HERWIRI1.0: MC Realization of IR-Improvement for DGLAP-CS Parton Showers

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Outline

- Introduction
- Review of Exact Amplitude-Based Resummation for QED$\otimes$QCD
- IR-Improved DGLAP-CS Theory: Parton Distributions, Kernels, Reduced Cross Sections with Shower/ME Matching
- MC Realization: IR-Improved Kernels in HERWIG6.5
- Conclusions

See B.F.L.W., S. Jadach and B.F.L. Ward, S. Jadach, *et al.*,B.F.L.W. and S. Yost, MPL A **14** (1999) 491, hep-ph/0205062; *ibid.* **12** (1997) 2425; *ibid.***19** (2004) 2113; hep-ph/0503189, 0508140,0509003, 0605054, 0607198, arxiv:0704.0294, 0707.2101, 0707:3424

Motivation

- FOR THE LHC/ILC, THE REQUIREMENTS ARE DEMANDING AND OUR $QED \otimes QCD$ SOFT n(G)-m(γ) MC RESUMMATION RESULTS WILL BE AN IMPORTANT PART OF THE NECESSARY THEORY – (YFS)RESUMMED $\mathcal{O}(\alpha_s^2)L^n, \mathcal{O}(\alpha_s\alpha)L^{n'}, \mathcal{O}(\alpha^2)L^{n''}, n = 0, 1, 2, n' = 0, 1, 2, n'' = 2, 1$, IN THE PRESENCE OF SHOWERS, ON AN EVENT-BY-EVENT BASIS, WITHOUT DOUBLE COUNTING AND WITH EXACT PHASE SPACE.

- HOW RELEVANT ARE QED HIGHER ORDER CORRECTIONS WHEN QCD IS CONTROLLED AT $\sim 1\%$ PRECISION?

- CROSS CHECK OF QCD LITERATURE:

  1. PHASE SPACE – CATANI, CATANI-SEYMOUR, ALL INITIAL PARTONS MASSLESS
  2. RESUMMATION – STERMAN, CATANI ET AL., BERGER ET AL., ...
  3. NO-GO THEOREMS–Di'Lieto et al.,Doria et al.,Catani et al.,Catani
  4. IR QCD EFFECTS IN DGLAP-CS THEORY

- CROSS CHECK OF QED-EW LITERATURE:

  1. ESTIMATES BY SPIESBERGER, STIRLING, ROTH and WEINZIERL, BLUMLEIN and KAWAMURA – FEW PER MILLE EFFECTS FROM QED CORRECTIONS TO STR. FN. EVOLUTION.

  2. WELL-KNOWN POSSIBLE ENHANCEMENT OF QED CORRECTIONS AT THRESHOLD, ESPECIALLY IN RESONANCE PRODUCTION

  3. *See for example, A. Kulesza et al., S. Pozzorini et al., A. Denner et al., in "Proc. RADCOR07", for large (Sudakov log, etc.) EW effects in hadron-hadron scattering – at 1TeV, W's and Z's are almost massless!*

  $\Rightarrow$ HOW TO BEST REALIZE THESE EFFECTS AT THE LHC?

- TREAT QED AND QCD SIMULTANEOUSLY IN THE (YFS) RESUMMATION TO OBTAIN THE ROLE OF THE QED-EW AND TO REALIZE AN APPROACH TO SHOWER/ME MATCHING.

- CURRENT STATE OF AFFAIRS: see N. Adam et al., arxiv.org:0802.3251 – Using MC@NLO and FEWZ, HORACE, PHOTOS, etc., $(4.1 \pm 0.3)\% = (1.51 \pm 0.75)\%(QCD) \oplus 3.79(PDF) \oplus 0.38 \pm 0.26(EW)\%$ accuracy on single Z to leptons at LHC was found($\sim 5.7\%$ for W, see 0808.0758), but no exclusive hard gluon/quark radiation phase space available – the latter are truly needed for realistic theoretical results. They are our goal, at $\lesssim 1\%$.

PRELIMINARY STUDIES

- REPRESENTATIVE PROCESSES
  $pp \rightarrow V + m(\gamma) + n(G) + X \rightarrow \bar{\ell}\ell' + m'(\gamma) + n(G) + X$,
  where $V = W^{\pm}, Z$,and $\ell = e, \mu,\ \ell' = \nu_e, \nu_\mu(e, \mu)$
  respectively for $V = W^{+}(Z)$, and $\ell = \nu_e, \nu_\mu,\ \ell' = e, \mu$
  respectively for $V = W^{-}$.

- Realize IR-improved kernels in state-of-the-art MC environment:
  HERWIG-6.5 => HERWIRI1.0

Recapitulation of QED⊗QCD Resummation

In hep-ph/0210357(ICHEP02), Acta Phys.Polon.B33,1543-1558,2002, Phys.Rev.D52(1995)108;ibid. 66 (2002) 019903(E);PLB342 (1995) 239; Ann.Phys.323(2008)2147;Phys. Rev.D78(2008)056001; Adv. High Energy Phys. 2008 (2008): 682312, we have extended the YFS theory to QCD:

$$\begin{aligned}
d\hat\sigma_{\rm exp} &= \sum_n d\hat\sigma^n \\
&= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}} \\
&* \tilde{\bar\beta}_n(k_1,\ldots,k_n) \frac{d^3 P_2}{P_2^{\,0}} \frac{d^3 Q_2}{Q_2^{\,0}}
\end{aligned} \tag{1}$$

where the new hard gluon residuals $\tilde{\bar\beta}_n(k_1,\ldots,k_n)$ defined by

$$\tilde{\bar\beta}_n(k_1,\ldots,k_n) = \sum_{\ell=0}^{\infty} \tilde{\bar\beta}_n^{(\ell)}(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$. $\Rightarrow$

Simultaneous exponentiation of QED and QCD higher order effects,

hep-ph/0404087(MPLA19(2004)2119), arXiv:0704.0294(APPB38(2007)2395), arXiv:0808.3133, 0810.0723 ,

gives

$$\begin{aligned} B^{nls}_{QCD} &\to B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED}, \\ \tilde{B}^{nls}_{QCD} &\to \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED}, \\ \tilde{S}^{nls}_{QCD} &\to \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED} \equiv \tilde{S}^{nls}_{QCED} \end{aligned} \tag{2}$$

which leads to

$$\begin{aligned} d\hat{\sigma}_{\rm exp} &= e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \\ &\prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \\ &\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3 p_2}{p_2^{\,0}}\frac{d^3 q_2}{q_2^{\,0}}, \end{aligned} \tag{3}$$

where the new YFS residuals

$\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$, **with n hard gluons and m hard photons,** represent the successive application of the YFS expansion first for QCD and subsequently for QED.

The infrared functions are now

$$\begin{aligned}\mathrm{SUM_{IR}(QCED)} &= 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED}\\ D_{\mathrm{QCED}} &= \int \frac{dk}{k^0}\left(e^{-iky} - \theta(K_{max} - k^0)\right)\tilde{S}^{nls}_{QCED}\end{aligned} \quad (4)$$

where K_{max} is a dummy parameter – here the same for QCD and QED.

Infrared Algebra(QCED):

$x_{avg}(QED) \cong \gamma(QED)/(1+\gamma(QED))$

$x_{avg}(QCD) \cong \gamma(QCD)/(1+\gamma(QCD))$

$\gamma(A) = \frac{2\alpha_A \mathcal{C}_A}{\pi}(L_s - 1)$, $A = QED, QCD$

$\mathcal{C}_A = Q_f^2, C_F$, respectively, for $A = QED, QCD$

$\Rightarrow$ QCD dominant corrections happen an order of magnitude earlier than those for QED.

$\Rightarrow$ Leading $\tilde{\bar{\beta}}^{(0,0)}_{0,0}$-level gives a good estimate of the size of the effects we study: see arxiv.org: 0808.3133, and references therein.

Relationship to Sterman-Catani-Trentadue Soft Gluon Resummation

In Phys. Rev. D74 (2006) 074004[MADG], Abayat et al. apply the more familiar resummation for soft gluons to a general $2 \rightarrow n$ parton process [f] at hard scale Q,

$f_1(p_1,r_1)+f_2(p_2,r_2) \rightarrow f_3(p_3,r_3)+f_4(p_4,r_4)+\cdots+f_{n+2}(p_{n+2},r_{n+2})$**,**

where the p_i, r_i label 4-momenta and color indices respectively, with all parton masses set to zero to get

$$\begin{aligned}\mathcal{M}^{[f]}_{\{r_i\}} &= \sum_{L}^{C} \mathcal{M}^{[f]}_{L}(c_L)_{\{r_i\}} \\ &= J^{[f]} \sum_{L}^{C} S_{LI} H^{[f]}_{I}(c_L)_{\{r_i\}},\end{aligned} \tag{5}$$

$J^{[f]}$ **is the jet function**

S_{LI} **is the soft function which describes the exchange of soft gluons between the external lines**

$H_I^{[f]}$ **is the hard coefficient function**

infrared and collinear poles calculated to 2-loop order.

To make contact with our approach, identify in $\bar{Q}'Q \to \bar{Q}'''Q'' + m(G)$ **in (1)** $f_1 = Q, \bar{Q}', f_2 = \bar{Q}', f_3 = Q'', f_4 = \bar{Q}''', \{f_5, \cdots, f_{n+2}\} = \{G_1, \cdots, G_m\}$

$\Rightarrow n = m + 2$ **here.**

Observe the following:

- **By its definition in eq.(2.23) of [MADG], the anomalous dimension of the matrix** S_{LI} **does not contain any of the diagonal effects described by our infrared functions** $\Sigma_{IR}(QCD)$ **and** D_{QCD}**.**
- **By its definition in eqs.(2.5) and (2.7) of [MADG], the jet function** $J^{[f]}$ **contains the exponential of the virtual infrared function** $\alpha_s \Re B_{QCD}$**, so that we have to take care that we do not double count when we use (5) in (1) and the equations that lead thereto.**

$\Rightarrow$

We identify $\bar\rho^{(m)}$ in our theory as

$$\bar\rho^{(m)}(p_1,q_1,p_2,q_2,k_1,\cdots,k_m) = \overline{\sum}_{colors,spin}|\mathcal{M}'^{[f]}_{\{r_i\}}|^2$$

$$\equiv \sum_{spins,\{r_i\},\{r'_i\}} \mathfrak{h}^{cs}_{\{r_i\}\{r'_i\}}|\bar{J}^{[f]}|^2 \sum_{L=1}^{C}\sum_{L'=1}^{C} S^{[f]}_{LI}H^{[f]}_{I}(c_L)_{\{r_i\}}\left(S^{[f]}_{L'I'}H^{[f]}_{I'}(c_{L'})_{\{r'_i\}}\right)^{\dagger} \tag{6}$$

where here we defined $\bar{J}^{[f]} = e^{-\alpha_s \Re B_{QCD}} J^{[f]}$, and we introduced the color-spin density matrix for the initial state, $\mathfrak{h}^{cs}$.

