$\Upsilon(1S) --> \gamma(\eta', \eta, f_2(1270))$ decays

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Very small upper limits of the branching ratios of radiative $\Upsilon(1s)$ decays into η and η' have been reported by CLEO

$$B(\Upsilon(1s) \rightarrow \gamma\eta) < 1.0 \times 10^{-6},$$

$$B(\Upsilon(1s) \rightarrow \gamma\eta') < 1.9 \times 10^{-6}$$

at the 90%C.L. $B(\Upsilon(1S) \rightarrow \gamma\eta'$ are smaller than

$$B(J/\psi \rightarrow \gamma\eta') = (4.71 \pm 0.27) \times 10^{-3}$$

by almost three order of magnitudes. On the other hand,

$$B(\Upsilon(1S) \rightarrow \gamma f_2(1270)) = (10.2 \pm 0.8 \pm 0.7) \times 10^{-5},$$

$$B(\Upsilon(1S) \to \gamma f_2(1270)) = (10.5 \pm 1.6(stat)^{+1.9}_{-1.8}(syst)) \times 10^{-5}$$

$B(\Upsilon(1S) \to \gamma f_2(1270))$ is about one order of magnitude smaller than $B(J/\psi \to \gamma f_2(1270) = (1.43 \pm 0.11) \times 10^{-3}$. In this talk we try to understand why $B(\Upsilon(1s) \to \gamma\eta')$ is so small and why $B(\Upsilon(1S) \to \gamma f_2(1270))$ is not that small.

J/ψ and $\Upsilon(1s)$ are heavy vector mesons. It is known that the decay of heavy vector meson is a test ground of perturbative QCD(pQCD). In pQCD radiative decay of a heavy vector meson is described as $V \to \gamma gg$. The two gluons are color singlet and they can form $0^{++}, 0^{-+}$ and 2^{++} states. J/ψ radiative decay has long been regarded as a fertile hunting ground for glueballs. So far, the existence of glueball has not been established. The efforts in search for glueball are complicated by mixture between the components of gluons and quarks. The difference between the radiative decays of J/ψ and $\Upsilon(1s)$ is caused by the mass difference of c- and b-quark. There are different theoretical approaches in the study of the branching ratios of $\Upsilon(1s) \to \gamma(\eta', \eta)$: the Vector Meson Dominance Model, nonrelativistic quark model[7], QCD sum rule and the U(1) anomaly, etc..... In 1984 we have studied $J/\psi \to \gamma\eta, \gamma\eta'$. In this study η' and η are taken as mixing states of glueball and $q\bar{q}$ mesons and the gluon contents make dominant contributions to $J/\psi \to \gamma\eta, \gamma\eta'$. The ratio

$$\frac{\Gamma(J/\psi \to \gamma\eta')}{\Gamma(J/\psi \to \gamma\eta)} = 5.30 \pm 0.05$$

has been predicted, which agrees with data very well. The same approach is applied to study $\Upsilon \to \gamma\eta'(\eta)$.

The U(1) anomaly of η' meson[11] shows that there is strong coupling between η' and two gluons. It is known for a long time that η' contains substantial gluon content. In the decays of $J/\psi \to \gamma\eta'(\eta)$ there are two parts: the amplitude of $J/\psi \to \gamma gg$ is calculated by pQCD and the couplings $gg \to \eta'(\eta)$ are dominated by the gluon contents of η' and η. The flavor singlet part of η and η' is a mixing state of glueball and quark singlet

$$|\eta_0 >= sin\phi|G > +cos\phi|q\bar{q} >,$$

where ϕ is the mixing angle. It is argued in Ref.[10] that the quark contents of η and η' are suppressed in $J/\psi \to \gamma\eta(\eta')$ by $O(\alpha_s^2)$. Therefore, only the glueball content is taken into account in the calculations of $\Gamma(J/\psi \rightarrow \gamma\eta(\eta'))$.

