

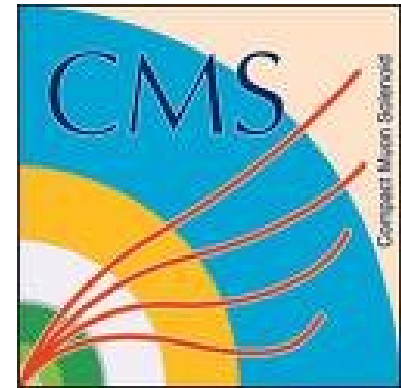
Probing the heavy flavor content in $t\bar{t}b\bar{a}r$ events & Using $t\bar{t}b\bar{a}r$ events as a calibration tool



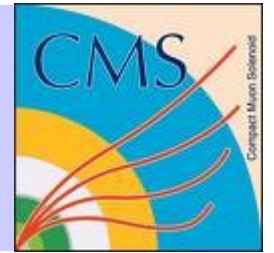
Roberta Volpe
Università degli Studi di Perugia
and INFN Perugia
on behalf of the CMS collaboration

DPF 2009

at



R measurement motivation



$$R = \frac{B(t \rightarrow Wb)}{B(t \rightarrow Wq)} \quad q = b, s, d$$

Standard Model (SM) predicts:

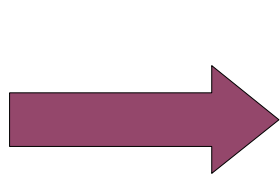
$$R = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}$$

▶ If 3 SM quark generations: Unitarity 3x3 CKM matrix $\Rightarrow R = |V_{tb}|^2$



R measurement \rightarrow **V_{tb} measurement**

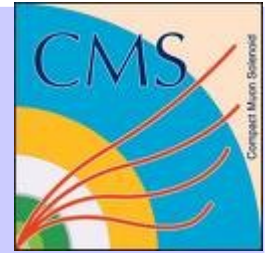
▶ If a 4th generation of quarks (t' and b') exists \rightarrow unitarity of a 4x4 matrix



- R measurement \rightarrow **constraints on V_{tb}**
- $R_{\text{SM}} - R > \mathcal{O}(10^{-1}) \rightarrow$ **clue of the existence of a 4th generation**

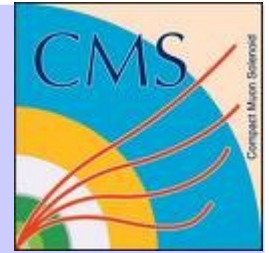
J. Alwall et al., "Is $V_{tb} = 1$?" *Eur. Phys. J. C* **49** (2007) 791–801

What we know about R and V_{tb}



- **From Standard Model :** $|V_{tb}| = 0.999133_{-0.000043}^{+0.000044}$
- **Last R measurements:**
 - CDF: $R > 0.61$ at 95 % C.L. with $L = 162 \text{ pb}^{-1}$
 - D0: $R = 0.97_{-0.08}^{+0.09}$ $R > 0.79$ at 95 % C.L.
Simultaneous measurement of R and $t\bar{t}$ cross section with $L = 900 \text{ pb}^{-1}$
- **V_{tb} Measurements from single top studies:**
 - CDF: $|V_{tb}| = 0.91 \pm 0.11(\text{stat} + \text{syst}) \pm 0.07(\text{theory})$
 $|V_{tb}| > 0.71$ at the 95% C.L. with $L = 3.2 \text{ fb}^{-1}$
 - D0: $|V_{tb}| > 0.78$ at 95% C.L. with $L = 2.3 \text{ fb}^{-1}$

Plans for R measurement with CMS experiment



Physical Observable: distribution of number of b-tagged jets ϵ_b = b-tagging efficiency
 b-tagging algorithm used relies on the track impact parameter ϵ_q = mis-tagging probability

Probability to find i b-tagged jets:

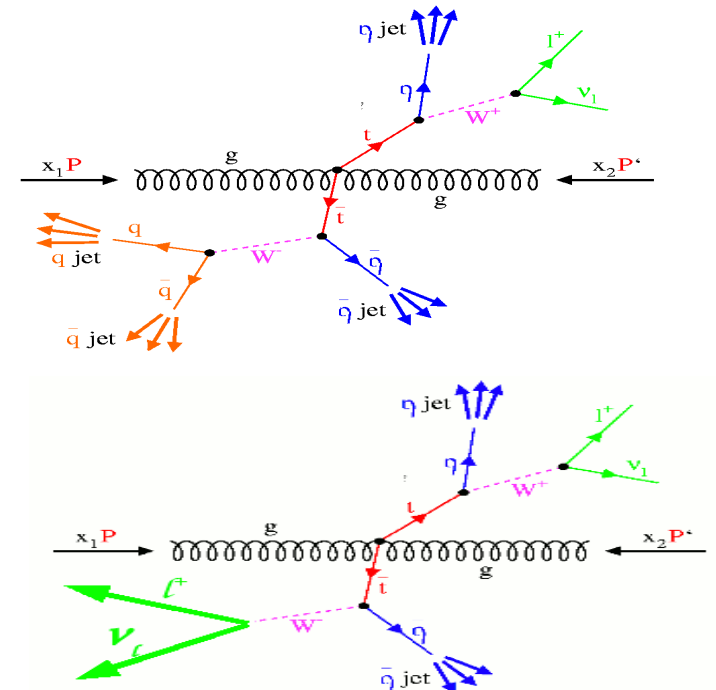
$$P_i(R; \epsilon_b, \epsilon_q) = R^2 P_i(tt \rightarrow WWbb) + 2R(1-R) P_i(tt \rightarrow WWbq) + (1-R)^2 P_i(tt \rightarrow WWqq)$$

with $q = s, d$ and $i =$ number of b-tagged jets

- **Semileptonic channel ($L = 1 \text{ fb}^{-1}$)**
 - ◆ **R measurement** *CMS PAS TOP-09-007*

- **Dileptonic channel $e \mu$ ($L = 250 \text{ pb}^{-1}$)**

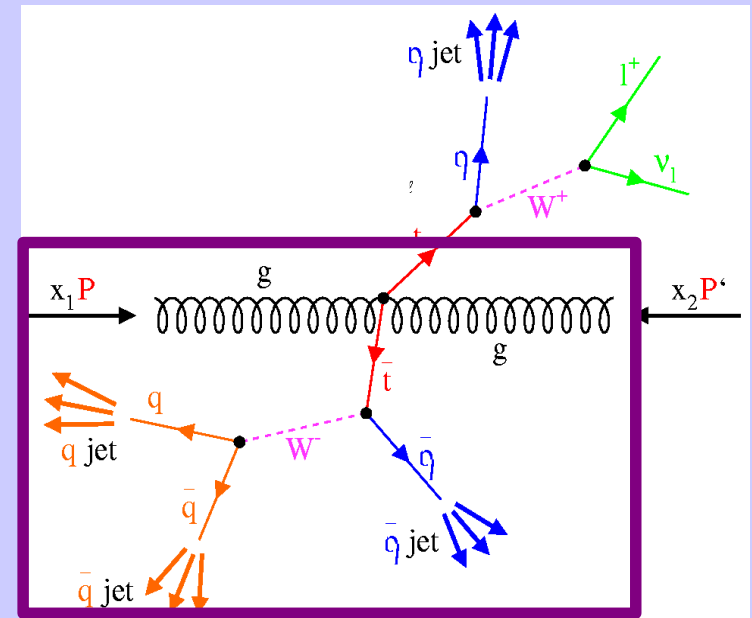
- ◆ **R measurement**
- ◆ **b-tagging efficiency measurement (by fixing R)**
CMS PAS TOP-09-001



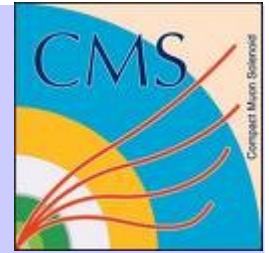
Semileptonic channel

$L = 1 \text{ fb}^{-1} @ 10 \text{ TeV}$

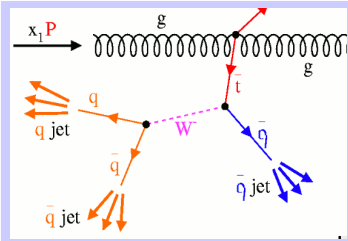
♦ *R measurement*



CMS PAS TOP-09-007



Selection and event reconstruction



1. Trigger request;
2. a single isolated and high energy lepton (electron or muon);
3. at least four selected jets;
4. $Centrality > 0.35$;
5. $|m_{ij} - m_{W had}| < \sigma(m_{W had})$;
6. $\chi^2 < 4$