Here, we recall (see Ann.Phys.323(2008)2147;Phys. Rev.D78(2008)056001; Adv. High Energy Phys. 2008 (2008): 682312, for example) that in our theory, we have

$$d\hat\sigma^n = \frac{e^{2\alpha_s ReB_{QCD}}}{n!}\int \prod_{m=1}^{n}\frac{d^3k_m}{(k_m^2+\lambda^2)^{1/2}}\delta(p_1+q_1-p_2-q_2-\sum_{i=1}^{n}k_i)$$

$$\bar\rho^{(n)}(p_1,q_1,p_2,q_2,k_1,\cdots,k_n)\frac{d^3p_2 d^3q_2}{p_2^0 q_2^0}, \tag{7}$$

for n-gluon emission. ⇒ Repeat usual steps to get our formula (1), no double counting of effects - in progress. Today, we show platform for this progress.

IR-Improved DGLAP-CS Theory: Parton Distributions, Kernels, Reduced Cross Sections with Shower/ME Matching

IR-Improved DGLAP-CS Theory

Exponentiation of QCD higher order effects: Where to apply?

Ann.Phys.323(2008)2147;Phys. Rev.D78(2008)056001; Adv. High Energy Phys. 2008 (2008): 682312,

consider

$$\frac{dq^{NS}(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y} q^{NS}(y,t) P_{qq}(x/y) \tag{8}$$

where the well-known result for the kernel $P_{qq}(z)$ is, for $z < 1$,

$$P_{qq}(z) = C_F \frac{1+z^2}{1-z}, \tag{9}$$

$t = \ln \mu^2/\mu_0^2$ for some reference scale μ_0. $\Rightarrow$

Unintegrable singularity at $z = 1$, usually regularized by

$$\frac{1}{(1-z)} \rightarrow \frac{1}{(1-z)_+} \tag{10}$$

with $\frac{1}{(1-z)_+}$ such that

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}. \tag{11}$$

$\Rightarrow$

$$\frac{1}{(1-z)_+} = \frac{1}{(1-z)}\theta(1-\epsilon-z) + \ln\epsilon\,\delta(1-z) \tag{12}$$

with the understanding that $\epsilon \downarrow 0$.

Require

$$\int_0^1 dz P_{qq}(z) = 0, \tag{13}$$

$\Rightarrow$ **add virtual corrections to get**

$$P_{qq}(z) = C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right). \tag{14}$$

Observations

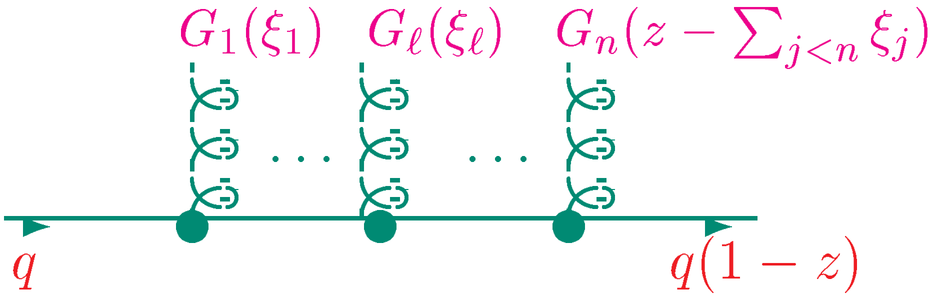
- Smooth, divergent $1/(1-z)$ behavior as $z \to 1$ replaced with a mathematical artifact: the regime $1-\epsilon < z < 1$ now has no probability at all; at $z = 1$ we have a large negative integrable contribution $\Rightarrow$ a finite (zero) value for the total integral of $P_{qq}(z)$
- LEP1,2 experience: such mathematical artifacts, while correct, impair precision.

Why set $P_{qq}(z)$ to 0 for $1-\epsilon < z < 1$ where it actually has its largest values?

- USE EXPERIENCE FROM LEP1,2: $\frac{1}{(1-z)_+}$ SHOULD BE EXPONENTIATED –SEE CERN YELLOW-BOOKS, CERN-89-08., YIELDING FROM (1) THE REPLACEMENT

$$
\begin{aligned}
P_{BA} &= \frac{1}{2} z(1-z)\overline{\sum_{spins}} \frac{|V_{A\rightarrow B+C}|^2}{p_\perp^2} \\
&\Rightarrow \\
P_{BA} &= \frac{1}{2} z(1-z)\overline{\sum_{spins}} \frac{|V_{A\rightarrow B+C}|^2}{p_\perp^2} z^{\gamma_q} F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q}
\end{aligned}
\tag{15}
$$

WHERE $A = q$, $B = G$, $C = q$ AND $V_{A\rightarrow B+C}$ IS THE LOWEST ORDER AMPLITUDE FOR $q \rightarrow G(z) + q(1-z)$.



$$q \rightarrow q(1-z) + G_1 + \cdots + G_n$$

$\Rightarrow$

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+z^2}{1-z}(1-z)^{\gamma_q} \tag{16}$$

where

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0} \tag{17}$$

$$\delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}) \tag{18}$$

and

$$F_{YFS}(\gamma_q) = \frac{e^{-C_E \gamma_q}}{\Gamma(1+\gamma_q)}. \tag{19}$$

Note: $\int_{k_0} dz/z = C_0 - \ln k_0$

is experimentally distinguishable from

$\int_{k_0} dz/z^{1-\gamma} = C'_0 - k_0^{\gamma}/\gamma$.

- **NORMALIZATION CONDITION (13) $\Rightarrow$:**

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right] \qquad (20)$$

where

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{\gamma_q+1} + \frac{1}{\gamma_q+2}. \qquad (21)$$

- **THIS IS OUR IR-IMPROVED P_{qq} DGLAP-CS KERNEL.**

$\Rightarrow$ STANDARD DGLAP-CS THEORY:

for $z < 1$, we have

$$P_{Gq}(z) = P_{qq}(1-z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}. \qquad (22)$$

$\Rightarrow$ TEST OF NEW THEORY – QUARK MOMENTUM SUM RULE:

$$\int_0^1 dz z \left(P_{Gq}(z) + P_{qq}(z)\right) = 0. \qquad (23)$$

$\Rightarrow$ **CHECK VANISHING OF**

$$I = \int_0^1 dz z \left(\frac{1+(1-z)^2}{z} z^{\gamma_q} + \frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right). \quad (24)$$

NOTE,

$$\frac{z}{1-z} = \frac{z-1+1}{1-z} = -1 + \frac{1}{1-z}. \quad (25)$$

$\Rightarrow$

$$\begin{aligned} I = \int_0^1 dz\{ &((1+(1-z)^2)z^{\gamma_q} - (1+z^2)(1-z)^{\gamma_q} \\ &+ \frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z)\} \\ = 0 & \end{aligned}$$

QUARK MOMENTUM SUM RULE IS SATISFIED.

- **For $P_{qG}(z),\ P_{GG}(z)$, we get, with the replacement $C_F \rightarrow C_G$ in the IR algebra, that the usual results**

$$\begin{aligned} P_{GG}(z) &= 2C_G\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right) \\ P_{qG}(z) &= \frac{1}{2}\left(z^2+(1-z)^2\right) \end{aligned} \tag{26}$$

become

$$\begin{aligned} P_{GG}(z) &= 2C_G F_{YFS}(\gamma_G)e^{\frac{1}{2}\delta_G}\left\{\frac{1-z}{z}z^{\gamma_G}+\frac{z}{1-z}(1-z)^{\gamma_G}\right. \\ &\left.+\frac{1}{2}\left(z^{1+\gamma_G}(1-z)+z(1-z)^{1+\gamma_G}\right)-f_G(\gamma_G)\delta(1-z)\right\}, \end{aligned} \tag{27}$$

$$P_{qG}(z) = F_{YFS}(\gamma_G)e^{\frac{1}{2}\delta_G}\frac{1}{2}\left\{z^2(1-z)^{\gamma_G}+(1-z)^2 z^{\gamma_G}\right\}, \tag{28}$$

where

$$\gamma_G = C_G\frac{\alpha_s}{\pi}t = \frac{4C_G}{\beta_0} \tag{29}$$

$$\delta_G = \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}), \tag{30}$$

$$f_G(\gamma_G) = \frac{n_f}{C_G}\frac{1}{(1+\gamma_G)(2+\gamma_G)(3+\gamma_G)} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} \tag{31}$$

$$+ \frac{1}{(1+\gamma_G)(2+\gamma_G)} + \frac{1}{2(3+\gamma_G)(4+\gamma_G)} \tag{32}$$

$$+ \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)}. \tag{33}$$

THE GLUON MOMENTUM SUM RULE HAS BEEN USED.

- **THIS DEFINES THE NEW IR-IMPROVED DGLAP-CS THEORY.**

IR-IMPROVED DGLAP-CS KERNELS

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right], \tag{34}$$

$$P_{Gq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}, \tag{35}$$

$$\begin{aligned} P_{GG}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{ &\frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G} \\ &+ \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z)\}, \end{aligned} \tag{36}$$

$$P_{qG}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \}. \tag{37}$$

Higher Order DGLAP-CS Kernels

Connection with the exact $\mathcal{O}(\alpha_s^2),\ \mathcal{O}(\alpha_s^3)$ kernel results of Curci, Furmanski and Petronzio, Floratos et al., Moch et al., etc., is immediate:

For example, non-singlet case, using standard notation,

$$P_{ns}^{+} = P_{qq}^{v} + P_{q\bar{q}}^{v} \equiv \sum_{n=0}^{\infty} (\frac{\alpha_s}{4\pi})^{n+1} P_{ns}^{(n)+} \qquad (38)$$

where at order $\mathcal{O}(\alpha_s)$ we have

$$P_{ns}^{(0)+}(z) = 2C_F\{\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\} \qquad (39)$$

$\Rightarrow P_{ns}^{(0)+}(z)$ **agrees with the unexponentiated result for P_{qq} except for an overall factor of 2. Floratos et al., etc., have exact result for $P_{ns}^{(1)+}(z)$, and Moch et al. have** exact results for $P_{ns}^{(2)+}(z)$. Applying (1) to $q \to q + X$, $\bar{q} \to q + X'$, we get

$$P_{ns}^{+,exp}(z) = (\frac{\alpha_s}{4\pi})2P_{qq}^{exp}(z) + F_{YFS}(\gamma_q)e^{\frac{1}{2}\delta_q}\Big[(\frac{\alpha_s}{4\pi})^2\{(1-z)^{\gamma_q}\bar{P}_{ns}^{(1)+}(z) + \bar{B}_2\delta(1-z)\} + (\frac{\alpha_s}{4\pi})^3\{(1-z)^{\gamma_q}\bar{P}_{ns}^{(2)+}(z) + \bar{B}_3\delta(1-z)\}\Big] \quad (40)$$

where $P_{qq}^{exp}(z)$ is given above and the resummed residuals $\bar{P}_{ns}^{(i)+}$, $i = 1, 2$ are related to the exact results for $P_{ns}^{(i)+}$, $i = 1, 2$, as follows:

$$\bar{P}_{ns}^{(i)+}(z) = P_{ns}^{(i)+}(z) - B_{1+i}\delta(1-z) + \Delta_{ns}^{(i)+}(z) \quad (41)$$

where

$$\begin{aligned}\Delta_{ns}^{(1)+}(z) &= -4C_F\pi\delta_1\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\}\\ \Delta_{ns}^{(2)+}(z) &= -4C_F(\pi\delta_1)^2\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\}\\ &\quad - 2\pi\delta_1\bar{P}_{ns}^{(1)+}(z)\end{aligned} \quad (42)$$

and

$$\bar{B}_2 = B_2 + 4C_F\pi\delta_1 f_q$$
$$\bar{B}_3 = B_3 + 4C_F(\pi\delta_1)^2 f_q - 2\pi\delta_1\bar{B}_2. \tag{43}$$