The coupling between state-G and two gluons is expressed as

$$< G|T\{A^a_\alpha(x_1)A^b_\beta(x_2)\}|0 >= \frac{\delta_{ab}}{\sqrt{2E_G}}\epsilon_{\alpha\beta\mu\nu}(x_1-x_2)^\mu p^\nu f_G(x_1-x_2)e^{\frac{i}{2}p_G(x_1+x_2)}. \tag{1}$$

The function $f_G(x_1-x_2)$ is unknown. In order to see the effect of this function $f_G(x_1-x_2)$ is taken as a harmonic function at first to calculate the decay width then instead $f_G(x_1-x_2)$ $f_G(0)$ is taken. $\Gamma(J/\psi \rightarrow \gamma\eta(\eta'))$ obtained from these two expressions of $f_G(x_1-x_2)$ are different by about 10%. The decay widths are derived as

$$\Gamma(J/\psi \rightarrow \gamma\eta') = cos^2\theta sin^2\phi\frac{2^{11}}{81}\alpha\alpha_s^2(m_c)\psi_J^2(0)f_G^2\frac{1}{m_c^8}\frac{(1-\frac{m^2_{\eta'}}{m^2_J})^3}{\{1-2\frac{m^2_{\eta'}}{m^2_J}+\frac{4m_c^2}{m_J^2}\}^2}$$

$$\{2m_J^2-3m^2_{\eta'}(1+\frac{2m_c}{m_J})-16\frac{m_c^3}{m_J}\}^2,$$

$$\Gamma(J/\psi \rightarrow \gamma\eta) = sin^2\theta sin^2\phi\frac{2^{11}}{81}\alpha\alpha_s^2(m_c)\psi_J^2(0)f_G^2\frac{1}{m_c^8}\frac{(1-\frac{m^2_{\eta}}{m^2_J})^3}{\{1-2\frac{m^2_{\eta}}{m^2_J}+\frac{4m_c^2}{m_J^2}\}^2}$$

$$\{2m_J^2-3m^2_{\eta}(1+\frac{2m_c}{m_J})-16\frac{m_c^3}{m_J}\}^2,$$

$$\psi_J^2(0) = \frac{27}{64\pi\alpha^2}m_J^2\Gamma_{J/\psi\rightarrow ee^+}.$$

In the study of $J/\psi \to \gamma + f(1273)$ it is found that $m_c = 1.3GeV$ fits the data very well and this value is in the range of

$$m_c = 1.25 \pm 0.09GeV.$$

$$\theta = -10.75^0 \pm 0.05^0.$$

It is predicted

$$\frac{\Gamma(J/\psi \to \gamma\eta')}{\Gamma(J/\psi \to \gamma\eta)} = 5.30 \pm 0.05.$$

The prediction agrees with current experimental value $4.81(1 \pm 0.15)$.

The factor $\frac{1}{m_c^8}$ shows strong dependence of the decay rate on m_c. Because of the cancellation between m_J and m_c in the factors

$$\{2m_J^2 - 3m_{\eta'}^2(1+\frac{2m_c}{m_J}) - 16\frac{m_c^3}{m_J}\}^2, \quad \{2m_J^2 - 3m_\eta^2(1+\frac{2m_c}{m_J}) - 16\frac{m_c^3}{m_J}\}^2$$

the decay rates are very sensitive to the value of m_c.

By changing corresponding quantities, the decay rates of $\Upsilon(1s) \to \gamma\eta', \gamma\eta$ are obtained. The ratio is determined to be

$$R_{\eta'} = \frac{B(\Upsilon \to \gamma\eta')}{B(J/\psi \to \gamma\eta')}$$

$$= \frac{1}{4}\frac{\alpha_s^2(m_b)}{\alpha_s^2(m_c)}\frac{\psi_\Upsilon^2(0)}{\psi_J^2(0)}\frac{m_c^8}{m_b^8}\frac{(1-\frac{m_{\eta'}^2}{m_\Upsilon^2})^3}{(1-\frac{m_{\eta'}^2}{m_J^2})^2}\frac{(1-2\frac{m_{\eta'}^2}{m_J^2}+4\frac{m_c^2}{m_J^2})^2}{(1-2\frac{m_{\eta'}^2}{m_\Upsilon^2}+4\frac{m_b^2}{m_\Upsilon^2})^2}$$

$$\frac{\{2m_\Upsilon^2-3m_{\eta'}^2(1+\frac{2m_b}{m_\Upsilon})-16\frac{m_b^3}{m_\Upsilon}\}^2}{\{2m_J^2-3m_{\eta'}^2(1+\frac{2m_c}{m_J})-16\frac{m_c^3}{m_J}\}^3}\frac{\Gamma_{J/\psi}}{\Gamma_\Upsilon},$$

where

$$\frac{\psi_\Upsilon^2(0)}{\psi_J^2(0)} = 4\frac{\Gamma_{\Upsilon\to ee^+}}{\Gamma_{J/\psi\to ee^+}}\frac{m_\Upsilon^2}{m_J^2}.$$

