New limit of pion form factor at very large Q^2

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Pion form factor is a very important quantity in hadron physics.

In 1973 the quark counting rule predicts

$$F_\pi(Q^2) \sim \frac{1}{Q^2}.$$

Perturbative QCD predicts that dominance of one gluon exchange at large Q^2

$$F_\pi(Q^2)|_{Q^2\to\infty} = 4\pi\alpha_s(Q^2) f_\pi^2/Q^2,$$

The issue in the study of $F_\pi(Q^2)$ is that the pion wave function or the distribution amplitude is a quantity of nonperturbative QCD. Other different pion distribution amplitudes are discussed in the light-cone formalism too. For example,

$$\{x(1-x)\}^{\frac{1}{2}}$$

this new function increases the the value of $F_\pi(Q^2)$ at $Q^2 \to \infty$ by $\frac{16}{9}$ relative to the prediction obtained by x(1-x). Different distribution amplitude leads to different asymptotic value of $F_\pi(Q^2)$ at $Q^2 \to \infty$. The determination of pion wave function or distribution amplitude is still a open question. In this talk another BS wave function of pion is introduced to calculate $F_\pi(Q^2)$ at large Q^2.

According to Mandelstam's representation, the current matrix element of pion is written as

$$< \pi^+|j_\mu(0)|\pi^+ >= \int d^4k_2 \int d^4k_1 Tr\{\phi_\pi(k_1,p_f)T_H(k_1,k_2,p_f,p_i)_\mu \phi_\pi(k_2,p_i)\}$$

$$= F_\pi(Q^2)P_\mu.$$

The same pion wave function should determine $F_\pi(Q^2)$ at both low Q^2 and high Q^2.

Current algebra is successful in studying hadron physics at lower energies, in which chiral symmetry plays essential role. Based on chiral symmetry and current algebra an effective chiral Lagrangian of pseudoscalar, vector, and axial-vector mesons is constructed[7] as

$$\mathcal{L} = \bar{\psi}(x)(i\gamma\cdot\partial + \gamma\cdot v + \gamma\cdot a\gamma_5 - mu(x))\psi(x) - \bar{\psi(x)}M\psi(x)$$

$$+\frac{1}{2}m_0^2(\rho_i^\mu\rho_{\mu i} + \omega^\mu\omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu)$$

where $a_\mu = \tau_i a_\mu^i + f_\mu$, $v_\mu = \tau_i\rho_\mu^i + \omega_\mu$, $u = exp\{i\gamma_5(\tau_i\pi_i + \eta)\}$, and M is the matrix of current quark masses. m is the constituent quark mass and it originates in quark condensation. Therefore, this theory has dynamical chiral symmetry breaking. Integrating out the quark fields, the Lagrangian of mesons is obtained. The tree diagrams of mesons are at leading order in N_C expansion and loop diagrams of mesons are at higher order. In the limit, $m_q \to 0$, explicit chiral symmetry is recovered. It is known that chiral symmetry, quark condensation and N_C expansion are from nonperturbative QCD. Meson physics at lower energies has been extensively studied by this Lagrangian. Theory agrees with data very well.

The pion decay constant is defined

$$f_\pi^2 = F^2(1 - \frac{2c}{g}),$$

$$\frac{F^2}{16} = \frac{N_C}{(4\pi)^2} \int d^4k \frac{m^2}{(k^2+m^2)^2},$$

c is determined to be $c = \frac{f_\pi^2}{2gm_\rho^2}$,

$$g^2 = \frac{2}{3}\frac{N_C}{(4\pi)^4} \int d^4k \frac{1}{(k^2+m^2)^2} = \frac{1}{6}\frac{F^2}{m^2}.$$

f_π and g are the two parameters of this theory. $g = 0.39$ is determined from the decay rate of $\rho \rightarrow ee^+$. The cut-off Λ of the integrals is determined to be $\Lambda = 1.8GeV$.