	$\sigma \cdot BR (pb)$	N_{ev}
$t\bar{t}$ Semilep	182	2650
other $t\bar{t}$	232	109
$W + jets$	$4 \cdot 10^4$	260
$Z + jets$	$3.7 \cdot 10^3$	52
tW	29	52
QCD	10^9	56

$L=1 \text{ fb}^{-1}$

$$N(\text{Semilept } t\bar{t}b\bar{a}) / N(\text{Bkg}) \sim 5$$

Centrality:

Sums on every jet
$$\frac{\sum E_T}{\sqrt{(\sum E)^2 - (\sum p_z)^2}}$$

Hadronic W reconstruction:

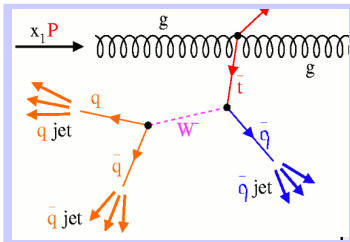
4 good jets with largest E_T : Invariant mass M_{ij} for each pair of jets, choose (i, j) in order to have minimum $\Delta M_W = |M_{ij} - M_{W had}|$

χ^2 based on tops masses:

2 jets not associated to W_{had} : k, p

$$\chi^2 = \left(\frac{m_{ijk} - m_{tHad}}{\sigma(m_{tHad})} \right)^2 + \left(\frac{m_{lvp} - m_{tLep}}{\sigma(m_{tLep})} \right)^2$$

choose k and p in order to have the minimum χ^2

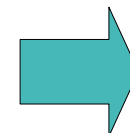
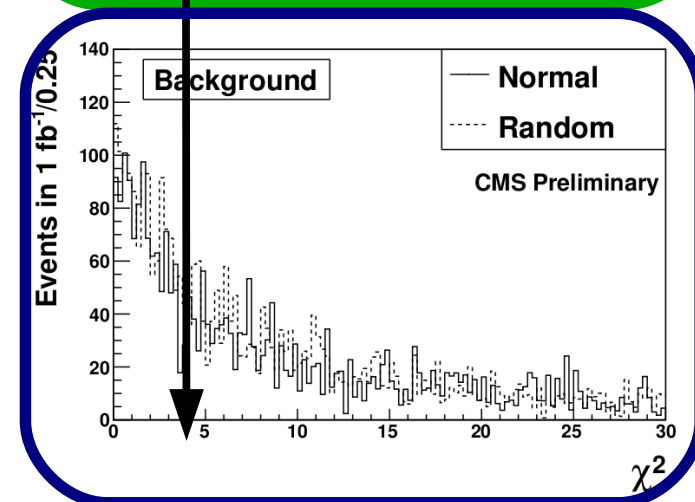
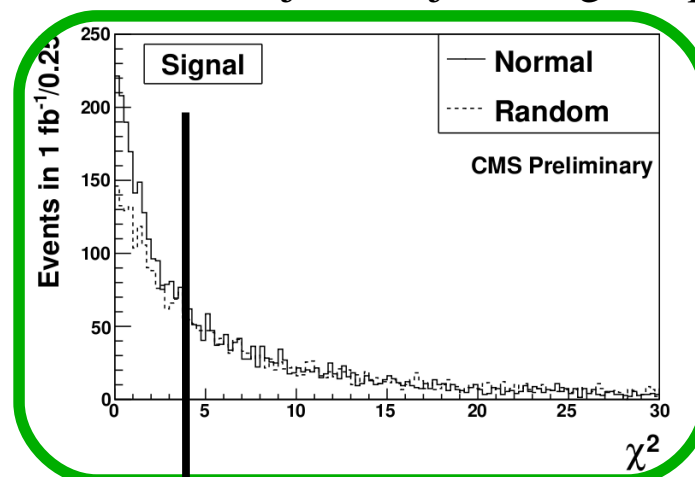
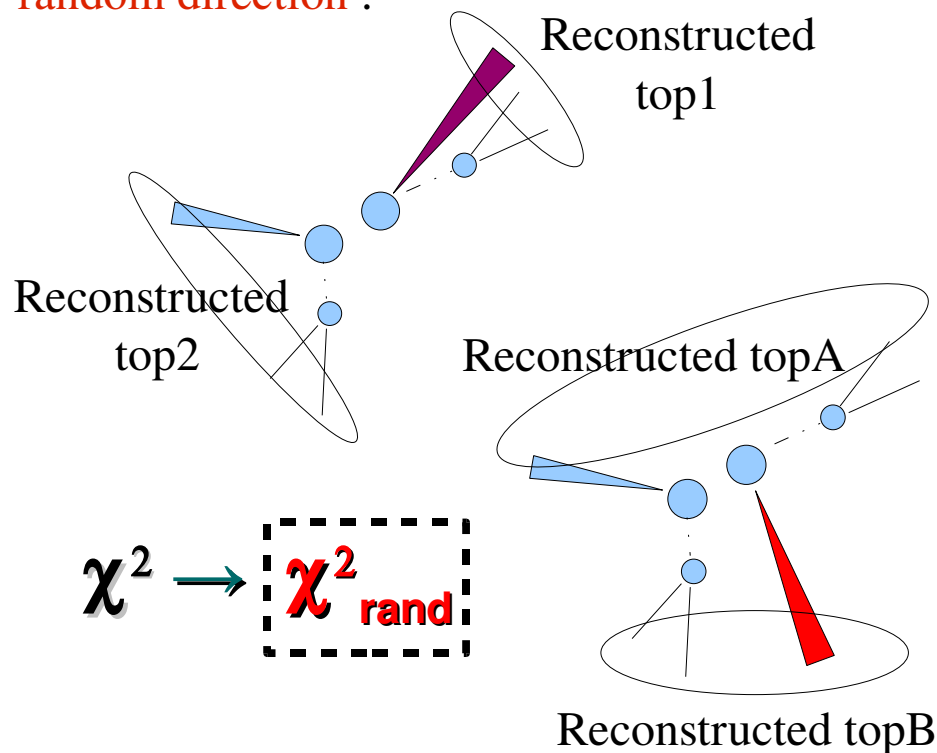


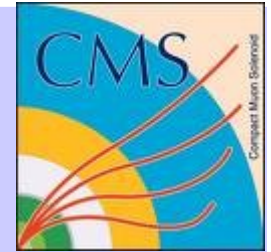
Background Subtraction



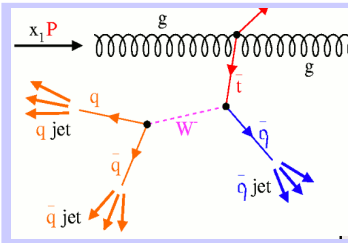
Signal: semilept $t\bar{t}$ events with 2 jets well matched to partons coming from the direct decays of top
Background: semilept $t\bar{t}$ other than Signal ; dilep and hadr $t\bar{t}$, W +jets, Z +jets, single top, QCD.

Change the momentum direction of the **jet** with larger E_t coming from the top decay to a **random direction** :