If $m_J = 2m_c$ and $m_\Upsilon = 2m_b$ are taken

$$R_{\eta'} = \frac{B(\Upsilon\to\gamma\eta')}{B(J/\psi\to\gamma\eta')} = \frac{\Gamma(\Upsilon\to\gamma\eta')/\Gamma(\Upsilon\to light\ hadrons)}{\Gamma(J/\psi\to\gamma\eta')/\Gamma(J/\psi\to light\ hadrons)}$$

$$\frac{B(\Upsilon\to light\ hadrons)}{B(J/\psi\to light\ hadrons)}$$

$$= \frac{\alpha_s(m_c)}{\alpha_s(m_b)}(\frac{m_c}{m_b})^7\frac{1-\frac{m_{\eta'}^2}{4m_b^2}}{1-\frac{m_{\eta'}^2}{4m_c^2}}\frac{B(\Upsilon\to light\ hadrons)}{B(J/\psi\to light\ hadrons)}\frac{\Gamma_{\Upsilon\to ee}}{\Gamma_{J/\psi\to ee}}$$

$$= 0.29\frac{\alpha_s(m_c)}{\alpha_s(m_b)}(\frac{m_c}{m_b})^7.$$

Comparing with all other studies, stronger dependence on quark masses and small coefficient are obtained by this approach.

In this paper $m_b = 4.7$ GeV and $m_c = 1.3$GeV are taken. Inputting

$B(J/\psi \to \gamma\eta')$, it is obtained

$$B(\Upsilon \to \gamma\eta') = R_{\eta'}B(J/\psi \to \gamma\eta') = 1.04\times^{-7}$$

$$B(\Upsilon \to \gamma\eta) = 0.022B(\Upsilon \to \gamma\eta') = 0.23 \times 10^{-8}$$

Both branching ratios are less than the experimental upper limits.

The approach has been applied to study $J/\psi, \Upsilon(1s) \to \gamma f_2(1270)$, in which the vector bosons decays to γgg and $f_2(1270)$ is coupled to two gluons. The tensor meson $f_2(1270)$ contains glueball components

$$|f_2 >= cos\phi|q\bar{q} > +sin\phi|gg > .$$

The glueball component of f_2 is dominant in the decay $J/\psi, \Upsilon(1S) \to \gamma f_2$. We have studied $J/\psi \to \gamma f_2(1270)$ long time ago. The S-matrix element of the decay is defined as

$$< f_{\lambda_2}\gamma_{\lambda_1}|S|J_\lambda >= (2\pi)^4\delta(p_J - p_\gamma - p_f)e(8\omega_\gamma E_f E_J)^{-\frac{1}{2}}T_{\lambda_2},$$

where T_{λ_2} is the helicity amplitude which is calculated in the rest frame of the f-meson,

$$T_0 = -\frac{2}{\sqrt{6}}(A_2 + p^2 A_1),$$

$$T_1 = -\frac{\sqrt{2}}{m_J}(EA_2 + m_f p^2 A_3),$$

$$T_2 = -2A_2,$$

where

$$A_1 = -a\frac{2m_f^2 - m_J(m_J - 2m_c)}{m_c m_J[m_c^2 + \frac{1}{4}(m_J^2 - 2m_f^2)]},$$

$$A_2 = -a\frac{1}{m_c}\{\frac{m_f^2}{m_J} - m_J + 2m_c\},$$

$$A_3 = -a\frac{m_f^2 - \frac{1}{2}(m_J - 2m_c)^2}{m_c m_J[m_c^2 + \frac{1}{4}(m_J^2 - 2m_f^2)]},$$

$$a = \frac{16\pi}{3\sqrt{3}}\alpha_s(m_c)G(0)\psi_J(0)\frac{\sqrt{m_J}}{m_c^2},$$

$$E = \frac{1}{2m_f}(m_J^2 + m_f^2), \quad p = \frac{1}{2m_f}(m_J^2 - m_f^2),$$

where G(0) and $\psi_J(0)$ are the wave functions at origin of the glueball and J/ψ.

The decay width of $J/\psi \to \gamma f_2$ is derived as

$$\Gamma(J/\psi \to \gamma f_2) = \frac{128\pi\alpha}{81} sin^2\phi \alpha_s^2(m_c) G^2(0)\psi_J^2(0)\frac{1}{m_c^4}(1 - \frac{m_f^2}{m_J^2})\{T_0^2 + T_1^2 + T_2^2\}.$$

The ratios of the helicity amplitudes are defined as

$$x = \frac{T_1}{T_0}, \quad y = \frac{T_2}{T_0}.$$

It is obtained

$$y = 0.$$

It agrees with data.

