
PRESYMMETRY BEYOND THE STANDARD MODEL

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Outline

- **Quark-lepton charge symmetry**
- **Statement of principles**
- **Prequarks, preleptons, presymmetry**
- **Cancellation of gauge anomalies**
- **Presymmetry beyond the SM**
- **Conclusions**

Presymmetry*

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Even in the presence of external fields, space-time symmetry implies nontrivial relations between observables at one time, i.e., kinematical relations. Symmetry operations at one time—translations, rotations, and (for Galilei symmetry) velocity shifts—can be performed on observation-producing and on state-producing instruments, regardless of the existence of an external field. Furthermore, it is possible to give an operational definition of every initial state intrinsically, i.e., regardless of the external field. The precise statement of this empirical fact explains, for example, why a particle in an external field has integral or half-integral eigenvalues of the spin, why a Hamiltonian exists even in the presence of a time-dependent external field, and why (for Galilei symmetry) the canonical commutation relations are still valid, although the full space-time symmetry from which these results can be derived has been destroyed. It is pointed out that the rigorous validity of kinematical relations, in spite of strong breaking of the underlying space-time symmetry, is analogous to the rigorous validity of equal-time current commutation rules, in spite of the breaking of the underlying $U(3)$ symmetry.

I. INTRODUCTION

THE exploration of the constraints imposed on quantum mechanics by space-time symmetry has been very successful, both in predicting power and in the elimination of redundant assumptions. The prime example for the first kind of capability is the prediction that spins can be only integral or half-integral. Perhaps the most spectacular, but not highly advertised, success of symmetry theory in decreasing the number of independent assumptions is the derivation of the canonical commutation relations from the theory of the Galilei group.^{1,2}

by themselves lead to observable results. The most striking example for the affirmative answer is the historical fact that the canonical commutation relations were originally suggested to Heisenberg by the Thomas-Kuhn sum rules and that, conversely, the canonical commutation relations imply these sum rules regardless of the Hamiltonian.³

Presymmetry—the survival of some results of space-time symmetry even when this is broken by an external field—should be deducible from prime principles. It will be shown that the accepted principles are in fact sufficient *if they are explicitly stated*. In discussing observations to test space-time symmetry, a number of

Standard Model

Fermions	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\begin{pmatrix} u_{aL} \\ d_{aL} \end{pmatrix}$	3	2	1/3
u_{aR}	3	1	4/3
d_{aR}	3	1	-2/3
$\begin{pmatrix} \nu_{aL} \\ e_{aL} \end{pmatrix}$	1	2	-1
ν_{aR}	1	1	0
e_{aR}	1	1	-2

Quark-lepton charge symmetry

$$Y(u_{aL}) = 1/3 = -1 + 4/3 = Y(\nu_{aL}) - 4/3 (3B-L)(\nu_{aL})$$

$$Y(u_{aR}) = 4/3 = 0 + 4/3 = Y(\nu_{aR}) - 4/3 (3B-L)(\nu_{aR})$$

$$Y(d_{aL}) = 1/3 = -1 + 4/3 = Y(e_{aL}) - 4/3 (3B-L)(e_{aL})$$

$$Y(d_{aR}) = -2/3 = -2 + 4/3 = Y(e_{aR}) - 4/3 (3B-L)(e_{aR})$$

$$Y(\nu_{aL}) = -1 = 1/3 - 4/3 = Y(u_{aL}) - 4/3 (3B-L)(u_{aL})$$

$$Y(\nu_{aR}) = 0 = 4/3 - 4/3 = Y(u_{aR}) - 4/3 (3B-L)(u_{aR})$$

$$Y(e_{aL}) = -1 = 1/3 - 4/3 = Y(d_{aL}) - 4/3 (3B-L)(d_{aL})$$

$$Y(e_{aR}) = -2 = -2/3 - 4/3 = Y(d_{aR}) - 4/3 (3B-L)(d_{aR})$$

Are these charge relations real or accidental?

Statement of principles

- **Principle of electroweak quark-lepton symmetry:** There exists a hidden discrete Z_2 symmetry in the electroweak interactions of quarks and leptons.
- **Principle of weak topological-charge confinement:** Observable particles have no weak topological charge.
 - Particles with fractional charge have hidden charge structure and nontrivial topology.
 - Topological quarks have a topological bookkeeping Z_3 charge, with +1 (-1) in quarks (antiquarks).

