DPF 2009

2009 Meeting of the Division of Particles and Fields of the American Physical Society (DPF 2009)

26-31 JULY 2009

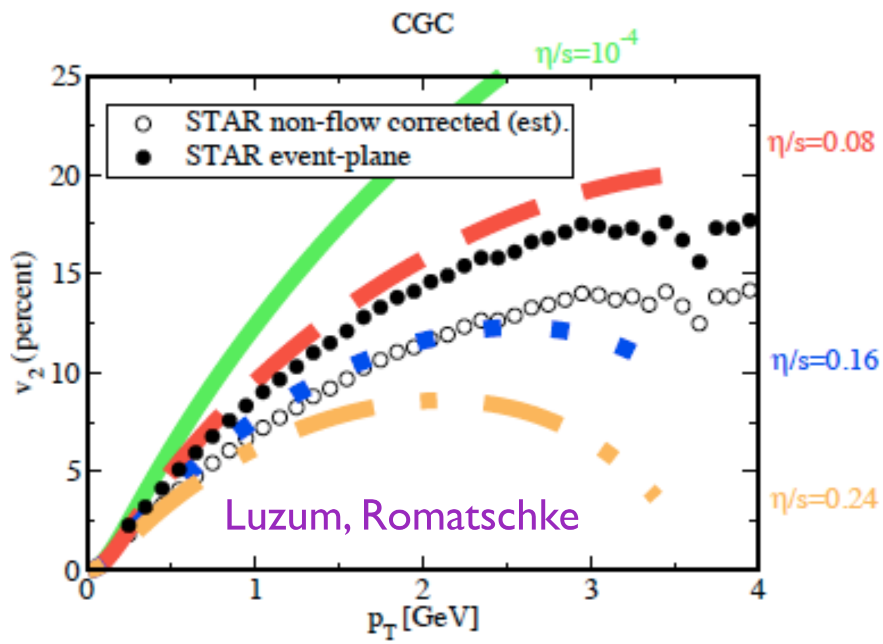
Wayne State University, Detroit, MI

v_2 from viscous hydrodynamics

Kevin Dusling,
Guy Moore & Derek Teaney

BROOKHAVEN NATIONAL LABORATORY

Viscosity at RHIC



1. Do we understand the uncertainties in this figure?
2. What is the connection to energy loss? $v_2(p_T)$ vs $R_{AA}(\phi)$

Viscous Corrections

1. Viscous correction to equations of motion

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu} - \eta \langle \partial^\mu u^\nu \rangle$$

2. Viscous correction to spectra

$$f \to f_0 + \delta f$$

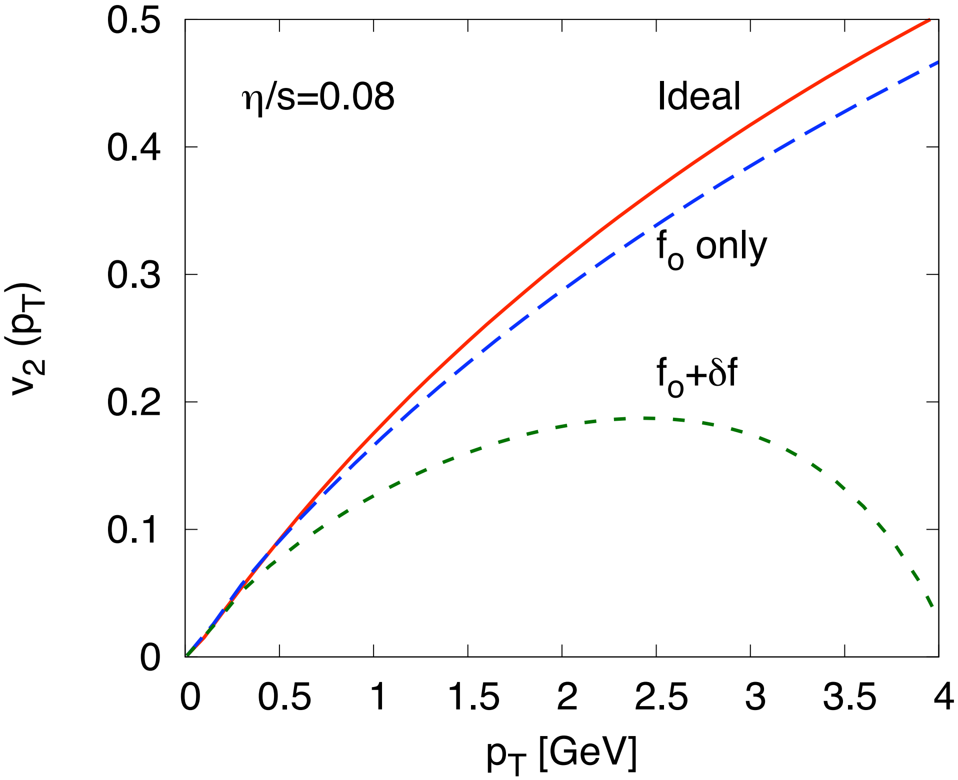
δf must be a scalar proportional to strains

$$\delta f = \chi(p) \times f_0(p) \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle$$

To date every group has taken the “quadratic ansatz”

$$\chi(p) \propto p^2$$

Role played by δf



We need to have a quantitative understanding of the δf and the quadratic ansatz.

Outline

- Let's look at various models of energy loss
  - Relaxation time approximation
  - Weakly coupled pure glue
    - Connection with energy loss
  - Weakly coupled QCD

- Phenomenology
  - Two component meson / baryon gas
  - "Viscous Scaling" of elliptic flow

Relaxation time approximation

- Boltzmann eqn:

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} f = -\frac{f(p) - f_0(p)}{\tau_R(E_p)}$$

- Substitute $f(p) = f_0(p) + \delta f(p)$ where $f_0(p) \propto \exp(-p^\mu u_\mu / T)$

$$\delta f \propto \frac{\tau_R(E_p)}{E_p} f_0(p) p^i p^j \langle \partial_i u_j \rangle$$

- Get back quadratic ansatz when $\tau_R \propto E_p$
  but what about $\tau_R \propto (E_p)^\beta$?