By changing corresponding quantities, the helicity amplitudes and the decay width of $\Upsilon(1s) \rightarrow \gamma f_2$ are obtained. The ratio of the wave functions of Υ or J/ψ at the origin is expressed as

$$\frac{\psi^2_\Upsilon(0)}{\psi^2_J(0)} = 4\frac{\Gamma_{\Upsilon\rightarrow ee^+}}{\Gamma_{J/\psi\rightarrow ee^+}}\frac{m^2_\Upsilon}{m^2_{J/\psi}}.$$

The ratios of the helicity amplitudes are obtained

$$x^2 = 0.058, \quad y^2 = 5.9\times 10^{-3}.$$

They are consistent with experimental values

$$x^2 = 0.00^{+0.02+0.01}_{-0.00-0.00}, \quad y^2 = 0.09^{+0.08+0.04}_{-0.07-0.03}.$$

In order to show the quark mass dependence of the ratio, $\frac{B(\Upsilon\to\gamma f_2)}{B(J/\psi\to\gamma f_2)}$, the decay rates of J/ψ and Υ are simplified. For $J/\psi \to \gamma f_2$ the value of m_c makes $T_2 \sim 0$ and $A_2 = 0$ is taken. For $\Upsilon \to \gamma f_2$ because of small x^2 and y^2 then T_1 and T_2 are ignored. In QCD $J/\psi,\ \Upsilon \to \mathit{light\ hadrons}$ are described as $J/\psi, \Upsilon \to 3g$ whose decay width is proportional to $\alpha_s^3 m_V$, where m_V is the mass of $J/\psi, \Upsilon$ respectively. The ratio is expressed as

$$R = \frac{B(\Upsilon\to\gamma f_2)}{B(J/\psi\to\gamma f_2)} = \frac{\Gamma(\Upsilon\to\gamma f_2)}{\Gamma(J/\psi\to\gamma f_2)}\frac{\Gamma(J/\psi\to lh)}{\Gamma(\Upsilon\to lh)}\frac{B(\Upsilon\to lh)}{B(J/\psi\to lh)}$$

$$= 1.06\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\frac{p_\Upsilon^4}{p_J^4}\frac{m_J m_c^6}{m_\Upsilon m_b^6}\frac{[m_c^2+\frac{1}{4}(m_J^2-2m_f^2)]^2}{[m_b^2+\frac{1}{4}(m_\Upsilon^2-2m_f^2)]^2}\frac{(1-\frac{m_f^2}{m_\Upsilon^2})}{(1-\frac{m_f^2}{m_\Upsilon^2})}$$

$$\frac{\{2m_f^2-m_\Upsilon(m_\Upsilon-2m_b)\}^2}{\{2m_f^2-m_J(m_J-2m_c)\}^2+6\frac{m_f^2}{m_J^2}\{m_f^2-\frac{1}{2}(m_J-2m_c)^2\}^2},$$

where $p_\Upsilon = \frac{m_\Upsilon^2}{2m_f}(1-\frac{m_f^2}{m_\Upsilon^2})$ and $p_J = \frac{m_J^2}{2m_f}(1-\frac{m_f^2}{m_J^2})$. Two factors affect the quark mass dependence of the ratio R. It is the same as $B(\Upsilon \to \gamma\eta'(\eta))$, in the amplitude $(\Upsilon, J/\psi) \to \gamma gg, gg \to f_2$ $(\Upsilon, J/\psi) \to \gamma gg$ is strongly suppressed by heavy quark mass. The second one is that d-wave dominates both the decay amplitudes of $(\Upsilon, J/\psi) \to \gamma f_2$. The s-wave is only related to A_2 which is very small and can be ignored in both cases. The enhancement factor, $\frac{p_\Upsilon^4}{p_J^4}$, is the consequence of the d-wave dominance. This factor increases with b-quark mass dramatically. The combination of these two effects makes the dependence of $\frac{B(\Upsilon\to\gamma f_2)}{B(J/\psi\to\gamma f_2)}$ on quark masses much weaker than the ratio $\frac{B(\Upsilon\to\gamma\eta'(\eta)}{B(J/\psi\to\gamma\eta'(\eta))}$. d-wave dominance in $\Upsilon,\ J/\psi \to \gamma f_2$ leads to larger $B(\Upsilon \to \gamma f_2)$.

$m_c = 1.29GeV$ and $m_b = (5.04 \pm 0.075 \pm 0.04)GeV$ are chosen to fit the experimental data of $B(\Upsilon(1S) \to \gamma f_2(1270))$.