Testing the viscous hydrodynamic paradigm for RHIC

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reflects long history of collaboration with Pasi Huovinen:

PRL94, 012302 (nucl-th/0404065)
PRC79, 014906 (arXiv:0808.0953)
JPG35, 104125 (arXiv:0806.1367)
arXiv:0907.5014 + work in progress

Outline

- **I. The viscous hydrodynamics paradigm for RHIC**

  - hydrodynamics and thermalization at RHIC
  - viscosity and hydrodynamics

- **II. Testing the paradigm**

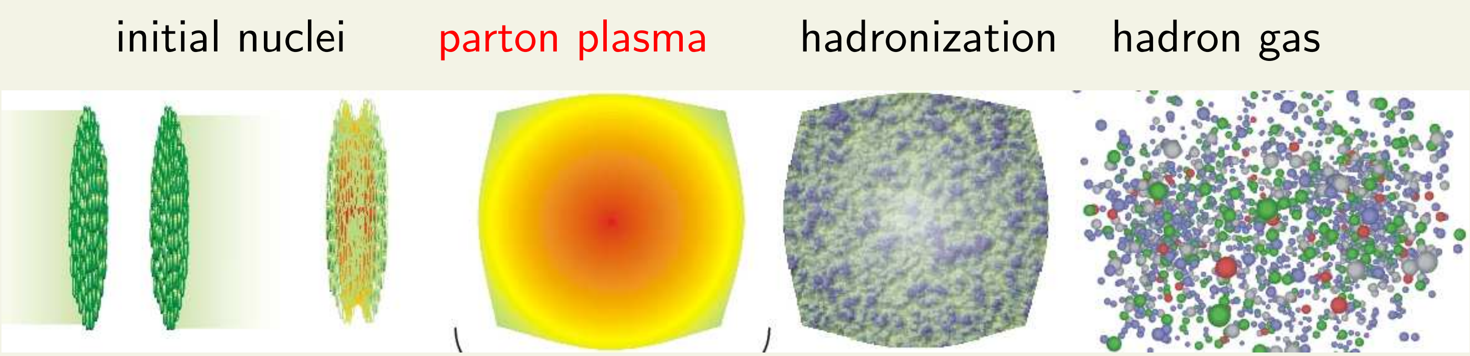
  - region of validity through with covariant transport
  - influence of bulk viscosity

- **III. Next steps / "To do" and wish lists**

  - viscosity and equation of state from the data
  - other observables - conical flow

Heavy ion collisions

goal: experimentally test QCD at finite T - equation of state, viscosity, ...



$\uparrow$ $\sim 10^{-14}$ **m** **= 10 fermis** $\downarrow$

<—— $\sim 10^{-23}$ **sec = 10 yochto(!)-secs** ——>

- **rapid expansion, inhomogeneities (time and length scales comparable)**
- **early and late dynamics out of equilibrium**

does it thermalize?

We think it does thermalize to a large degree, based on

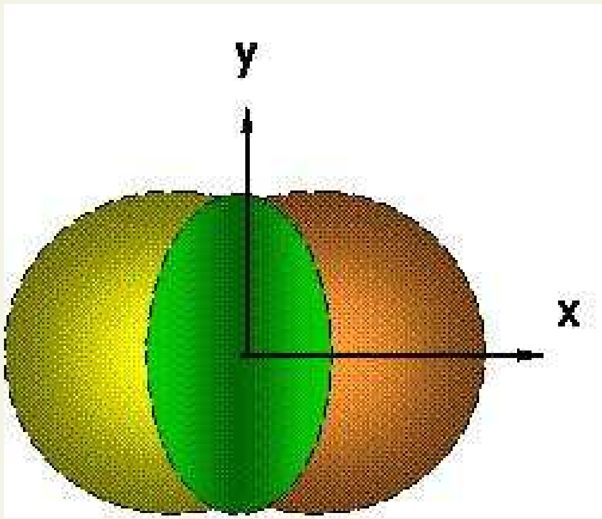
- **yields and spectra of particles look thermal**

  $T_{chem} \sim 165$ MeV, $T_{kin} \sim 110$ MeV, $\langle v_T \rangle \sim 0.5c$

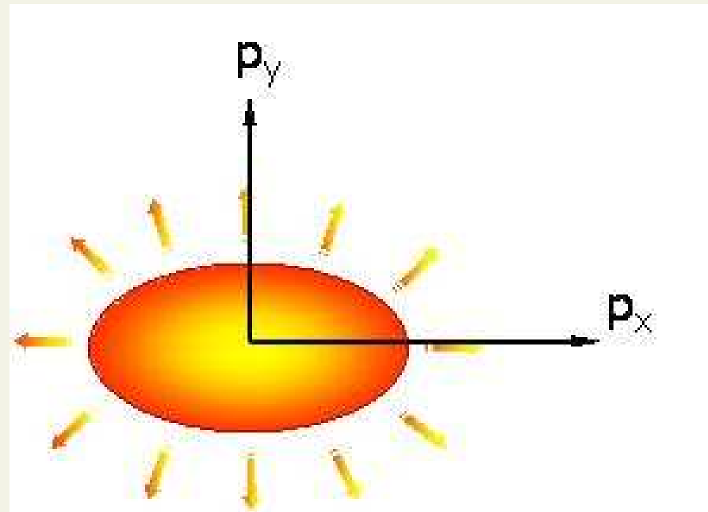
- **large azimuthal anisotropy ("elliptic flow")**
- **strong suppression of high-momentum probes, even heavy quarks**
- **remarkable success of ideal hydrodynamics**

Elliptic flow (v_2)

- striking collective behavior $v_2 \sim 0.1 - 0.2$



$\longrightarrow$



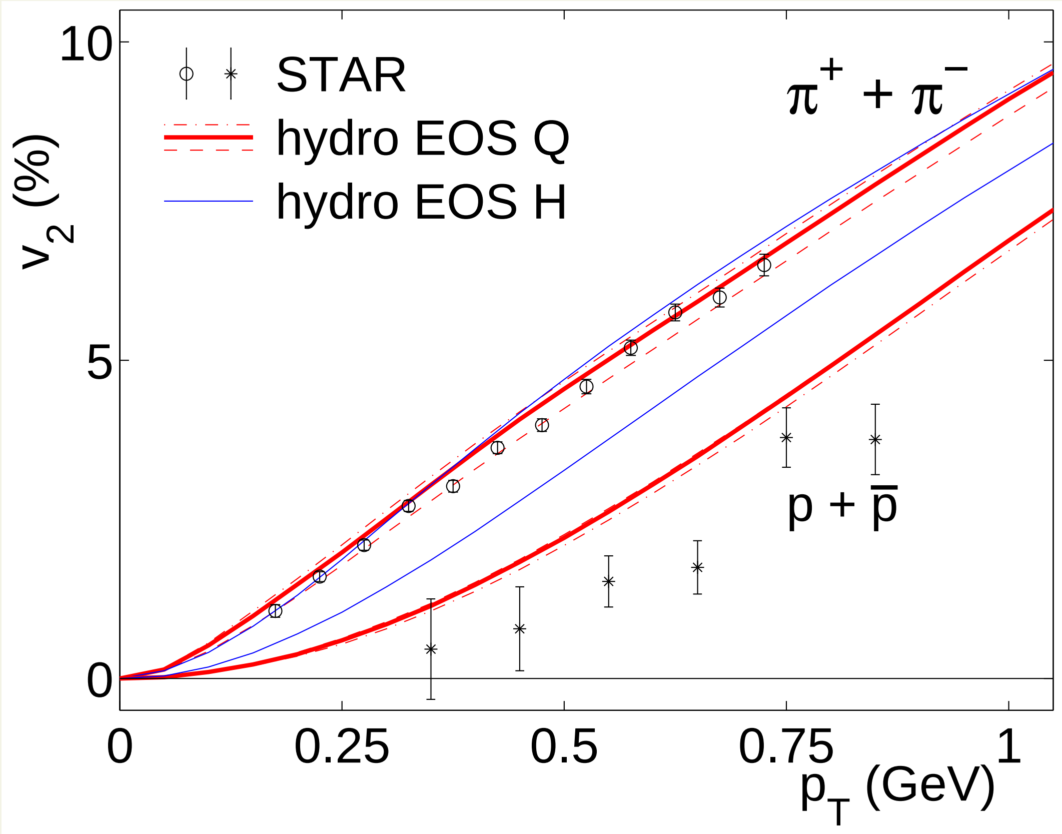
$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} \qquad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

sensitive to pressure gradients and therefore speed of sound $c_s^2 = \partial p / \partial e$

Success of ideal hydrodynamics

$\partial_\mu T^{\mu\nu}(x) = 0$, $\partial_\mu N^\mu_B = 0$, $+\ p(e, n_B)$ **equation of state (local equil.)**

minimum-bias Au+Au at RHIC Kolb, Heinz, Huovinen et al ('01), nucl-th/0305084



$\tau_{therm} = 0.6$ **fm** , $\langle e \rangle_{init} \sim 5GeV/fm^3$, $T_{freezeout} = 120$ **MeV**

pion-proton splitting favors equation of state with QGP phase transition

WHY does hydro work??

rather nontrivial - local equilibrium is a moving target (system expands)

competing ideas:

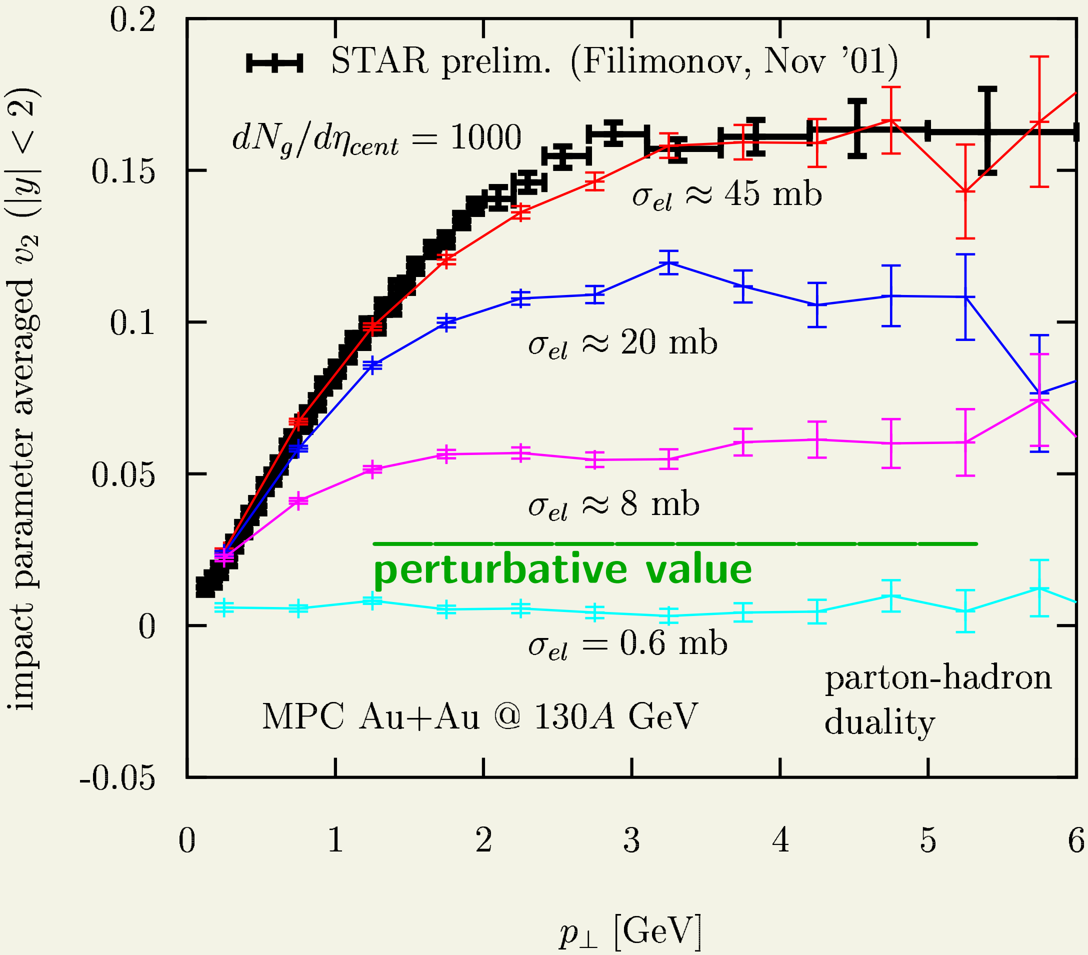
- perturbative angle: radiative $3 \leftrightarrow 2$ processes thermalize
- strongly-coupled angle: analogy with $\mathcal{N} = 4$ SYM through AdS/CFT
  nonperturbative, minimally viscous $\eta/s = \mathcal{O}(1)/(4\pi)$
- classical field angle: plasma instabilities - still too slow

  Mrowczynski, Arnold, Moore, ..., Strickland, Dumitru et al

[• elliptic flow jumps up at hadronization (coalescence) - largely ruled out]

Zimanyi, Biro, Levai... Mueller, Bass... Ko, Greco... Voloshin... DM, Lin...

$v_2(p_T, \chi)$ from $2 \to 2$ parton kinetic theory DM & Gyulassy, NPA 697 ('02)



$$p^\mu \partial_\mu f(x, \vec{p}) = C_{2\to 2}[f](x, \vec{p})$$

well-defined hydrodynamic limit

can thermalize, in principle

$e \approx 3p$ **roughly on lattice, too**

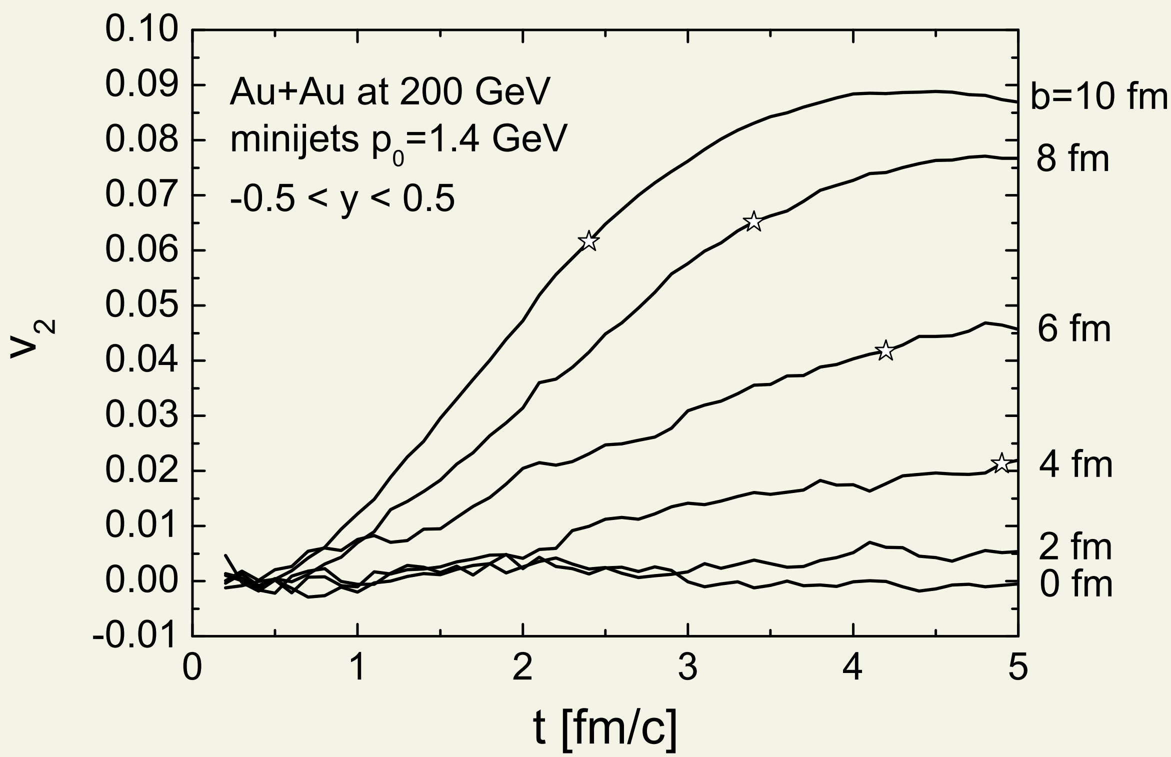
but perturbative $2 \to 2$ rates are not enough, need $15\times$ higher

radiative $ggg \leftrightarrow gg$ can provide the rate enhancement Baier, Mueller, Son... ('01), Greiner, Xu... ('05)

$$p^\mu \partial_\mu f(x,\vec{p}) = C_{2\to 2}[f](x,\vec{p}) + C_{3\leftrightarrow 2}[f](x,\vec{p})$$

allows for large-angle scattering and thus can generate large elliptic flow

El, Xu & Greiner, NPA 785 ('07): **perturbative calculation with $2 \to 2$ and $3 \leftrightarrow 2$**



$\Rightarrow$ **the opposite of the strongly-coupled nonperturbative plasma paradigm**

strongly coupled $\mathcal{N} = 4$ SYM in 4D $\Leftrightarrow$ weakly-coupled gravity on AdS$_5$

key insights, in large 't Hooft coupling $\lambda \equiv g_s^2 N_c \to \infty$ limit

only modest change to ideal gas thermodynamics Gubser et al ('96), Buchel et al ('08):

$$\frac{S}{S_{SB}} = \frac{3}{4}\left[1 + 4\delta + \mathcal{O}(\frac{\sqrt{\lambda}}{N_c^2}) + \mathcal{O}(\frac{1}{\lambda^{3/2}})\right]$$