The pion form factor is derived up to the fourth order in covariant derivatives

$$|F_\pi(q^2)|^2 = f_{\rho\pi\pi}^2(q^2)\frac{m_\rho^4 + q^2\Gamma_\rho^2(q^2)}{(q^2-m_\rho^2)^2 + q^2\Gamma_\rho^2(q^2)},$$

$$f_{\rho\pi\pi}(q^2) = 1 + \frac{q^2}{2\pi^2 f_\pi^2}[(1-\frac{2c}{g})^2 - 4\pi^2c^2],$$

$$\Gamma_\rho(q^2) = \frac{f_{\rho\pi\pi}^2(q^2)}{12\pi g^2}(1-\frac{4m_\pi^2}{q^2})^{\frac{3}{2}}.$$

In these equations there is no new adjustable parameter. At $q^2 = m_\rho^2$ $\Gamma_\rho = 150MeV$ which is consistent with data. $F_\pi(q^2)$ consists of two parts: the ρ

pole and an intrinsic form factor $f_{\rho\pi\pi}(q^2)$ which is obtained from quark loop. The intrinsic form factor is a new result of this theory. The ρ-pole form factor of pion has shortcomings: in space-like region it decreases too slow and in time-like region it decreases too fast. The intrinsic form factor redeems these two problems. The comparison between the theory and the data is shown The radius of charged pion is obtained

$$< r^2 >_\pi = \frac{6}{m_\rho^2} + \frac{3}{\pi^2 f_\pi^2}\{(1 - \frac{2c}{g})^2 - 4\pi^2 c^2\} = 0.452 fm^2. \quad (1)$$

The experimental data is $(0.439 \pm 0.03) fm^2$. The contribution of the ρ pole to $< r^2 >_\pi$ is $0.395 fm^2$.

In the leading order in N_C expansion(ignore the loop diagrams of mesons) a pion is made of constituent quark pair with ρ meson cloud. The intrinsic form factor is obtained from the quark pair component and the $\rho - pole$ of $F_\pi(q^2)$ is from the ρ meson cloud.

The BS wave function of pion can be defined in this theory. The pion fields have two sources: $u = exp\{i\gamma_5\pi\} = 1 + i\gamma_5\pi + ...$ and the shifting $a_\mu \rightarrow$