Background Subtraction

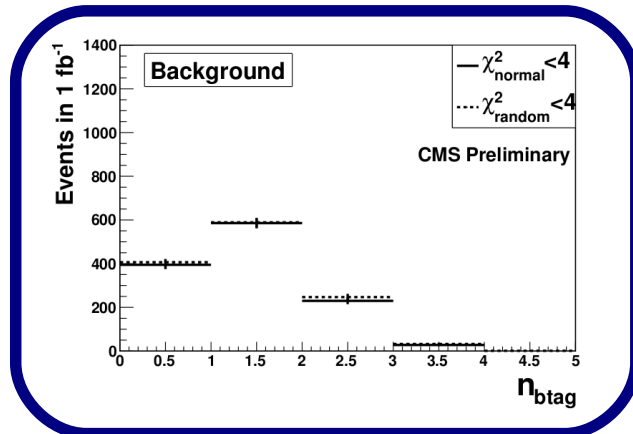
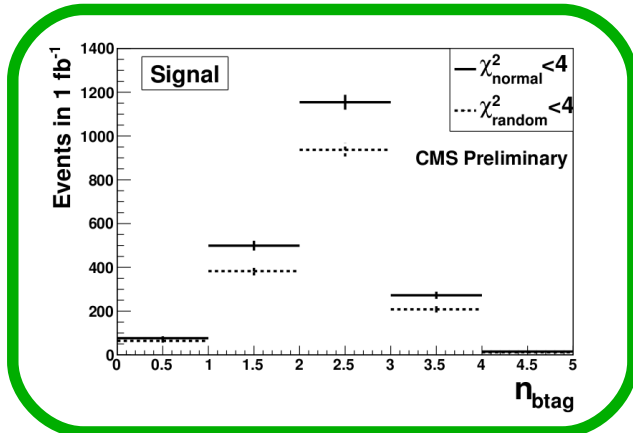


$$\chi^2_{\text{norm}} < 4$$

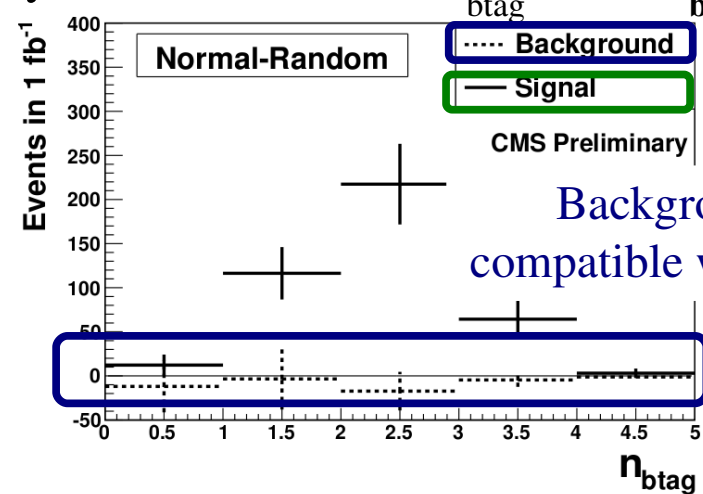
$$\rightarrow n_{\text{btag}}^{\text{norm}}$$

$$\chi^2_{\text{rand}} < 4$$

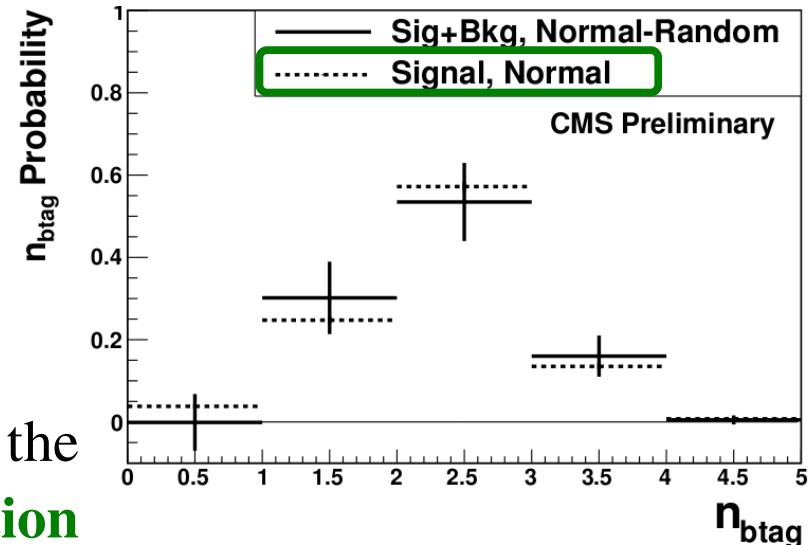
$$\rightarrow n_{\text{btag}}^{\text{rand}}$$



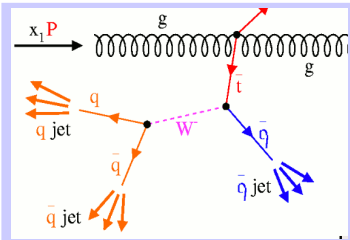
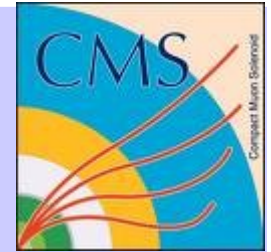
Bin by bin subtraction $n_{\text{btag}}^{\text{norm}} - n_{\text{btag}}^{\text{rand}}$



Background compatible with zero



The resulting distribution is proportional to the **signal n_{btag} distribution**



R measurement method

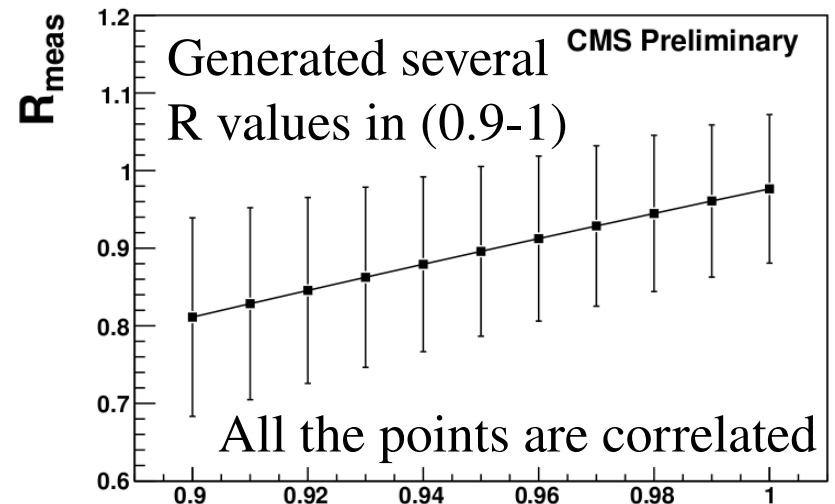
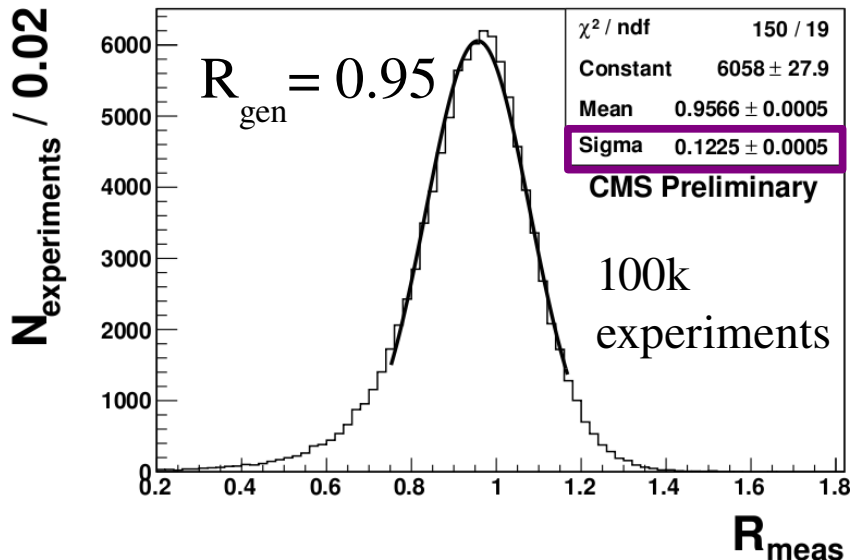
$n_{btag}^{norm} - n_{btag}^{rand}$ (all data) is to be fitted with:

$$P_i(R; \epsilon_b, \epsilon_q) = R^2 P_i(bb) + 2R(1 - R)P_i(bq) + (1 - R)^2 P_i(qq)$$

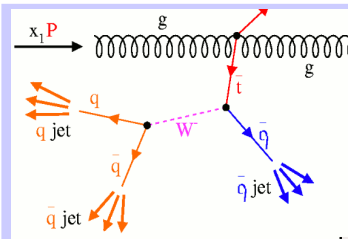
$P_i(\dots) = f(\epsilon_b, \epsilon_q)$ R and ϵ_b are fully correlated \rightarrow ϵ_b is fixed
Free parameters: R and ϵ_q