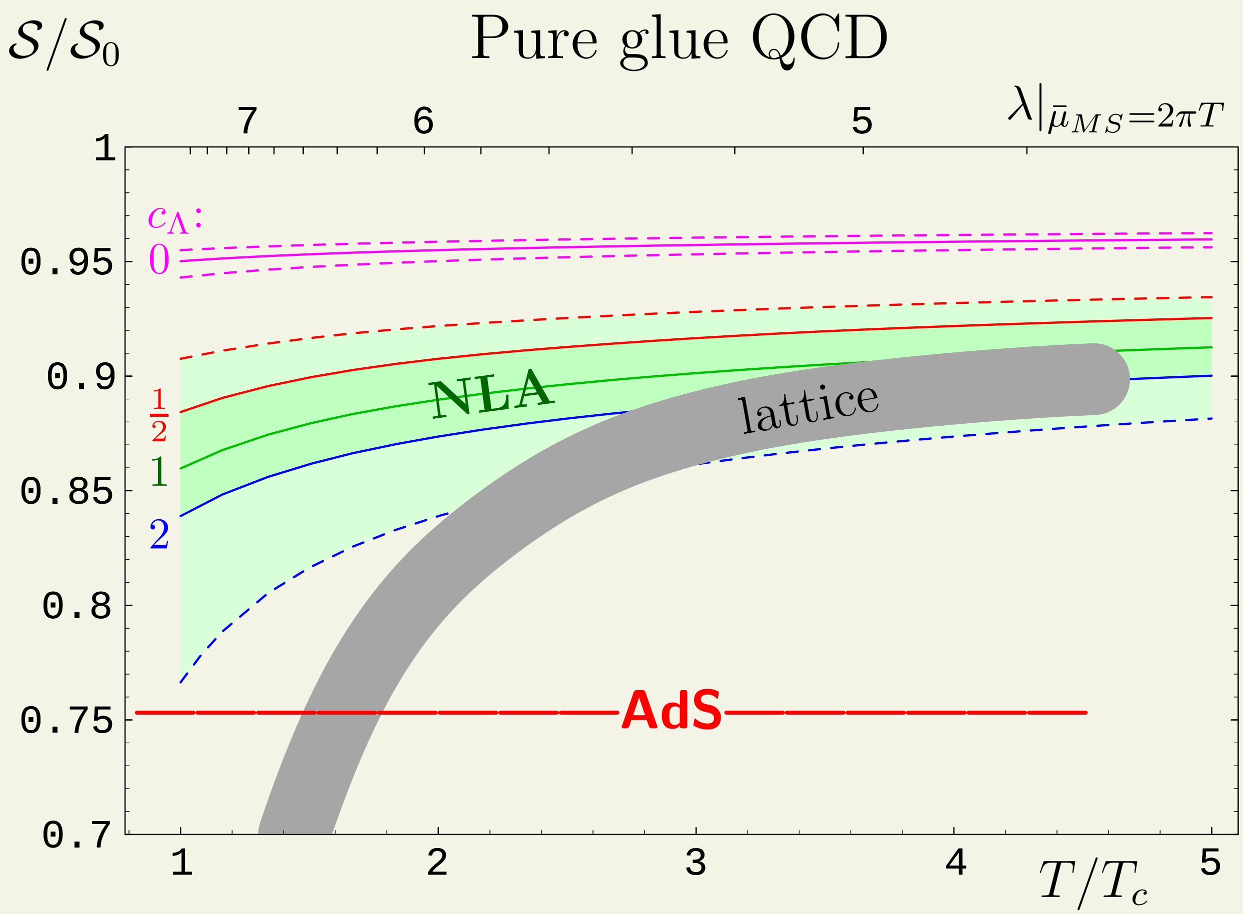
very small, but nonzero shear viscosity Son, Policastro et al ('02), ('04), Buchel et al ('09):

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \delta + \mathcal{O}(\frac{1}{\lambda^{3/2}})\right]$$

short relaxation times Baier et al arxiv:0712.2451, Chesler and Yaffe, arxiv:0812.2053,

$$\tau_\pi = \frac{2 - \ln 2}{2\pi T} \qquad\qquad \tau_{th} \sim \frac{\mathcal{O}(1)}{T_{eff}}$$

(adapted from Blaizot et al, JHEP0706, 035 ('07))

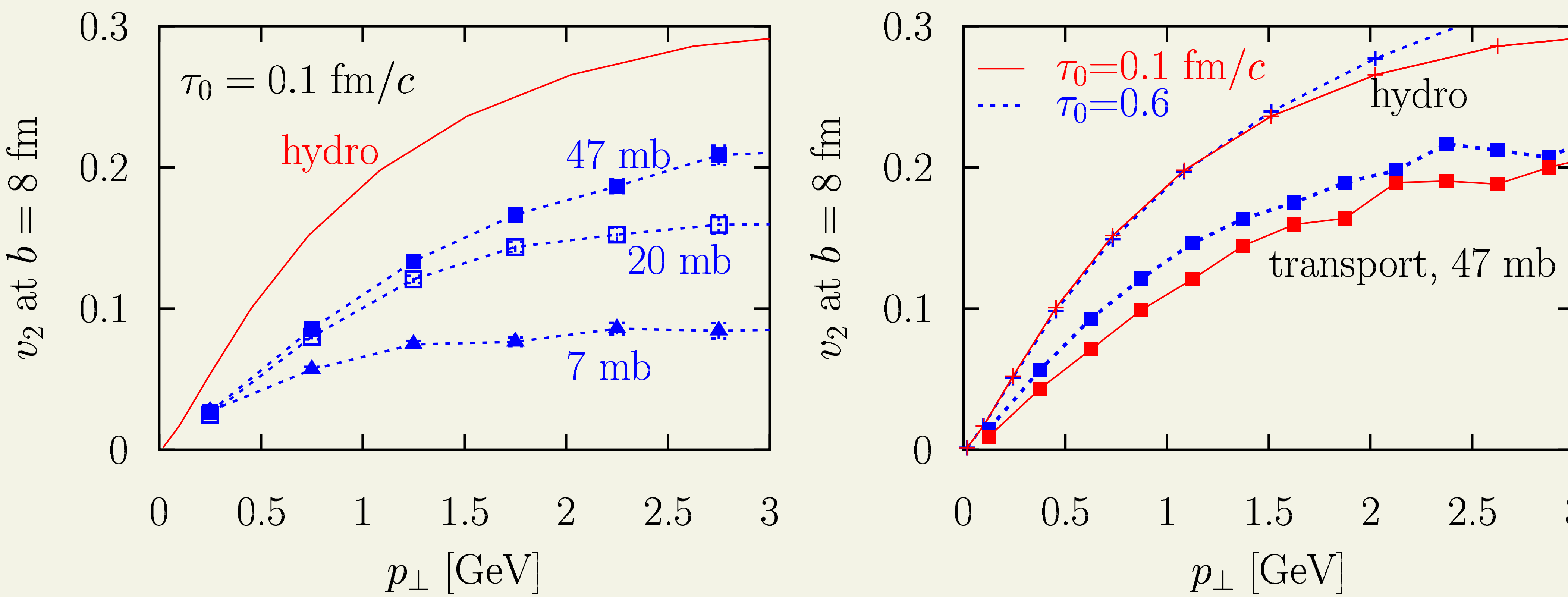


which of the two limits applies better at RHIC? $(g \approx 2,\ SYM \neq QCD)$

$\Rightarrow$ **unclear, should pursue BOTH limits**

does either of these limits support ideal hydrodynamics?

DM & Huovinen, PRL94 ('05): **hydro vs $2 \to 2$ transport**



transport calculations that reproduce RHIC data still show $20 - 30$% corrections to ideal hydro

Dissipative hydrodynamics

relativistic Navier-Stokes hydro: small corrections linear in gradients

$$T^{\mu\nu}_{NS} = T^{\mu\nu}_{ideal} + \overbrace{\eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha)}^{\text{shear stress } \pi^{\mu\nu}} + \overbrace{\zeta(\partial^\alpha u_\alpha)}^{-\Pi \text{ bulk pressure}}\Delta^{\mu\nu}$$

$$N^\mu_{NS} = N^\mu_{ideal} - \frac{n}{e+p}\underbrace{\kappa\nabla^\mu T}_{q^\mu \text{ - heat flow}}$$

where $\Delta^{\mu\nu} \equiv u^\mu u^\nu - g^{\mu\nu}$, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

η, ζ **shear and bulk viscosities,** κ **heat conductivity**

two problems:

parabolic equations $\rightarrow$ acausal Müller ('76), Israel & Stewart ('79) ...

instabilities Hiscock & Lindblom, PRD31, 725 (1985) ...

Second-order theories

gradient expansion in kinetic theory Chapman-Enskog, ... de Groot, ... Moore and York ('08)

$$f(x,p) = f_{eq}(x,p) + \lambda f_1(x,p) + \lambda^2 f_2(x,p) + ..., \qquad (\lambda \text{ counts derivatives})$$

$\rightarrow$ **1st order gives Navier-Stokes (acausal, unstable)**

moment expansion in kinetic theory Grad..., Israel, Stewart, ...

$$f(x,p) = f_{eq}(x,p)\,(1 + p_\alpha C^\alpha(x) + p_\alpha p_\beta C^{\alpha\beta}(x) + ...)$$

$\rightarrow$ **2nd order gives Israel-Stewart hydrodynamics**

generalized thermodynamics Israel, Stewart

$$S^\mu = s_{eq}u^\mu + q^\mu \frac{\mu n}{T(e+p)} + all\ quadratic\ terms\ with\ \Pi, \pi_{\mu\nu}, q_\mu + ...$$

match most general equation of motion to correlators Baier, Romatschke et al...

$\Rightarrow$ **basically all convert Navier-Stokes to relaxation equations**

Complete set of Israel-Stewart equations of motion

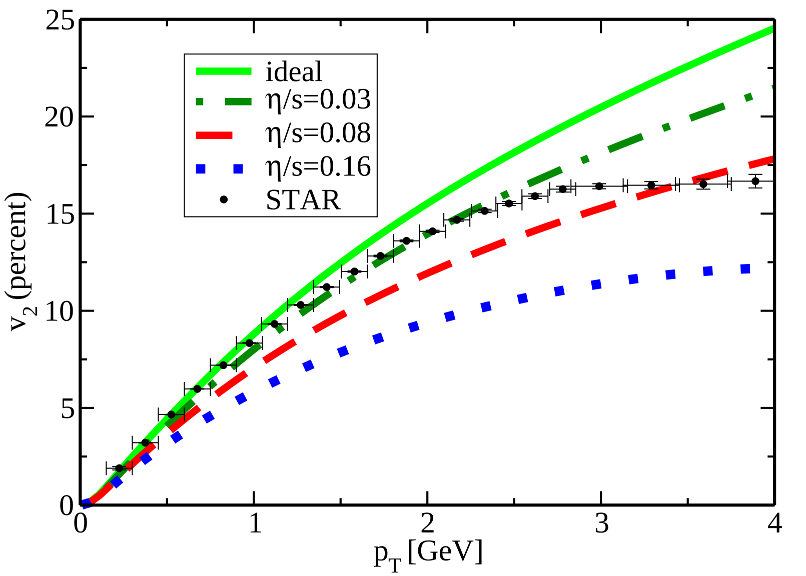
$$
\begin{aligned}
D\Pi &= -\frac{1}{\tau_\Pi}\left(\Pi + \zeta\nabla_\mu u^\mu\right) \\
&\quad -\frac{1}{2}\Pi\left(\nabla_\mu u^\mu + D\ln\frac{\beta_0}{T}\right) \\
&\quad +\frac{\alpha_0}{\beta_0}\partial_\mu q^\mu - \frac{a_0'}{\beta_0}q^\mu D u_\mu
\end{aligned} \quad (1)
$$

$$
\begin{aligned}
Dq^\mu &= -\frac{1}{\tau_q}\left[q^\mu + \kappa_q\frac{T^2 n}{\varepsilon + p}\nabla^\mu\left(\frac{\mu}{T}\right)\right] - u^\mu q_\nu D u^\nu \\
&\quad -\frac{1}{2}q^\mu\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_1}{T}\right) - \omega^{\mu\lambda}q_\lambda \\
&\quad -\frac{\alpha_0}{\beta_1}\nabla^\mu\Pi + \frac{\alpha_1}{\beta_1}(\partial_\lambda\pi^{\lambda\mu} + u^\mu\pi^{\lambda\nu}\partial_\lambda u_\nu) + \frac{a_0}{\beta_1}\Pi D u^\mu - \frac{a_1}{\beta_1}\pi^{\lambda\mu}D u_\lambda
\end{aligned} \quad (2)
$$

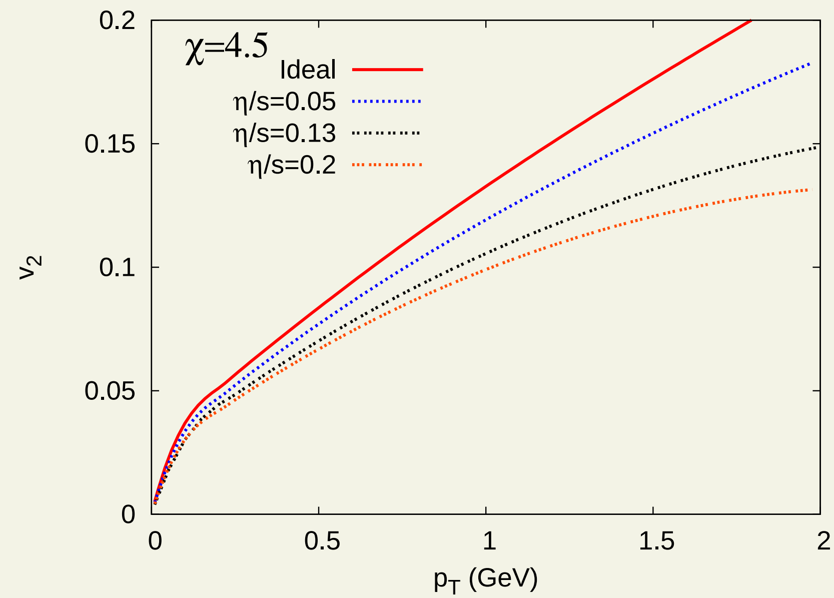
$$
\begin{aligned}
D\pi^{\mu\nu} &= -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu)D u_\lambda \\
&\quad -\frac{1}{2}\pi^{\mu\nu}\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_2}{T}\right) - 2\pi_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda} \\
&\quad -\frac{\alpha_1}{\beta_2}\nabla^{\langle\mu}q^{\nu\rangle} + \frac{a_1'}{\beta_2}q^{\langle\mu}D u^{\nu\rangle}\ .
\end{aligned} \quad (3)
$$

where $A^{\langle\mu\nu\rangle} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(A_{\alpha\beta}+A_{\beta\alpha}) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}A^{\alpha\beta}$, $\quad\omega^{\mu\nu} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\beta u_\alpha - \partial_\alpha u_\beta)$

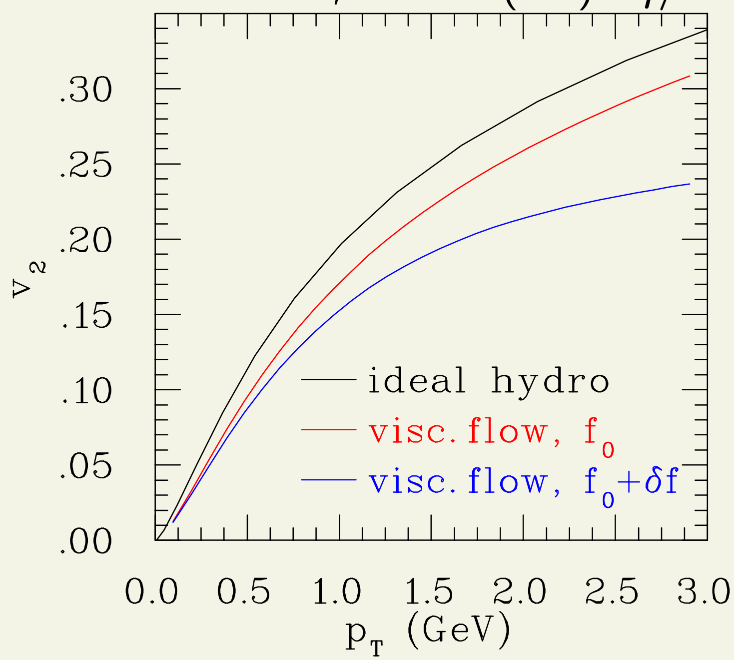
Romatschke & Romatschke, PRL99 ('07)



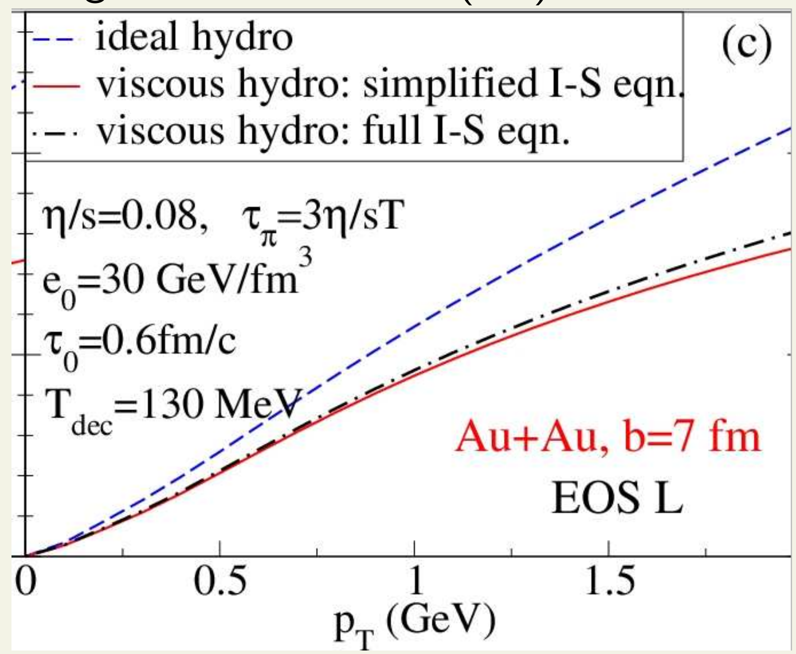
Dusling & Teaney, PRC77



Huovinen & DM, JPG35 ('08): $\eta/s \approx 1/4\pi$



Song & Heinz, PRC78 ('08)



All groups find $\sim 20-30$% effects for $\eta/s = 1/(4\pi)$ in Au+Au at RHIC.