Generalization of quadratic ansatz

- Most general form is $\delta f = -\chi(\tilde{p}) \times f_0(1 \pm f_0)\hat{p}^i\hat{p}^j\langle\partial_i u_j\rangle$
- From now on take $\chi(\tilde{p}) = C(\alpha)\tilde{p}^{2-\alpha}$ where $\tilde{p} \equiv \dfrac{p}{T}$ and $\hat{p}^i \equiv \dfrac{p^i}{|\mathbf{p}|}$
- Constant determined from first moment of δf

$$\delta T^{ij} \equiv -\eta\langle\partial^i u^j\rangle = \int_{\mathbf{p}} d\Gamma p^i p^j \delta f(p)$$

$$\chi(\tilde{p}) = \frac{120}{\Gamma(6-\alpha)} \times \frac{\eta}{sT} \times \tilde{p}^{2-\alpha}$$

Two Extreme Limits

- Quadratic (α=0): Relaxation time growing with parton energy

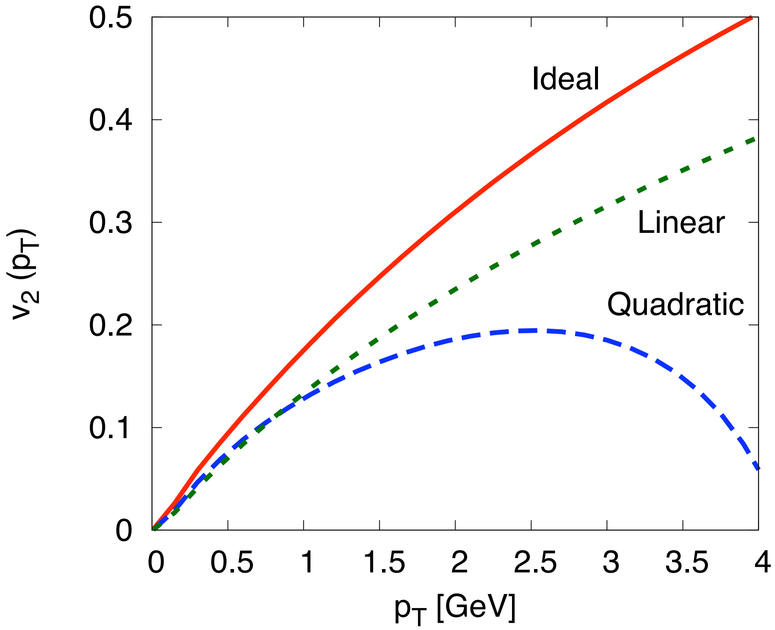
$$\tau_R \propto E_p \qquad \frac{dp}{dt} \propto \text{const.} \qquad \chi(p) \propto p^2$$

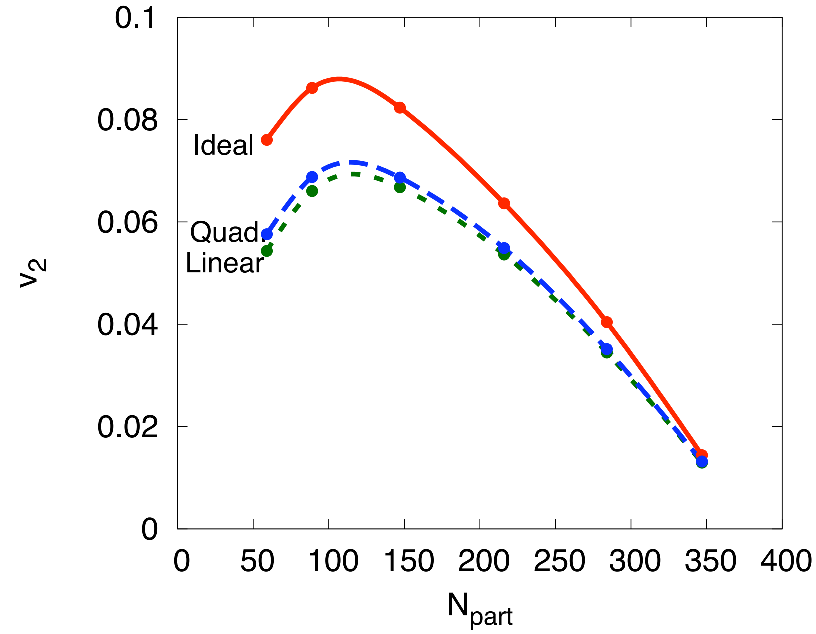
- Linear (α=1): Relaxation time independent of parton energy

$$\tau_R \propto \text{const.} \qquad \frac{dp}{dt} \propto p \qquad \chi(p) \propto p$$