CMS will use methods to evaluate ϵ_b from data with an independent sample

Fit Validation with a Toy Monte Carlo experiment:



$$\sigma_{stat}(R) \sim 0.12 \text{ for every } R_{gen}$$



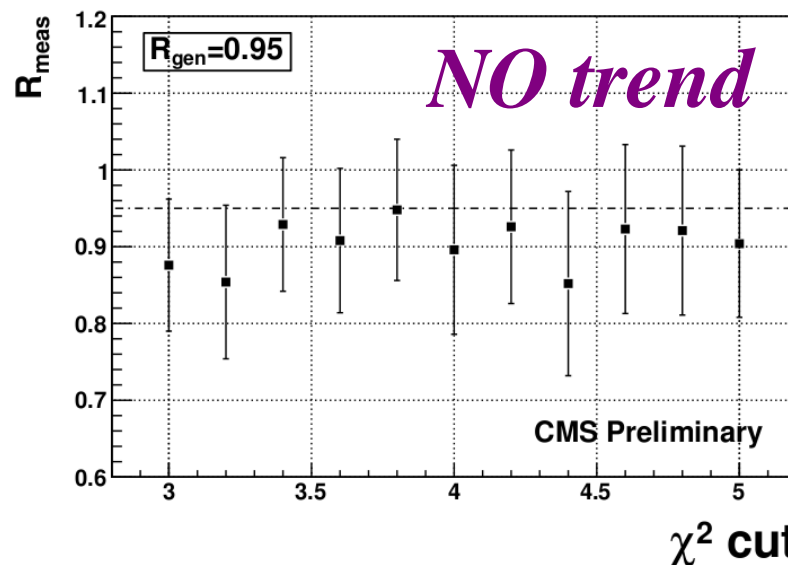
Expected uncertainties



Systematics evaluation:

Stability under changes
of χ^2 cut:

systematics	σ_{sys}
b tagging efficiency	0.04
b tagging efficiency bias	0.04
Jet Energy Scale	0.09
χ^2 cut	0.02
Selection efficiency	0.006
total	0.11



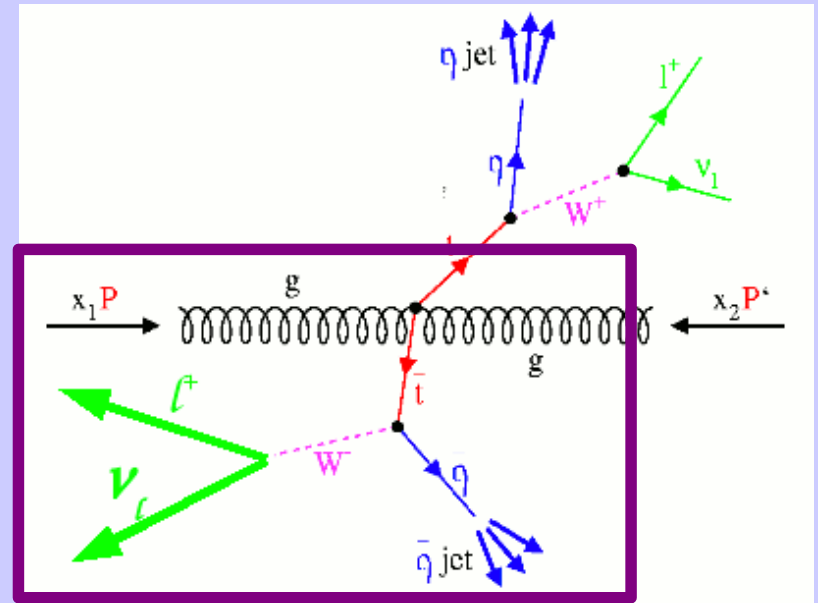
The main systematics sources
(JES and ϵ_b) will be reduced
once the detector is well understood

- σ_R (stat) = 0.12
- σ_R (sys) = 0.11

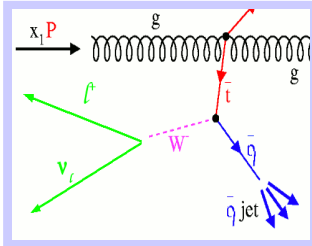
Dileptonic channel

$L = 250 \text{ pb}^{-1} @ 10 \text{ TeV}$

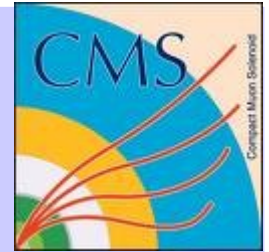
- ♦ R measurement
- ♦ ϵ_b measurement



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Analysis strategy



The nbtag distribution depends mainly on:

- ϵ_b and ϵ_q
- R

• The probability to reconstruct and select jets from top decays: α

Distribution considered

in order to evaluate α : M_{lb}

l and b lepton and jet coming from the same top

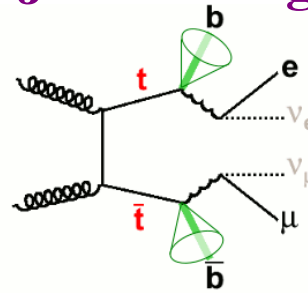
Selection
Triggered
≥ 2 leptons (>20 GeV/c)
1 e and 1 μ
≥ 2 jets (>30 GeV)
$\cancel{E}_T \geq 30$ GeV
Opp. sign leptons

	$\sigma \cdot BR$ (pb)	N_{ev}
$t\bar{t}$ Dilep	41	787
Single top	144	29
other $t\bar{t}$	372	14
Di-boson	40	11
W/Z+ jets	$4.4 \cdot 10^4$	27
V+QQ	290	1
QCD	10^9	0

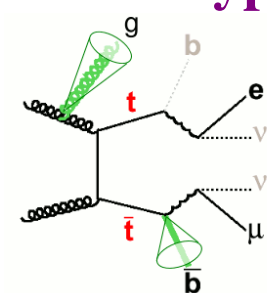
$L=250 \text{ pb}^{-1}$

S/B~10

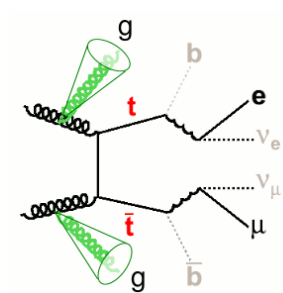
Jet misassignment event type contributions:



$$\alpha_2 = \alpha^2$$



$$\alpha_1 = 2(1-\alpha)\alpha^2$$

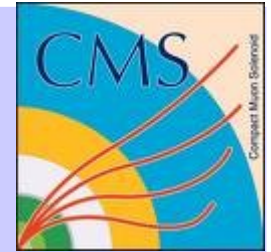


$$\alpha_0 = (1-\alpha)^2$$

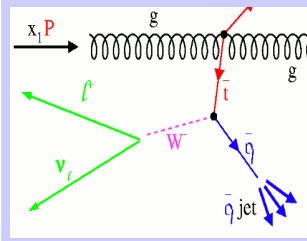
2 methods to estimate from data each event type contribution:

➤ **Swap**: pair all leptons from one event with all jets from a different event

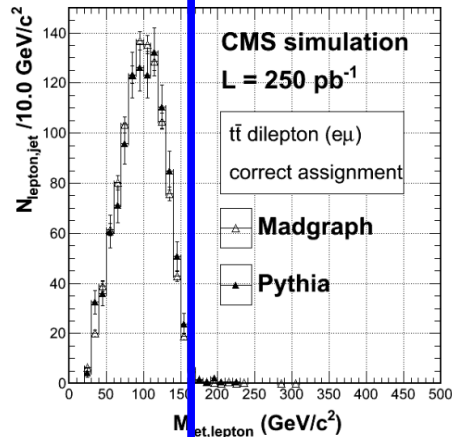
➤ **Random rotation**: randomize the direction of all leptons before pairing with all jets