Part II. Testing the hydro paradigm

Two kinds of questions:

- **how well it works against data**
- **accuracy / region of validity**

 generally: long wavelength, late time

 quantitative answer: requires comparison to a nonequilibrium theory

especially important because for typical RHIC initial conditions $T_0\tau_0 \sim 1$**, dissipative corrections to pressure are fairly large already when** $\eta/s = 1/(4\pi)$

$$\frac{\delta T^{\mu\nu}_{NS}}{p} \approx \left(2\frac{\eta_s}{s}\frac{\nabla^{\langle\mu}u^{\nu\rangle}}{T} + \frac{\zeta}{s}\frac{\nabla_\alpha u^\alpha}{T}\right)\frac{\varepsilon + p}{p} \sim \mathcal{O}\left(\frac{8\eta_s}{s}, \frac{4\zeta}{s}\right) . \quad (4)$$

Transport vs dissipative hydro

Huovinen & DM, PRC 79, 014906

- **covariant $2 \to 2$ transport vs complete set of Israel-Stewart equations**

- **near hydro limit of transport (transport cross section $\sigma_{tr} \to \infty$)**

$$e = 3p \ , \quad \eta \approx \frac{4T}{5\sigma_{tr}} \ , \quad \tau_\pi \approx 1.2\lambda_{tr} \equiv \frac{1.2}{n\sigma_{tr}} \ , \quad \zeta = 0$$

- **the main challenge for hydro in heavy-ion collisions is to keep up with the rapid longitudinal dilution of the system (Bjorken expansion)**

most relevant parameter is the inverse Knudsen number

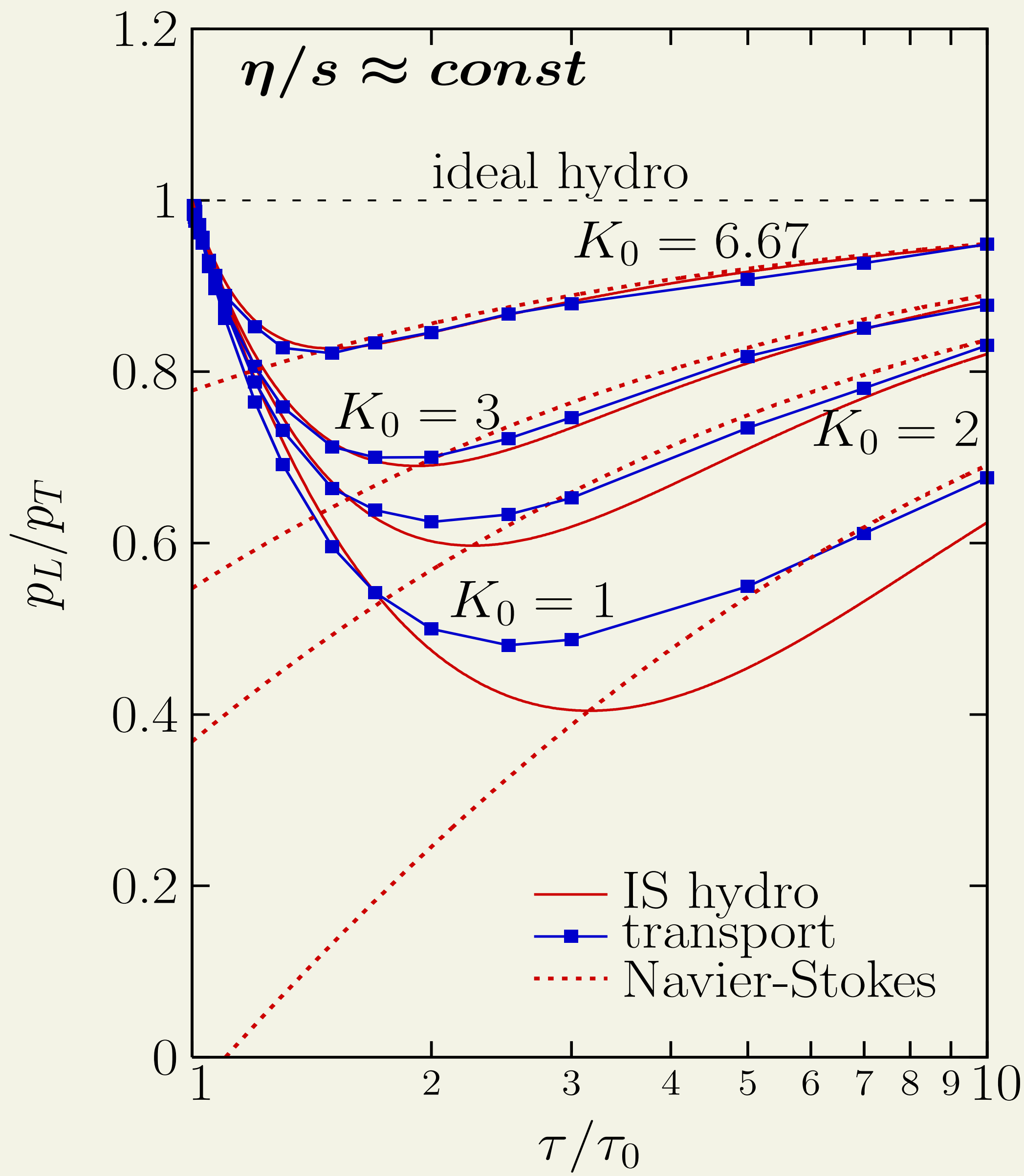
$$K(\tau) \equiv \frac{\tau}{\lambda_{tr}(\tau)} = \frac{\tau_{exp}}{\tau_{scatt}} \approx \frac{6\tau_{exp}}{5\tau_\pi} \qquad (5)$$

$K \to \infty$ - **hydro,** $\qquad K \to 0$ - **free streaming**

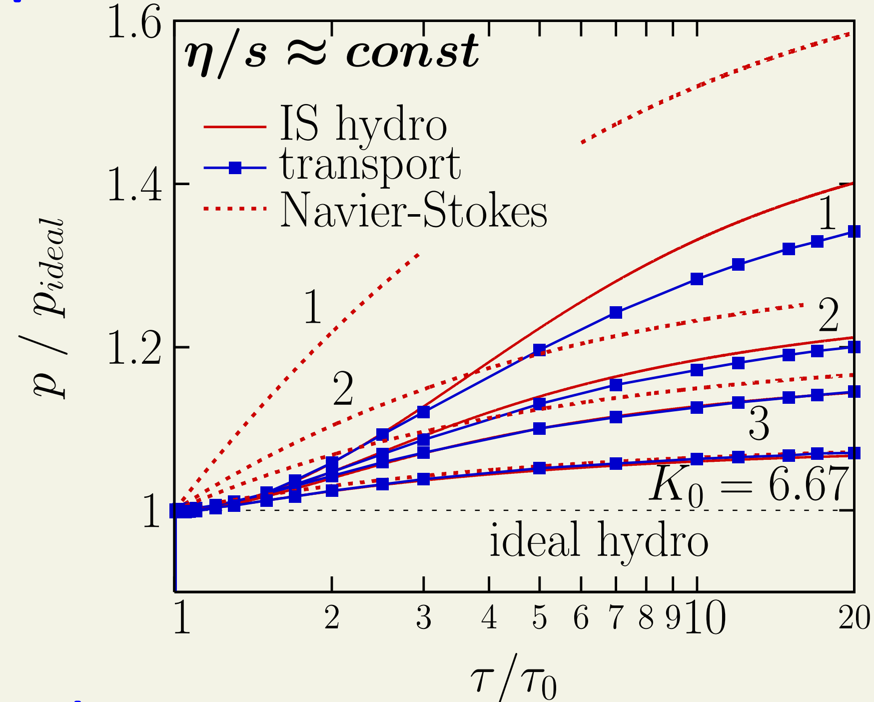
for conformal $\eta/s = const$ **dynamics:** $\lambda \sim 1/T \sim \tau^{-1/3} \Rightarrow K(\tau) \sim \tau^{2/3}$

for $\eta/s \approx const$ dynamics:

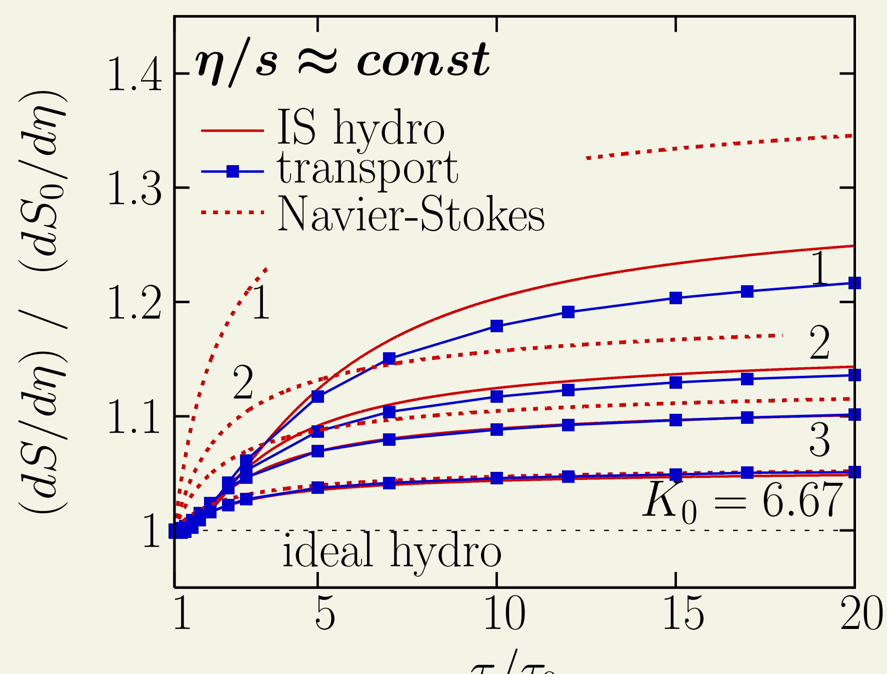
pressure anisotropy T_{zz}/T_{xx}



pressure



entropy



IS hydro 10% accurate when $K_0 \gtrsim 2$

Connection to viscosity

$$K_0 \approx \frac{T_0 \tau_0}{5} \frac{s_0}{\eta_{s,0}} \approx 12.8 \times \left(\frac{T_0}{1\ \mathrm{GeV}}\right) \left(\frac{\tau_0}{1\ \mathrm{fm}}\right) \left(\frac{1/(4\pi)}{\eta_0/s_0}\right) \tag{6}$$

For typical RHIC hydro initconds $T_0 \tau_0 \sim 1$, therefore

$$K_0 \gtrsim 2 - 3 \qquad \Rightarrow \qquad \frac{\eta}{s} \lesssim \frac{1-2}{4\pi} \tag{7}$$

I.e., shear viscosity cannot be many times more than the conjectured bound, for IS hydro to be applicable.

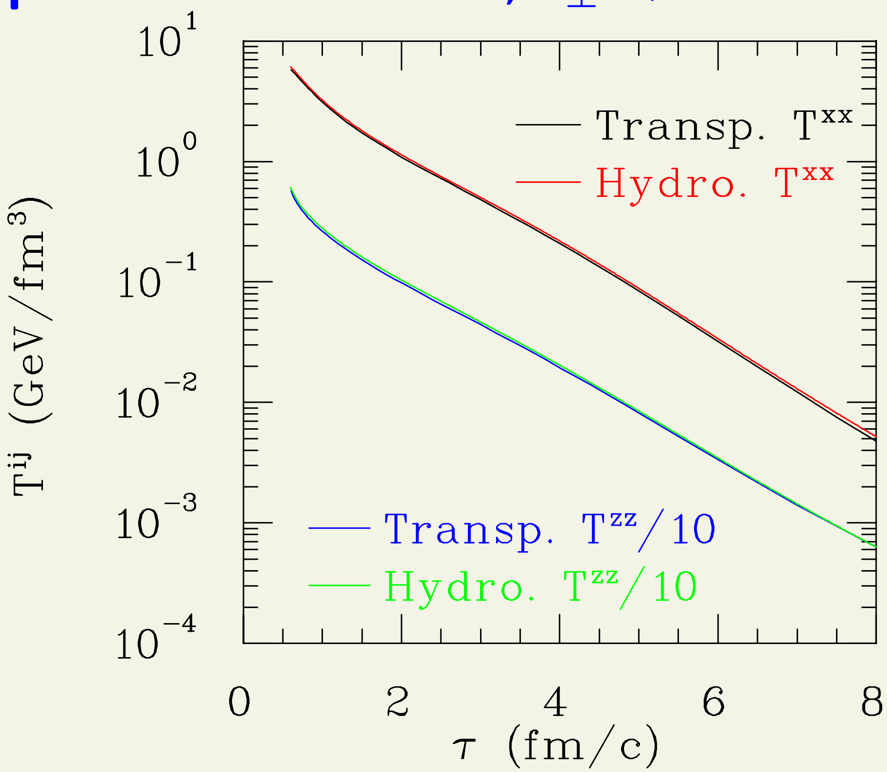
When IS hydro is accurate, dissipative corrections to pressure and entropy do not exceed 20% significantly (a necessary condition). This holds for a wide range $[0.476, 1.697]$ of initial pressure anisotropies.

Test of IS hydro in 2+1D

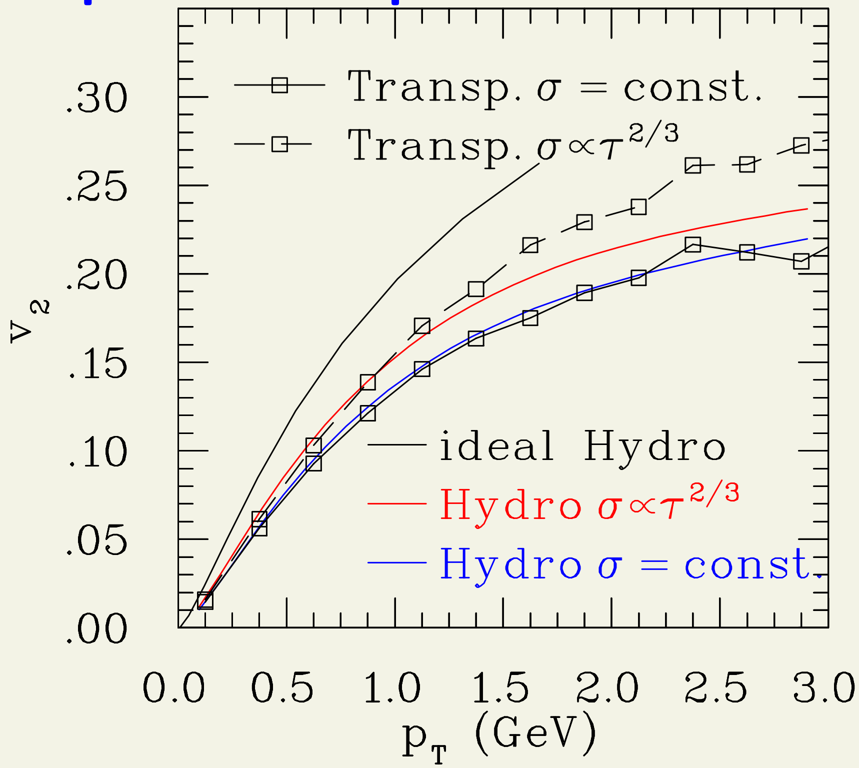
Au+Au at RHIC, $b = 8$ fm

Huovinen & DM, JPG35, 104125

pressure in the core, $r_\perp < 1$ fm



elliptic flow vs pT



$\boldsymbol{\eta/s \approx 1/(4\pi)}$ $(\sigma \propto \tau^{2/3})$

- $\sigma =$ **const** $\sim 47mb$
- $\eta/s \approx 1/(4\pi)$**, i.e.,** $\sigma \propto \tau^{2/3}$

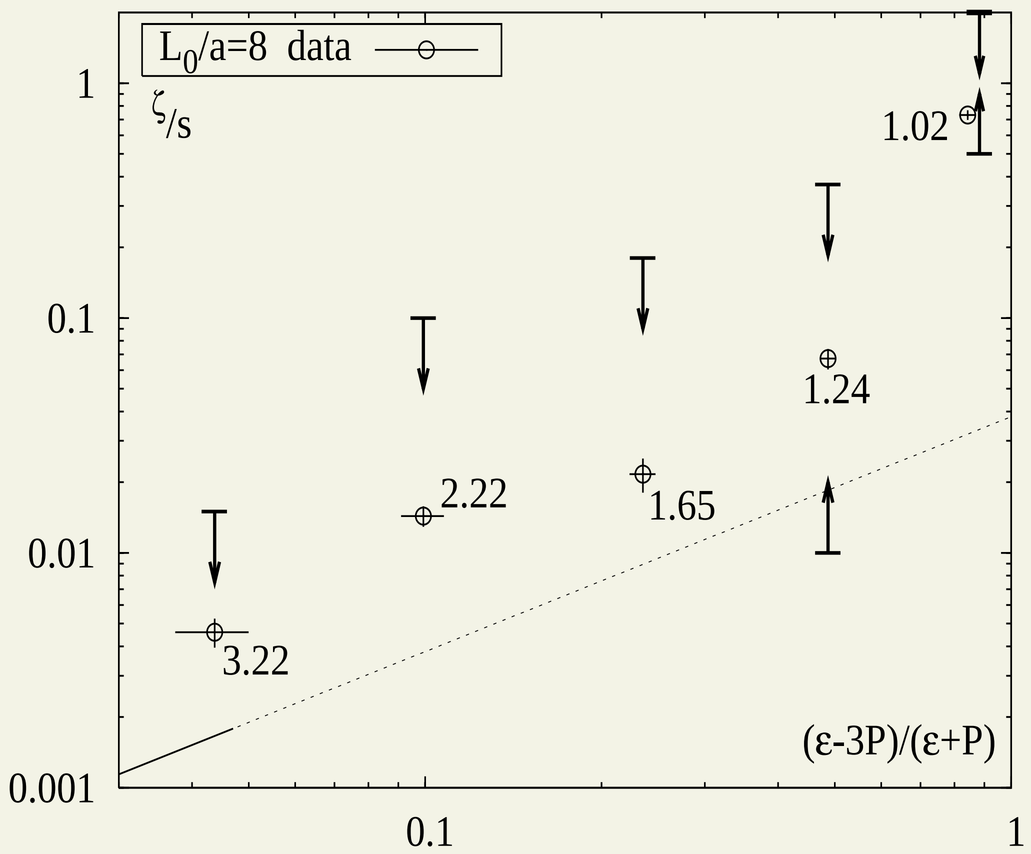
$\Rightarrow$ **if** $\eta/s \approx 1/(4\pi)$**, or** $\sigma \sim 47mb$**, IS hydro is a good approximation**