The constants $B_i,\ i = 2, 3$ are given by

$$\begin{aligned}
B_2 &= 4C_GC_F(\frac{17}{24} + \frac{11}{3}\zeta_2 - 3\zeta_3) - 4C_Fn_f(\frac{1}{12} + \frac{2}{3}\zeta_2) + 4C_F^2(\frac{3}{8} - 3\zeta_2 + 6\zeta_3)\\
B_3 &= 16C_GC_Fn_f(\frac{5}{4} - \frac{167}{54}\zeta_2 + \frac{1}{20}\zeta_2^2 + \frac{25}{18}\zeta_3)\\
&\quad + 16C_GC_F^2(\frac{151}{64} + \zeta_2\zeta_3 - \frac{205}{24}\zeta_2 - \frac{247}{60}\zeta_2^2 + \frac{211}{12}\zeta_3 + \frac{15}{2}\zeta_5)\\
&\quad + 16C_G^2C_F(-\frac{1657}{576} + \frac{281}{27}\zeta_2 - \frac{1}{8}\zeta_2^2 - \frac{97}{9}\zeta_3 + \frac{5}{2}\zeta_5)\\
&\quad + 16C_Fn_F^2(-\frac{17}{144} + \frac{5}{27}\zeta_2 - \frac{1}{9}\zeta_3)\\
&\quad + 16C_F^2n_F(-\frac{23}{16} + \frac{5}{12}\zeta_2 + \frac{29}{30}\zeta_2^2 - \frac{17}{6}\zeta_3)\\
&\quad + 16C_F^3(\frac{29}{32} - 2\zeta_2\zeta_3 + \frac{9}{8}\zeta_2 + \frac{18}{5}\zeta_2^2 + \frac{17}{4}\zeta_3 - 15\zeta_5).
\end{aligned} \tag{44}$$

Contact with Wilson Expansion

N-th moment of the invariants $T_{i,\ell},\ i=L,2,3,\ \ell=q,G,$ of the forward Compton amplitude in DIS:(Gorishni et al.)

$$\mathcal{P}_N \equiv \left[\frac{q^{\{\mu_1}\cdots q^{\mu_N\}}}{N!}\frac{\partial^N}{\partial p^{\mu_1}\cdots\partial p^{\mu_N}}\right]|_{p=0}, \tag{45}$$

$x_{Bj}=Q^2/(2qp)$ in the standard DIS notation – Projects the coefficient of $1/(2x_{Bj})^N$. Terms which we resum here $\Leftrightarrow$ Formally γ_q-dependent anomalous dimensions associated with the respective coefficient, not in Wilson's expansion by usual definition.:

LARGE λ NOT ALL ON TIP OF LIGHTCONE.

COMMENTS

$(*)$ **IRI-DGLAP-CS RESUMS IR SINGULAR ISR;BY FACTORIZATION THIS IS NOT CONTAINED IN ANY RESUMMATION OF HARD SHORT-DISTANCE COEFFICIENT FN CORRECTIONS AS IN THE STERMAN, CATANI-TRENTADUE, COLLINS ET AL. FORMULAS**

$(**)$ **WE DO NOT CHANGE THE PREDICTED HADRON CROSS SECTION:**

$$\begin{aligned}\sigma &= \sum_{i,j}\int dx_1dx_2F_i(x_1)F_j(x_2)\hat\sigma(x_1x_2s)\\ &= \sum_{i,j}\int dx_1dx_2{F'}_i(x_1){F'}_j(x_2)\hat\sigma'(x_1x_2s)\end{aligned} \tag{46}$$

ORDER BY ORDER IN PERTURBATION THEORY.

$\{P^{exp}\}$ **factorize** $\hat\sigma_{\text{unfactorized}} \Rightarrow \hat\sigma'$ **– NEW SCHEME**

$\{P\}$ **factorize** $\hat\sigma_{\text{unfactorized}} \Rightarrow \hat\sigma$

$(***)$ **QUARK NUMBER CONSERVATION AND CANCELLATION OF IR SINGULARITIES IN XSECTS: Quaranteed by fundamental quantum field theoretic principles: Global Gauge Invariance, Unitarity – Everybody may use these principles.**

Effects on Parton Distributions

Moments of kernels ⇔ Logarithmic exponents for evolution

$$\frac{dM_n^{NS}(t)}{dt} = \frac{\alpha_s(t)}{2\pi} A_n^{NS} M_n^{NS}(t) \tag{47}$$

where

$$M_n^{NS}(t) = \int_0^1 dz z^{n-1} q^{NS}(z,t) \tag{48}$$

and the quantity A_n^{NS} is given by

$$\begin{aligned} A_n^{NS} &= \int_0^1 dz z^{n-1} P_{qq}(z), \\ &= C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)] \end{aligned} \tag{49}$$

where $B(x,y)$ is the beta function given by

$$B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$$

.

Compare the usual result

$$A_n^{NS^o} \equiv C_F\left[-\frac{1}{2}+\frac{1}{n(n+1)}-2\sum_{j=2}^{n}\frac{1}{j}\right]. \qquad (50)$$

- **ASYMPTOTIC BEHAVIOR: IR-improved goes to a multiple of $-f_q$, consistent with** $\lim_{n\to\infty} z^{n-1}=0$ **for** $0\le z<1$**; usual result diverges as** $-2C_F\ln n$**.**
- **Different for finite n as well: for** $n=2$ **we get, for example, for** $\alpha_s\cong .118$**,**

$$A_2^{NS}=\begin{cases}C_F(-1.33) & ,\ \textbf{un-IR-improved}\\ C_F(-0.966) & ,\ \textbf{IR-improved}\end{cases} \qquad (51)$$

- **For completeness we note**

$$\begin{aligned} M_n^{NS}(t) &= M_n^{NS}(t_0)e^{\int_{t_0}^{t} dt' \frac{\alpha_s(t')}{2\pi} A_n^{NS}(t')} \\ &= M_n^{NS}(t_0)e^{\bar{a}_n[Ei(\frac{1}{2}\delta_1\alpha_s(t_0)) - Ei(\frac{1}{2}\delta_1\alpha_s(t))]} \\ &\underset{t,t_0\ \text{large with}\ t>>t_0}{\Longrightarrow} M_n^{NS}(t_0)\left(\frac{\alpha_s(t_0)}{\alpha_s(t)}\right)^{\bar{a'}_n} \end{aligned} \tag{52}$$

where $Ei(x) = \int_{-\infty}^{x} dre^r/r$ is the exponential integral function,

$$\begin{aligned} \bar{a}_n &= \frac{2C_F}{\beta_0} F_{YFS}(\gamma_q)e^{\frac{\gamma_q}{4}}[B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)] \\ \bar{a'}_n &= \bar{a}_n\left(1 + \frac{\delta_1}{2}\frac{(\alpha_s(t_0) - \alpha_s(t))}{\ln(\alpha_s(t_0)/\alpha_s(t))}\right) \end{aligned} \tag{53}$$

with

$\delta_1 = \frac{C_F}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right)$.

Compare with un-IR-improved result where last line in eq.(52) holds exactly with $\bar{a'}_n = 2A_n^{NS^o}/\beta_0$.

- For $n = 2$, taking $Q_0 = 2$GeV and evolving to $Q = 100$GeV, with $\Lambda_{QCD} \cong .2GeV$ and $n_f = 5$ for illustration, (52,53) $\Rightarrow$ a shift of evolved NS moment by $\sim\ 5\%$, of some interest in view of the expected HERA precision (see for example, T. Carli et al., Proc. HERA-LHC Wkshp, 2005).

ANOTHER EXAMPLE: THRESHOLD CORRECTIONS

We have applied the new simultaneous QED$\otimes$QCD exponentiation calculus to the single Z production with leptonic decay at the LHC (and at FNAL) to focus on the ISR alone, for definiteness. See also the work of Baur *et al.*, Dittmaier and Kramer, Zykunov for exact $\mathcal{O}(\alpha)$ results and Hamberg *et al.*, van Neerven and Matsuura and Anastasiou *et al.* for exact $\mathcal{O}(\alpha_s^2)$ results.

For the basic formula

$$d\sigma_{exp}(pp \to V+X \to \bar{\ell}\ell'+X') = \sum_{i,j}\int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s), \tag{54}$$

we use the result in (3) here with semi-analytical methods and structure functions from Martin *et al.*. A MC realization will appear, but see below.

SHOWER/ME MATCHING

- Note the following: In (54) WE DO NOT ATTEMPT *HERE* TO REPLACE HERWIG and/or PYTHIA – WE INTEND *HERE* TO COMBINE OUR EXACT YFS CALCULUS, $d\hat{\sigma}_{exp}(x_i x_j s)$, WITH HERWIG and/or PYTHIA BY USING THEM/IT TO GENERATE A PARTON SHOWER STARTING FROM (x_1, x_2) AT FACTORIZATION SCALE μ AFTER THIS POINT IS PROVIDED BY $\{F_i\}$: THERE ARE TWO APPROACHES TO THE MATCHING UNDER STUDY, ONE BASED ON p_T-MATCHING AND ONE BASED ON SHOWER-SUBTRACTED RESIDUALS $\{\hat{\bar{\bar{\beta}}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\}$, WHEREIN THE SHOWER FORMULA AND THE $QED \otimes QCD$ EXPONENTIATION FORMULA CAN BE EXPANDED IN PRODUCT AND REQUIRED TO MATCH THE GIVEN EXACT RESULT TO THE SPECIFIED ORDER – SEE hep-ph/0509003.
- THIS COMBINATION OF THEORETICAL CONSTRUCTS CAN BE SYSTEMATICALLY IMPROVED WITH EXACT RESULTS ORDER-BY-ORDER IN α_s, α, WITH EXACT PHASE SPACE.
- THE RECENT ALTERNATIVE PARTON EVOLUTION ALGORITHM BY JADACH and SKRZYPEK,Acta. Phys. Pol.B35, 745 (2004), CAN ALSO BE USED.
- LACK OF COLOR COHERENCE $\Rightarrow$ ISAJET NOT CONSIDERED HERE.