Charge of topological quarks and leptons

$$Y(q) = Y(\hat{q}) - \frac{4}{3}(B - 3L)(\hat{q})$$

$q = \text{topological quark}$
 $\hat{q} = \text{prequark}$

$$Y(\ell) = Y(\hat{\ell}) - \frac{4}{3}(B - 3L)(\hat{\ell})$$

$\hat{\ell} = \text{prelepton}$
 $= \text{topological lepton}$

Constraints :

$$Y(\hat{q}) = Y(\ell) \quad (\hat{q}, \ell : \text{trivial topology})$$

$$Y(\hat{\ell}) = Y(q) \quad (q, \hat{\ell} : \text{nontrivial topology})$$

$$(B - 3L)(\hat{q}) = (3B - L)(\ell) = -(3B - L)(q)$$

$$(B - 3L)(\hat{\ell}) = (3B - L)(q) = -(3B - L)(\ell)$$

Prequark quantum numbers

Prequark	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\begin{pmatrix} \hat{u}_{aL} \\ \hat{d}_{aL} \end{pmatrix}$	3	2	-1
\hat{u}_{aR}	3	1	0
\hat{d}_{aR}	3	1	-2

Prelepton quantum numbers

Prelepton	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\begin{pmatrix} \hat{\nu}_{aL} \\ \hat{e}_{aL} \end{pmatrix}$	1	2	1/3
$\hat{\nu}_{aR}$	1	1	4/3
\hat{e}_{aR}	1	1	-2/3

Presymmetry

- Quark-prelepton:

$$u_{aL}^i \leftrightarrow \hat{V}_{aL}$$

$$u_{aR}^i \leftrightarrow \hat{V}_{aR}$$

$$d_{aL}^i \leftrightarrow \hat{e}_{aL}$$

$$d_{aR}^i \leftrightarrow \hat{e}_{aR}$$

- Lepton-prequark:

$$\nu_{aL} \leftrightarrow \hat{u}_{aL}^i$$

$$\nu_{aR} \leftrightarrow \hat{u}_{aR}^i$$

$$e_{aL} \leftrightarrow \hat{d}_{aL}^i$$

$$e_{aR} \leftrightarrow \hat{d}_{aR}^i$$

Gauge anomalies

- Gauge current

$$\hat{J}_Y^\mu = \hat{q}_{aL} \gamma^\mu \frac{Y}{2} \hat{q}_{aL} + \hat{q}_{aR} \gamma^\mu \frac{Y}{2} \hat{q}_{aR} + \bar{l}_{aL} \gamma^\mu \frac{Y}{2} l_{aL} + \bar{l}_{aR} \gamma^\mu \frac{Y}{2} l_{aR}$$

- Gauge anomaly

$$\partial_\mu \hat{J}_Y^\mu = -\frac{g^2}{32\pi^2} \left(\sum_{\hat{q}_{L^L}} \frac{Y}{2} \right) \text{Tr} W_{\mu\nu} \tilde{W}^{\mu\nu} - \frac{g'^2}{48\pi^2} \left(\sum_{\hat{q}_{L^L}} \frac{Y^3}{2^3} - \sum_{\hat{q}_{R^R}} \frac{Y^3}{2^3} \right) F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- Topological current

$$\partial_\mu \hat{J}_Y^\mu = -N_{\hat{q}} \partial_\mu J_T^\mu \quad N_{\hat{q}} = 12N_g$$

$$J_T^\mu = \frac{1}{4N_{\hat{q}}} K^\mu \sum_{\hat{q}_{L^L}} Y + \frac{1}{16N_{\hat{q}}} L^\mu \left(\sum_{\hat{q}_{L^L}} Y^3 - \sum_{\hat{q}_{R^R}} Y^3 \right) = -\frac{1}{6} K^\mu + \frac{1}{8} L^\mu$$

$$K^\mu = \frac{g^2}{8\pi^2} \varepsilon^{\mu\nu\lambda\rho} \text{Tr} (W_\nu \partial_\lambda W_\rho - \frac{2}{3} ig W_\nu W_\lambda W_\rho)$$

$$L^\mu = \frac{g'^2}{12\pi^2} \varepsilon^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho$$