Tests with shear and bulk viscosity

bulk viscosity - largely unknown at $T \sim 200 - 400$ MeV relevant for RHIC

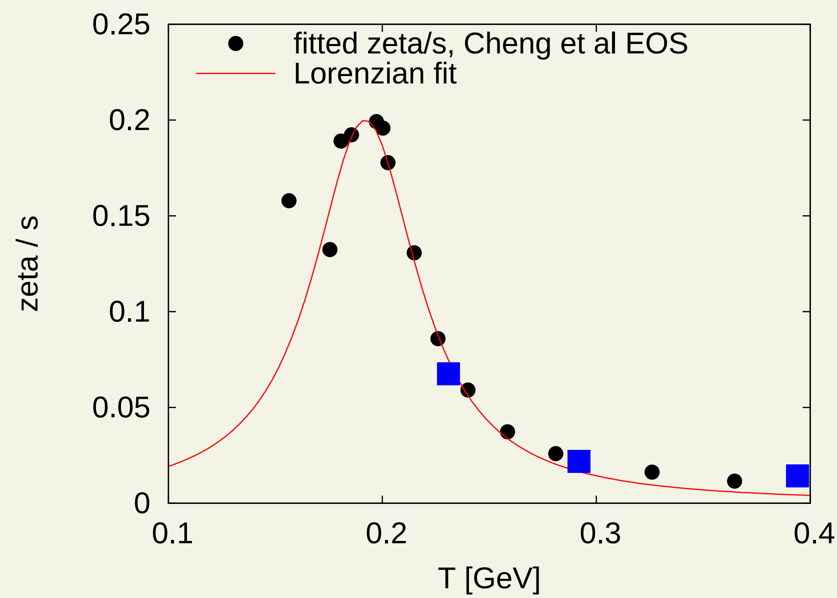
perturbatively tiny: $\zeta/s \sim 0.02\alpha_s^2$,
Arnold, Dogan, Moore, PRD74 ('06)

on lattice: preliminary estimate
Meyer, arXiv:0710.3717



- **no quarks (gluons only)**
- **crude lattices**

implies peak near T_c



study range of validity as a function of initial shear stress, bulk pressure, and thermalization time DM & Huovinen, arxiv:0907.5014

local equilibrium: $\Pi = 0 = \pi_L$

Navier-Stokes: $\pi_L = -4\eta/(3\tau)$, $\Pi = \zeta/\tau$

color glass condensate: $\pi_L \approx -p$, $\Pi \approx 0$

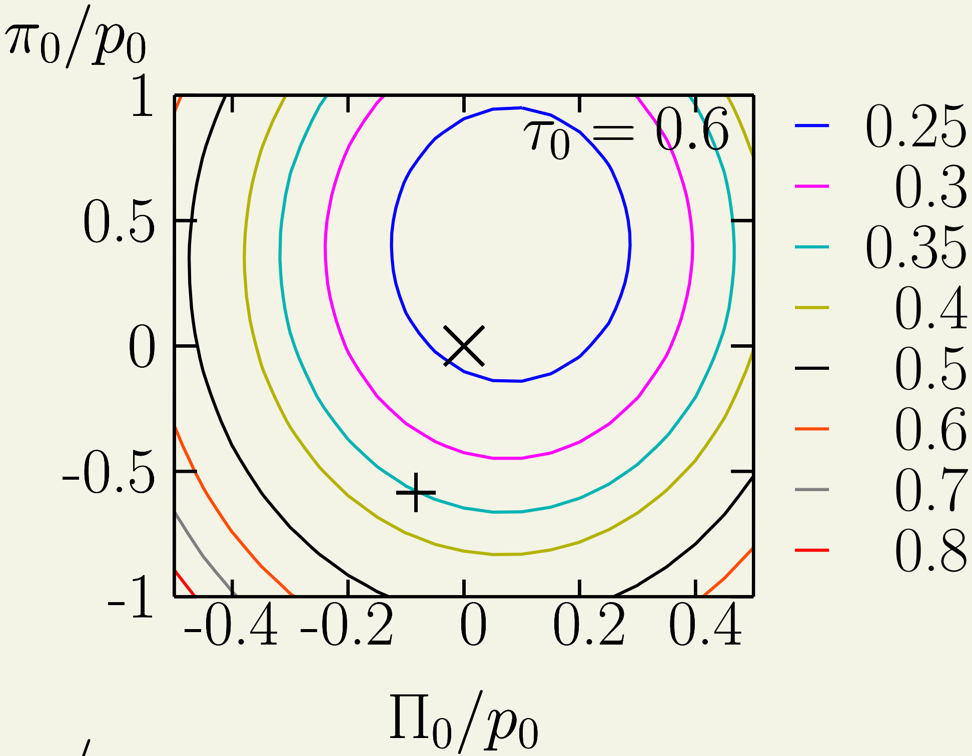
strategy: rely on the 20% rule of thumb, and demand

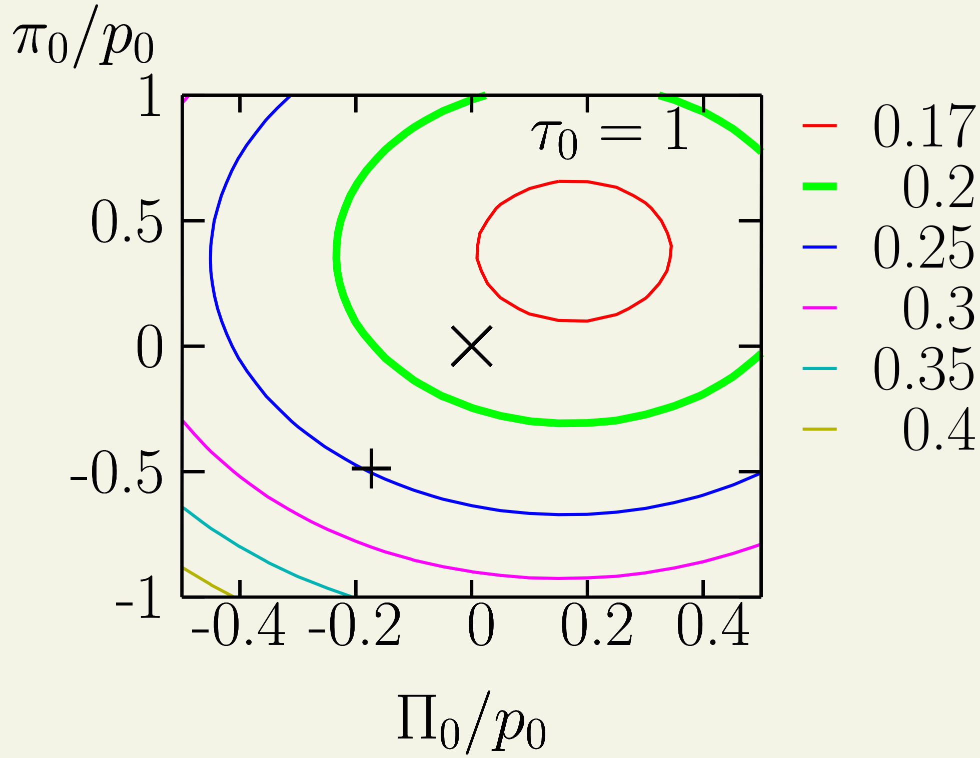
$$\frac{\Delta S}{S_0} \equiv \frac{S(T_f = 180 \text{ MeV}) - S_0}{S_0} \leq 0.2$$

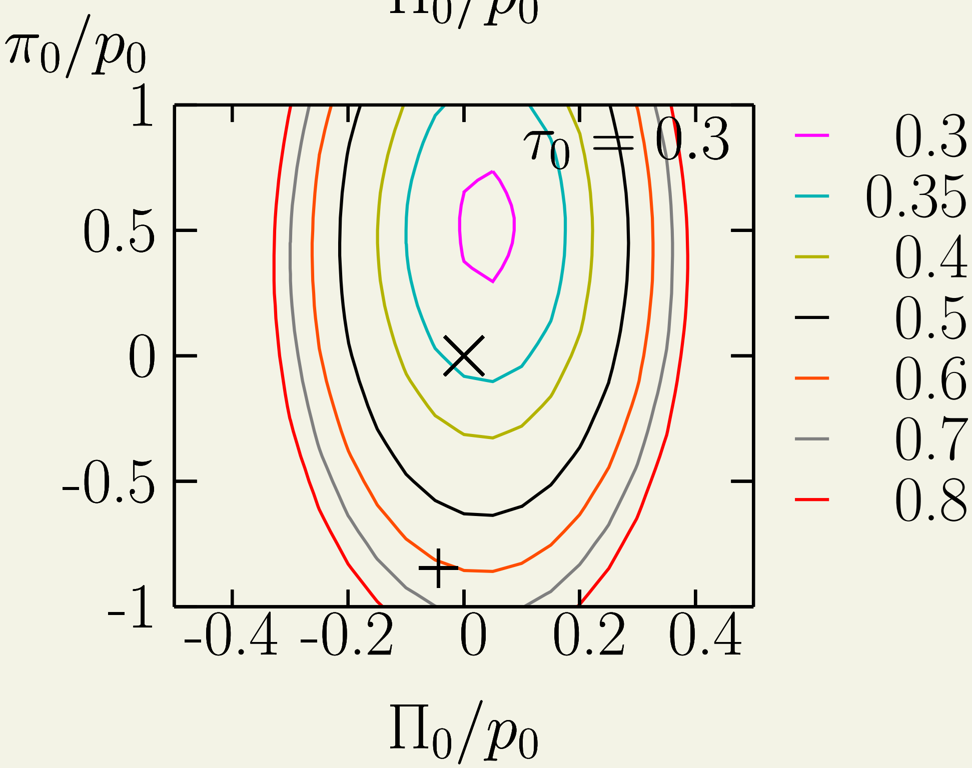
(direct hydro-transport comparison unfeasible - from transport $\zeta/\eta \lesssim 0.01$)

ingredients and initial conditions (in 0+1D):

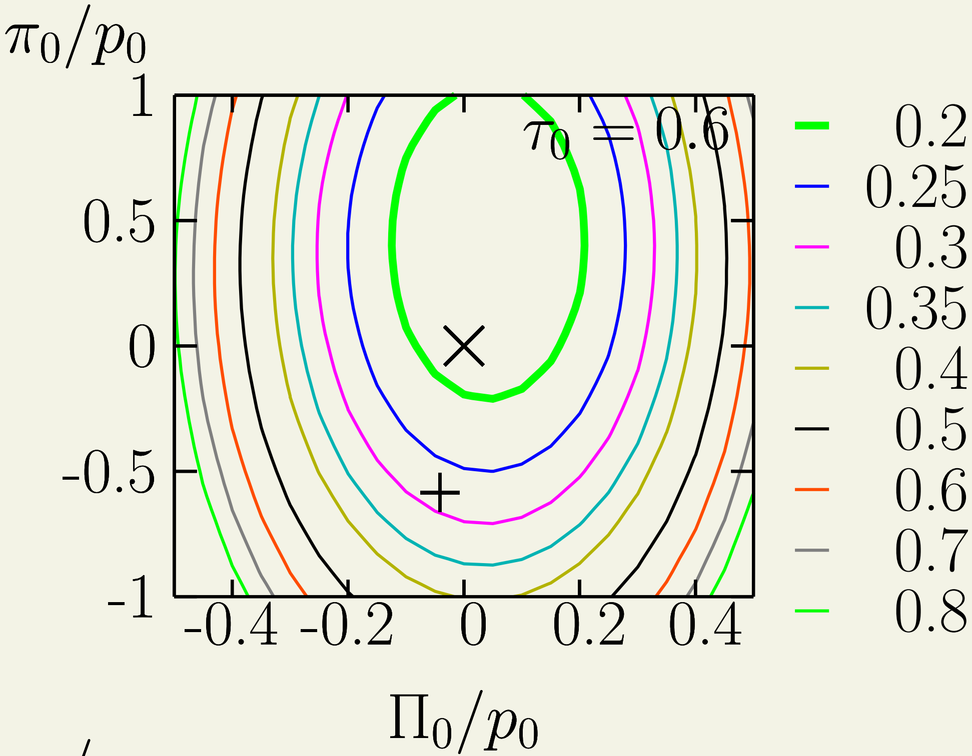
- **lattice EOS, minimal** $\eta = s/(4\pi)$, $\tau_\pi = \frac{6}{T}\frac{s}{\eta} = \tau_\Pi$
- $e_0 = 15$ **GeV/fm**^{3} **at** $\tau_0 = 0.6$ **fm, scale to other** τ_0 **via entropy conservation**