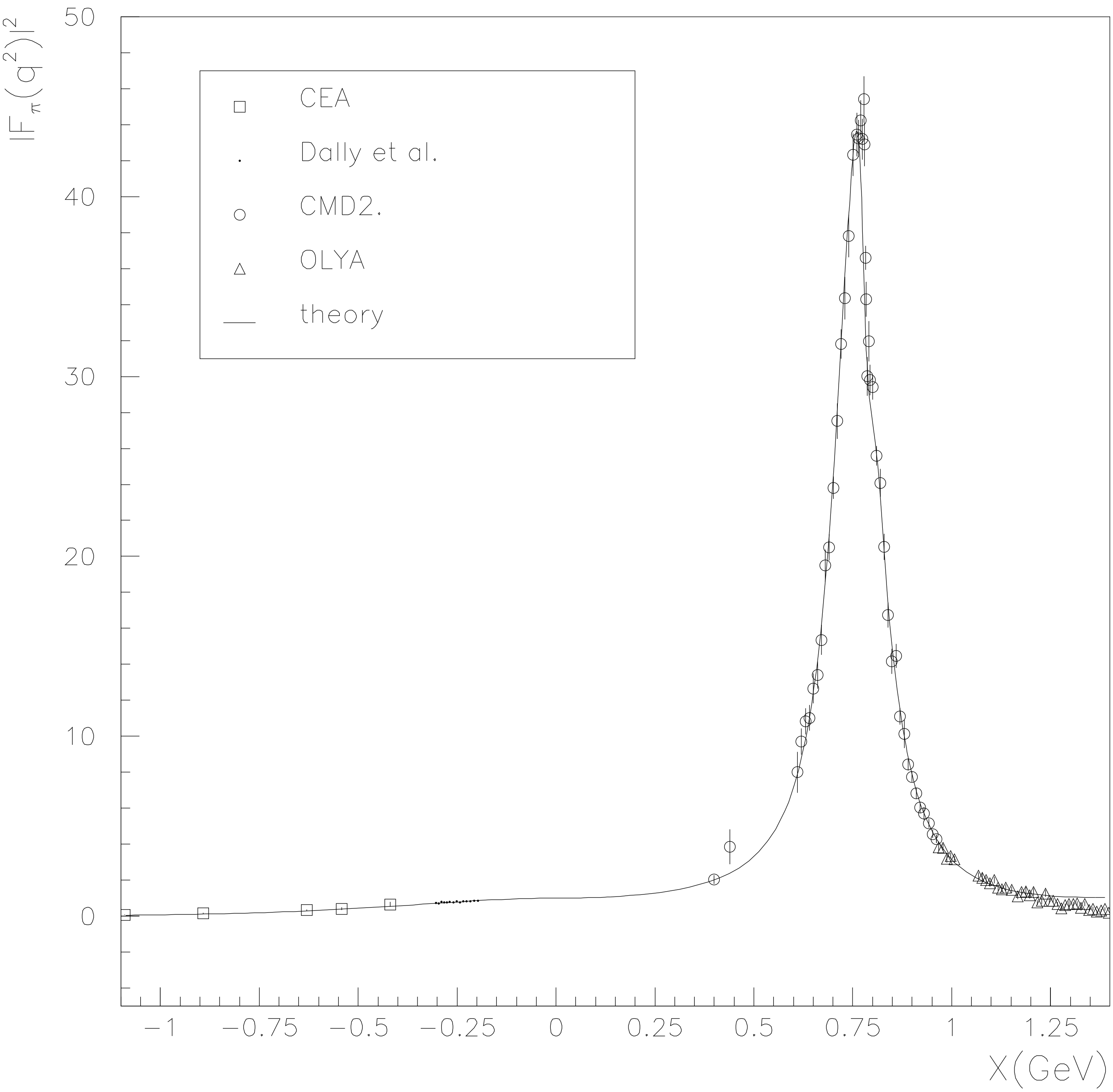

FIG. .

$a_\mu(physical) - \frac{c}{g}\partial_\mu\pi$, which is caused by the mixing term $a^i_\mu\partial_\mu\pi^i$ obtained from quark loop diagram. Combining these two sources together, the vertex related to pion is obtained as

$$\mathcal{L}_\pi = -\frac{2im}{f_\pi}\bar{\psi}\tau_i\gamma_5(1 + i\frac{c}{g}\frac{\gamma\cdot\partial}{m})\psi\pi_i,$$

The wave function of pion is derived as

$$\phi(z,p) =< 0|\{\psi(\frac{z}{2})\bar{\psi}(-\frac{z}{2})\}|\pi(p) >$$

$$= \frac{2\sqrt{2}m}{f_\pi}\frac{1}{(2\pi)^4}\int d^4k \frac{e^{-ikz}}{(k^2-m^2)((k-p)^2-m^2)}(\gamma\cdot k + m)\gamma_5$$
$$(1 + \frac{c}{g}\frac{\gamma\cdot p}{m})(\gamma\cdot k - \gamma\cdot p + m).$$

f_π can be derived by the pion wave function too

$$Tr\phi(0,p)\gamma_\mu\gamma_5 = \frac{i}{\sqrt{2}}f_\pi p_\mu.$$

The pion mass has been derived as

$$m^2_\pi = -\frac{4}{f^2_\pi} < \bar{\psi}\psi > (m_u + m_d).$$

In the chiral limit, $m_q \to 0$, $m^2_\pi \to 0$. Goldstone theorem is satisfied. The three form factors of $\pi^- \rightarrow e\gamma\nu$, $\pi-\pi$ scattering, $\pi^0 \rightarrow \gamma\gamma$, and many other processes in which pion is involved are successfully studied by this theory.