With this said, we compute , with and without QED, the ratio

$r_{exp} = \sigma_{exp}/\sigma_{Born}$

to get the results (We stress that we *do not* use the narrow resonance approximation here.)

$$r_{exp} = \begin{cases} 1.1901 & \text{, QCED} \equiv \text{QCD+QED, LHC} \\ 1.1872 & \text{, QCD, LHC} \\ 1.1911 & \text{, QCED} \equiv \text{QCD+QED, Tevatron} \\ 1.1879 & \text{, QCD, Tevatron} \end{cases} \quad (55)$$

$\Rightarrow$

QED IS AT .3% AT BOTH LHC and FNAL.

$\ast$***THIS IS STABLE UNDER SCALE VARIATIONS.***

$\ast$***WE AGREE WITH BAUR ET AL., HAMBERG ET AL., van NEERVEN and ZIJLSTRA–NOTE THAT AS WE HAVE AN EXPONENTITED FORMULA, IT MAKES SENSE TO COMPARE WITH THE LATTER.***

$\ast$***QED EFFECT SIMILAR IN SIZE TO STR. FN. RESULTS.***

$\ast$***DGLAP-CS SYNTHESIZATION HAS NOT COMPROMISED THE NORMALIZATION.***

QUARK MASSES and RESUMMATION in PRECISION QCD THEORY

(PHYS. REV.D78(2008)056001)

- Di'Lieto et al.(NPB183(1981)223), Doria et al.(*ibid.*168(1980)93), Catani et al.(*ibid.*264(1986)588;Catani(ZPC37(1988)357): IN ISR, BLOCH-NORDSIECK CANCELLATION FAILS AT $\mathcal{O}(\alpha_s^2)$ for $m_q \neq 0$.
- FOR $q + q' \rightarrow q'' + q''' + V + X$, THEY GET

$$\text{flux}\ \frac{d\sigma}{d^3Q} = \frac{-g^4\bar{H}}{(d-4)32\pi^2}\left(\frac{1-\beta}{\beta}\right)\left(\frac{1}{\beta}\ln(\frac{1+\beta}{1-\beta}) - 2\right) \tag{56}$$

- HERE, $\bar{H}$ IS THE HARD PROCESS DRESSED AS

$$F_1 = C_2(G)H_{ab}^{\alpha\beta}(T_i)^{\beta\alpha}(T_i)_b a \tag{57}$$

FOR

$$f_{ijk}f_{ijl} = C_2(G)\delta_{kl}$$
$$(T_iT_i)_{ab} = C_2(F)I_{ab}.$$

THEY EVALUATE THE GRAPHS IN FIG.1 USING MUELLER'S THM.

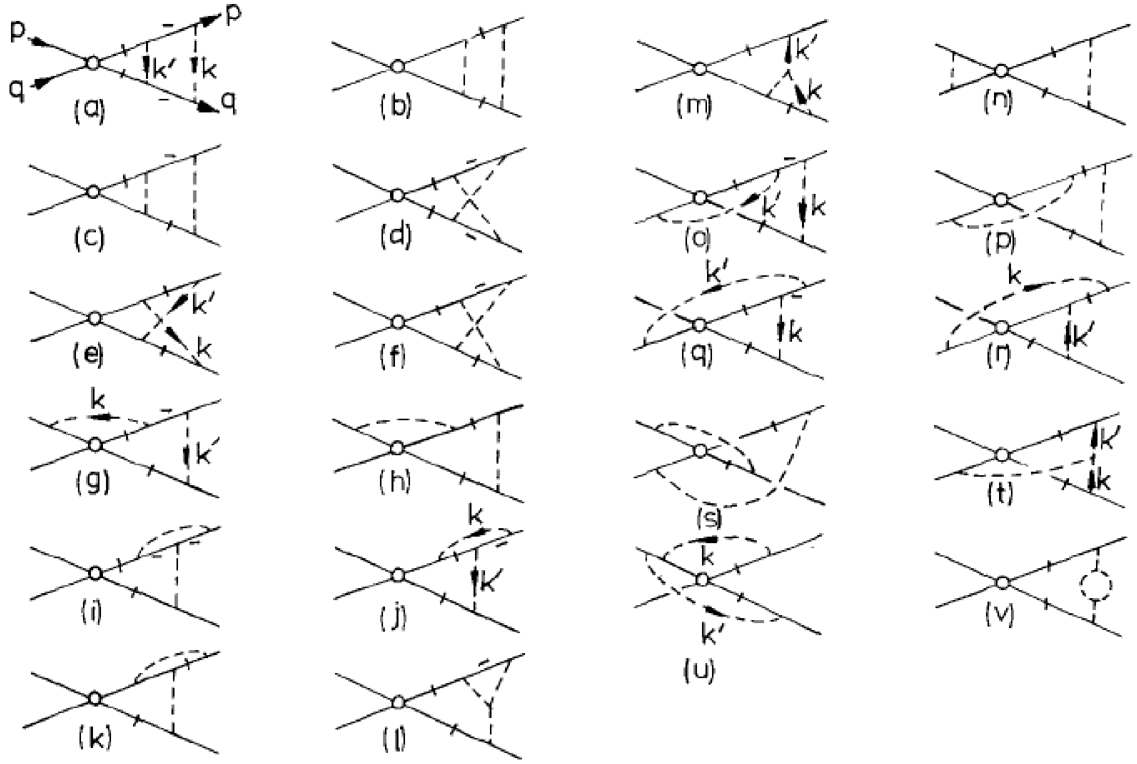


Figure 1. Graphs evaluated in Ref. [2] (see the first paper therein especially) in arriving at the result in (3) using Mueller's theorem for the respective cross section. The usual Landau-Bjorken-Cutkosky (LBC) [10] rules obtain so that a slash puts the line on-shell and a dash changes the iε-prescription; and, graphs that have cancelled or whose contributions are implied by those in the figure are not shown explicitly.

- SINCE BN VIOLATION VANISHES FOR $m_q \to 0$, MUST SET $m_q = 0$ IN ISR for $\mathcal{O}(\alpha_s^n),\ n \geq 2$: NOTE, $m_b \cong 5$GeV.
- SOURCE OF BN-VIOLATION: LOOK AT CONTRIBUTION OF DIAGRAMS (q-o) IN FIG.1:

$$A_{q-o} = \frac{1}{\beta^2}\int \frac{d^3k d^3k' 2k_z}{(k_z + k'_z + i\epsilon)(\beta^2 k_z^2 - \mathbf{k}^2)(\beta^2 k_z^2 - \mathbf{k'}^2 + i\epsilon)(k_z^2 + \epsilon^2)} \tag{58}$$

UV-REGULATED RESULT: USE THE REGULATOR $e^{-\mathbf{k}^2/\Lambda^2}$,

$\Rightarrow$

$$A_{q-o}|_{UV-reg} = \frac{4\pi^{n+1}(\Lambda^2)^{n-3}}{\beta^2}\left\{\frac{1}{(n-3)^2} + \frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right\}. \tag{59}$$

$\Rightarrow$

$$F_{nbn} = \frac{(1-\beta)(\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta)}{\ln\left(\frac{1+\beta}{1-\beta}\right)} \tag{60}$$

IS FRACTION OF SINGLE-POLE TERM UN-CANCELLED.

- ANALYSIS IN PRD 78 (2008) 056001 $\Rightarrow$ REAL EMISSION IN A_{q-o} SATURATES SINGLE IR POLE.
- THUS, WE WRITE

$$\text{flux}\ \frac{d\sigma}{d^3Q} = \frac{-g^4\bar{H}}{64\pi^6} F_{nbn} A_{q-o}|_{\mathfrak{R},\text{real rad., IR pole part}}, \tag{61}$$

WHERE FROM PRD78(2008)056001 WE HAVE

$$A_{q-o}|_{\mathfrak{R},\text{real rad., IR pole part}} = \frac{4\pi^4}{\beta^2}\left(\frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right). \tag{62}$$

- APPLY QCD RESUMMATION TO REAL EMISSION IN $A_{q-o}|_{\mathfrak{R}}$: APPLY IT TO THE FRACTION F_{nbn}; REMAINING $1-F_{nbn}$ CANCELLED BY VIRTUAL CORRECTIONS

- USING

$$
\begin{aligned}
d\hat{\sigma}_{\rm exp} &= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-p_X-\sum k_j)} \\
&\quad * e^{D_{\rm QCD}} \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3p_2}{p_2^{\,0}} \frac{d^3q_2}{q_2^{\,0}} \frac{d^3p_X}{p_X^{\,0}}
\end{aligned} \tag{63}
$$

WE GET

$$
\begin{aligned}
F_{nbn} A_{q-o}\big|_{\Re,\text{real rad., resummed}} &= F_{nbn} \Re \frac{-i\pi^2}{\beta^2} \int d^2k_\perp \int_0^{\sqrt{\epsilon}} dk_z F_{YFS}(\bar{\gamma}_q) e^{\bar{\delta}_q/2} \\
&\quad (\beta k_z)^{\bar{\gamma}_q} \left(-\ln(k_z+i\epsilon-\beta k_z)+\ln(k_z+i\epsilon+\beta k_z)\right) \\
&\quad \frac{1}{\beta^2 k_z^2-\mathbf{k}^2} \frac{2k_z}{k_z^2+\epsilon^2},
\end{aligned} \tag{64}
$$

WHERE WE HAVE DEFINED

$$\bar{\gamma}_q = 2C_F\frac{\alpha_s(Q^2)}{\pi}(\ln(s/m^2)-1) \tag{65}$$

$$\bar{\delta}_q = \frac{\bar{\gamma}_q}{2} + \frac{2\alpha_s C_F}{\pi}(\frac{\pi^2}{3}-\frac{1}{2}). \tag{66}$$

- USING THE SUBSTITUTION $k_z = \sqrt{\epsilon}\bar{k}_z$, WE HAVE

$$F_{nbn}A_{q-o}|_{\mathfrak{R},\text{real rad., resummed}} = F_{nbn}\Re\frac{-i\pi^2\epsilon^{\frac{\bar{\gamma}_q}{2}}}{\beta^2}\int d^2k_\perp\int_0^1 d\bar{k}_z F_{YFS}(\bar{\gamma}_q)e^{\bar{\delta}_q/2}$$
$$(\beta\bar{k}_z)^{\gamma_q}\left(-\ln(\bar{k}_z+i\sqrt{\epsilon}-\beta\bar{k}_z)+\ln(\bar{k}_z+i\sqrt{\epsilon}+\beta\bar{k}_z)\right)$$
$$\frac{1}{-(1-\beta^2)\epsilon\bar{k}_z^2-\mathbf{k}_\perp{}^2}\frac{2\bar{k}_z}{\bar{k}_z^2+\epsilon}. \tag{67}$$

THE RHS OF THIS LAST EQUATION VANISHES AS $\epsilon \longrightarrow 0$, REMOVING THE VIOLATION OF BLOCH-NORDSIECK CANCELLATION IN (56).