- As we will show reality is probably somewhere between

Two Extreme Limits





$$\eta \propto \int_{\mathbf{p}} d\Gamma \; p \; f_0 \chi(p)$$

$$\delta\overline{v_2} \propto \int_{\mathbf{p}} d\Gamma \; p^2 \; f_0 \chi(p)$$

Weakly coupled pure-glue QCD

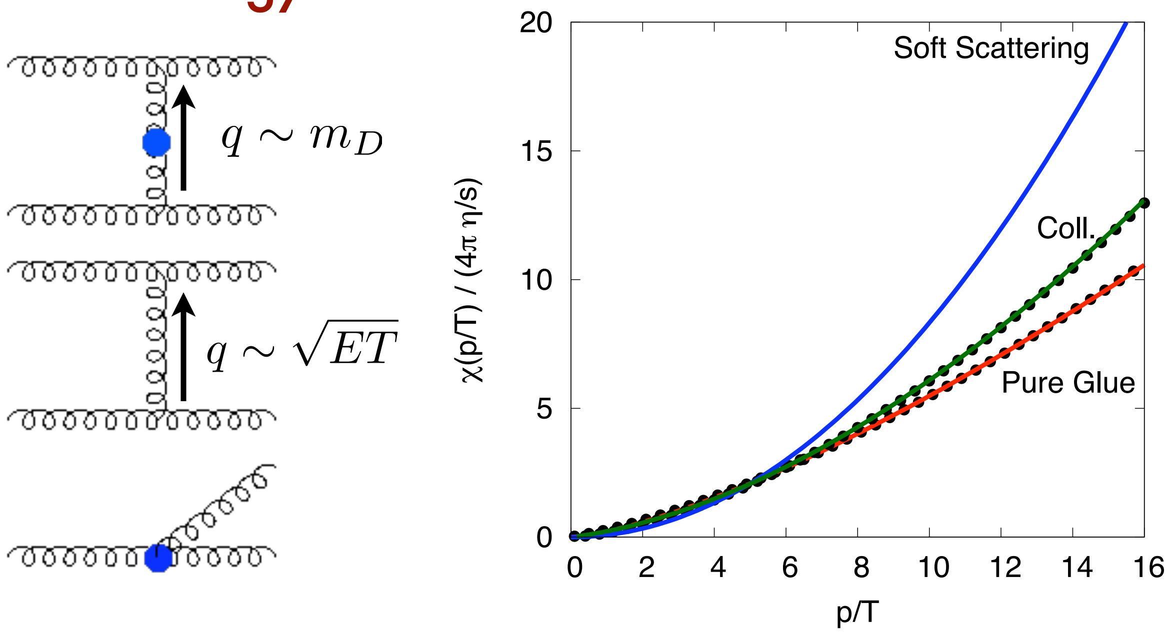
- Boltzmann eqn:

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} f = -\mathcal{C}^{2\leftrightarrow 2}[f] - \mathcal{C}^{1\leftrightarrow 2}[f]$$

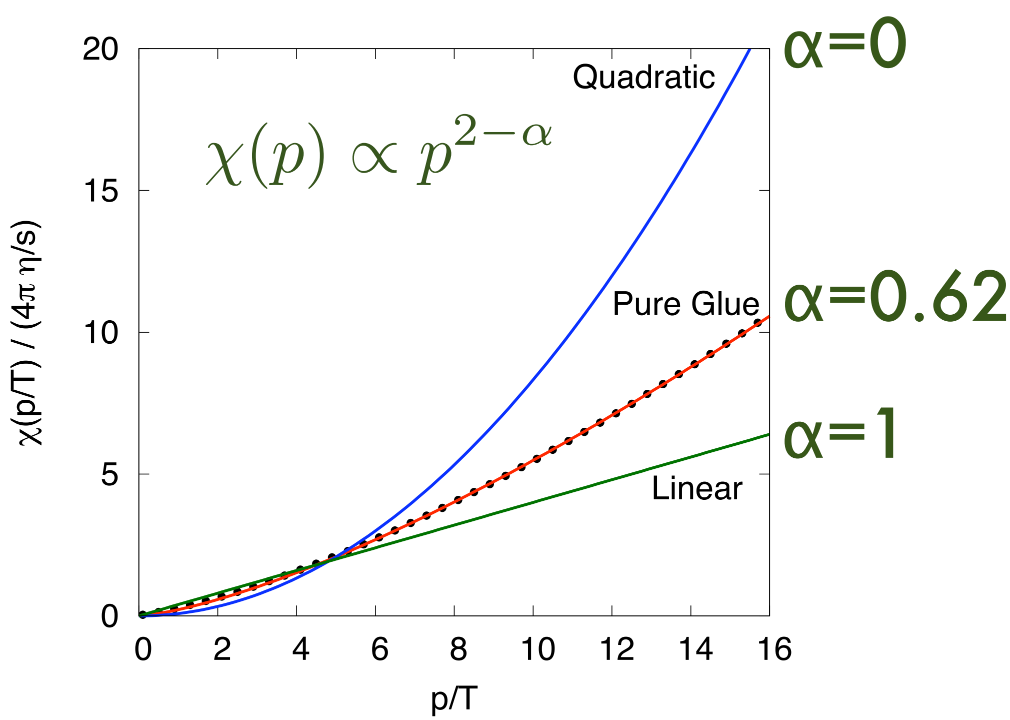
- Substitute $f(p) = f_0(p) + \delta f(p)$ and invert numerically for δf

- Three different modes of energy loss

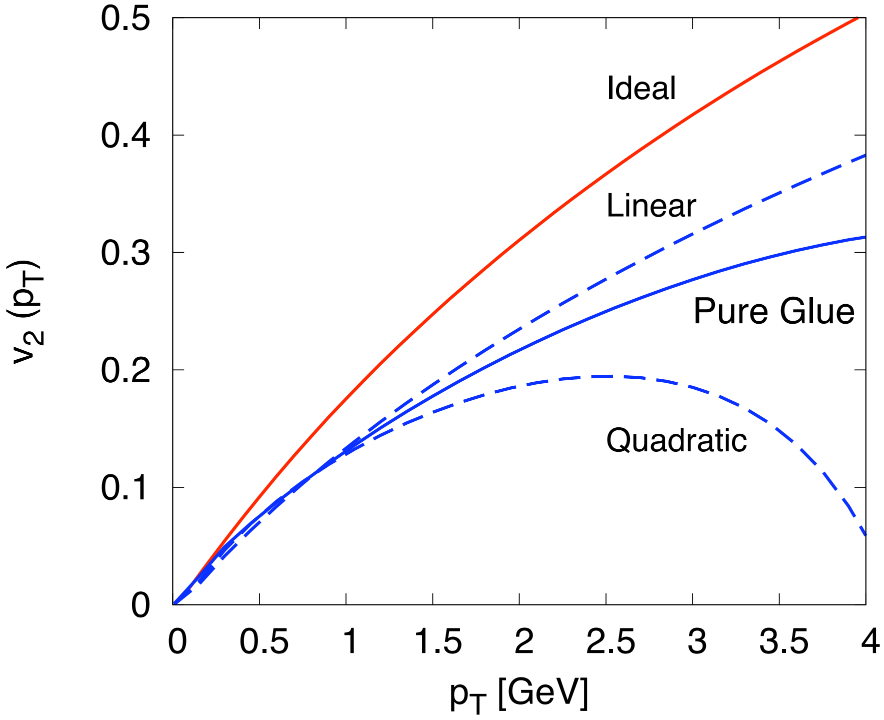
1. Soft Scattering
2. + Collisional
3. + Radiative