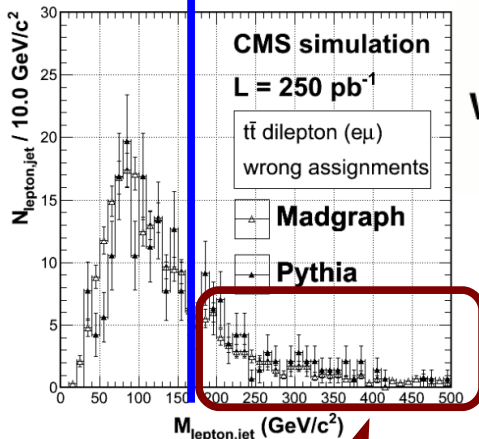
Jet misassignment estimation from data



$$M_{l,b}^{max} \equiv \sqrt{m_t^2 - m_W^2} = 156 \text{ GeV}/c^2$$

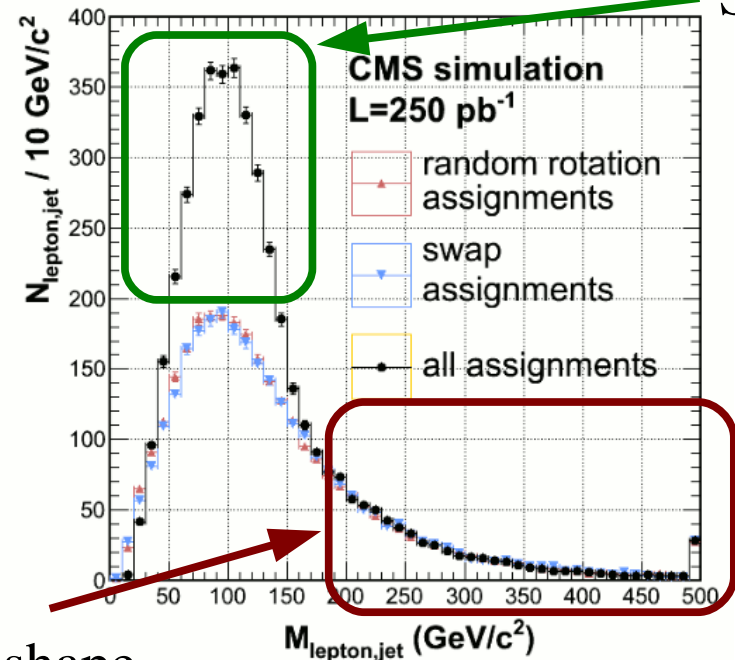


Correct assignments
have $M_{ij} < 156 \text{ GeV}/c^2$



Wrong assignments
are at large M_{ij}

Scale the shape to match the data for $M_{lep,jet} > 190 \text{ GeV}/c^2$



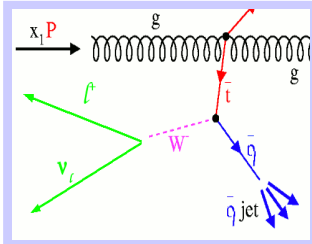
Signal

From the normalization factor **find α**

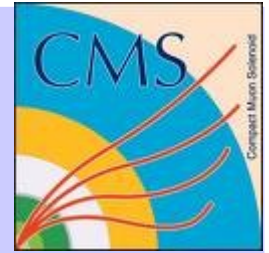
The average of the **swap** and **random** results is used

Method	$N_{mis}^{M>190} / N_{mis}$	α
<i>t\bar{t}</i> events from MADGRAPH		
average	0.21 ± 0.01	0.82 ± 0.04
MC truth	0.20 ± 0.01	0.80 ± 0.01

Only wrong assignment



Fit strategy



$$P_i(R; \epsilon_b, \epsilon_q) = R^2 P_i(bb) + 2R(1 - R)P_i(bq) + (1 - R)^2 P_i(qq) \quad \alpha \text{ from data} \rightarrow \alpha_2$$

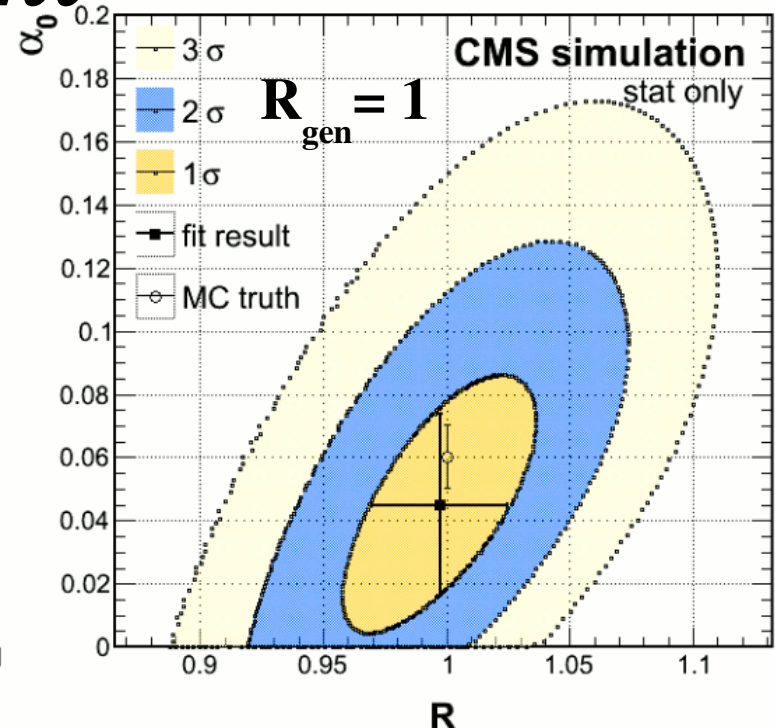
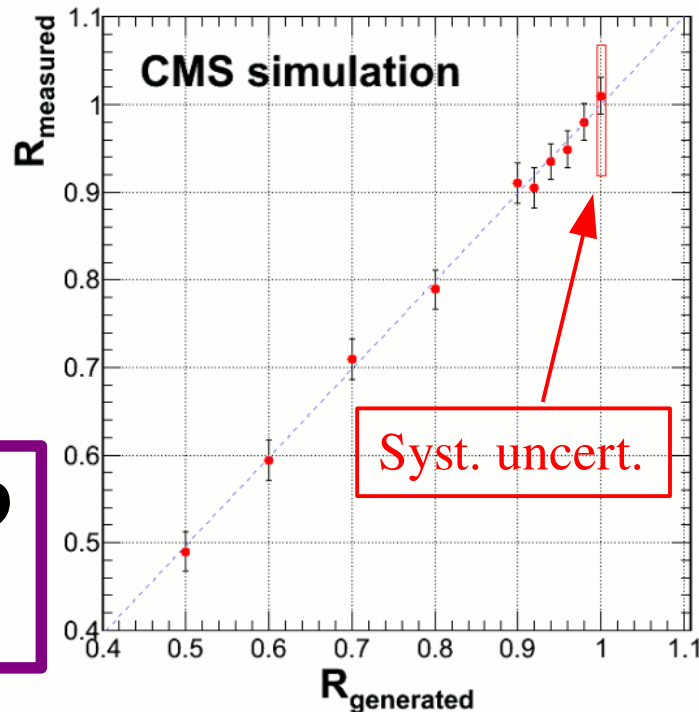
$$P_i(\dots) = f(\epsilon_b, \epsilon_q, \alpha_0, \alpha_1, \alpha_2) \quad \alpha_1 \text{ from normalization}$$

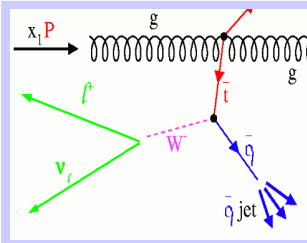
B-tagging working point slightly different from semileptonic analysis

R measurement

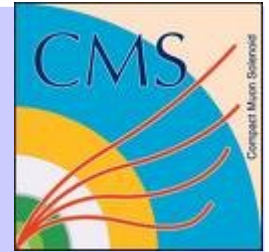
ϵ_b, ϵ_q fixed
(from other measurements)
free parameters:
R and α_0

$\sigma_{\text{stat+sys}}(\mathbf{R}) = 0.09$
for $L = 250 \text{ pb}^{-1}$

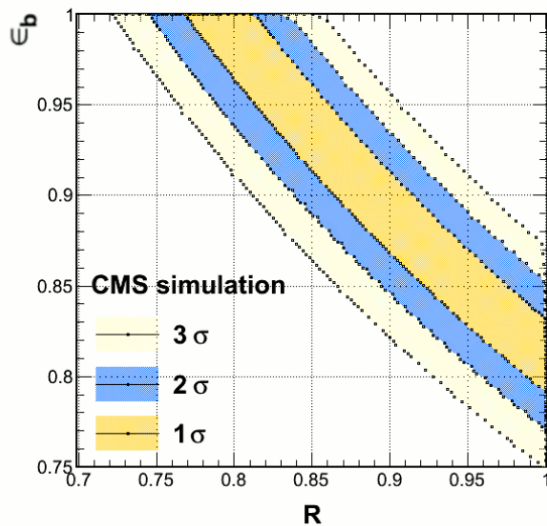




Fit strategy



ϵ_b and \mathbf{R} are fully correlated:

