{-5}\,\mathrm{eV}^2$$

$$\Delta m^2_{13} = \begin{cases} -2.37^{+0.43}_{-0.46} \times 10^{-3}\,\mathrm{eV}^2 \text{ (inverted hierarchy)} \\ 2.46^{+0.47}_{-0.42} \times 10^{-3}\,\mathrm{eV}^2 \text{ (normal hierarchy).} \end{cases}$$



Generating Small Neutrino Masses

- Dirac neutrinos: add right handed neutrino to SM:

$$m_D \nu_L \overline{N}_R \qquad m_{ij} = Y_{ij} < H >$$

but very small Yukawa coupling (~ 10^{-12})

- Majorana neutrinos: seesaw mechanism

(Minkowski; Gell-Mann, Ramond, Slansky; Yanagida)

Effective Majorana neutrino mass terms (LL couplings)

$$(m_D M_{maj}^{-1} m_D^T)_{ij} L_i L_j$$

Flavor Symmetry

- Postulate a flavor symmetry (continuous or discrete) to explain mixings and masses.
  - Use symmetry to forbid mass term at renormalized level and generate mass at higher level through SSB of a flavon field.

$$T \cong A_4 \qquad\qquad I \cong A_5$$

$$\frac{1}{\sqrt{2}} \qquad\qquad A_4 \subseteq A_5 \qquad\qquad \phi = \frac{1+\sqrt{5}}{2}$$

$$ArcTan\left(\frac{1}{\sqrt{2}}\right) = 35.2644^\circ \qquad\qquad ArcTan\left(\frac{1}{\phi}\right) = 31.7175^\circ$$

Y. Kajiyama, M. Raidal, and A. Strumia ($Z_2 \otimes Z_2$);
L.L. Everett and A. J. Stuart (A_5);

A. Adulpravitchai, A. Blum, W. Rodejohann (D_{10})

Icosahedral Group is not crystallographic point group so there was work to be done.

The Icosahedral Group, I

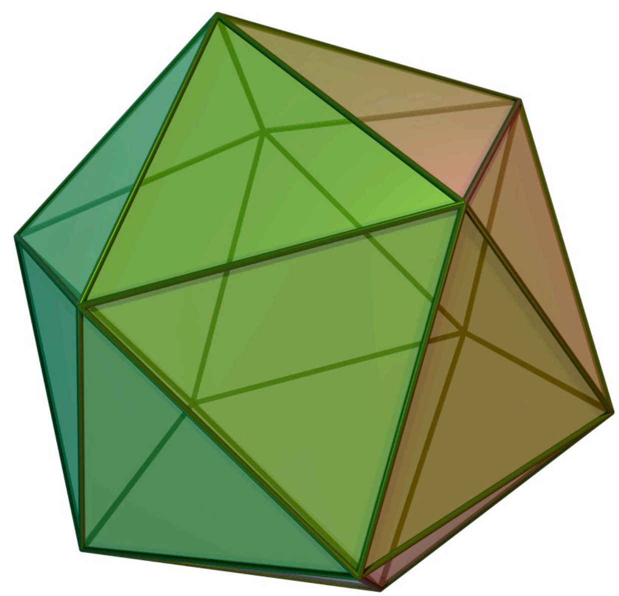
- An icosahedron is the Platonic solid that consists of 20 equilateral triangles. → f = 20
- 20 triangles each have 3 sides → 60 edges but 2 triangles/edge → 30 edges → e=30 I
- 20 triangles each have 3 vertices → 60 vertices but 5 vertices/edge → v= 12
- Are we right? $\chi(g) = 2 - 2g = v - e + f$

- Icosahedral group consists of all rotations

that take vertices to vertices on an icosahedron.

(i.e. $0, \pi, \frac{2\pi}{3}, \frac{2\pi}{5}, \frac{4\pi}{5}$)

$$I \cong A_5$$



http://upload.wikimedia.org/wikipedia/commons/e/eb/Icosahedron.jpg

Conjugacy Classes of I

Rotation by each angle forms its own conjugacy class.

Schoenflies Notation: C_n^k is a rotation by $\dfrac{2\pi k}{n}$

in front = # of elements in class

So for the Icosahedral group we have:

$$I, 12C_5, 12C_5^2, 20C_3, 15C_2$$

Note: $1+12+12+15+20=60=1^2+3^2+3^2+4^2+5^2$ **Two triplets.**

What do we do with this information?

The Icosahedral Character Table

- A character table is a table that gives the characters of the group elements as a function of the conjugacy class.

$\mathcal{I}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3'}$	$\mathbf{4}$	$\mathbf{5}$
e	1	3	3	4	5
$12C_5$	1	ϕ	$1-\phi$	-1	0
$12C_5^2$	1	$1-\phi$	ϕ	-1	0
$20C_3$	1	0	0	1	-1
$15C_2$	1	-1	-1	0	1

Kronecker Products of I

Use Character Table to easily obtain Kronecker Products (known)

All of this is abstract. We need actual representations.