CONCLUSION:

- RESUMMATION CURES LACK OF BN CANCELLATION IN MASSIVE QCD

MC Realization: IR-Improved Kernels in HERWIG6.5

- Approach:
  - Modify the kernels in the HWBRAN and Related Modules - (BW,MS)

$$\text{DGLAP-CS } P_{AB} \Rightarrow \text{IR-I DGLAP-CS } P^{exp}_{AB} \tag{68}$$

  - Leave Hard Processes Alone for the Moment:

    In progress (SY,BFLW,MH,SM,SJ)– include YFS synthesized EW modules from Jadach et al. MC's for HERWIG6.5,++ hard processes.
  - ISSUE: CTEQ and MRST BEST(after 2007) P.Dstrbn. Fns DO NOT INCLUDE PRECISION EW HO CORR.

Implementation Illustration

Probability that no branching occurs above virtuality cutoff Q_0^2 is $\Delta_a(Q^2,Q_0^2)$

⇒

$$d\Delta_a(t,Q_0^2) = \frac{-dt}{t}\Delta(t,Q_o^2)\sum_b \int dz \frac{\alpha_s}{2\pi}P_{ba}(z), \tag{69}$$

⇒

$$\Delta_a(Q^2,Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2}\frac{dt}{t}\sum_b \int dz \frac{\alpha_s}{2\pi}P_{ba}(z)\right]. \tag{70}$$

Non-branching probability appearing in the evolution equation is

$$\Delta(Q^2,t) = \frac{\Delta_a(Q^2,Q_o^2)}{\Delta_a(t,Q_o^2)}, \quad t = k_a^2 \text{ the virtuality of gluon } a. \tag{71}$$

Virtuality of parton a is generated with

$$\Delta_a(Q^2,t) = R, \tag{72}$$

where R is a random number uniformly distributed in $[0,1]$.

With

$$\alpha_s(Q) = \frac{2\pi}{b_0 \log\left(\frac{Q}{\Lambda}\right)}, \tag{73}$$

we get

$$\begin{aligned}\int_0^1 dz \frac{\alpha_s(Q^2)}{2\pi} P_{qG}(z) &= \frac{4\pi}{2\pi b_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)} \int_0^1 dz \frac{1}{2}\left[z^2 + (1-z)^2\right] \\ &= \frac{2}{3}\frac{1}{b_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}. \end{aligned} \tag{74}$$

⇒

$$I = \int_{Q_0^2}^{Q^2} \frac{1}{3}\frac{dt}{t}\frac{2}{b_0 \ln\left(\frac{t}{\Lambda^2}\right)},$$

$$\begin{aligned} I = & \frac{2}{3b_0} \ln\ln \frac{t}{\Lambda^2}\Big|_{Q_0^2}^{Q^2} \\ = & \frac{2}{3b_0}\left[\ln\left(\frac{\ln\left(\frac{Q^2}{\Lambda^2}\right)}{\ln\left(\frac{Q_0^2}{\Lambda^2}\right)}\right)\right]. \end{aligned} \quad (75)$$

Finally

$$\begin{aligned} \Delta_a(Q^2, Q_0^2) &= \exp\left[-\frac{2}{3b_0}\ln\left(\frac{\ln\left(\frac{Q^2}{\Lambda^2}\right)}{\ln\left(\frac{Q_0^2}{\Lambda^2}\right)}\right)\right] \\ &= \left[\frac{\ln\left(\frac{Q^2}{\Lambda^2}\right)}{\ln\left(\frac{Q_0^2}{\Lambda^2}\right)}\right]^{-\frac{2}{3b_0}}. \end{aligned} \quad (76)$$

Let $\Delta_a(Q^2,t) = R$, then

$$\left[\frac{\ln\left(\frac{t}{\Lambda^2}\right)}{\ln\left(\frac{Q^2}{\Lambda^2}\right)}\right]^{\frac{2}{3b_0}} = R \tag{77}$$

$\Rightarrow$

$$t = \Lambda^2\left(\frac{Q^2}{\Lambda^2}\right)^{R^{\frac{3b_0}{2}}} . \tag{78}$$

Recall

$$\begin{aligned} b_0 &= \left(\frac{11}{3}n_c - \frac{2}{3}n_f\right) \\ &= \frac{1}{3}\left(11n_c - 10\right), \quad n_f = 5 \\ &= \frac{2}{3}\mathbf{BETAF}. \end{aligned} \tag{79}$$

The momentum available after a $q\bar{q}$ split in HERWIG is given by

$$QQBAR = QCDL3\left(\frac{QLST}{QCDL3}\right)^{R^{BETAF}} \quad . \tag{80}$$

Let us now repeat the above calculation for the IR-Improved kernels.

$$P_{qG}(z)^{exp} = F_{YFS}(\gamma_G)e^{\frac{1}{2}\delta_G}\frac{1}{2}\left[z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G}\right] \tag{81}$$

so

$$\int_0^1 dz\frac{\alpha_s\left(Q^2\right)}{2\pi}P_{qG}(z)^{exp} = \frac{4F_{YFS}(\gamma_G)e^{\frac{1}{2}\delta_G}}{b_0\ln\left(\frac{Q^2}{\Lambda^2}\right)(\gamma_G+1)(\gamma_G+2)(\gamma_G+3)}. \tag{82}$$

$\Rightarrow$

$$I=\int_{Q_0^2}^{Q^2}\frac{dt}{t}\frac{4F_{YFS}(\gamma_G)e^{\frac{1}{2}\delta_G}}{b_0\ln\left(\frac{t}{\Lambda^2}\right)(\gamma_G+1)(\gamma_G+2)(\gamma_G+3)},$$

$$I=\quad\frac{4F_{YFS}(\gamma_G)e^{0.25\gamma_G}}{b_0\,(\gamma_G+1)(\gamma_G+2)(\gamma_G+3)}Ei\left(1,\frac{8.369604402}{b_0\ln\left(\frac{t}{\Lambda^2}\right)}\right)\Bigg|_{Q_0^2}^{Q^2} \quad (83)$$

Where we have used

$$\delta_G=\frac{\gamma_G}{2}+\frac{\alpha_s C_G}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right), \quad (84)$$

with $C_G=3$ the gluon quadratic Casimir invariant. So finally

$$\Delta_a(Q^2,t)=\exp\left[-\left(F\left(Q^2\right)-F\left(t\right)\right)\right], \quad (85)$$

where

$$F(Q^2)=\frac{4F_{YFS}(\gamma_G)e^{0.25\gamma_G}}{b_0\,(\gamma_G+1)(\gamma_G+2)(\gamma_G+3)}Ei\left(1,\frac{8.369604402}{b_0\ln\left(\frac{Q^2}{\Lambda^2}\right)}\right), \quad (86)$$

and Ei is the exponential integral fn.

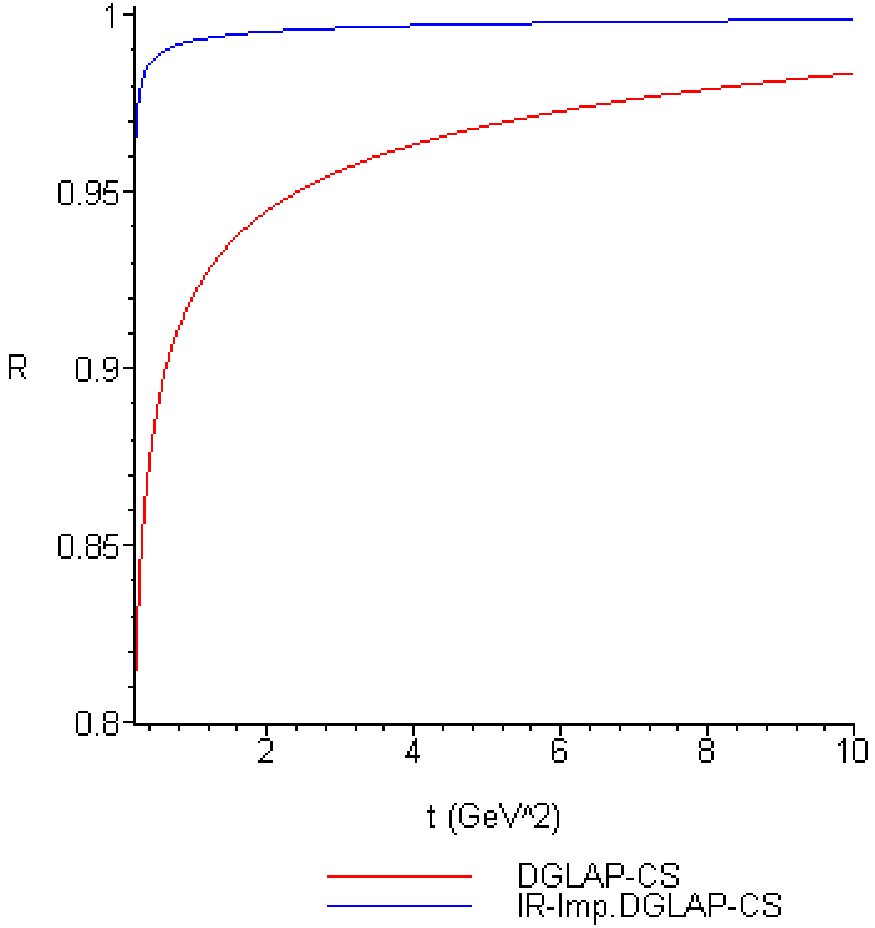


Figure 1: Graph of $\Delta_a(Q^2,t)$ for the DGLAP-CS and IR.Imp.DGLAP-CS kernels (76, 85)

RESULTS

We have preliminary results on IR-Improved Showers in HERWIG6.5: we compare the z-distributions, p_T-dist. etc., of the IR-Improved and usual DGLAP-CS showers in the following figs.

NOTE: SIMILAR RESULTS FOR PYTHIA and MC@NLO IN PROGRESS.