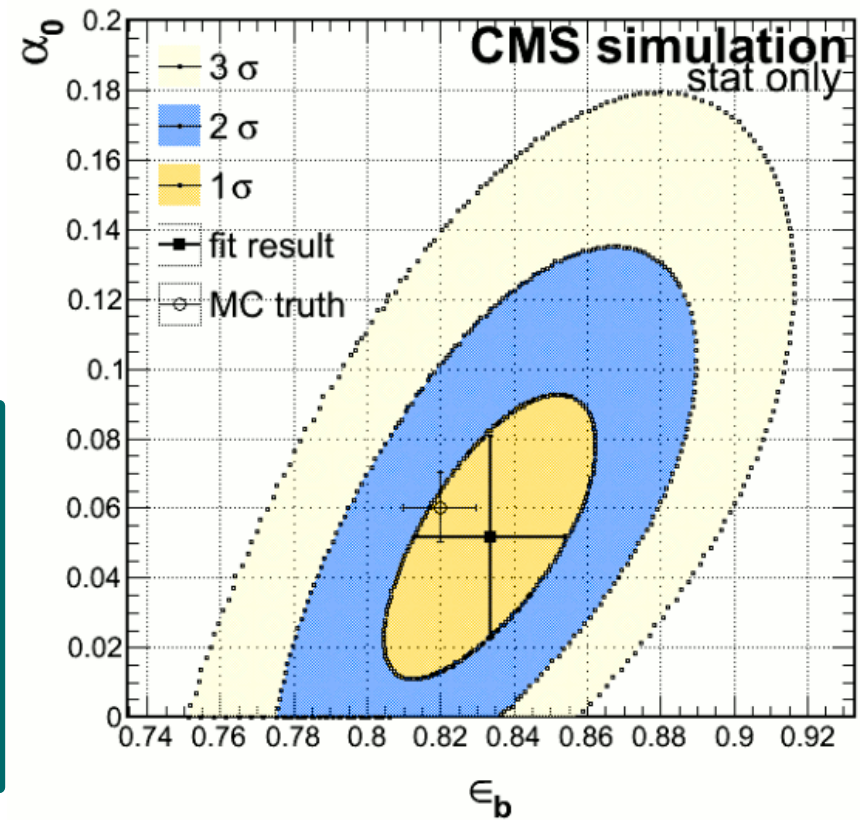


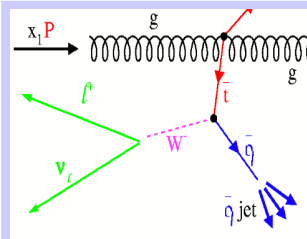
\mathbf{R} fixed from Standard Model

$0.79 < \epsilon_b < 0.83$
in agreement with MC

\mathbf{R} fixed from SM
 ϵ_q fixed from other source
 free parameters: α_0 and ϵ_b
 (constraint $0 < \epsilon_b < 1$)

ϵ_b measurement



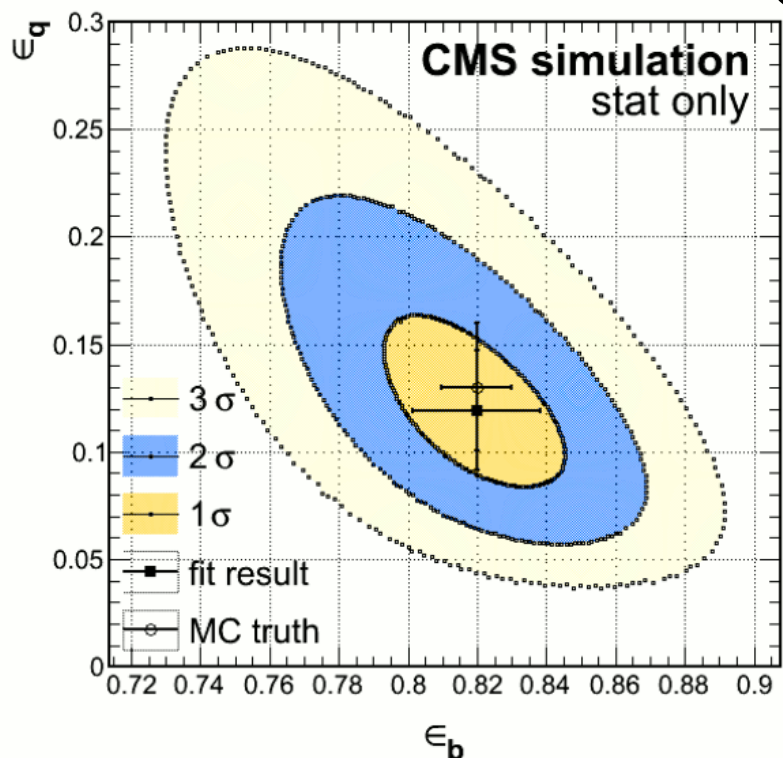


Fit strategy: ϵ_b measurement



2 different b-tagging algorithms, each for 3 different working points

algorithm	working point	ϵ_b (MC truth)	ϵ_b
Jet Probability	loose	0.82 ± 0.01	0.81 ± 0.02
	medium	0.63 ± 0.01	0.63 ± 0.02
	tight	0.41 ± 0.01	0.41 ± 0.02
Track Counting	loose	0.80 ± 0.01	0.82 ± 0.02
	medium	0.65 ± 0.01	0.65 ± 0.02
	tight	0.40 ± 0.01	0.41 ± 0.02



Closure test:
fitting ϵ_b, ϵ_q

$$\epsilon_b = 0.82 \pm 0.02$$

$$\epsilon_q = 0.12 \pm 0.03$$

Syst. Uncert. ~ 4 %
for $L = 250 \text{ pb}^{-1}$

Conclusions



- **R measurement strategies (data-driven background evaluation) have been developed in both semileptonic and dileptonic $t\bar{t}$ events**

$$\begin{array}{l} \text{Semilep} \quad \sigma_{\text{stat}}(\mathbf{R}) = 0.12 \\ (\mathbf{L} = 1 \text{ fb}^{-1}) \quad \sigma_{\text{sys}}(\mathbf{R}) = 0.11 \end{array}$$

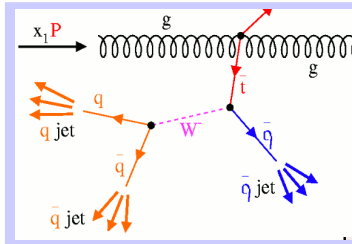
$$\begin{array}{l} \text{Dilep} (\mathbf{L} = 250 \text{ pb}^{-1}) \\ \sigma_{\text{stat+sys}}(\mathbf{R}) = 0.09 \end{array}$$

- **B-tagging efficiency measurement from data using dileptonic $t\bar{t}$ events is possible already with early LHC data**

$$\begin{array}{l} \sigma_{\text{stat}}(\epsilon_{\text{b}}) = 0.02 \\ \sigma_{\text{sys}}(\epsilon_{\text{b}}) \sim 0.04 \end{array}$$

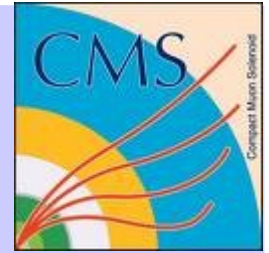
$$\text{Dilep} (\mathbf{L} = 250 \text{ pb}^{-1})$$

Back-up



Selection (semil)

Physics Objects Reconstruction



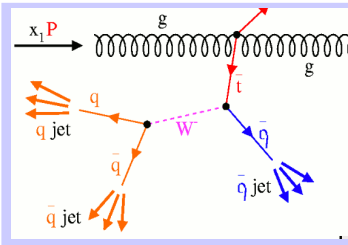
- **HLT:** single lepton trigger (single electron || single muon)
- **Good Jets:**
 - $|\eta| < 2.4, E_T^{corr} > 40 \text{ GeV}$
 - $\Delta R(Jet, Lep) > 0.5, EmFr < 1$
- **Missing Transverse Energy:**
 - Vectorial sum of calorimetric energy
 - deposits (JES and muon energy)
 - deposits considered)
- **Good Lepton (e or μ):**
lepton ID, Isolation cut, $p_t(lep) > 30 \text{ GeV}, |\eta| < 2.4$

Preliminary selection:

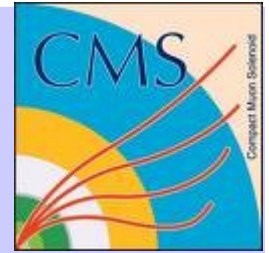
- ✓ HLT
- ✓ one single lepton
- ✓ at least 4 jets