Weakly coupled pure-glue QCD



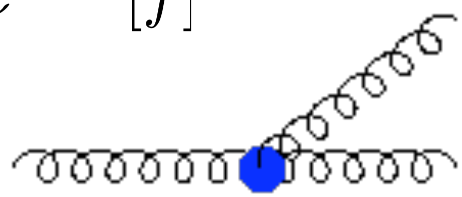
Weakly coupled pure-glue QCD



Connection to energy loss

- Let's look at asymptotically large, $\ln^{-1}(\tilde{p}) \ll 1$, energies

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} f = -\mathcal{C}^{1\to 2}[f]$$



- Only contribution is from radiation

$$\mathcal{C}^{1\to 2} \propto \int_0^1 dx\ \gamma(p; xp, (1-x)p) \left[\chi_p - \chi_{xp} - \chi_{(1-x)p}\right]$$

- where in the deep LPM regime (P. Arnold, C. Dogan, arXiv:0804:3359)

$$\gamma \propto \alpha_s C_A d_A \sqrt{p\hat{q}} \frac{[1 - x(1-x)]^{5/2}}{[x(1-x)]^{3/2}}$$

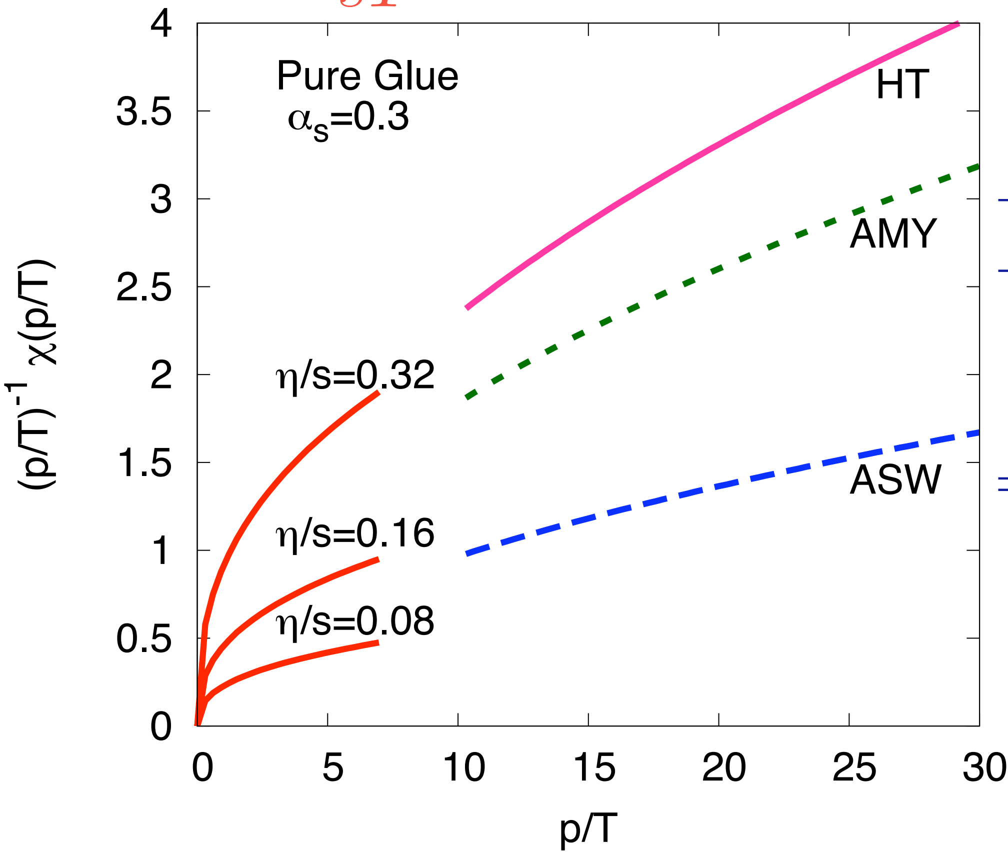
- Finally get the semi-analytic result:

$$\chi_g(p) \approx \frac{0.7}{\alpha_s T \sqrt{\hat{q}}} p^{3/2}$$

Connection to energy loss

$$\chi_g(p) \approx 2.84 \frac{\eta}{sT} \tilde{p}^{3/2}$$

$$\chi_g(p) \approx \frac{0.7}{\alpha_s T \sqrt{\hat{q}}} p^{3/2}$$



Model	$\hat{q}_0$ [GeV^2/fm]	$\hat{q}/T^3$
ASW	20	59
HT	3.4	10
AMY	5.5	16.3

Extracted from fits to R_{AA} by
Bass, Gale, Majumder, Nonaka, Qin, Renk, Ruppert,
PRC 79, 024901, 2009.