Our work: Identified useful group presentation where golden ratio is manifest:

K. Shirai, J. Phys. Soc. Jpn. **61** 2735 (1992).

Explicitly constructed C-G Series in Shirai basis.

$$3 \otimes 3 = 1 \oplus 3 \oplus 5$$
$$3' \otimes 3' = 1 \oplus 3' \oplus 5$$
$$3 \otimes 3' = 4 \oplus 5$$
$$3 \otimes 4 = 3' \oplus 4 \oplus 5$$
$$3' \otimes 4 = 3 \oplus 4 \oplus 5$$
$$3 \otimes 5 = 3 \oplus 3' \oplus 4 \oplus 5$$
$$3' \otimes 5 = 3 \oplus 3' \oplus 4 \oplus 5$$
$$4 \otimes 4 = 1 \oplus 3 \oplus 3' \oplus 4 \oplus 5$$
$$4 \otimes 5 = 3 \oplus 3' \oplus 4 \oplus 5 \oplus 5$$
$$5 \otimes 5 = 1 \oplus 3 \oplus 3' \oplus 4 \oplus 4 \oplus 5 \oplus 5$$

Presentation of Icosahedral Group

- Presentation is rules along with elements that together generate the group.
- Tetrahedral group only has one presentation.
- Icosahedral group(A_5) has many. We'll focus on one:

$$< a, b | a^2 = b^3 = (ab)^5 = e >$$

Hamilton

$$S = a \qquad\qquad T = bab$$

$$< S, T | S^2 = T^5 = (T^2 S T^3 S T^{-1} S T S T^{-1})^3 = e >$$

Kramer and Haase, Shirai, Hoyle

Explicit Generators

(From Shirai)

$$S_3 = \frac{1}{2}\begin{pmatrix} -1 & \phi & \frac{1}{\phi} \\ \phi & \frac{1}{\phi} & 1 \\ \frac{1}{\phi} & 1 & -\phi \end{pmatrix} \qquad T_3 = \frac{1}{2}\begin{pmatrix} 1 & \phi & \frac{1}{\phi} \\ -\phi & \frac{1}{\phi} & 1 \\ \frac{1}{\phi} & -1 & \phi \end{pmatrix}$$

$$S_{3'} = \frac{1}{2}\begin{pmatrix} -\phi & \frac{1}{\phi} & 1 \\ \frac{1}{\phi} & -1 & \phi \\ 1 & \phi & \frac{1}{\phi} \end{pmatrix} \qquad T_{3'} = \frac{1}{2}\begin{pmatrix} -\phi & -\frac{1}{\phi} & 1 \\ \frac{1}{\phi} & 1 & \phi \\ -1 & \phi & -\frac{1}{\phi} \end{pmatrix}$$

Tensor Product Decomposition

$$3 \otimes 3$$

$$3 = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$3 \otimes 3'$$

$$1 = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$3 = (a_3 b_2 - b_2 a_3, a_1 b_3 - a_3 b_1, a_2 b_1 - a_1 b_2)^T$$

$$5 = \begin{pmatrix} a_2 b_2 - a_1 b_1 \\ a_2 b_1 + a_1 b_2 \\ a_3 b_2 + a_2 b_3 \\ a_1 b_3 + a_3 b_1 \\ -\frac{1}{\sqrt{3}}(a_1 b_1 + a_2 b_2 - 2 a_3 b_3) \end{pmatrix}$$

$$4 = \begin{pmatrix} \frac{1}{\phi} a_3 b_2 - \phi a_1 b_3 \\ \phi a_3 b_1 + \frac{1}{\phi} a_2 b_3 \\ -\frac{1}{\phi} a_1 b_1 + \phi a_2 b_2 \\ a_2 b_1 - a_1 b_2 + a_3 b_3 \end{pmatrix}$$

$$5 = \begin{pmatrix} \frac{1}{2}(\phi^2 a_2 b_1 + \frac{1}{\phi^2} a_1 b_2 - \sqrt{5} a_3 b_3) \\ -(\phi a_1 b_1 + \frac{1}{\phi} a_2 b_2) \\ \frac{1}{\phi} a_3 b_1 - \phi a_2 b_3 \\ \phi a_3 b_2 + \frac{1}{\phi} a_1 b_3 \\ \frac{\sqrt{3}}{2}(\frac{1}{\phi} a_2 b_1 + \phi a_1 b_2 + a_3 b_3) \end{pmatrix}$$

How to Build an A_5 Flavor Model

Seek scenario with: maximal θ_{23}, zero θ_{13}, and "golden" θ_{12}.