	$\epsilon(HLT) \%$	$\epsilon(Lep) \%$	$\epsilon(Jets) \%$	$\epsilon(Centr) \%$	$\epsilon(\Delta M_W) \%$	$\epsilon(\chi^2) \%$	$N_{evts} \text{ with } 1 \text{ fb}^{-1}$
<i>t\bar{t} semil</i>	54	27.5	7.23	5.62	3.79	1.48	2650 ←
<i>t\bar{t} others</i>	22	7.9	$6.3 \cdot 10^{-1}$	$4.9 \cdot 10^{-1}$	$2.4 \cdot 10^{-1}$	$4.7 \cdot 10^{-2}$	109
<i>W + jets</i>	34	16	$9.5 \cdot 10^{-3}$	$6.8 \cdot 10^{-3}$	$3.7 \cdot 10^{-3}$	$6.5 \cdot 10^{-4}$	260
<i>Z + jets</i>	50	19	$1.9 \cdot 10^{-2}$	$1.4 \cdot 10^{-2}$	$7.4 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$	52
<i>tW</i>	35	18	1.2	$8.4 \cdot 10^{-1}$	$6.4 \cdot 10^{-1}$	$1.8 \cdot 10^{-1}$	52
<i>QCD</i>	2.7	$7.8 \cdot 10^{-3}$	$5.2 \cdot 10^{-6}$	$4.3 \cdot 10^{-6}$	$2.4 \cdot 10^{-6}$	$1.1 \cdot 10^{-6}$	56

selected **well reconstructed Signal events** = 2650 * 0.74 (well rec. fr.) = **1960**



Fitting function (semil)



The $n_{btag}^{norm} - n_{btag}^{rand}$ (sig+bkg) distribution is to be fitted with:

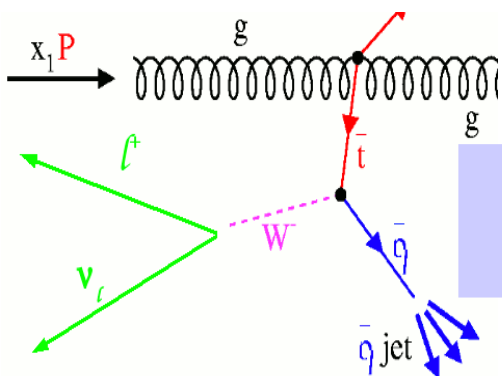
$$\mathbf{B} = \epsilon_b, \quad \mathbf{M} = \epsilon_q$$

$$\epsilon_i(R; B, M) = R^2 P_i(t\bar{t} \rightarrow bWbW) + 2R(1 - R)P_i(t\bar{t} \rightarrow bWqW) + (1 - R)^2 P_i(t\bar{t} \rightarrow qWqW)$$

With $P_i()$ from:

	P_0	P_1	P_2	P_3	P_4
bb	$(1 - B)^2(1 - M)^2$	$2B(1 - B)(1 - M)^2 + 2M(1 - B)^2(1 - M)$	$B^2(1 - M)^2 + 4BM(1 - M)(1 - B) + M^2(1 - B)^2$	$2B^2M(1 - M) + 2M^2B(1 - B)$	B^2M^2
bq	$(1 - B)(1 - M)^3$	$B(1 - M)^3 + (1 - B)3M(1 - M)^2$	$3BM(1 - M)^2 + 3M^2(1 - B)(1 - M)$	$M^3(1 - B) + 3M^2B(1 - M)$	BM^3
qq	$(1 - M)^4$	$4M(1 - M)^3$	$6M^2(1 - M)^2$	$4M^3(1 - M)$	M^4

Fixed parameter: B Free parameters: R and M



dilep.

Trigger, Reconstruction, Selection

- The dilepton channel ($e\mu$) event selection is used (cf. TOP-09-002)
 - 2 hard- p_T , prompt, isolated leptons

Trigger: Single electron or muon

Lepton Selection: $p_T > 20 \text{ GeV}/c$, $|\eta| < 2.4$
 $|d_0| < 400 \text{ mm}$, $I_{\text{trk}} > 0.9$, $I_{\text{calo}} > 0.9$ (0.8 for e)
 $\Delta(e, \mu) > 0.1$ Lepton id.

Jet Selection: **Calorimeter Jets** (SIS cone $\Delta R < 0.5$)
 2 constituents with $E_T > 2 \text{ GeV} + 1$ track
 $p_T^{\text{corr}} > 30 \text{ GeV}/c$, $|\eta| < 2.4$
 $\text{emf} < 0.98$, $\Delta R(\text{jet}, e/\mu) > 0.3$

Missing Et $> 30 \text{ GeV}$ (corrected for μ)

Event selection: results

Event yields for $L = 250 \text{ pb}^{-1}$

Table 1: Expected event yield for an integrated luminosity of $L=250 \text{ pb}^{-1}$ using a MADGRAPH sample. Only statistical uncertainties from the MC samples are shown.

Selection	Total	$t\bar{t}$ dileptons
Triggered	$(426 \pm 1) \cdot 10^6$	6251 ± 25
≥ 2 leptons ($>20 \text{ GeV}/c$)	$(204.7 \pm 0.5) \cdot 10^3$	2595 ± 16
1 e and 1 μ	2531 ± 32	1344 ± 12
≥ 2 jets ($>30 \text{ GeV}/c$)	1041 ± 12	914 ± 10
$\cancel{E}_T \geq 30 \text{ GeV}$	884 ± 10	789 ± 9
Opp. sign leptons	867 ± 10	787 ± 9

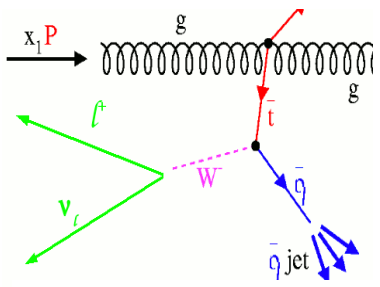
Background processes only

Table 2: Expected event yield for background processes for an integrated luminosity of $L=250 \text{ pb}^{-1}$. Only statistical uncertainties are shown. VQQ corresponds to the background from $W/Zb\bar{b}$ and $W/Zc\bar{c}$. All samples were generated with MADGRAPH except for QCD which was generated using PYTHIA.

Selection	Single top	other $t\bar{t}$	Di-boson	W/Z+jets	VQQ	QCD
Triggered	27563 ± 422	42303 ± 77	3400 ± 13	$(4338 \pm 2) \cdot 10^3$	38046 ± 64	$(422 \pm 1) \cdot 10^6$
≥ 2 leptons ($>20 \text{ GeV}/c$)	225 ± 10	66 ± 3	620 ± 5	$(193 \pm 0.4) \cdot 10^3$	8170 ± 26	401 ± 183
1 e and 1 μ	122 ± 8	34 ± 2	207 ± 2	749 ± 24	49 ± 2	26 ± 15
≥ 2 jets ($>30 \text{ GeV}/c$)	38 ± 4	27 ± 2	14 ± 1	45 ± 6	2.0 ± 0.4	-
$\cancel{E}_T \geq 30 \text{ GeV}$	29 ± 1	23 ± 2	11.6 ± 0.5	30 ± 5	0.7 ± 0.2	-
Opp. sign leptons	29 ± 1	14 ± 1	10.5 ± 0.5	26 ± 5	0.7 ± 0.2	-

dilep.