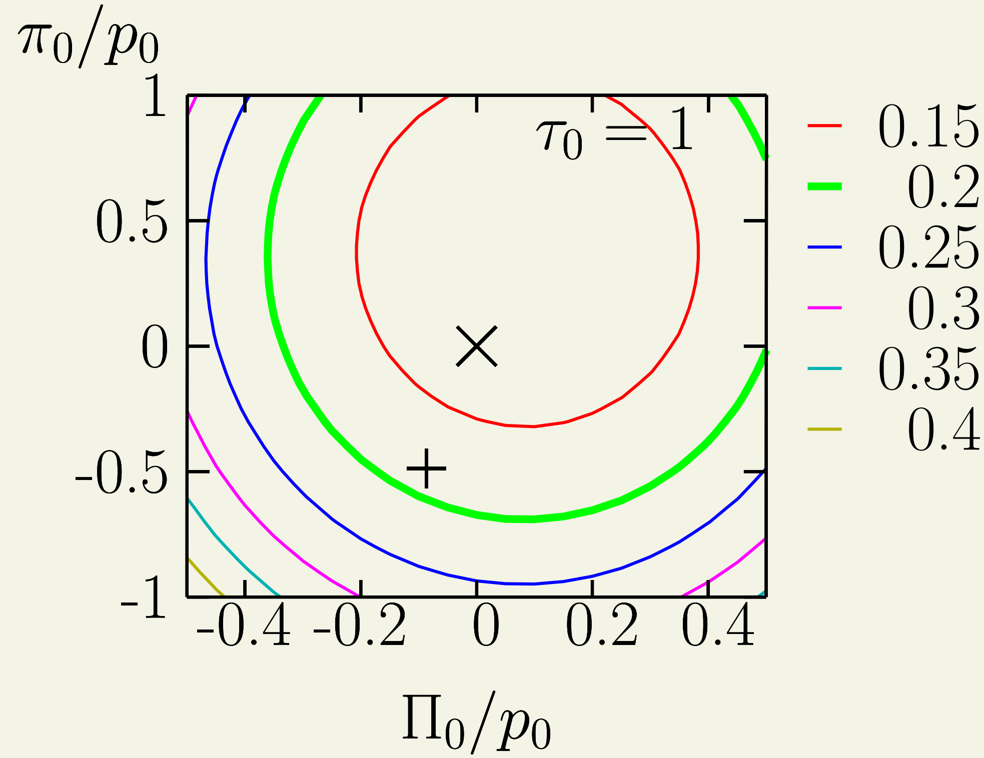


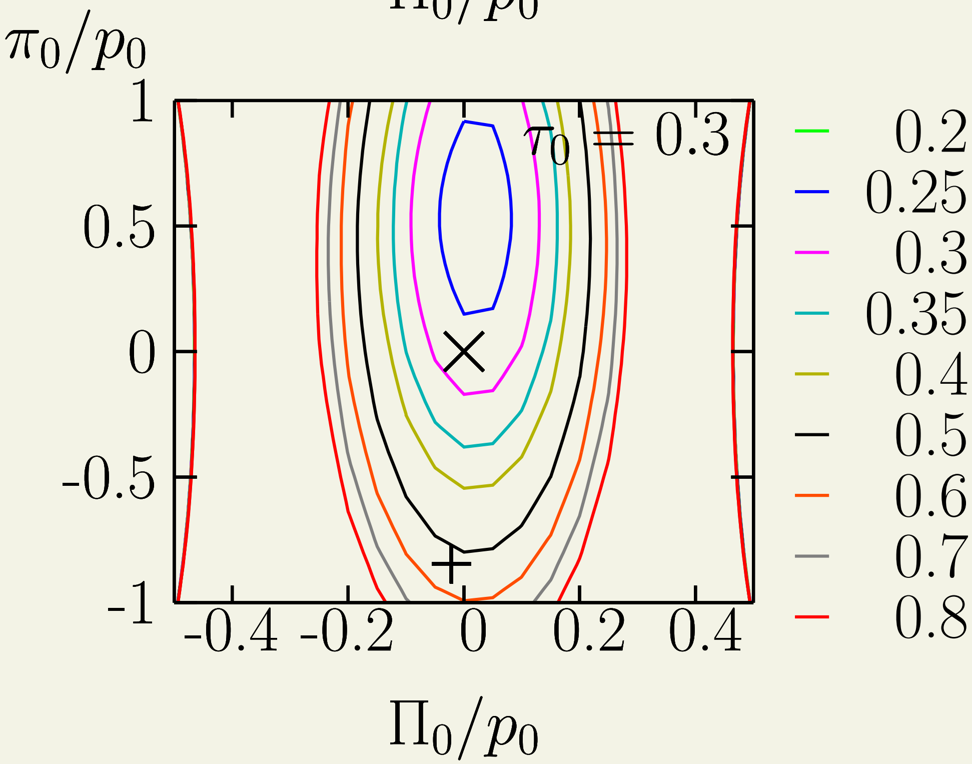




need $\tau_0 \gtrsim 1$ fm, even for ideal initconds

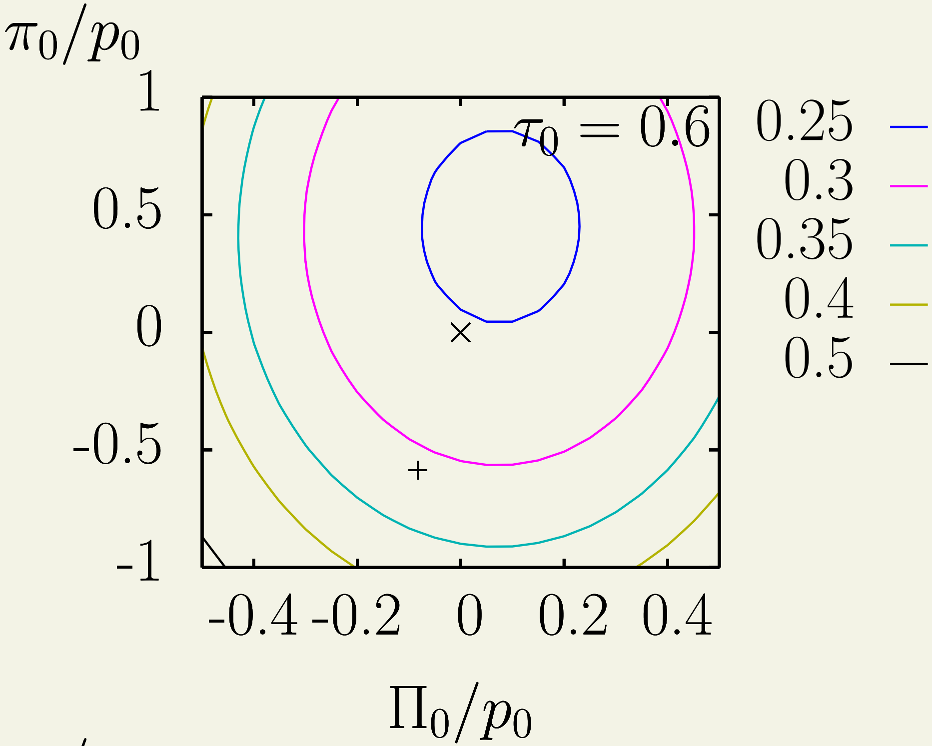


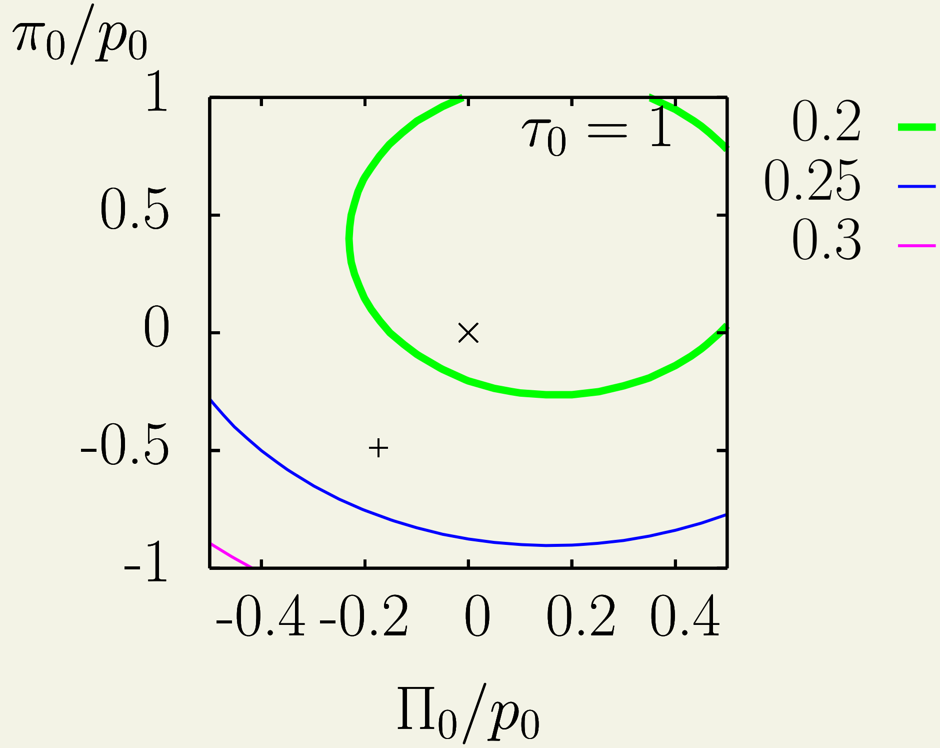


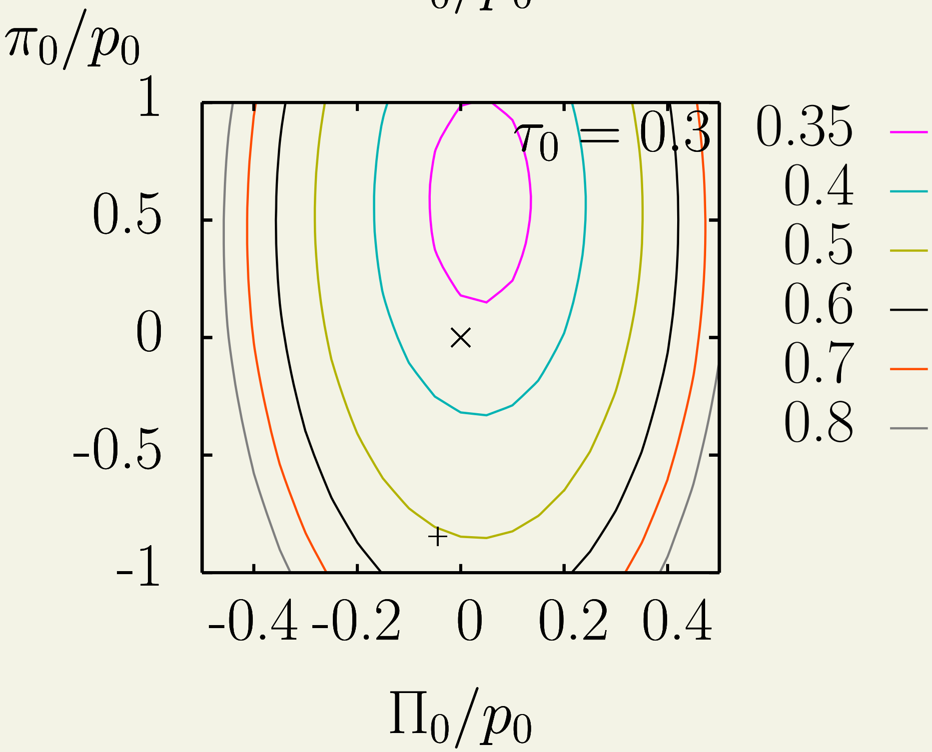


with halved bulk viscosity
$\boldsymbol{\zeta(T) \to 0.5 \times \zeta(T)}$

ideal initconds OK at $\tau_0 \gtrsim 0.6$ **fm**
Navier Stokes OK when $\tau_0 \gtrsim 1$ **fm**





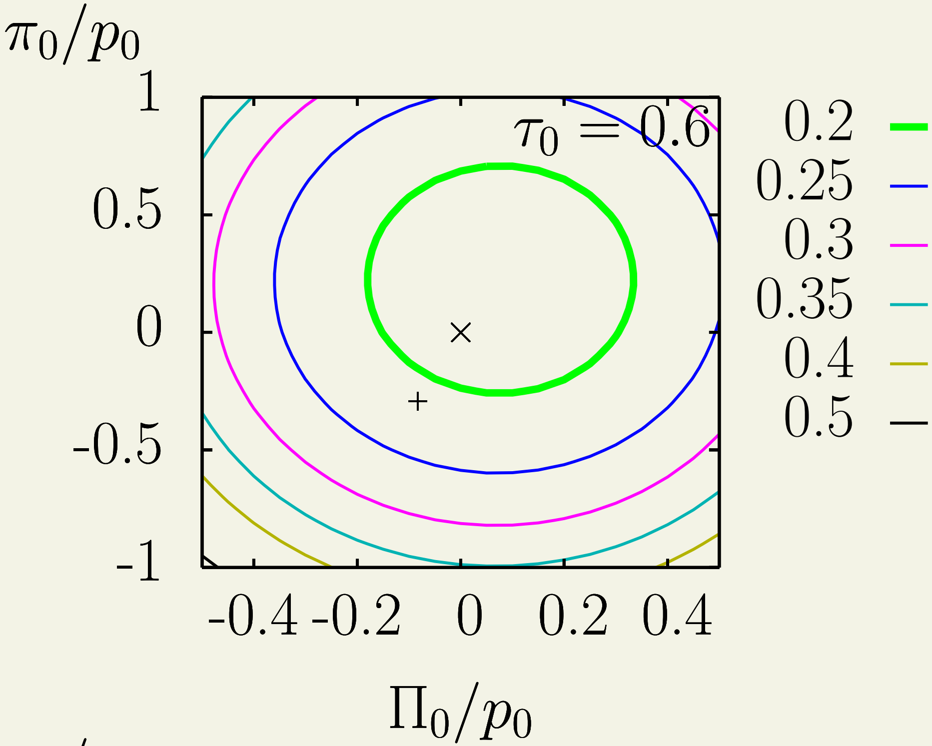


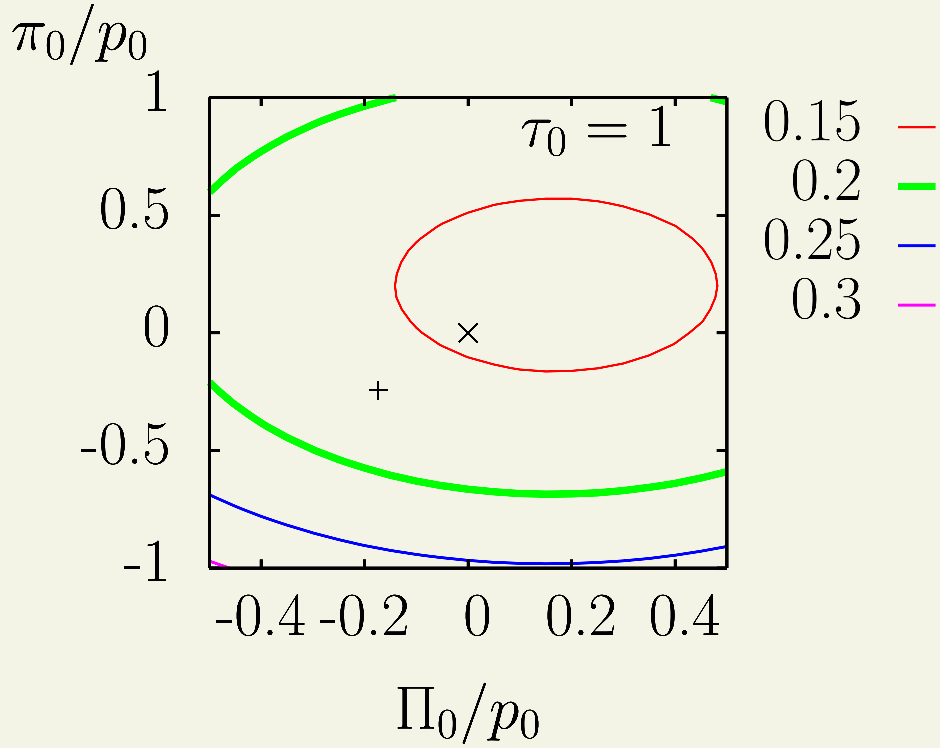
with halved relaxation times

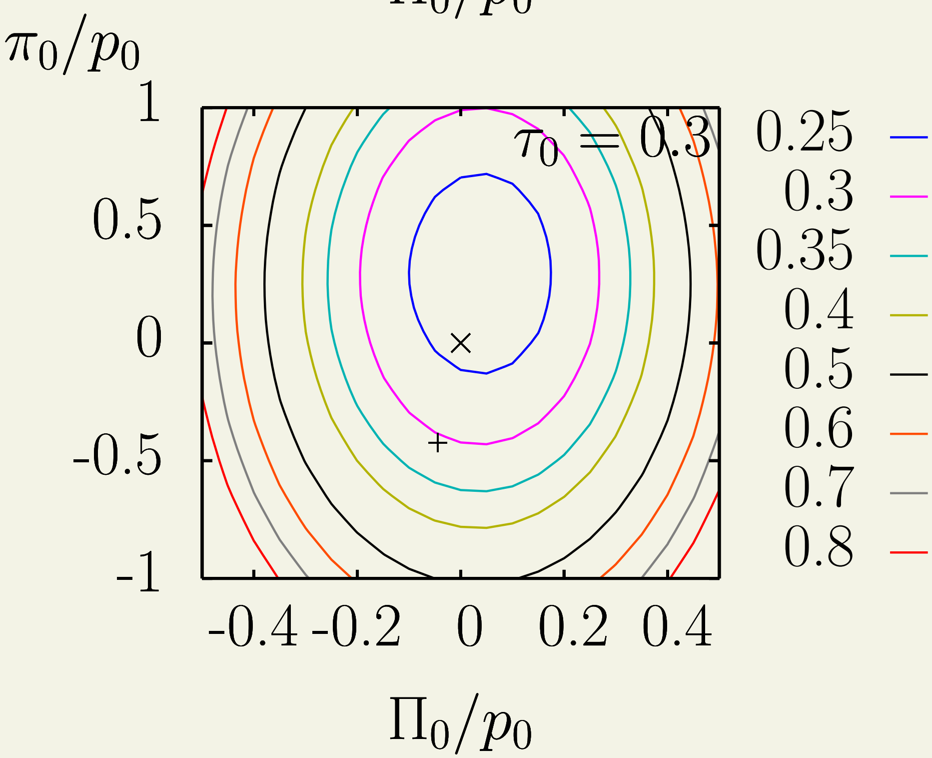
$\boldsymbol{\tau_{\pi,\Pi} \to 0.5 \times \tau_{\pi,\Pi}}$

not much help

reduces sensitivity to initial conditions (drives the system closer to Navier-Stokes evolution)







with halved shear viscosity

$\boldsymbol{\eta/s = 0.5 \times 1/(4\pi)}$

ideal, NS work for $\tau_0 \gtrsim 0.6$ **fm**

but CGC initconds still need $\tau_0 > 1$ **fm**

III. Next steps / wishlist

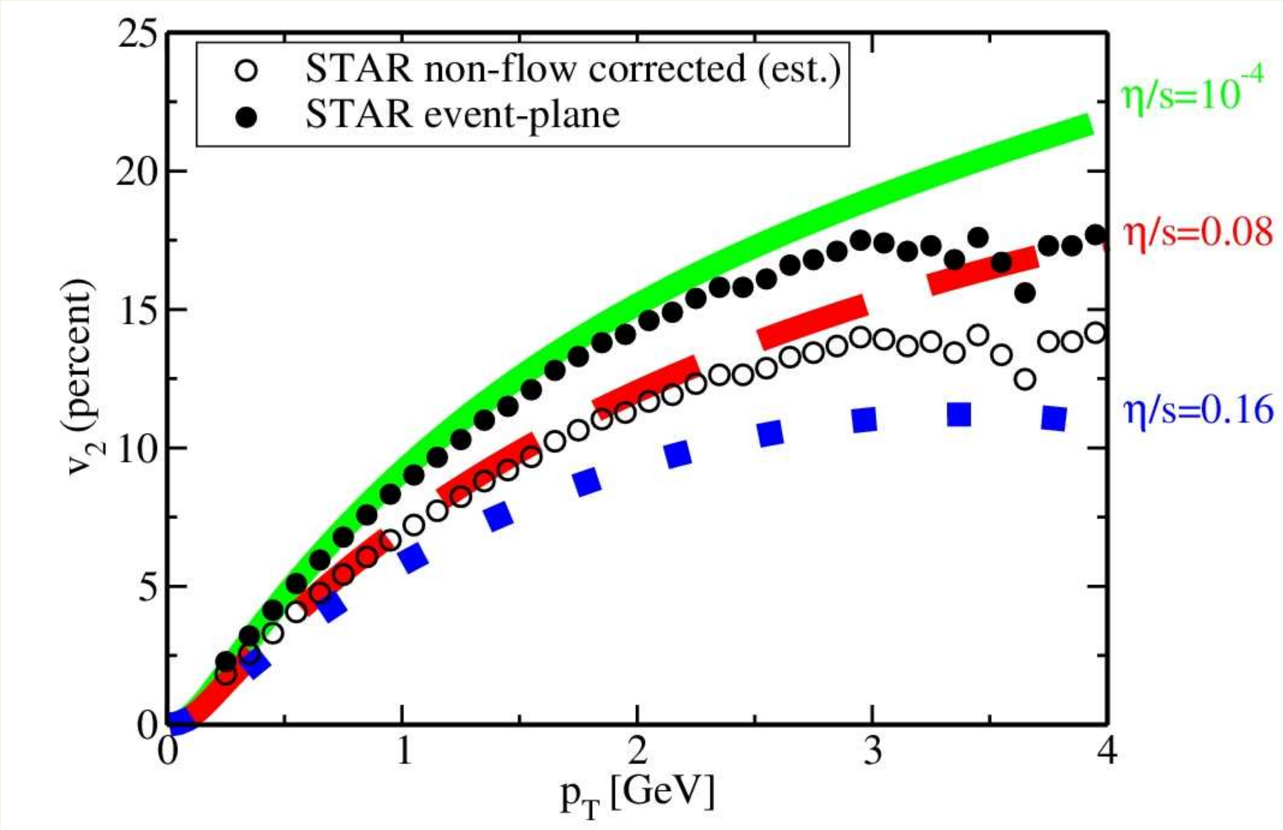
- compare with elliptic flow data

- cross-check with conical flow

[- viscous hydro vs transport studies with radiative transport]

Viscosity estimate at RHIC

Romatschke & Luzum, PRC78, 034915 ('08):