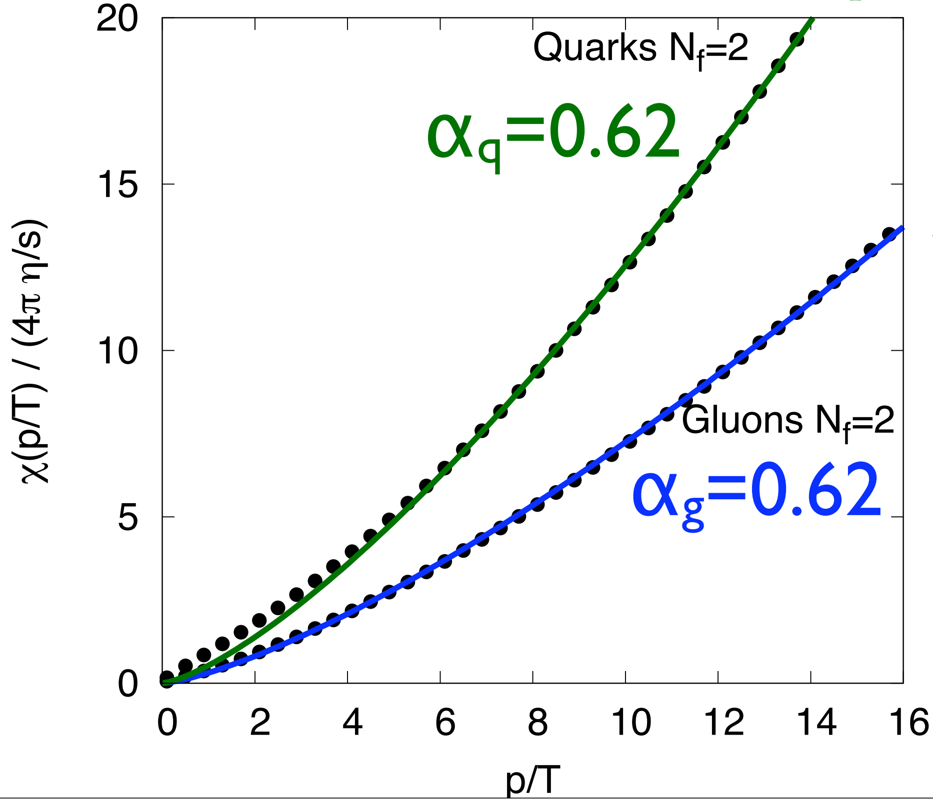
Two component QCD

- Consider a two component gas of quarks and gluons

$$\delta f_g(p) = n_p(1+n_p)\chi_g(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle$$
$$\delta f_q(p) = n_p(1-n_p)\chi_q(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle$$

- both go like $p^{1.38}$

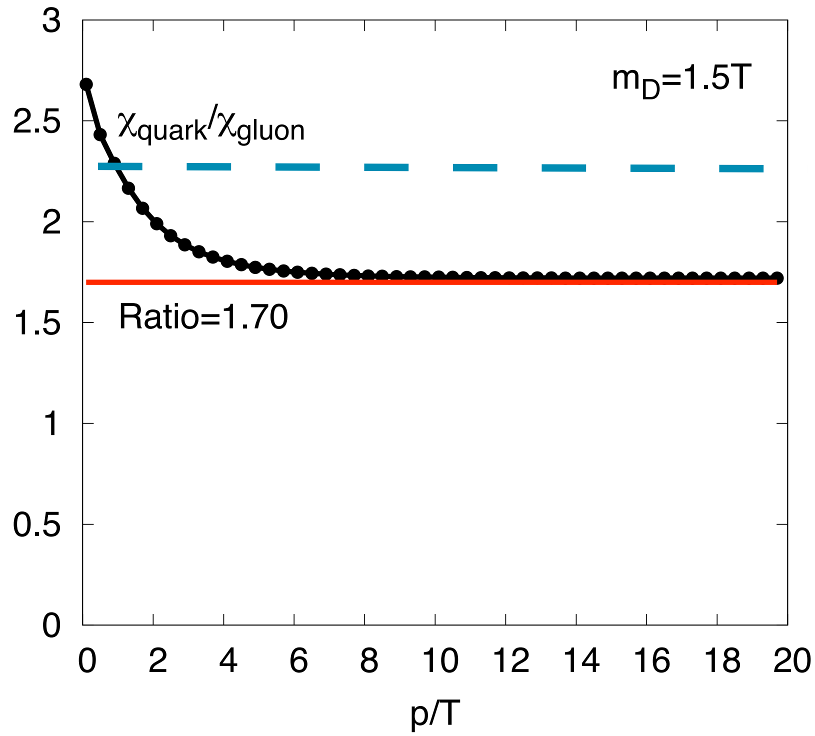
$$\chi_q(\tilde{p}) = C_q(\alpha_q)\tilde{p}^{2-\alpha_q}$$