The pion wave function determined by this theory is successful in studying pion physics at lower energies. Now this wave function is used to study pion form factor at large Q^2. At large Q^2 perturbative QCD is working. Using the pion wave function and T_H with one gluon exchange the matrix element of current of pion is written as

$$<\pi^+|j_\mu|\pi^+>=\frac{2m^2}{f_\pi^2}Tr\lambda^a\lambda^a g_s^2\int d^4k_1d^4k_2$$

$$\{\frac{1}{(k_1-k_2+p_i-p_f)^2}\frac{1}{(k_1+p_i-p_f)^2}Tr\gamma_\nu\phi_\pi(k_1,p_f)\gamma_\mu\gamma\cdot(k_1+p_i-p_2)\gamma_\nu\phi_\pi(k_2,p_i)$$
$$+\frac{1}{(k_1-k_2+p_i-p_f)^2}\frac{1}{(k_2+p_f-p_i)^2}Tr\gamma_\nu\phi_\pi(k_1,p_f)\gamma_\nu\gamma\cdot(k_2+p_f-p_i)\gamma_\mu\phi_\pi(k_2,p_i)\},$$

where $\phi_\pi(k,p)$ is the pion wave function in momentum space, k is the internal momentum and p is the momentum of the pion,

$$\phi_\pi(k_1,p_f)=\frac{1}{(2\pi)^4}\frac{1}{(k_1^2-m^2)[(k_1-p_f)^2-m^2]}[\gamma\cdot(k_1-p_f)+m]$$

$$\gamma_5(1-\frac{c}{g}\frac{\gamma\cdot p_f}{m})(\gamma\cdot k_1+m),$$

$$\phi_\pi(k_2,p_i) = \frac{1}{(2\pi)^4}\frac{1}{(k_1^2-m^2)[(k_2-p_i)^2-m^2]}(\gamma\cdot k_2+m)\gamma_5$$

$$(1+\frac{c}{g}\frac{\gamma\cdot p_i}{m})[\gamma\cdot(k_2-p_i)+m].$$

For $Q^2 >> \Lambda^2((1.8GeV)^2)$

$$<\pi^+|j_\mu|\pi^+>= \frac{2m^2}{f_\pi^2}Tr\lambda^a\lambda^a g_s^2\frac{1}{Q^4}\int d^4k_1 d^4k_2$$

$$\{Tr\gamma_\nu\phi_\pi(k_1,p_f)\gamma_\mu\gamma\cdot(k_1+p_i-p_2)\gamma_\nu\phi_\pi(k_2,p_i)$$

$$+Tr\gamma_\nu\phi_\pi(k_1,p_f)\gamma_\nu\gamma\cdot(k_2+p_f-p_i)\gamma_\mu\phi_\pi(k_2,p_i)\}$$

and the pion form factor at $Q^2 >> (1.8GeV)^2$ is obtained

$$F_\pi(Q^2) = 4\pi\alpha_s(Q^2)f_\pi^2\frac{1}{Q^2}\frac{1}{18}(1-\frac{2c}{g})^{-2}$$

$$\{\frac{2c^2}{g^2}+(1-\frac{c}{g})(1-\frac{4c}{g})-\frac{1}{4\pi^2g^2}(1-\frac{c}{g})(1-\frac{2c}{g})\}.$$

The numerical result is

$$F_\pi(Q^2) = 2.65\times10^{-2}4\pi\alpha_s(Q^2)f_\pi^2\frac{1}{Q^2}.$$

It is interesting to mention that at high Q^2 the ρ-pole with one gluon exchange behaves like $\frac{1}{Q^4}$. Therefore, at high Q^2 the contribution of ρ-pole can be ignored.

The numerical value of the coefficient of the new limit of the pion form factor is much smaller than previous one. It can be tested by experimental measurements of the $F_\pi(Q^2)$ at very large Q^2.