$\Rightarrow \eta/s \lesssim 0.5$

conservative, mainly because: assumed validity of hydro in hadronic phase, shear viscosity only, one-component gas of hadrons (one δf for all species)

must extend hydro freezeout to mixtures - also necessary in order to constrain equation of state (need to see $\pi - p$ difference)

Conical flow from transport

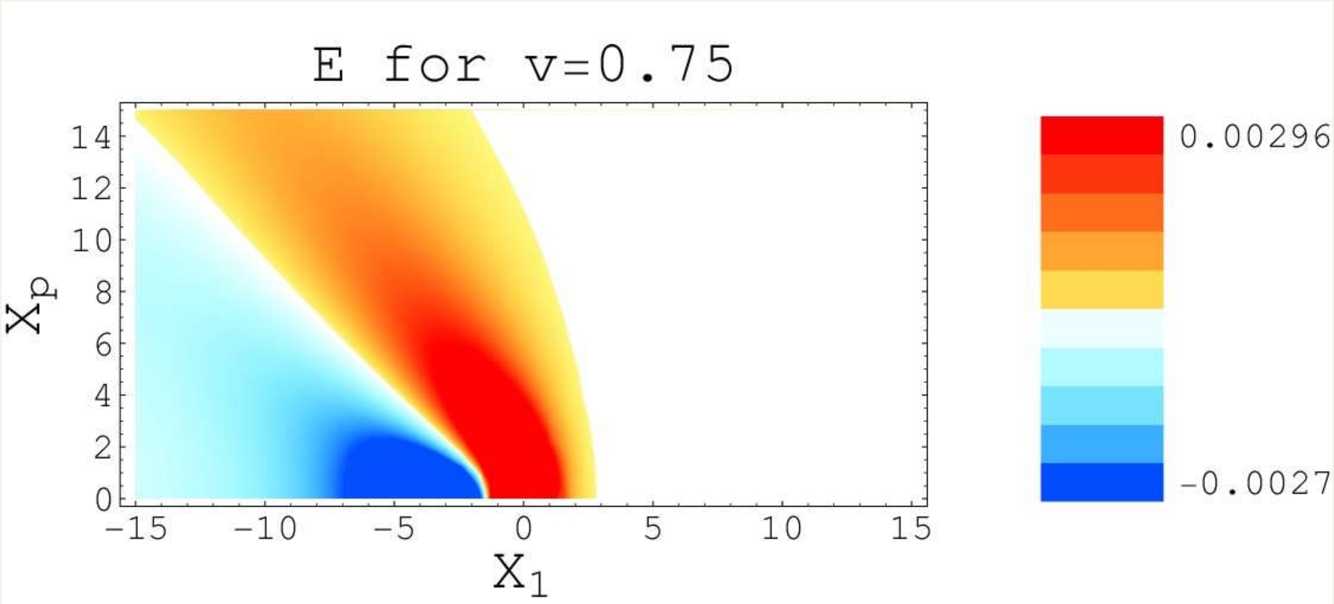
A sensitive probe of thermalization and low viscosity is conical flow.

Stöcker, Rischke... Shuryak, Teaney, Casalderrey-Solana... Neufeld & Mueller.. Betz et al ...

Why transport? - can treat full 3D (needed for jets in a heavy-ion collision)
- self-consistent jet+medium coupling
- it decouples, no Cooper-Frye prescription

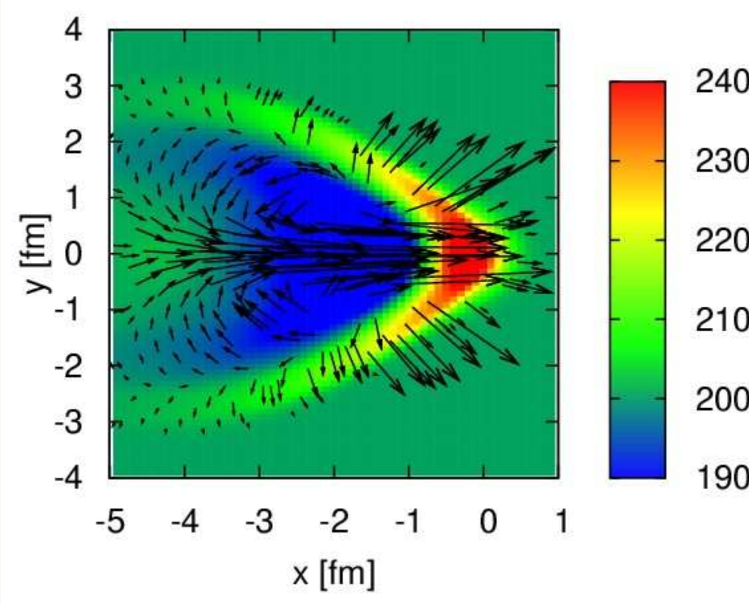
Heavy quark in AdS

Gubser, Yarom, Pufu, PRL100, 012301 ('08)



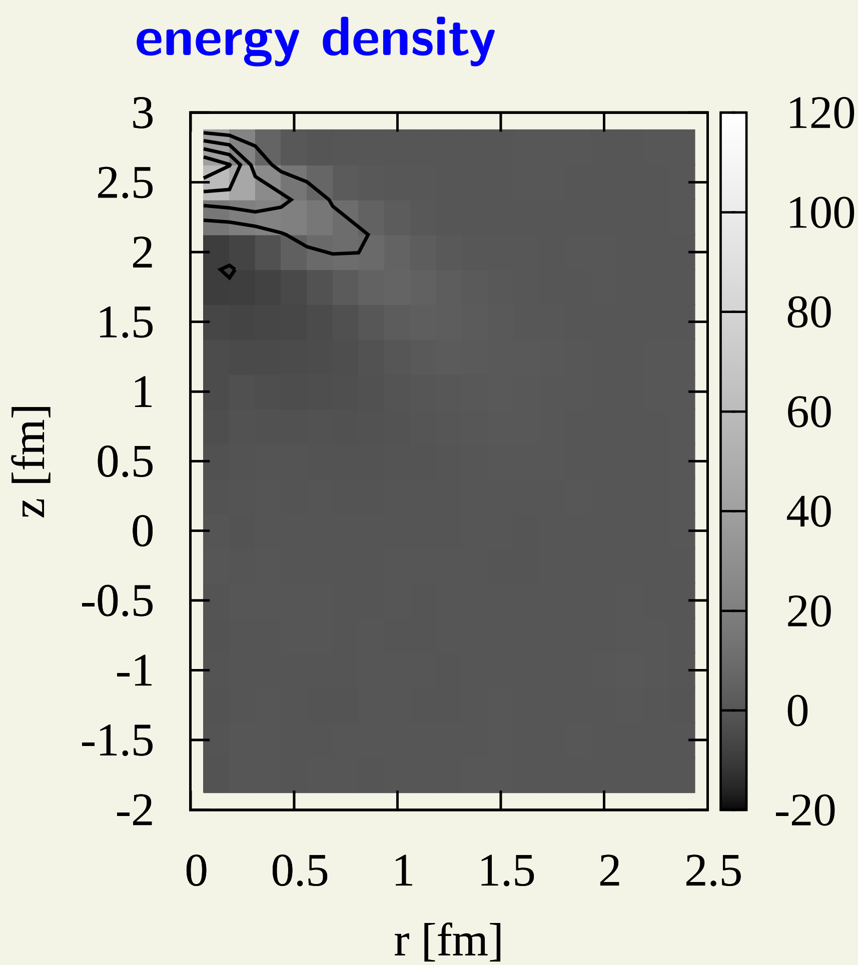
jet in ideal hydro

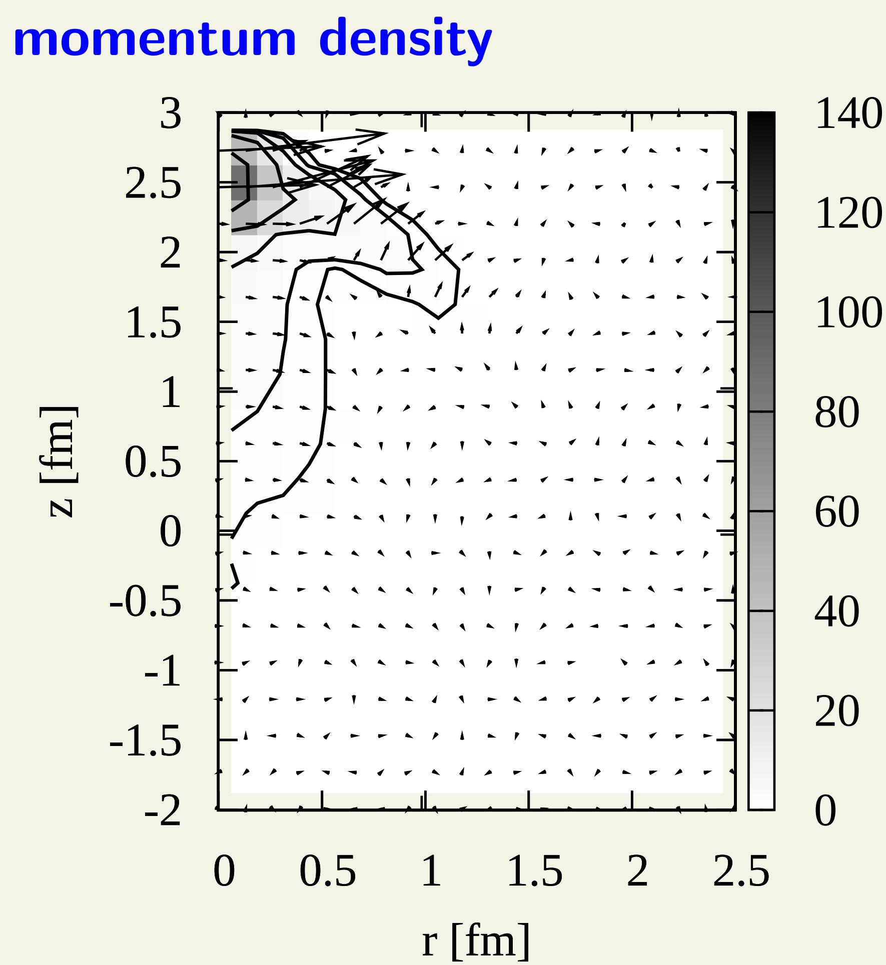
Betz et al, PRC79, 034902 ('09)



Feasibility study #1: external source moving along the z axis in static thermal bath, isotropic $2 \to 2$ transport

$dE/dx = 20$ **GeV/fm,** $\lambda_{MFP} = 0.125$ **fm,** $T = 0.385$ **GeV,** $v = 1$



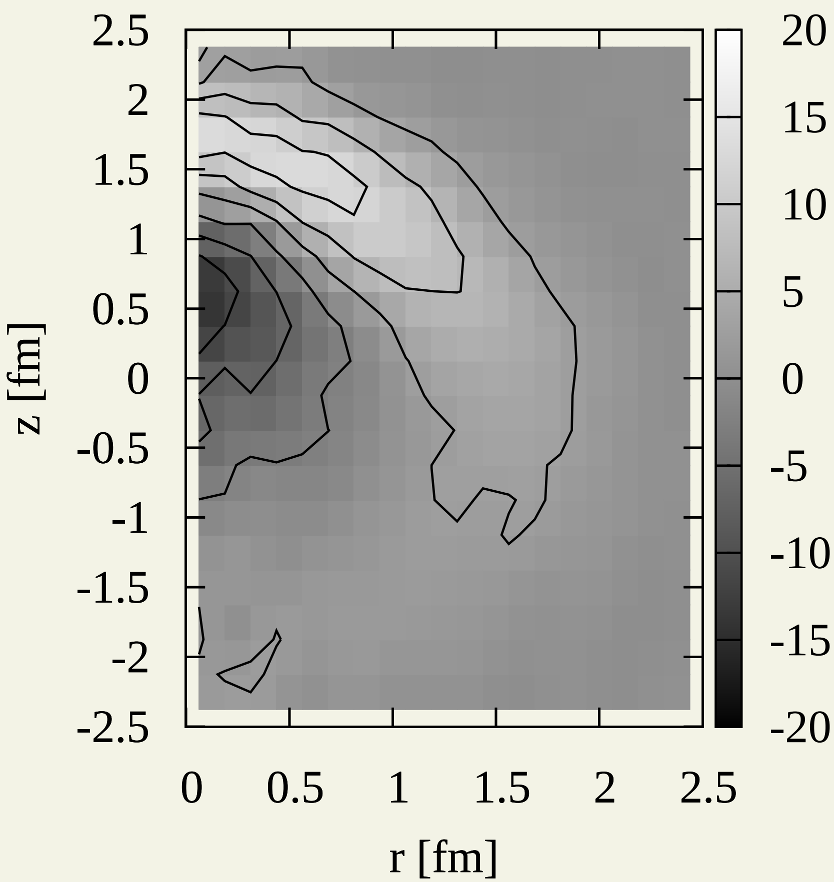


very little conical flow - 4-momentum is on the lightcone $dE/dz \approx dp_z/dz$ Betz et al ('08)**; moreover, disturbance has to be thermalized by the medium itself**

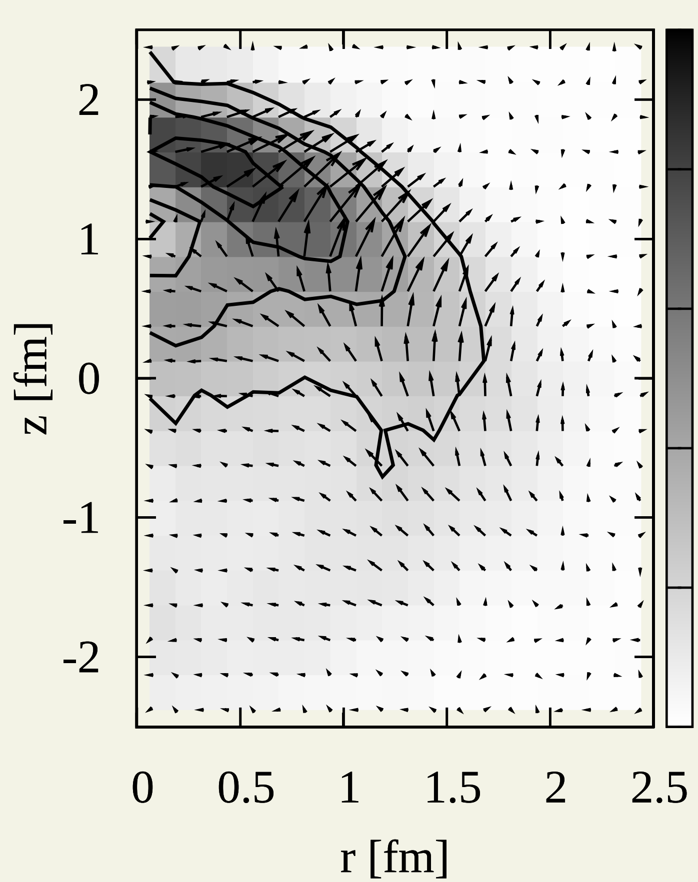
Feasibility study #2: pure energy deposition (thermalized) along the z axis in static thermal bath, isotropic $2 \to 2$ transport

$dE/dx = 68$ **GeV/fm,** $\lambda_{MFP} = 0.125$ **fm,** $T = 0.385$ **GeV,** $v = 0.9$

energy density



momentum density



much more pronounced conical flow component

(My) Wishlist

- an alternative solvable nonequilibrium theory (other than kinetic theory), with a hydrodynamic limit, ideally in full 3D

  $\longrightarrow$ would be highly useful for viscous hydro region of validity studies (different relaxation times, equation of state, ...)

- solutions for the evolution of the energy-momentum tensor of the medium, in some theory, in the case of a finite-mass source (quark) that is self-consistently decelerating as it loses energy

- a rigorous (and positive) lower bound on η/s in QCD at RHIC/LHC temperatures

...

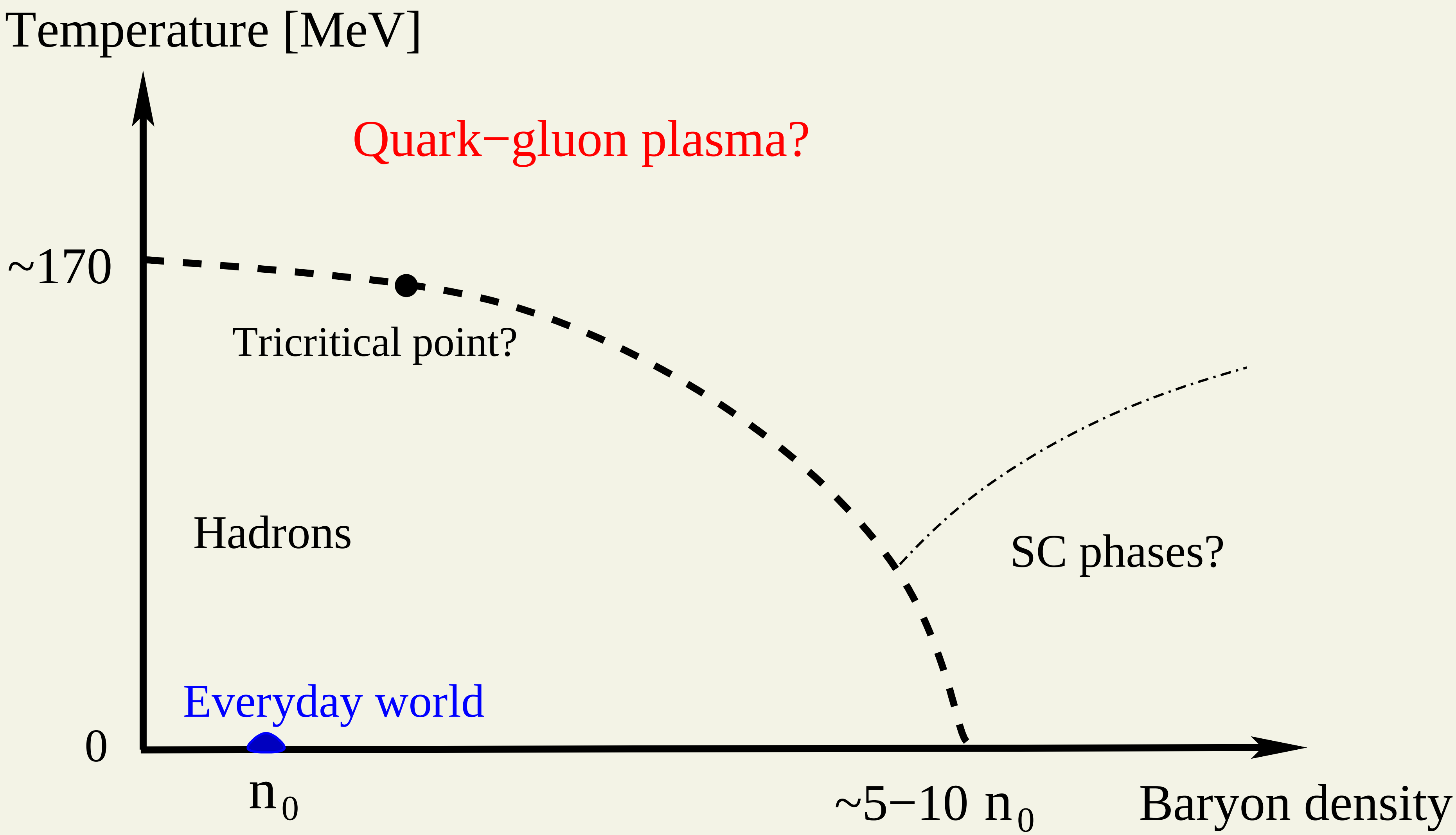
Conclusions

The rather successful ideal hydrodynamics paradigm for RHIC is being refined into a viscous hydrodynamics paradigm. The prospects so far look promising - based on comparison with covariant transport, Israel-Stewart hydrodynamics is a very good approximation at RHIC for a wide range of initial conditions even at early times $\tau_0 \sim 0.6$ fm, provided $\eta/s \lesssim (1.5-2)/4\pi$ and bulk viscosity is negligible. If bulk viscosity becomes as appreciable around T_c as in Meyer's estimate, good accuracy at such early times requires a shear viscosity that is significantly smaller than $s/(4\pi)$.