$$\chi_g(\tilde{p}) = C_g(\alpha_g)\tilde{p}^{2-\alpha_g}$$

Two component QCD

- Constants (C_g,C_q) constrained by
  1. Ratio of viscous correction

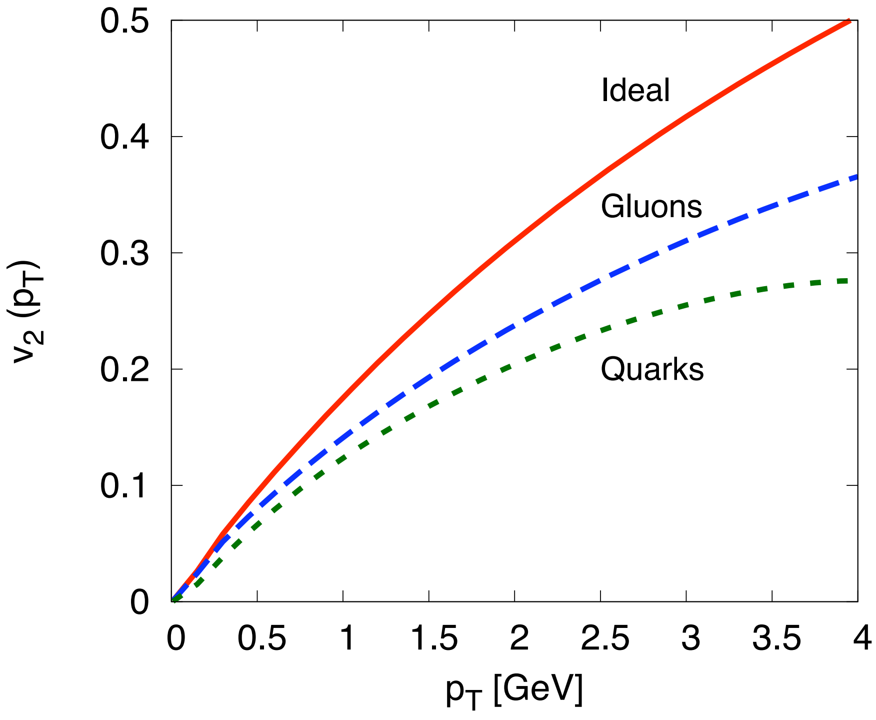


$$\frac{\chi_q}{\chi_g} \approx \frac{C_A}{C_F} = 2.25$$

  2. Shear viscosity of system

$$\eta = \frac{1}{15} \sum_{a=q,g} \nu_a C_a \int \frac{d^3p}{(2\pi)^3} p^{3-\alpha_a} n\,(1 \pm n)$$

Two component QCD



Two component meson / baryon gas

$$\begin{aligned} \delta f_m(p) &= n_p(1+n_p)\chi_m(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle \\ \delta f_b(p) &= n_p(1-n_p)\chi_b(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle \end{aligned}$$

- Characterize mesons and baryons by constants C_m and C_b

$$\begin{aligned} \chi_m(\tilde{p}) &= C_m(\alpha)\tilde{p}^{2-\alpha} \\ \chi_b(\tilde{p}) &= C_b(\alpha)\tilde{p}^{2-\alpha} \end{aligned}$$

- Consider two Ansatz for comparison to data
  1. Radiative: $p^{1.5}$ ($\alpha=0.5$)
  2. Quadratic: p^2 ($\alpha=0$)

Two component meson / baryon gas

- Constants (C_b,C_m) constrained by

1. Ratio of viscous correction

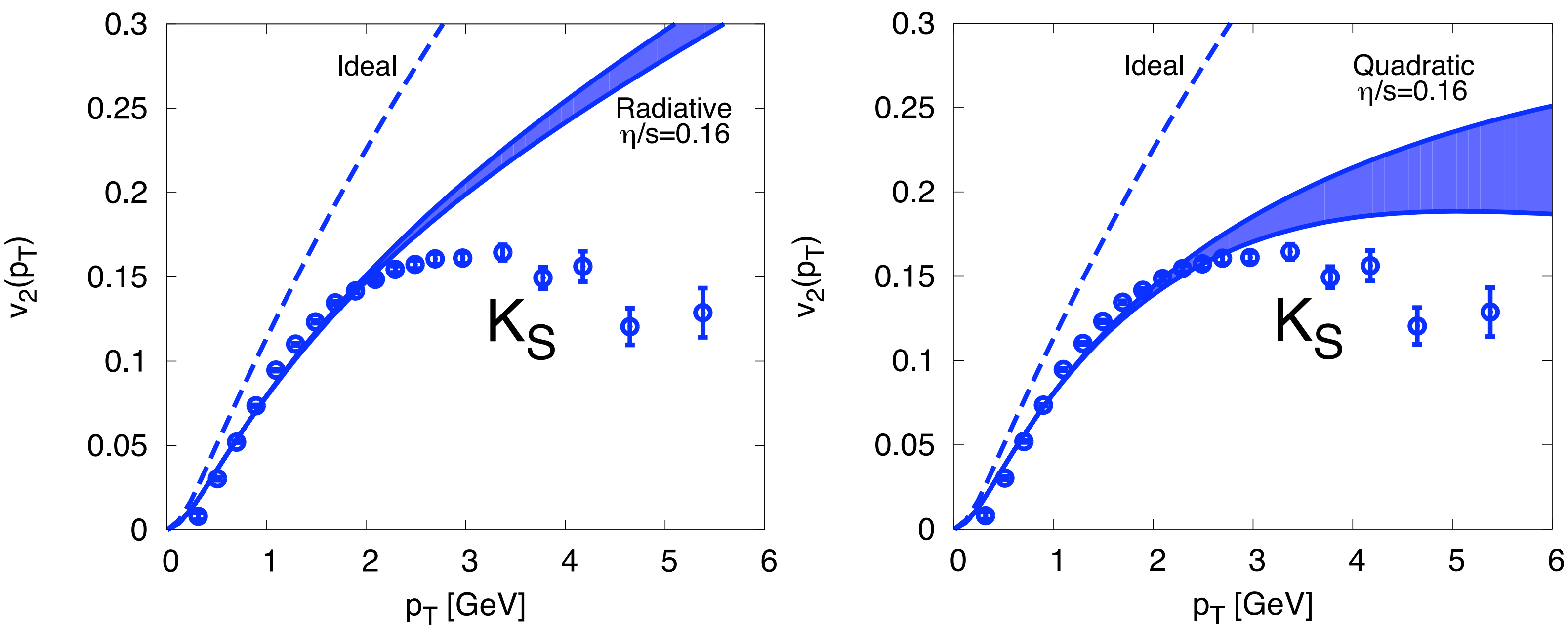
$$\frac{C_m}{C_b} = \begin{cases} 1.6 & \text{quadratic} \\ 1.4 & \text{radiative} \end{cases}$$