Misassignment models in background samples



All data

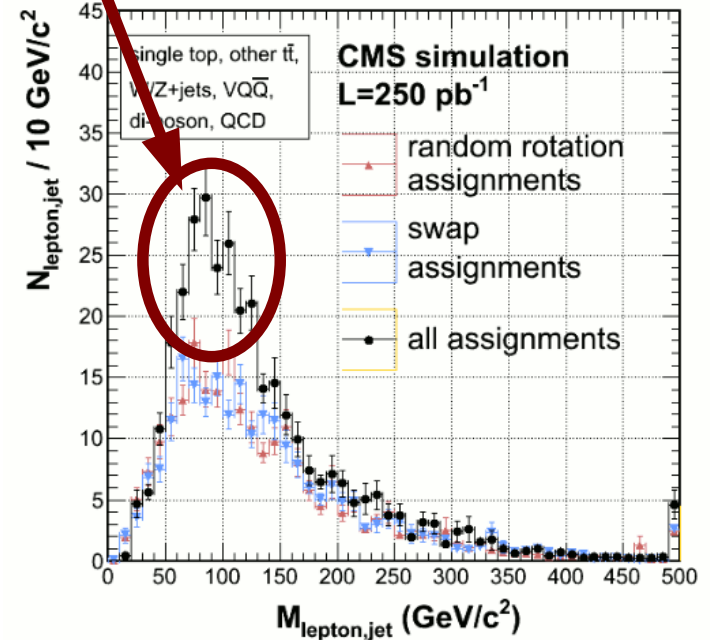
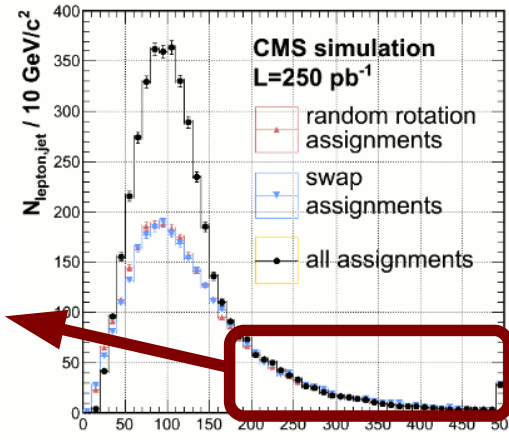
Contributions
with one top

Background

$$1 - \alpha = \frac{N_{mis}}{N} = 1 - \frac{N - N_{mis}}{2N_{evtS}}$$

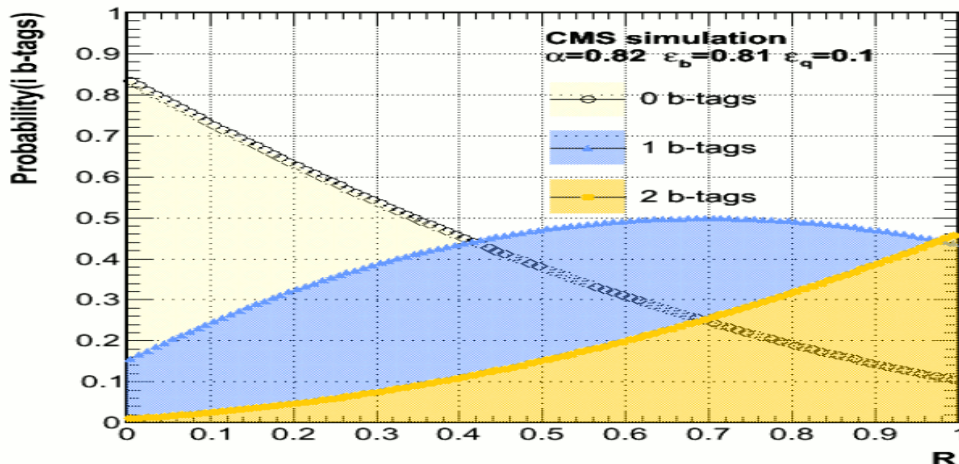
$$N_s = N_{\text{swap/random}} \quad N_{mis}^{M>z} = N_s^{M>z}$$

$$\frac{N_{mis}^{M>z}}{N_{mis}} = \frac{N_s^{M>z}}{N}$$



$$\alpha = \frac{N - N_{mis}}{2 N_{evtS}} = \frac{N}{2 N_{evtS}} \left(1 - \frac{N_{mis}}{N}\right) = \frac{N}{2 N_{evtS}} \left(1 - \frac{N^{|\nu|>z}}{N_s^{M>z}}\right)$$

Probabilistic model for the b-tagging multiplicity



example:

Expect **2 b-tags** in $t \bar{t}$ events from:

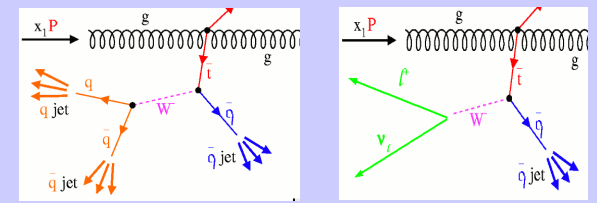
$$P(t \rightarrow Wb; t \rightarrow Wb) \propto R^2 \epsilon_b^2$$

$$+ P(t \rightarrow Wb; t \rightarrow Wq) \propto 2 R (1-R) \epsilon_b \epsilon_q$$

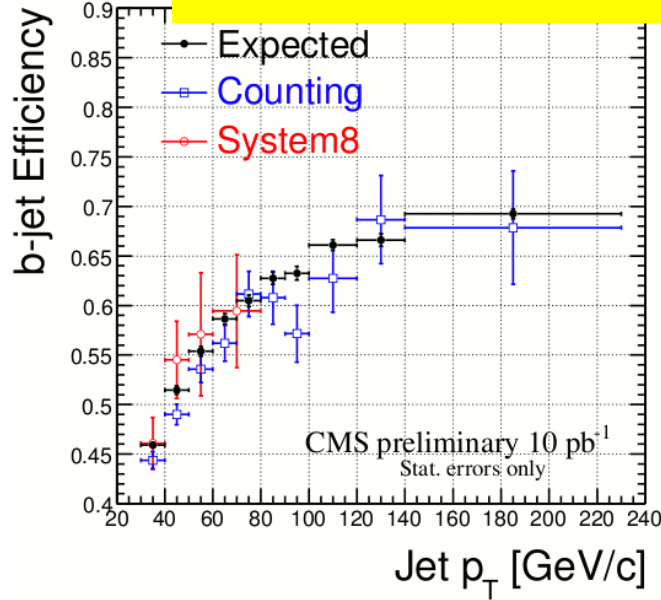
$$+ P(t \rightarrow Wq; t \rightarrow Wq) \propto (1-R)^2 \epsilon_q^2$$

(where $q = d, s$)

For R measurement, ϵ_b will be fixed to the value extracted from an independent sample: di-jet b bbar events



CMS PAS BTV-07-001



System8:

$$\begin{aligned}
 n &= n_b + n_{cl} \\
 p &= p_b + p_{cl} \\
 n^{tag} &= \epsilon_b^{tag} n_b + \epsilon_{cl}^{tag} n_{cl} \\
 p^{tag} &= \beta \epsilon_b^{tag} p_b + \alpha \epsilon_{cl}^{tag} p_{cl} \\
 n^\mu &= \epsilon_b^\mu n_b + \epsilon_{cl}^\mu n_{cl} \\
 p^\mu &= \epsilon_b^\mu p_b + \epsilon_{cl}^\mu p_{cl} \\
 n^{tag,\mu} &= \kappa_b \epsilon_b^{tag} \epsilon_b^\mu n_b + \kappa_{cl} \epsilon_{cl}^{tag} \epsilon_{cl}^\mu n_{cl} \\
 p^{tag,\mu} &= \kappa_b \beta \epsilon_b^{tag} \epsilon_b^\mu p_b + \kappa_{cl} \alpha \epsilon_{cl}^{tag} \epsilon_{cl}^\mu p_{cl}
 \end{aligned}$$

From data

From MC

$$\begin{aligned}
 \kappa_b &= \frac{\epsilon_b^{tag,\mu}}{\epsilon_b^{tag} \epsilon_b^\mu} & \kappa_{cl} &= \frac{\epsilon_{cl}^{tag,\mu}}{\epsilon_{cl}^{tag} \epsilon_{cl}^\mu} \\
 \beta &= \frac{\epsilon_{b,p}^{tag}}{\epsilon_{b,n}^{tag}} & \alpha &= \frac{\epsilon_{cl,p}^{tag}}{\epsilon_{cl,n}^{tag}}
 \end{aligned}$$

8 unknowns:

$$n_b, n_{cl}, p_b, p_{cl}, \epsilon_b^{tag}, \epsilon_b^{\mu}, \epsilon_{cl}^{tag}, \epsilon_{cl}^{\mu}$$

Counting:

Method relying on fit to the p_{Trel} distribution

operating point Luminosity (pb^{-1})	Loose		
	10	100	1000
Counting			
statistics data (%)	2.5	0.9	0.3
Template (%)	15	10	5
Mistag (%)	3.1	3.1	3.1
Total error (%)	16	11	6

operating point Luminosity (pb^{-1})	Loose		
	10	100	1000
Systematics (%)			
β	5.8	5.8	2.9
α	0.4	0.4	0.2
κ_b	3.4	3.4	1.7
κ_{cl}	0.2	0.2	0.1
p_{Trel}	2.8	2.8	2.8
statistics MC (%)	2.3	2.3	2.3
statistics data (%)	7.2	2.3	0.7
Total error (%)	10.5	8.0	6.4