- First, 2⟶2 hard processes at LHC

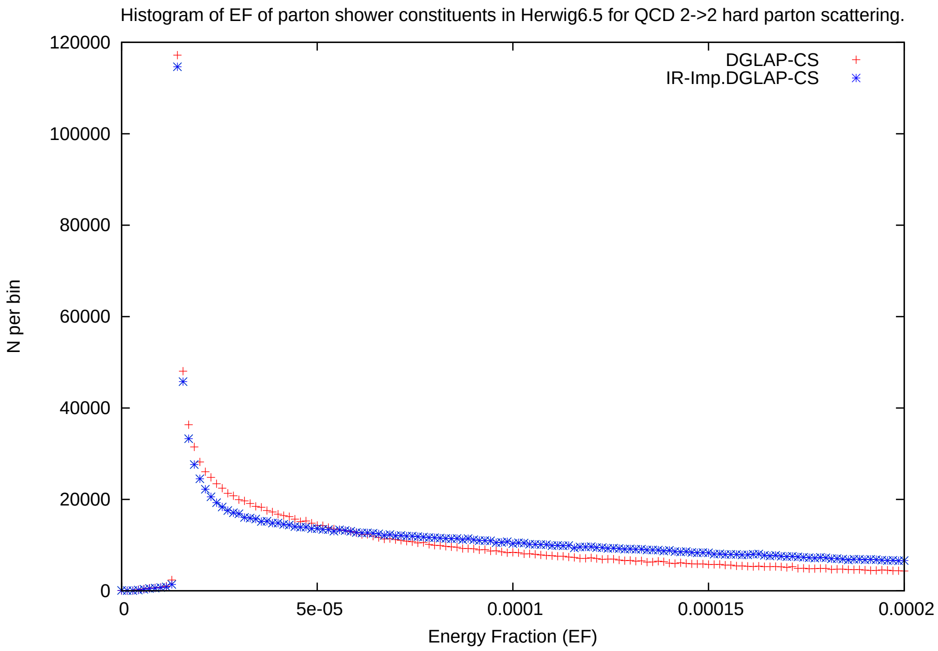


Figure 2: The z-distribution(ISR parton energy fraction) shower comparison in HERWIG6.5.

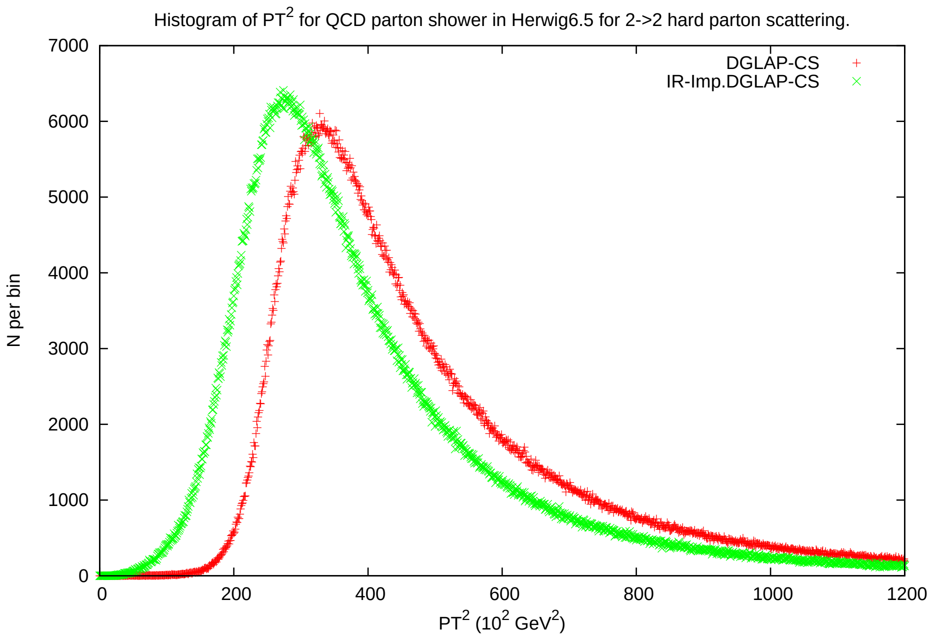


Figure 3: The P_T^2-distribution (ISR parton) shower comparison in HERWIG6.5.

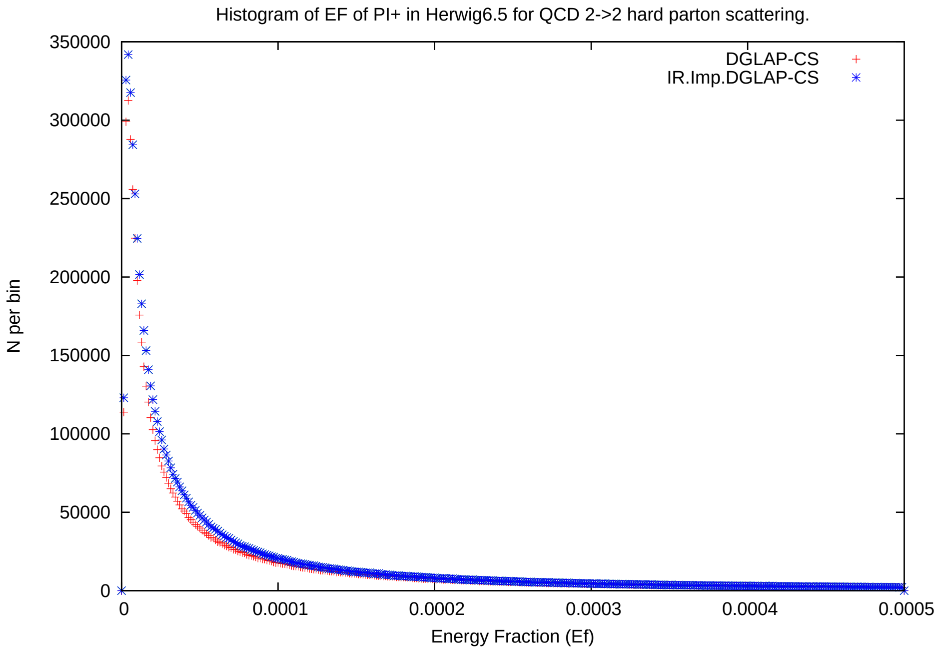


Figure 4: The π^+ energy fraction distribution shower comparison in HERWIG6.5.

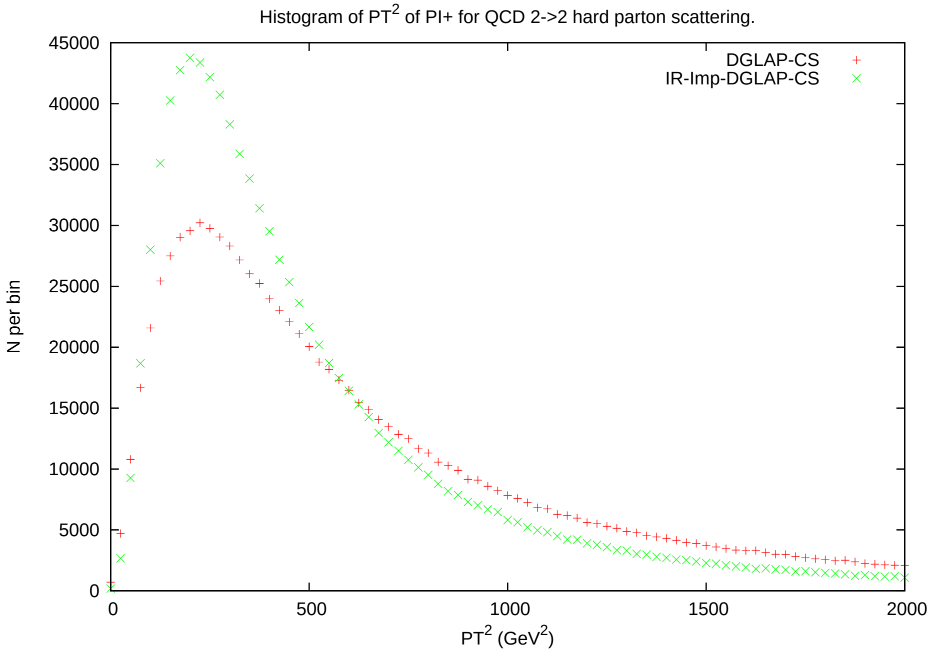


Figure 5: The π^+ P_T^2-distribution shower comparison in HERWIG6.5.

- **Single Z-production at LHC**

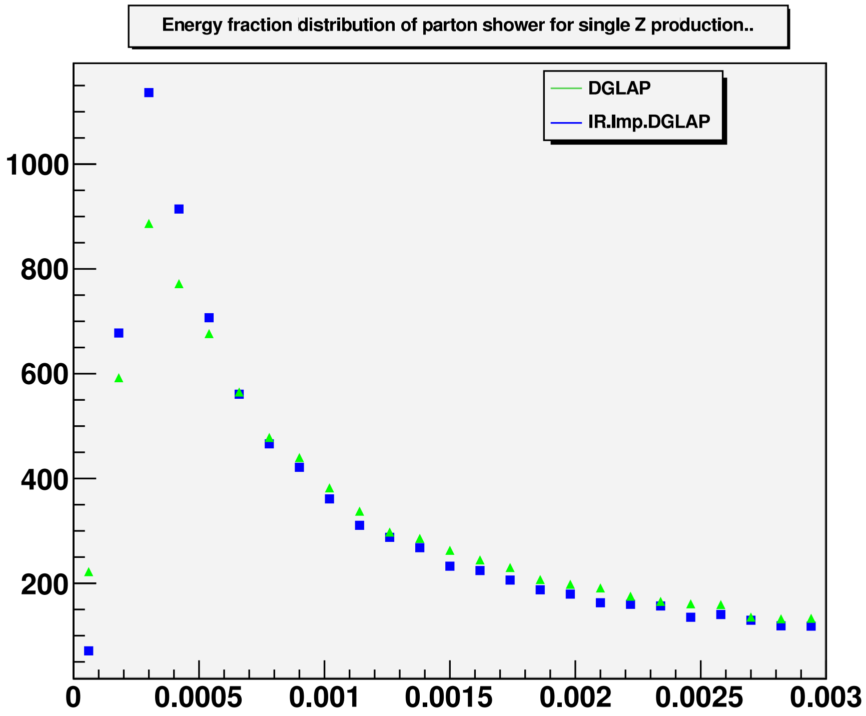


Figure 6: The z-distribution(ISR parton energy fraction) shower comparison in HERWIG6.5.