An aside: $\frac{\sigma_{NN}}{\sigma_{\pi N}} = 1.5$ in AQM

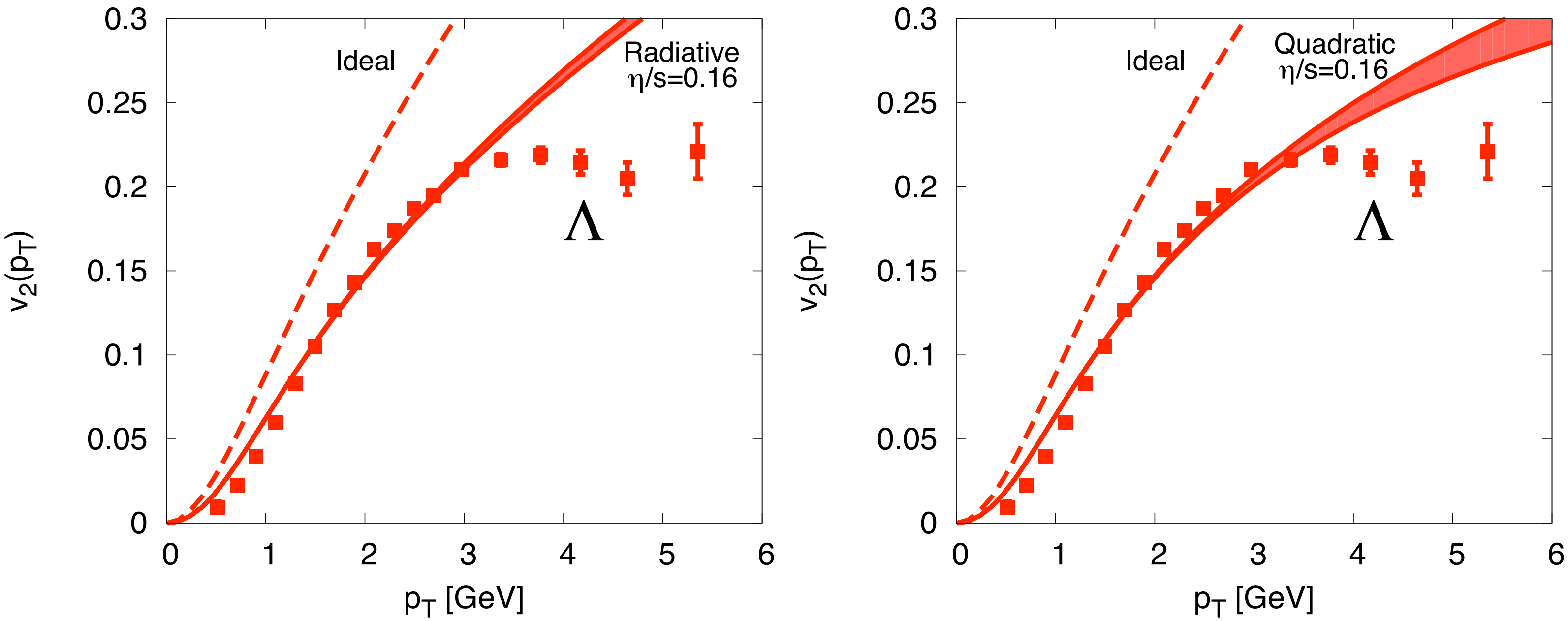
2. Shear viscosity of system

$$\eta = \frac{1}{15} \sum_{a=\pi,K,\ldots} \nu_a C_{m/b} \int \frac{d^3p}{(2\pi)^3 E_a} p^{4-\alpha} n(E_a) \left[1 \pm n(E_a)\right]$$