Gauge anomaly cancellation

- Local counterterm

$$\Delta\mathcal{L} = g' N_{\hat{q}} \mathbf{J}_T^\mu \mathbf{A}_\mu$$

- New anomaly-free gauge non-invariant current

$$\mathbf{J}_Y^\mu = \hat{\mathbf{J}}_Y^\mu + N_{\hat{q}} \mathbf{J}_T^\mu$$

$$\partial_\mu \mathbf{J}_Y^\mu = 0$$

- Charge nonconservation due to topological charge

$$Q_Y(t) = \int \mathbf{J}_Y^0 d^3 \mathbf{x} = \frac{N_{\hat{q}}}{6} n_w(t)$$

$$n_w(t) = \frac{1}{24\pi^2} \int \varepsilon^{ijk} \text{Tr}(\partial_i \mathbf{U} \mathbf{U}^{-1} \partial_j \mathbf{U} \mathbf{U}^{-1} \partial_k \mathbf{U} \mathbf{U}^{-1}) d^3 \mathbf{x}$$

$$\Delta Q_Y = \frac{N_{\hat{q}}}{6} [n_w(t = \infty) - n_w(t = -\infty)] = \frac{N_{\hat{q}}}{6} Q_T = \frac{N_{\hat{q}}}{6} Q^{(3)} n$$

$$Q_T = \int \partial_\mu \mathbf{K}^\mu d^4 \mathbf{x} = \frac{g^2}{16\pi^2} \int \text{Tr}(W_{\mu\nu} \tilde{W}^{\mu\nu})$$

Charge normalization

- Hypercharge shift

$$Y(\hat{q}) \rightarrow Y(\hat{q}) + \frac{n}{3} Q^{(3)}(\hat{q}) = Y(\hat{q}) - \frac{n}{3} (B - 3L)(\hat{q})$$

- Anomaly cancellation: $n=4$

$$\sum_{q_L^{\ell_L}} Y = 0 \quad \sum_{q_L^{\ell_L}} Y^3 - \sum_{q_R^{\ell_R}} Y^3 = 0$$

- Effective current that absorbs topological effects

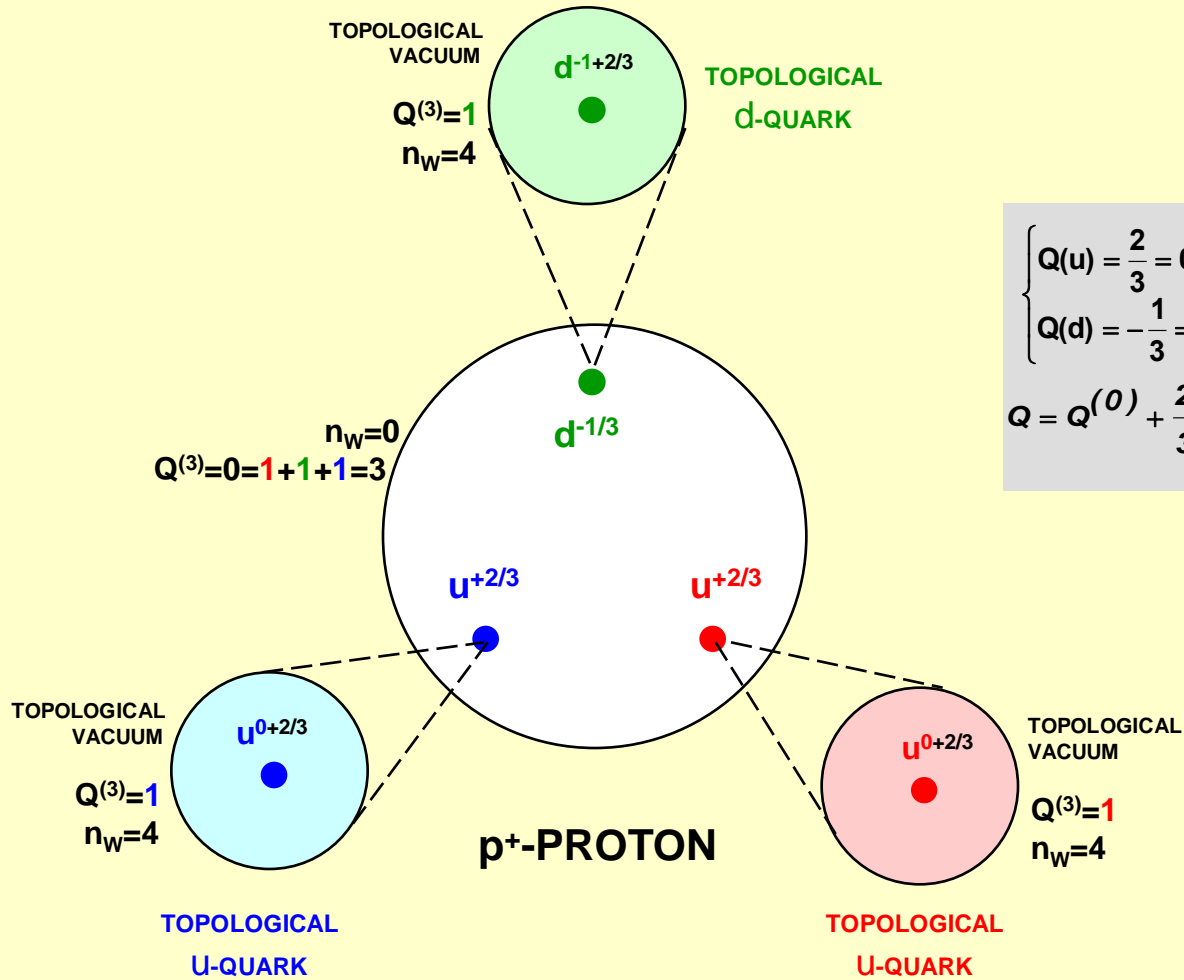
$$\hat{J}_{Y, \text{eff}}^\mu = -\frac{2}{3} [\bar{\hat{q}}_{aL} \gamma^\mu (B - 3L) \hat{q}_{aL} + \bar{\hat{q}}_{aR} \gamma^\mu (B - 3L) \hat{q}_{aR}]$$