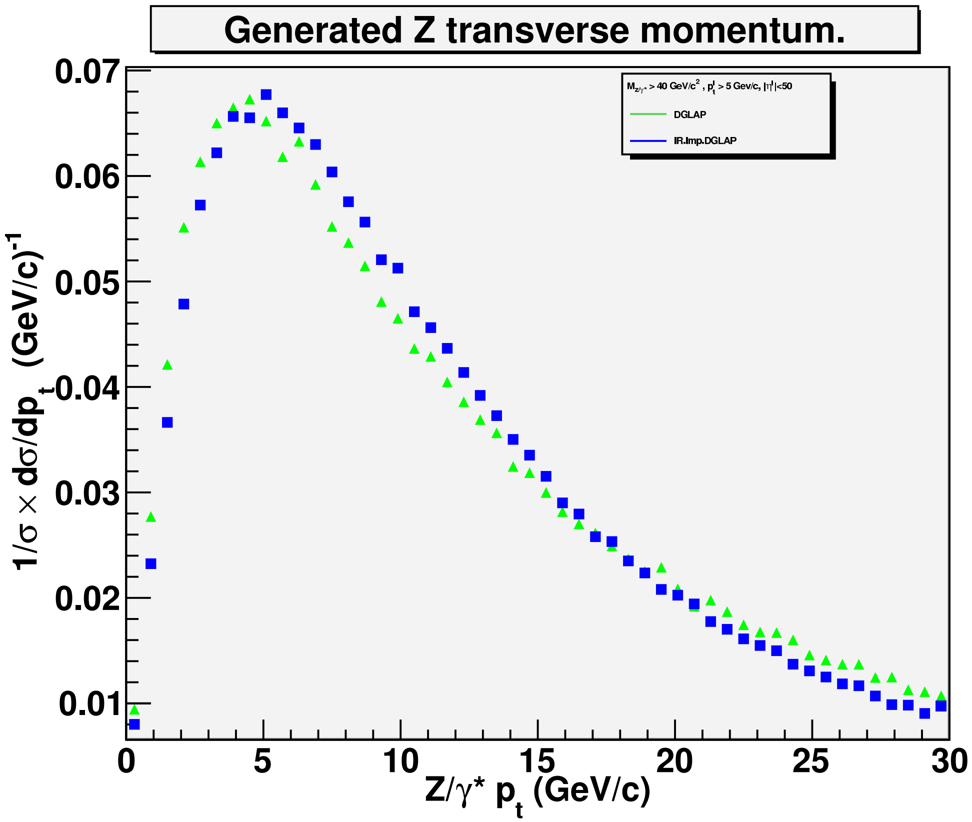


Figure 7: The Z p_T-distribution(ISR parton shower effect) comparison in HERWIG6.5.

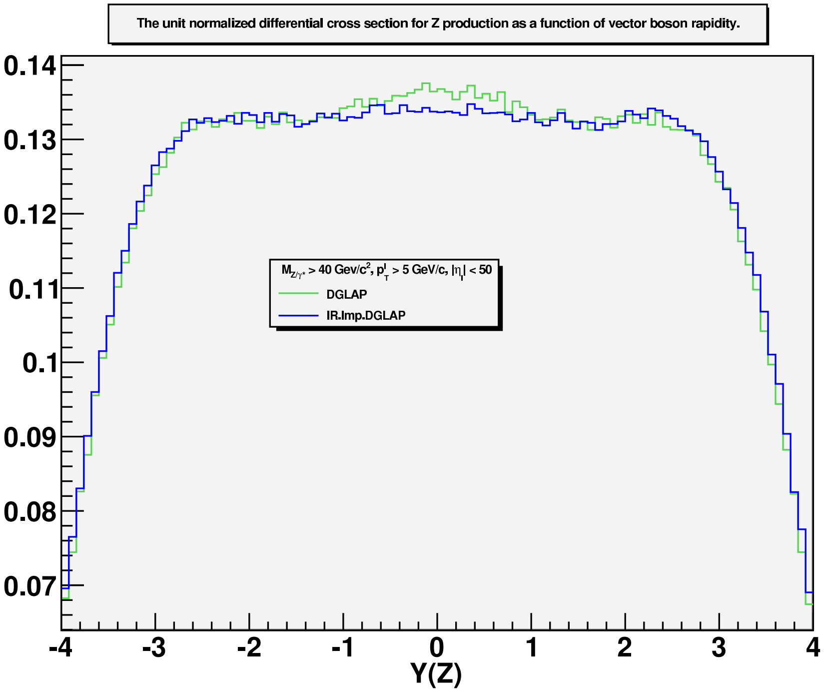


Figure 8: The Z rapidity-distribution(ISR parton shower) comparison in HERWIG6.5.

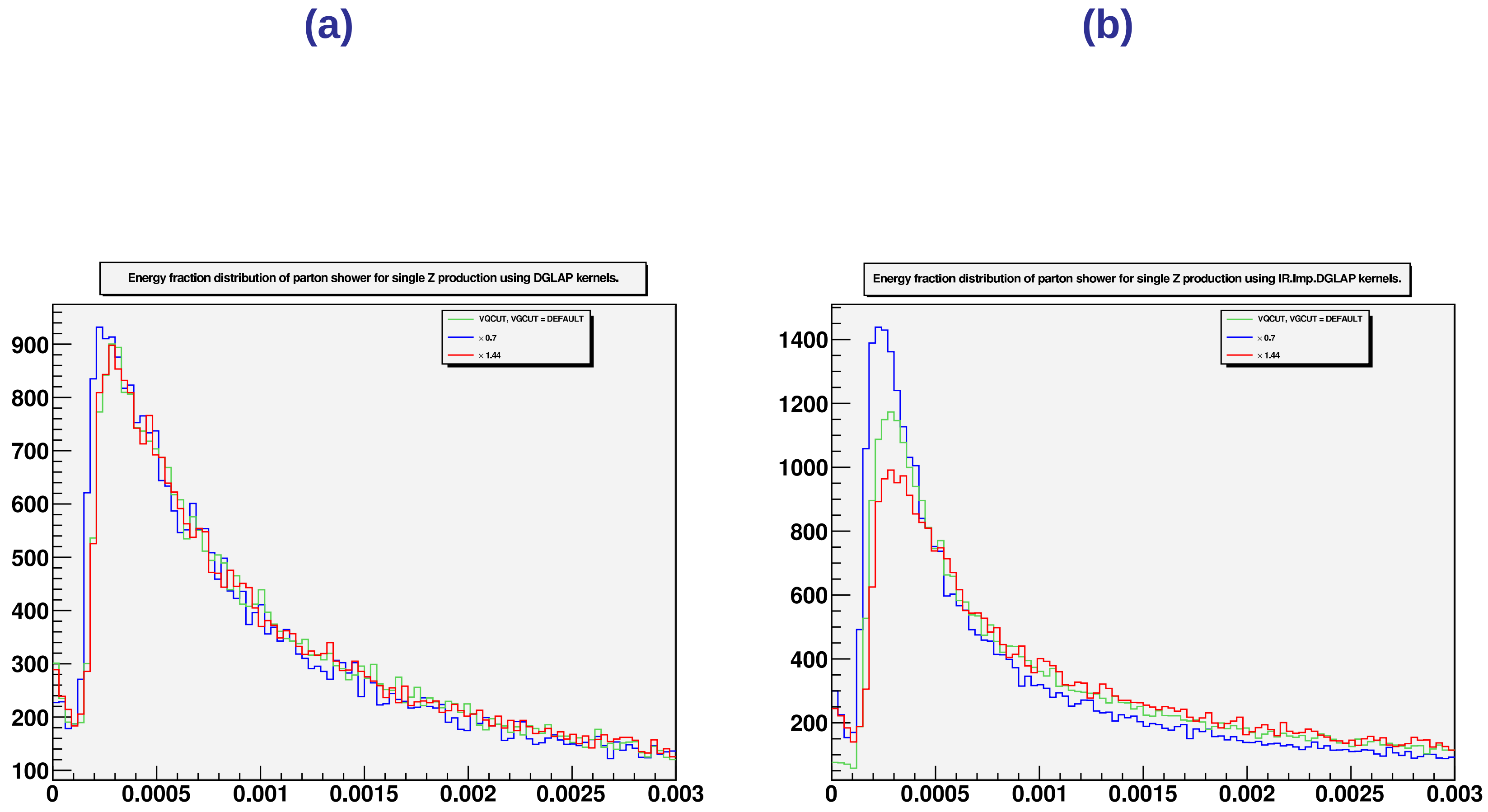


Figure 9: IR-cut-off sensitivity in z-distributions of the ISR parton energy fraction: (a), DGLAP-CS (b), IR-I DGLAP-CS – for the single Z hard subprocess in HERWIG-6.5 environment.

COMPARISON WITH DATA NOW FOLLOWS

(Galea, Proc. DIS 2008; Abasov et al., PRL100, 102002 (2008).)