Our approach: atmospheric mixing from the charged leptons, solar mixing from neutrinos

Choose to assign irreps to SM fields: L→3, $\bar{e}$→3'

Tree level: L $\bar{e}$ (3 x 3'): vanish
LL (3 x 3): degenerate

$$3 \otimes 3' = 4 \oplus 5 \quad 3 \otimes 3 = 1 \oplus 3 \oplus 5$$

How to Build an A_5 Flavor Model

Flavon sector can dramatically alter tree-level pattern:
Here, take flavon fields as $\xi, \chi, \text{and}\, \psi$

Charge assignment: $(L, \bar{e}, \xi, \chi, \psi) \sim (3, 3', 5, 4, 5')$

The invariant effective Lagrangian is:

$$-\mathcal{L}_{mass} = \frac{\alpha_{ijk}}{MM'} L_i H L_j H \xi_k + \frac{\beta_{ijk}}{M'} L_i \bar{e}_j H \psi_k + \frac{\gamma_{ijl}}{M'} L_i \bar{e}_j H \chi_l + \text{h.c.}$$

Spontaneous Symmetry Breaking

- **Assume flavon field vevs:**

$$\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \end{pmatrix} \longrightarrow \frac{\sqrt{3}}{2\alpha} \begin{pmatrix} \frac{1}{\sqrt{15}}(m_2 - m_1) \\ \frac{2}{\sqrt{15}}(m_2 - m_1) \\ 0 \\ 0 \\ -(m_1 + m_2) \end{pmatrix} \qquad \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \end{pmatrix} \longrightarrow \frac{1}{2\sqrt{6}\beta} \begin{pmatrix} -\sqrt{\frac{5}{3}} m_\tau \\ \frac{2}{\sqrt{3}}(-\phi\sqrt{2} m_e - \frac{1}{\phi} m_\mu) \\ -\frac{2}{\sqrt{3}}\phi m_\tau \\ -\frac{2}{\sqrt{3}}\phi m_\mu \\ m_\tau \end{pmatrix}$$

$$\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{pmatrix} \longrightarrow \frac{1}{3\sqrt{2}\gamma} \begin{pmatrix} -\frac{1}{\phi} m_\mu \\ \frac{1}{\phi} m_\tau \\ -\frac{\sqrt{2}}{\phi} m_e + \phi m_\mu \\ m_\tau \end{pmatrix}$$

Mass Matrices

- After spontaneous symmetry breaking, the mass matrices are:

Neutrinos: $$M_\nu = \frac{1}{\sqrt{5}} \begin{pmatrix} \phi m_1 + \frac{1}{\phi} m_2 & m_2 - m_1 & 0 \\ m_2 - m_1 & \frac{1}{\phi} m_1 + \phi m_2 & 0 \\ 0 & 0 & -\sqrt{5}(m_1 + m_2) \end{pmatrix}$$

Charged leptons: $$M_e = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} m_e & 0 & 0 \\ 0 & m_\mu & m_\tau \\ 0 & -m_\mu & m_\tau \end{pmatrix}$$

The MNSP Matrix

- By construction, we have:

$$U = U_{\mathrm{MNSP}} = U_e U_\nu^\dagger = \begin{pmatrix} \sqrt{\frac{\phi}{\sqrt{5}}} & \sqrt{\frac{1}{\sqrt{5}\phi}} & 0 \\ -\frac{1}{\sqrt{2}}\sqrt{\frac{1}{\sqrt{5}\phi}} & \frac{1}{\sqrt{2}}\sqrt{\frac{\phi}{\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\sqrt{\frac{1}{\sqrt{5}\phi}} & \frac{1}{\sqrt{2}}\sqrt{\frac{\phi}{\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathcal{P}$$

$$\mathcal{P} = \mathrm{Diag}(1, 1, i)$$

Features of the Model

- By design, we have: $\theta_{13} = 0^\circ$, $\theta_{23} = 45^\circ$,

$$\theta_{12} = ArcTan\left(\frac{1}{\phi}\right) = 31.7175^\circ$$

Predicts a normal hierarchy with: $m_3 = m_1 + m_2$

0νββ decay:

$$m_{\beta\beta} = \frac{m_1\phi}{\sqrt{5}} + \frac{m_2}{\phi\sqrt{5}}$$

- These results are well within the measurements. The solar angle is 2σ below best fit.

Outlook and Conclusion

- Summary:
  - Icosahedral (A_5) symmetry provides a rich setting for investigating the flavor puzzle.
  - "Golden Mean" models: intriguing alternative to tri-bimaximal mixing scenarios. Virtually unexplored!
- Where to next?
  - Analyze the flavon sector dynamics (in progress).
  - Study other lepton mixing scenarios, investigate quark sector, SUSY embeddings,…

For a more detailed discussion, arXiv: 0812.1057