- Full current

$$J_Y^\mu = \bar{\hat{q}}_{aL} \gamma^\mu \frac{Y - 4(B - 3L)/3}{2} \hat{q}_{aL} + \bar{\hat{q}}_{aR} \gamma^\mu \frac{Y - 4(B - 3L)/3}{2} \hat{q}_{aR} \\ + \bar{\ell}_{aL} \gamma^\mu \frac{Y}{2} \ell_{aL} + \bar{\ell}_{aR} \gamma^\mu \frac{Y}{2} \ell_{aR}$$

- Prequark-quark correspondence: $\hat{q} \rightarrow q$

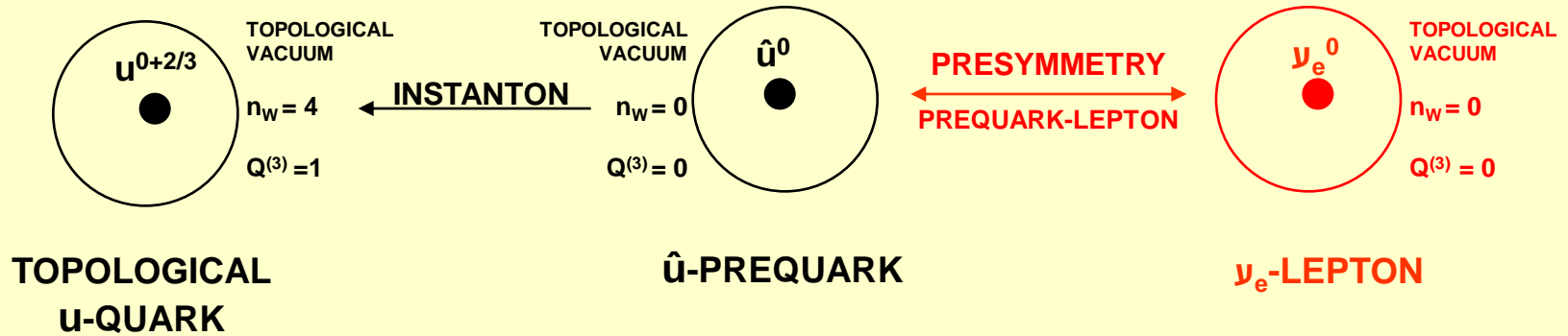
Topological quarks



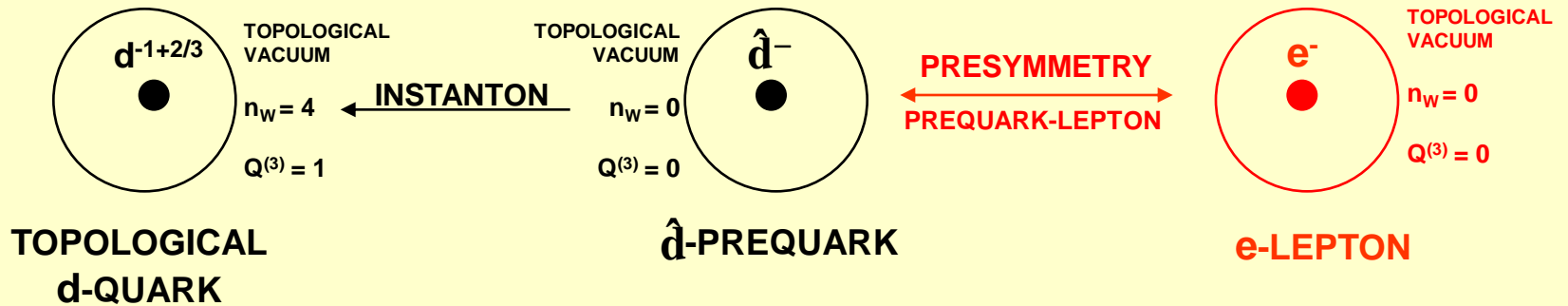
$$\begin{cases} Q(u) = \frac{2}{3} = 0 + \frac{2}{3} \\ Q(d) = -\frac{1}{3} = -1 + \frac{2}{3} \end{cases}$$

$$Q = Q^{(0)} + \frac{2}{3} = Q^{(0)} + \frac{n_W}{2N_C} Q^{(3)}$$

Presymmetry



instanton : $Q^{(3)} = -(B - 3L)$



Presymmetry beyond the Standard Model

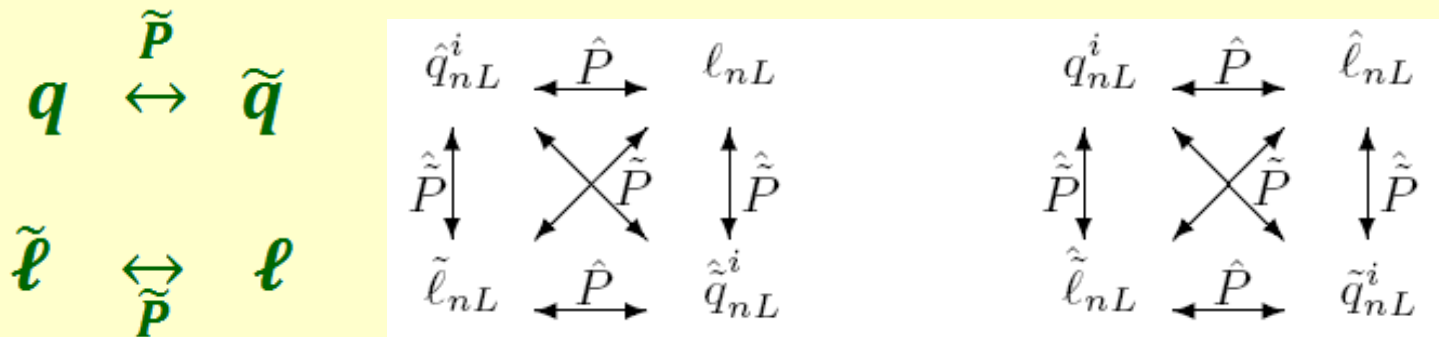
- **Generate residual presymmetry to avoid Occam's razor: "Entities should not be multiplied unnecessary."**