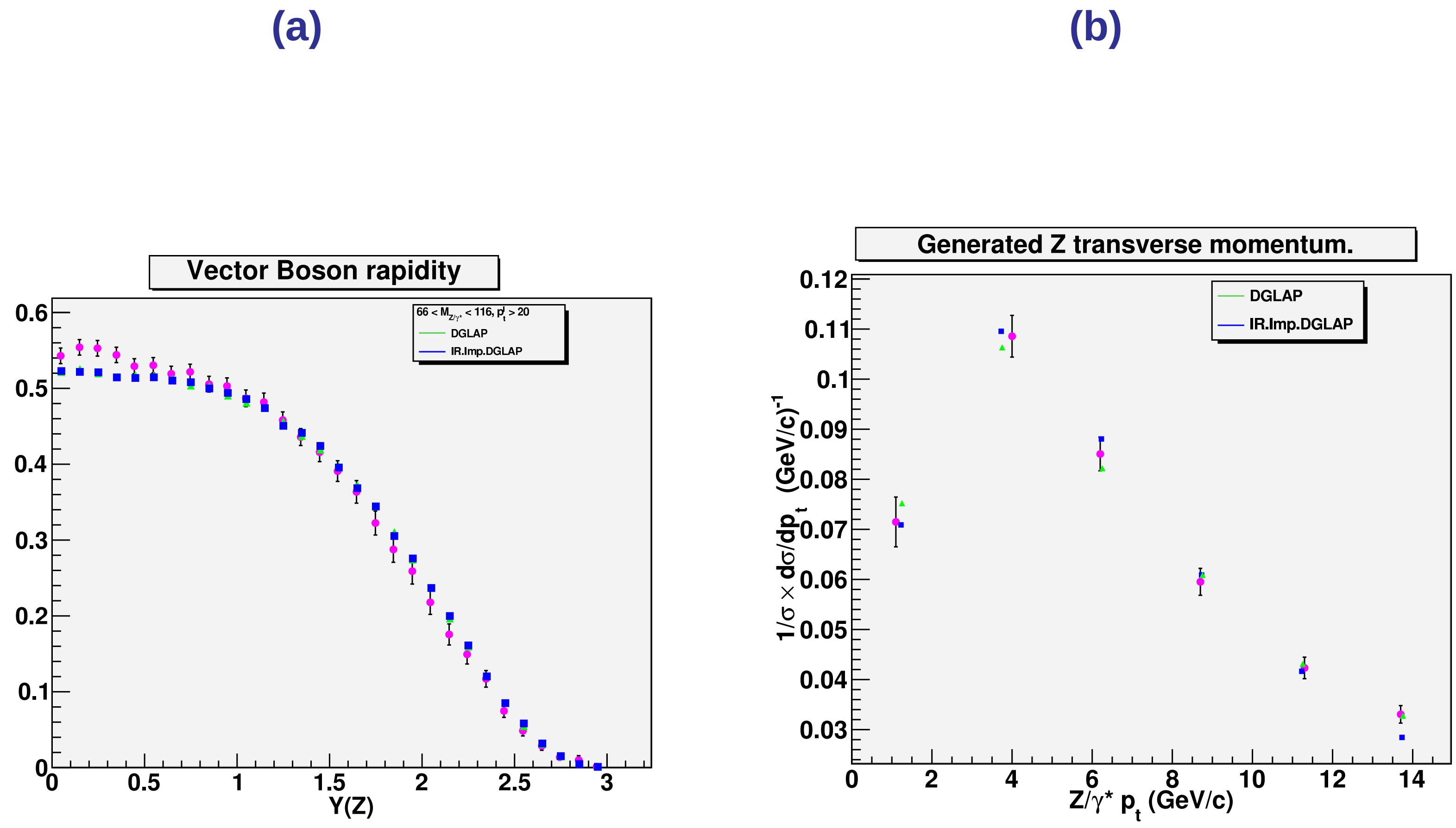
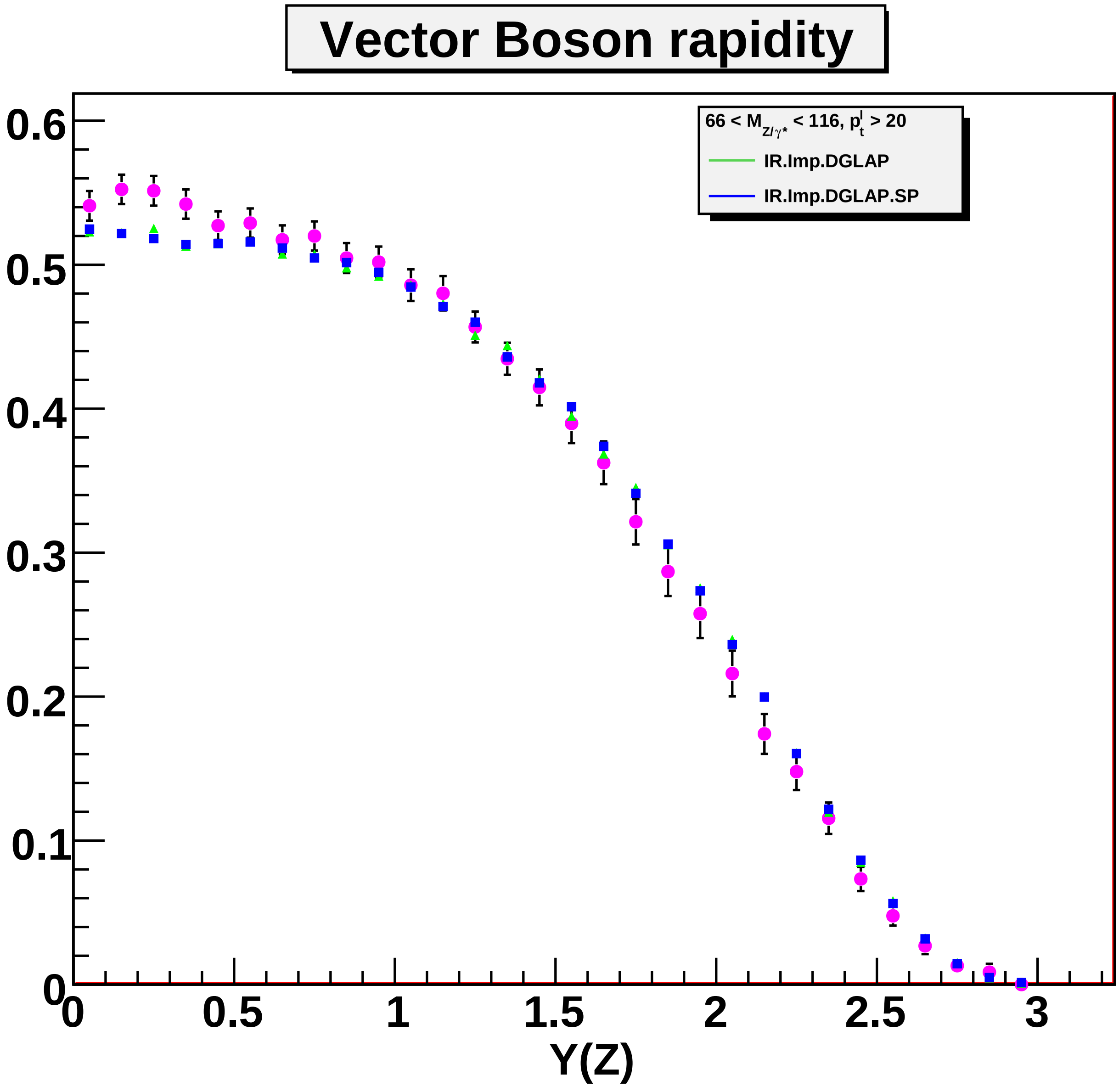
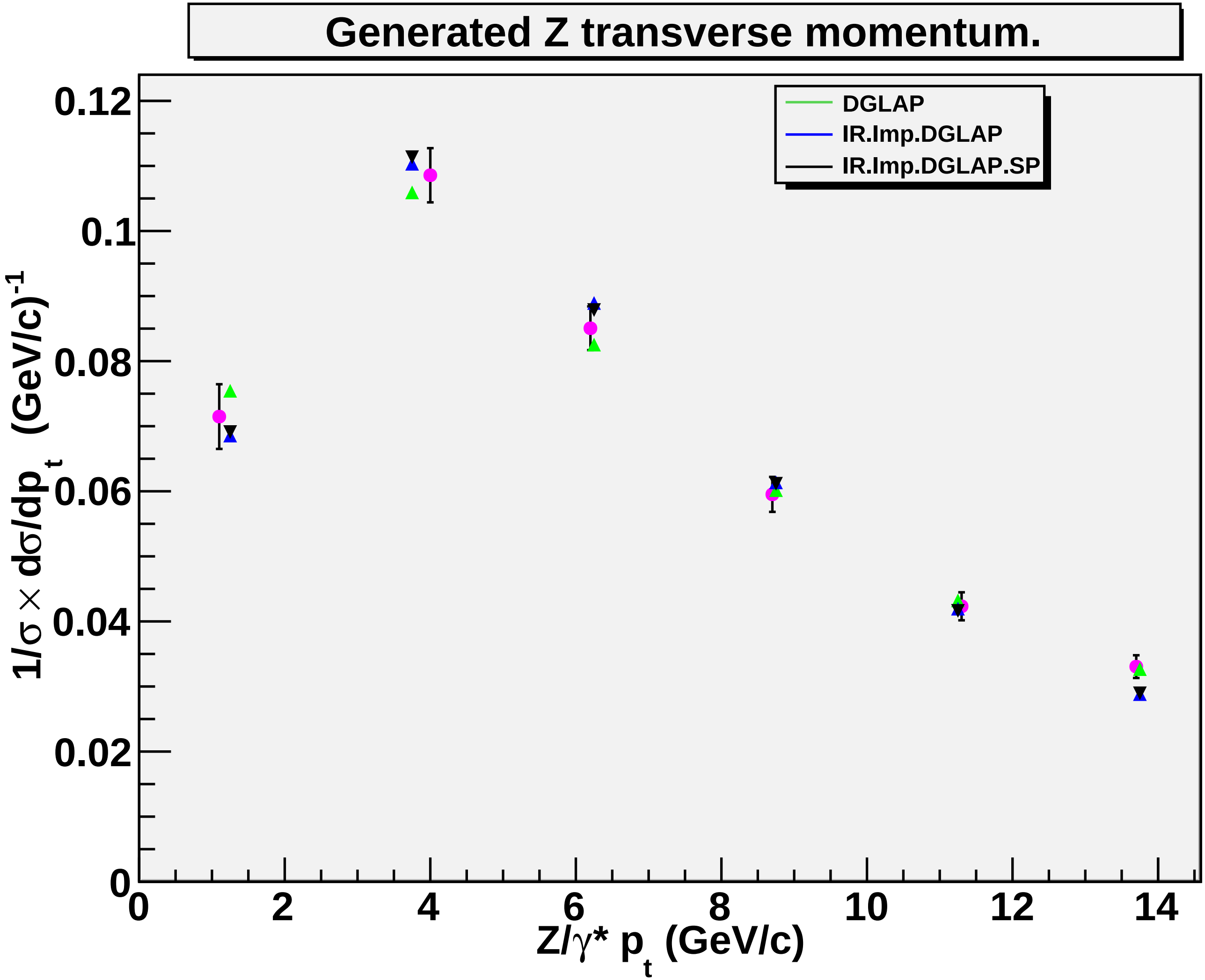


Figure 10: Comparison with FNAL data: (a), CDF rapidity data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data; (b), D0 p_T spectrum data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data.

For the D0 p_T data, we see that HERWIRI1.0 gives a better fit to the data compared to HERWIG6.5 for low p_T, (for $p_T < 12.5$GeV, the χ^2/d.o.f. are $\sim$ .29 and .40 respectively if we add the statistical and systematic errors), showing that the IR-improvement makes a better representation of QCD in the soft regime for a given fixed order in perturbation theory.


Vector Boson rapidity
66 < M_{Z/γ^*} < 116, p_t^l > 20
IR.Imp.DGLAP
IR.Imp.DGLAP.SP
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3
Y(Z)


Generated Z transverse momentum.
DGLAP
IR.Imp.DGLAP
IR.Imp.DGLAP.SP
$1/\sigma \times d\sigma/dp_t$ (GeV/c)$^{-1}$
0
0.02
0.04
0.06
0.08
0.1
0.12
0
2
4
6
8
10
12
14
Z/γ^* p_t (GeV/c)

Conclusions

YFS-TYPE METHODS (EEX AND CEEX) EXTEND TO NON-ABELIAN GAUGE THEORY AND ALLOW SIMULTANEOUS RESMN OF QED AND QCD WITH PROPER SHOWER/ME MATCHING BUILT-IN.

FOR QED$\otimes$QCD

- FULL MC EVENT GENERATOR REALIZATION OPEN.
- WE HAVE FIRST PHASE OF FULL MC REALIZATION: IR-IMPROVED HERWIG6.5 (HERWIRI1.0, arXiv:0906.0788, at http://thep03.baylor.edu)
- COMPARISON WITH THEORY ENCOURAGING: SOFTER SPECTRA, MORE ROBUSTNESS TO CUTS, ETC. – $\Delta\sigma_{Shower}$ IN PLAY
- COMPARISON WITH DATA – INITIAL FNAL RESULTS GOOD
- IMPLEMENTATION IN PYTHIA, HERWIG++, MC@NLO IN PROGRESS
- IMPLEMENTATION OF PRECISION EW MODULES (FROM JADACH ET AL.) IN HERWIG ALSO IN PROGRESS -- HERWIRI2.0.

- A FIRM BASIS FOR THE COMPLETE $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ MC RESULTS NEEDED FOR THE PRECISION FNAL/LHC/RHIC/ILC PHYSICS HAS BEEN DEMONSTRATED AND ALL THE LATTER IS IN PROGRESS, WITH M. Kalmykov, S. Majhi, S. Yost and S. Joseph.– SEE JHEP0702(2007)040,arxiv:0707.3654,0708.0803, 0810.3238, 0901.4716, 0902.1352, NEW RESULTS FOR HO F-Int's,etc. –no time to discuss here