The RHIC charged particle data indicate $\eta/s \lesssim 0.5$. Further cross-checks (energy loss, hydrodynamic excitations, ...) and theory developments (freezeout, initial conditions, thermalization...) are needed to reduce uncertainties. Some of these advances are also necessary in order to constrain the equation of state from the data.

Backup slides

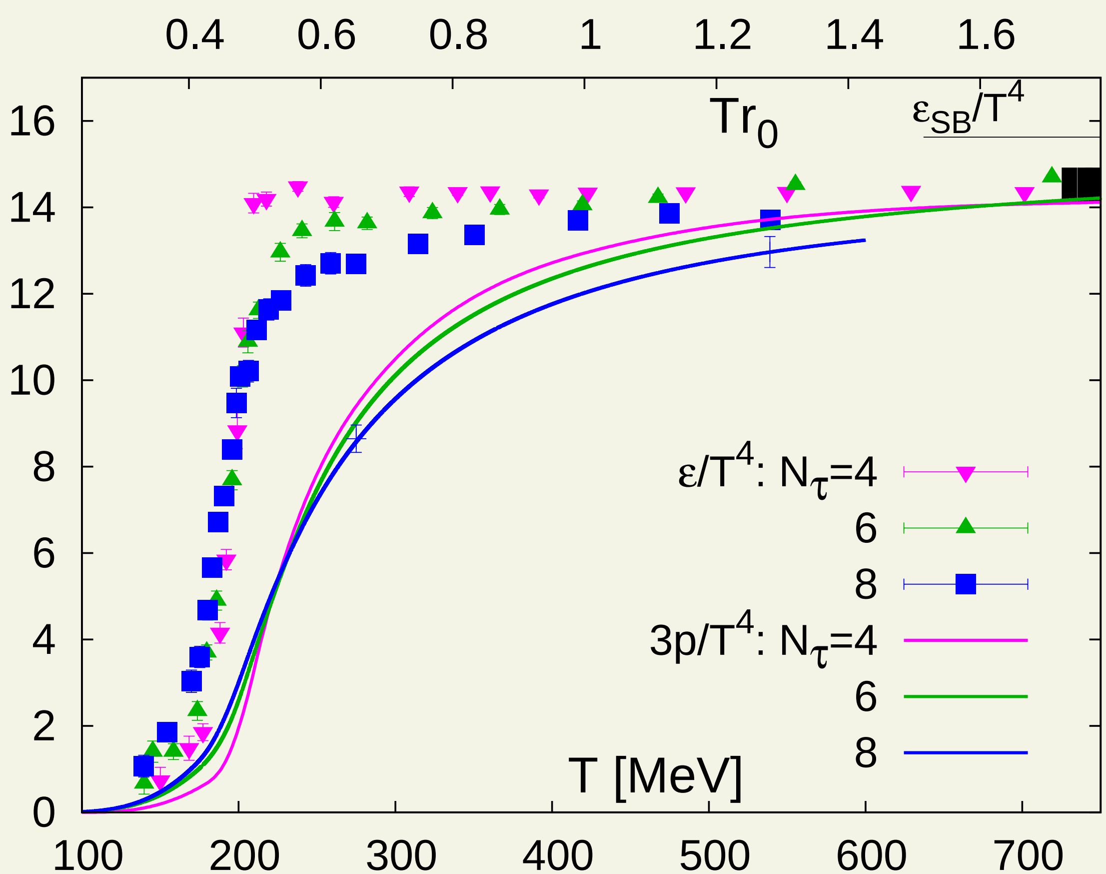
QCD phase diagram



Equation of state ($\mu_B = 0$)

lattice QCD - $Z = \int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}A\, e^{-S_E^{QCD}}$

A. Bazavov et al, arXiv:0903.4379



$$\frac{e_{SB}}{T^4} \approx \frac{d.o.f.}{3}$$

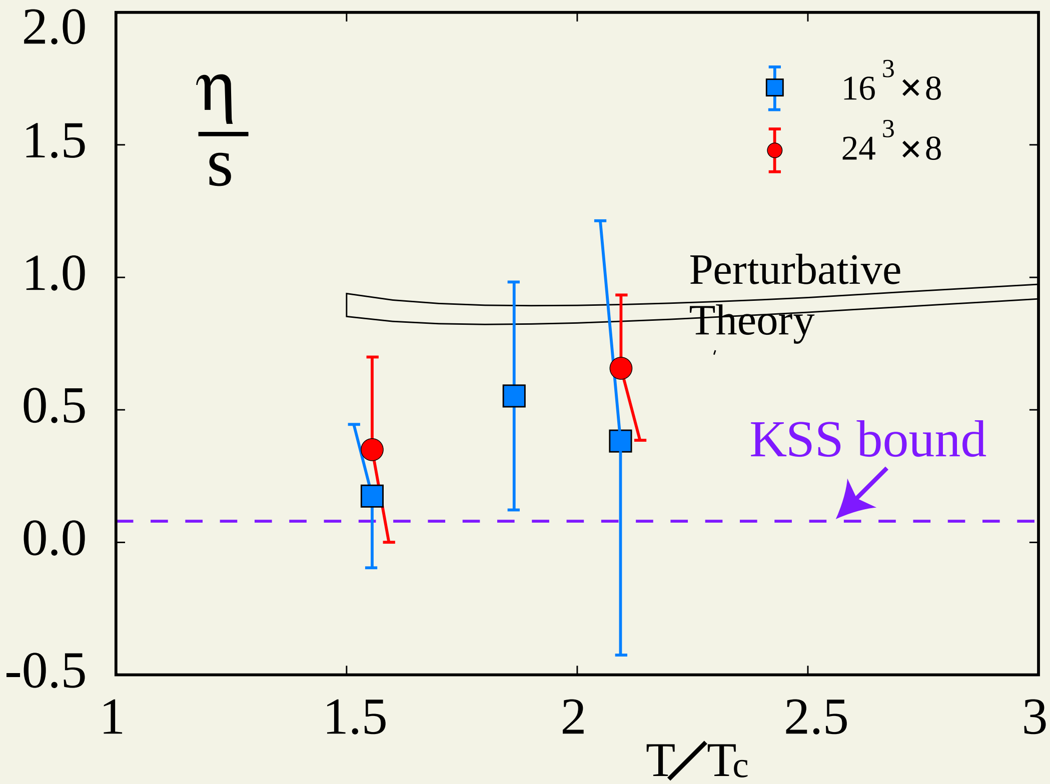
sharp rise in effective degrees of freedom around $T_c \approx 180 - 200$ MeV

Viscosity

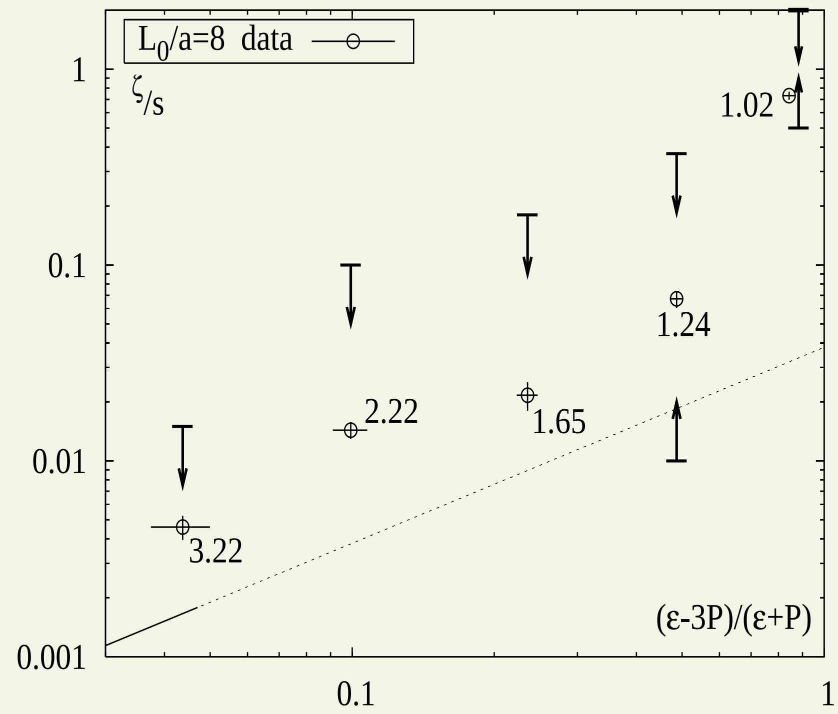
Largely unknown for $T \sim 200 - 400$ MeV relevant at RHIC.

perturbatively: $\eta/s \sim 1$, $\zeta/s \sim 0.02\alpha_s^2 \sim 0$ Arnold, Moore, Yaffe, JHEP 0305, 051 ('03); Arnold, Dogan, Moore, PRD74 ('06)

lattice: shear Nakamura & Sakai, NPA774, 775 ('06)

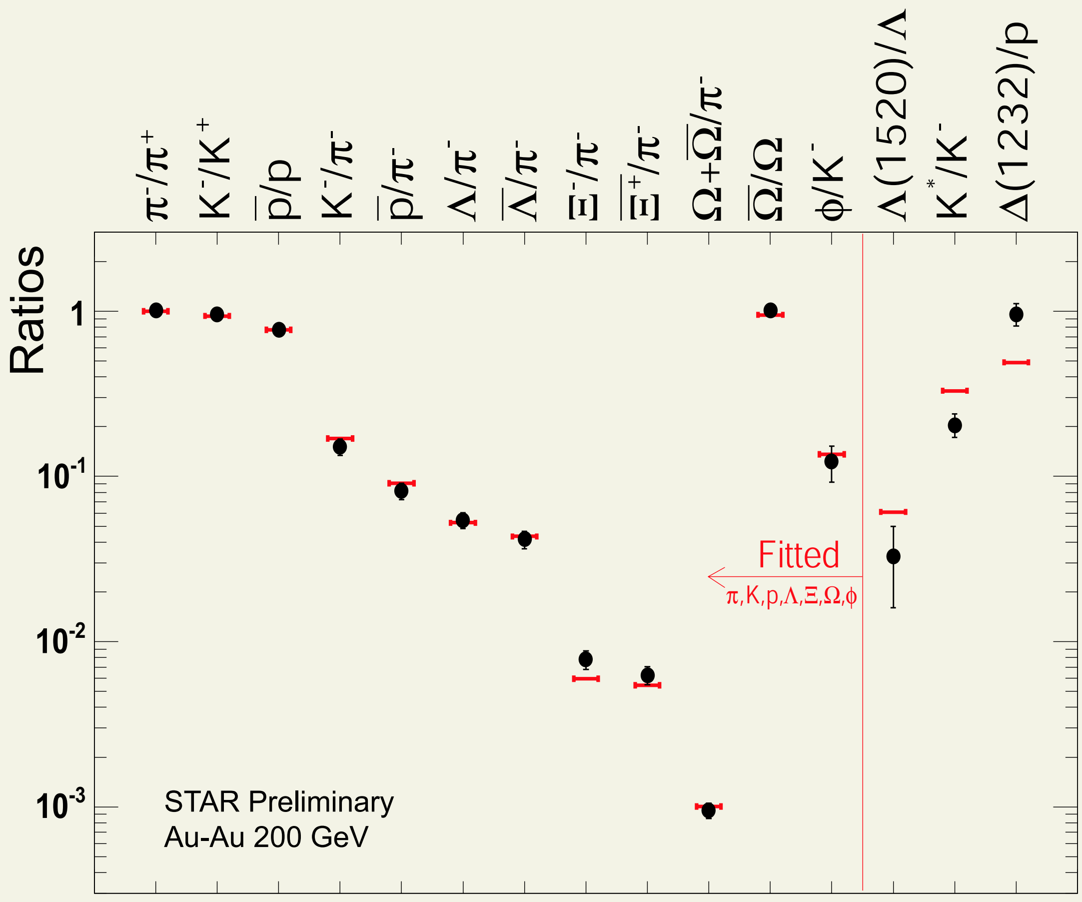


bulk estimate Meyer, PRD76, 101701 ('07)



particle ratios at RHIC

$$\frac{N_i}{V} = \frac{g_i}{2\pi^2} \int \frac{dp\ p^2}{\exp[E_i(p) - \mu_i]/T \pm 1}$$



central Au+Au:

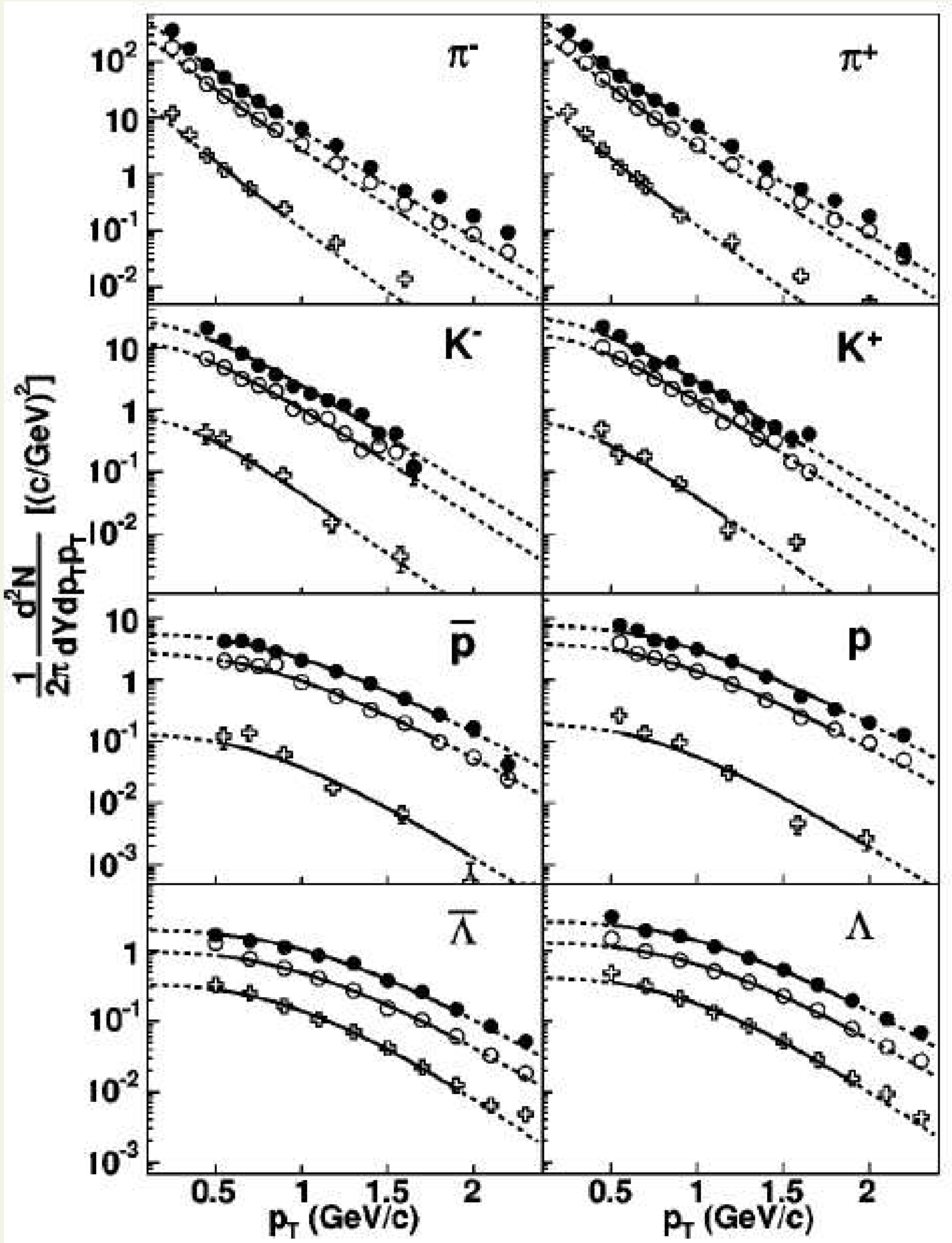
$T_{chem} = 160 \pm 5$ **MeV**

$\mu_B = 24 \pm 4$ **MeV**

$\mu_s = 1.4 \pm 1.6$ **MeV**

[STAR, JPG31, 101 (2005)]

spectra at RHIC - collection of boosted thermal sources



"Blast-wave" model

[Schnedermann, Sollfrank, & Heinz, PRC48, 2462 ('94)]