- **Residual presymmetry requires partners of SM particles.**
- **New families must be nonsequential, duplicating gauge groups.**
- **If presymmetry is transverse to everything, it extends to forces doubling gauge symmetry.**
- **Extend presymmetry to strong sector.**

Exotic duplication of the Standard Model

Fermions	SU(3) _c	SU(2) _L	U(1) _Y
$\begin{pmatrix} u_{aL} \\ d_{aL} \end{pmatrix}$	3	2	1/3
u_{aR}	3	1	4/3
d_{aR}	3	1	-2/3
$\begin{pmatrix} \tilde{\nu}_{aL} \\ \tilde{e}_{aL} \end{pmatrix}$	1	2	-1
$\tilde{\nu}_{aR}$	1	1	0
\tilde{e}_{aR}	1	1	-2

Fermions	SU(3) _c	SU(2) _L	U(1) _Y
$\begin{pmatrix} \tilde{u}_{aL} \\ \tilde{d}_{aL} \end{pmatrix}$	3	2	1/3
\tilde{u}_{aR}	3	1	4/3
\tilde{d}_{aR}	3	1	-2/3
$\begin{pmatrix} \nu_{aL} \\ e_{aL} \end{pmatrix}$	1	2	-1
ν_{aR}	1	1	0
e_{aR}	1	1	-2