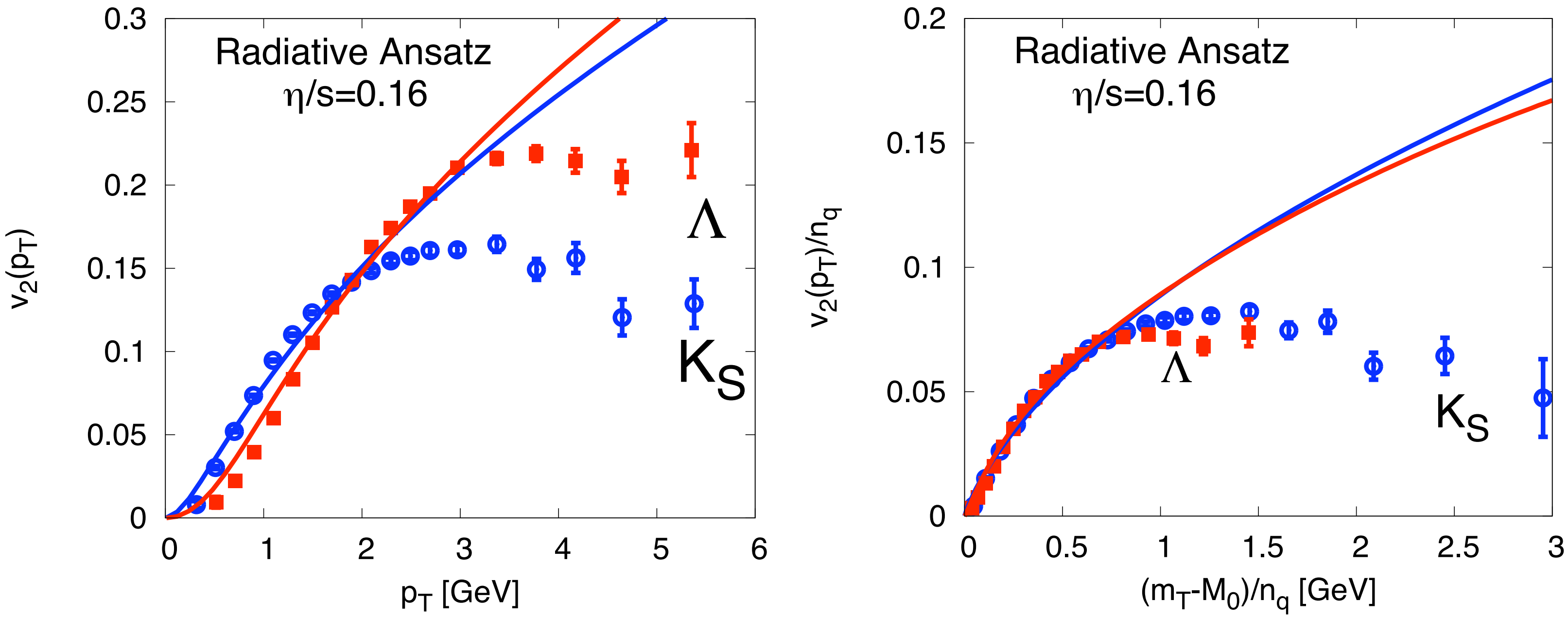
Two component meson / baryon gas



Two component meson / baryon gas



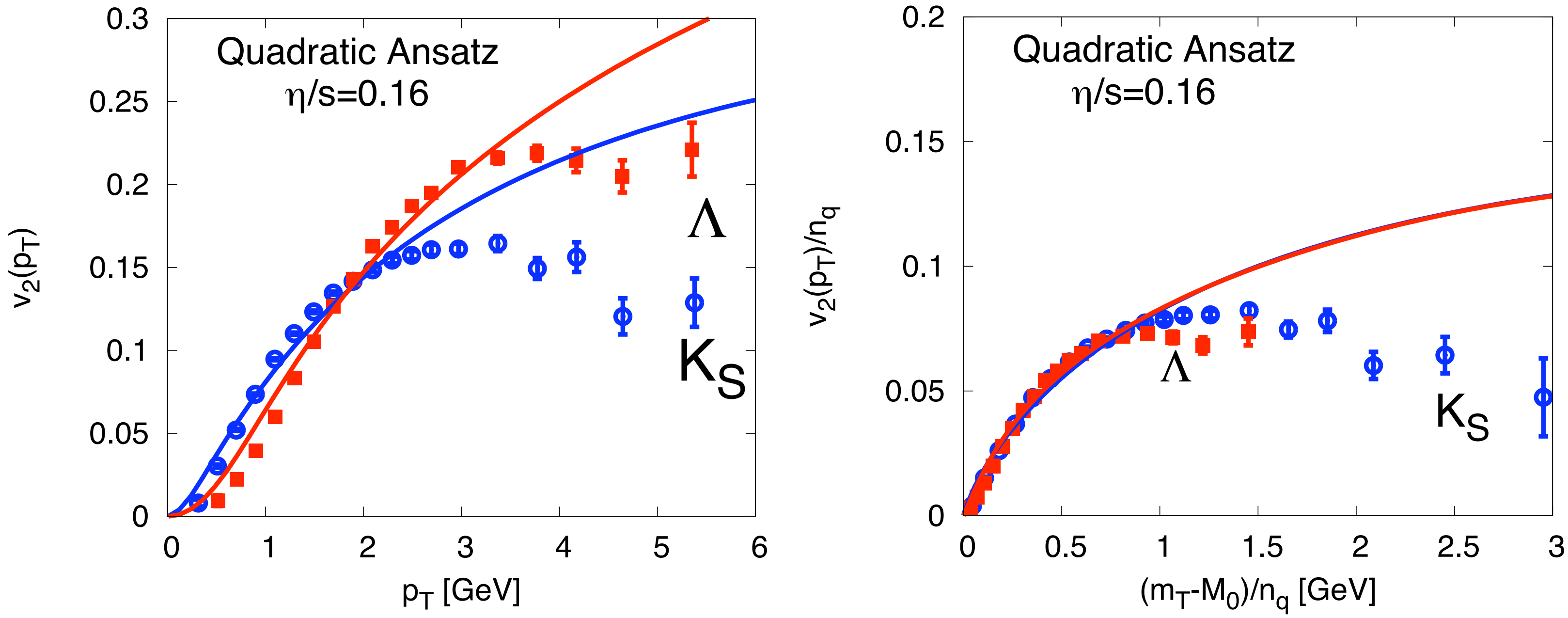
“Viscous Scaling”



Conclusions

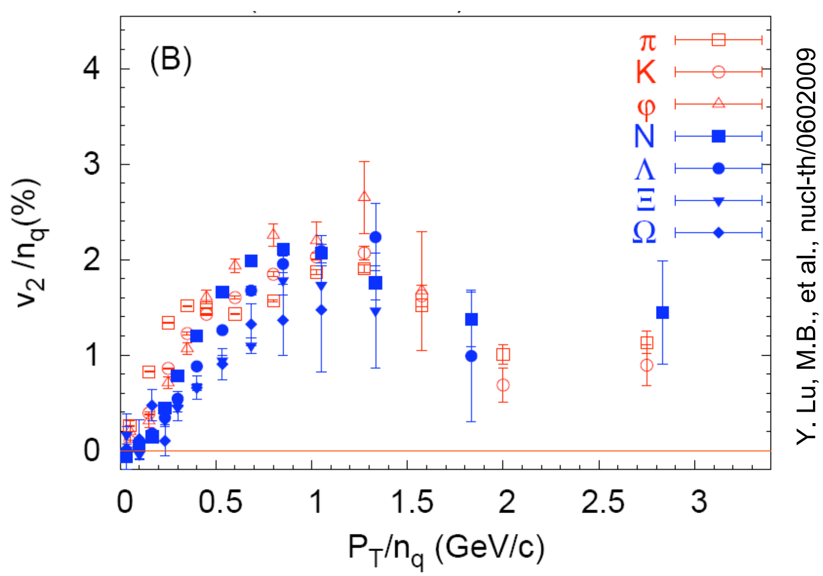
- Studied the kinetics of quarks and gluons and their elliptic flow
- Radiative energy loss increases elliptic flow at *larger* p_T
- Made the connection between energy loss and viscosity
- Observed "Viscous Scaling" of mesons and baryons
  - A model having constituent quark scaling without constituent quarks

“Viscous Scaling”



Scaling of v2 in UrQMD

From talk of Marcus Bleicher, HQ06



AQM cross sections:

Nπ: 26 mb
Λπ: 23 mb
Ξπ: 20 mb
Ωπ: 16 mb

ππ: 18 mb
Kπ: 14 mb

~3:2