Exotic residual presymmetry



Symmetry breaking

$$[\mathbf{SU}(3)_c]^2 \times [\mathbf{SU}(2)_L]^2 \times [\mathbf{U}(1)_Y]^2 \times \tilde{P}$$

$$\downarrow$$

$$[\mathbf{SU}(3)_c]^2 \times \mathbf{SU}(2)_L \times \mathbf{U}(1)_Y$$

$$\downarrow$$

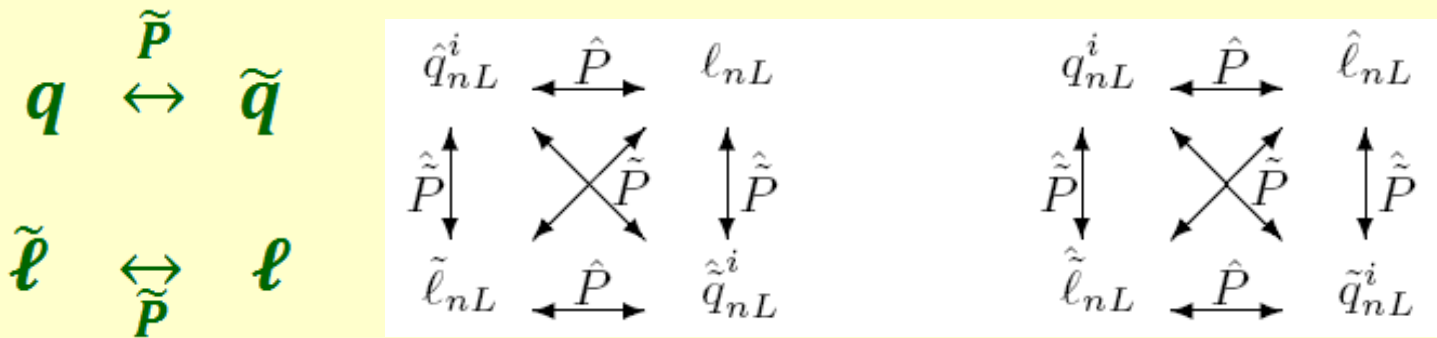
$$[\mathbf{SU}(3)_c]^2 \times \mathbf{U}(1)_{em}$$

Hidden copy of the Standard Model

Fermions	SU(3) _c	SU(2) _L	U(1) _Y
$\begin{pmatrix} u_{aL} \\ d_{aL} \end{pmatrix}$	3	2	1/3
u_{aR}	3	1	4/3
d_{aR}	3	1	-2/3
$\begin{pmatrix} \nu_{aL} \\ e_{aL} \end{pmatrix}$	1	2	-1
ν_{aR}	1	1	0
e_{aR}	1	1	-2

Fermions	SU(3) _c	SU(2) _L	U(1) _Y
$\begin{pmatrix} \tilde{u}_{aL} \\ \tilde{d}_{aL} \end{pmatrix}$	3	2	1/3
\tilde{u}_{aR}	3	1	4/3
\tilde{d}_{aR}	3	1	-2/3
$\begin{pmatrix} \tilde{\nu}_{aL} \\ \tilde{e}_{aL} \end{pmatrix}$	1	2	-1
$\tilde{\nu}_{aR}$	1	1	0
\tilde{e}_{aR}	1	1	-2

Hidden residual presymmetry



Symmetry breaking

$$[\mathbf{SU}(3)_c]^2 \times [\mathbf{SU}(2)_L]^2 \times [\mathbf{U}(1)_Y]^2 \times \tilde{P}$$

$$\downarrow$$

$$[\mathbf{SU}(3)_c]^2 \times \mathbf{SU}(2)_L \times \mathbf{U}(1)_Y$$

$$\downarrow$$

$$[\mathbf{SU}(3)_c]^2 \times \mathbf{U}(1)_{em}$$

Conclusions

- **Presymmetry remains hidden if there is no copy of SM.**
- **Partners and their symmetry appear as manifestations of residual presymmetry and its extension from matter to forces.**
- **Duplication of SM particles keeps spin and is nondegenerated about TeV scale.**

- **Majorana neutrinos are excluded.**
- **Number of fermion families and number of colors are equal.**
- **New quark-lepton families are nonsequential.**