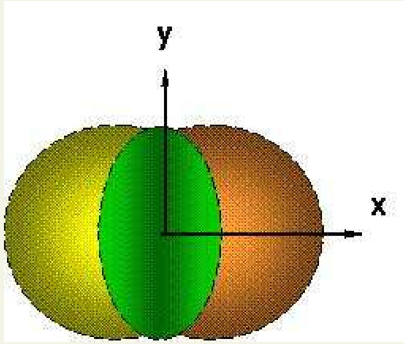
fits for Au+Au at RHIC

[Retiere & Lisa, PRC70, 044907 ('04)]

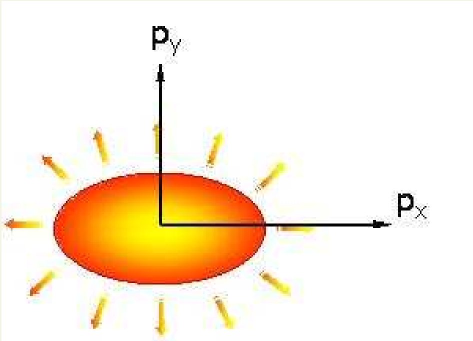
average radial velocity
$\langle v_r \rangle \sim 0.5c$**(!)**

kinetic freezeout temperature
$T_{fo} \sim 110$ **MeV**

elliptic flow: conversion of spatial eccentricity to momentum anisotropy



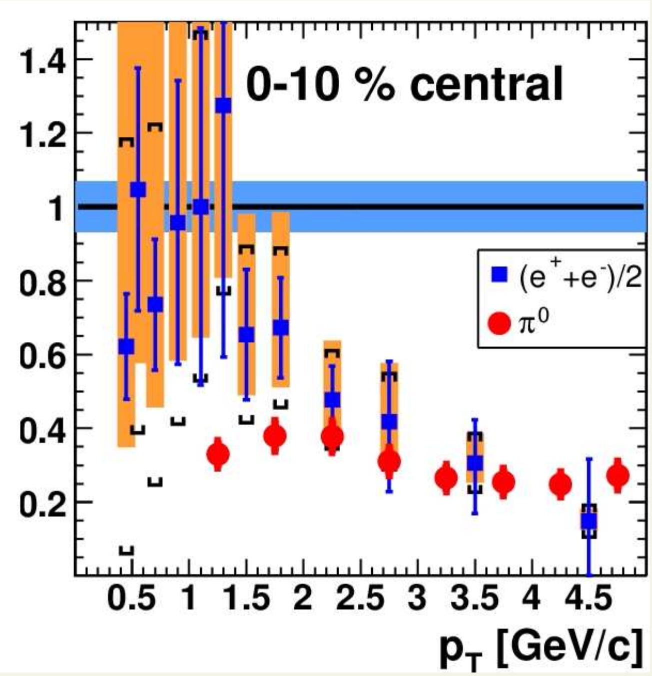
$\longrightarrow$



"elliptic flow"

$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} \qquad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \equiv \langle \cos 2\phi_p \rangle \qquad \sim 0.1 - 0.2 \text{ large(!)}$$

very opaque - large energy loss, even heavy quarks are suppressed PHENIX, PRL96, 032301 ('06)



Ideal (Euler) hydrodynamics

local, macroscopic variables: energy density $e(x)$
pressure $p(x)$
baryon density $n_B(x)$
flow velocity $u^\mu(x)$ $\quad (u^\mu u_\mu = 1)$

energy-momentum and charge conservation: $\partial_\mu T^{\mu\nu}(x) = 0$
$\partial_\mu N^\mu(x) = 0$

local equilibrium: $T^{\mu\nu} = (e+p)u^\mu u^\nu - p g^{\mu\nu} \quad , \quad N^\mu = n_B u^\mu$

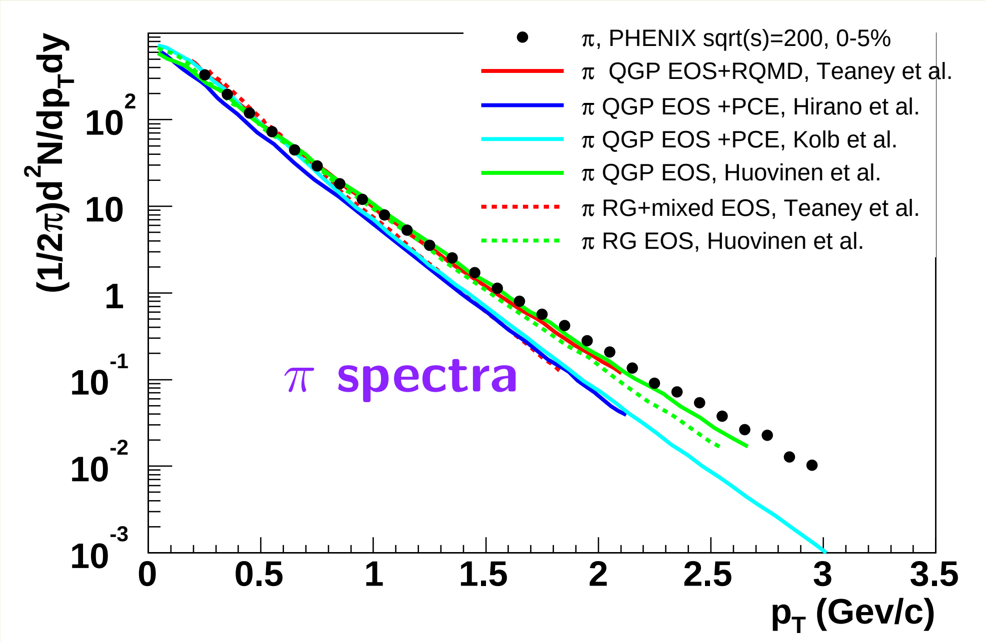
$$[T^{\mu\nu}_{LR} = diag(e,p,p,p) \quad , \quad N^\mu_{LR} = (n_B, \vec{0})]$$

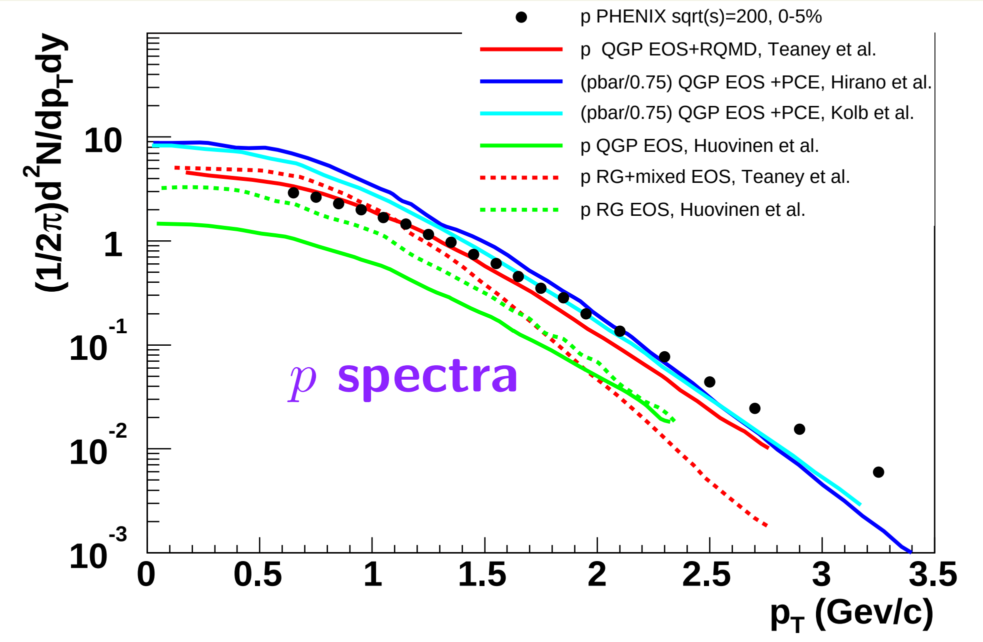
conserves entropy, NO dissipation $\partial_\mu(s u^\mu) = 0$

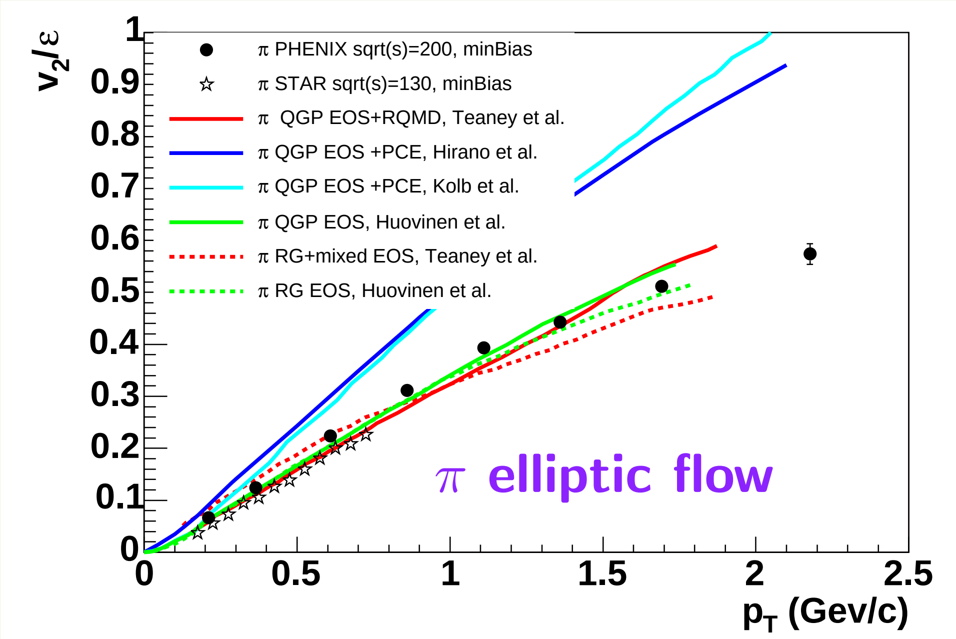
matter characterized by: equation of state $p(e, n_B)$

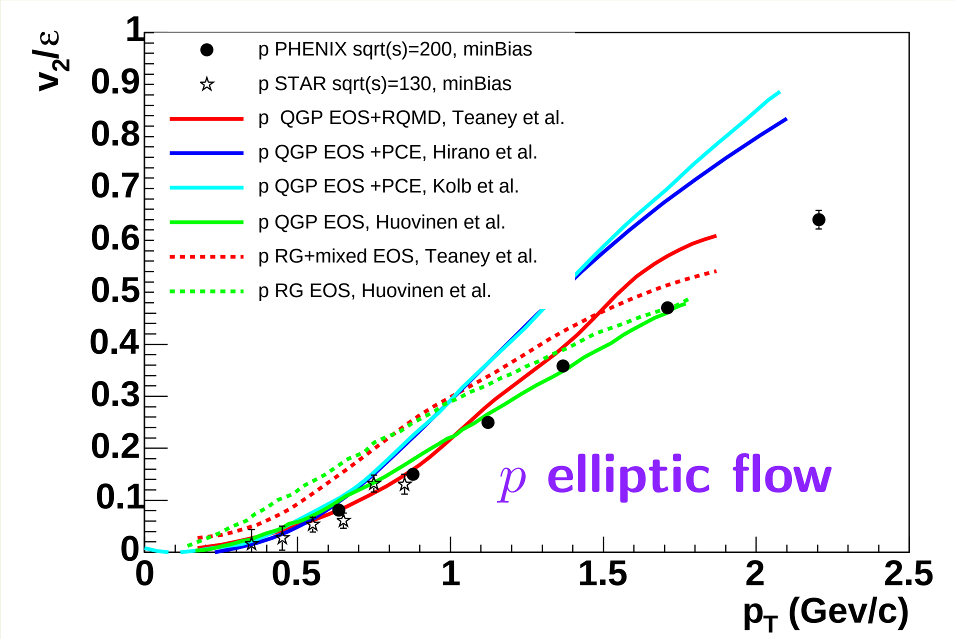
requires initial conditions and final break-up (freezeout) prescription

All hangs together, with a small correction (PHENIX White Paper, NPA757, 184 ('05))**:**
Ideal hydro alone fails. What works is ideal hydro + hadronic transport.









Two options to study dissipative dynamics

- causal viscous hydrodynamics (e.g., Israel-Stewart formulation)

Israel, Stewart; ... Muronga, Rischke; Teaney et al; Romatschke et al; Heinz et al; DM, Huovinen; Kodama et al...

- covariant transport

Israel, de Groot,... Zhang, Gyulassy, DM, Pratt, Xu, Greiner...

[- apart, of course, from the AdS / CFT world]

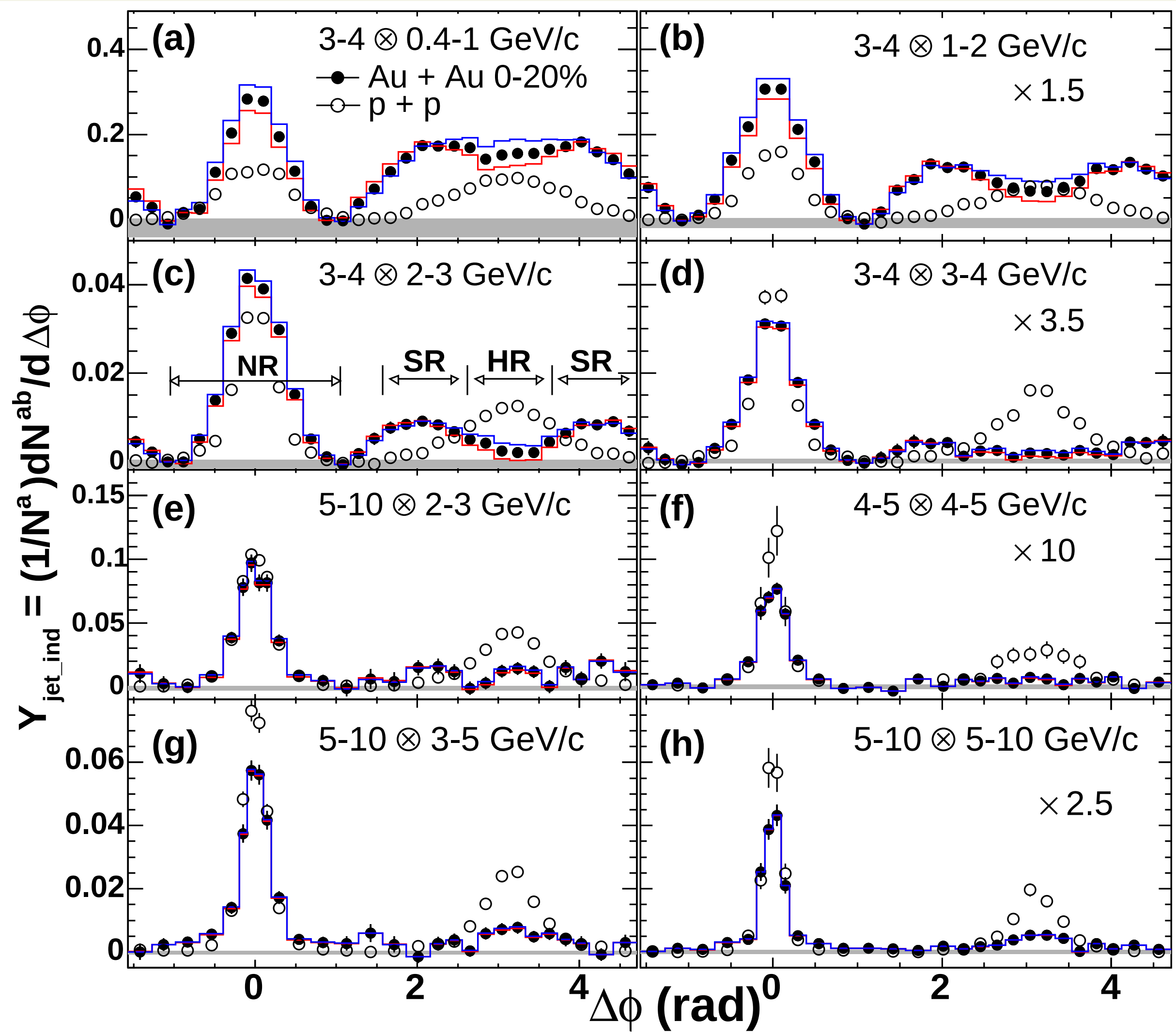
Son et al, Yanik, Gubser et al, Yarom et al, Noronha, Yaffee, Chestler, ...

Advantages of transport:

i) numerical solution techniques are readily available in full 3+1D spacetime

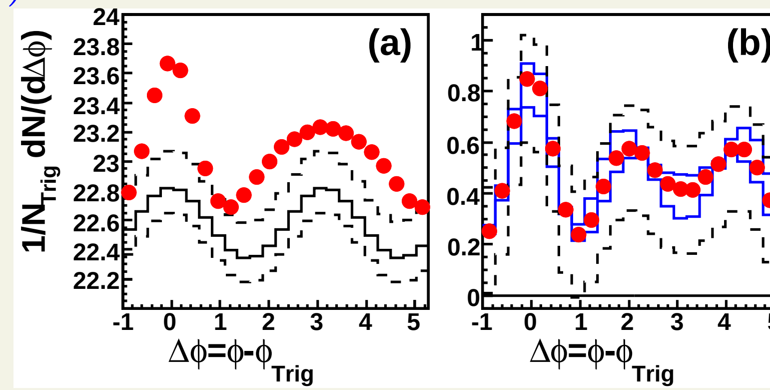
ii) naturally incorporates thermalization and decoupling

PHENIX, PRC78 ('08):

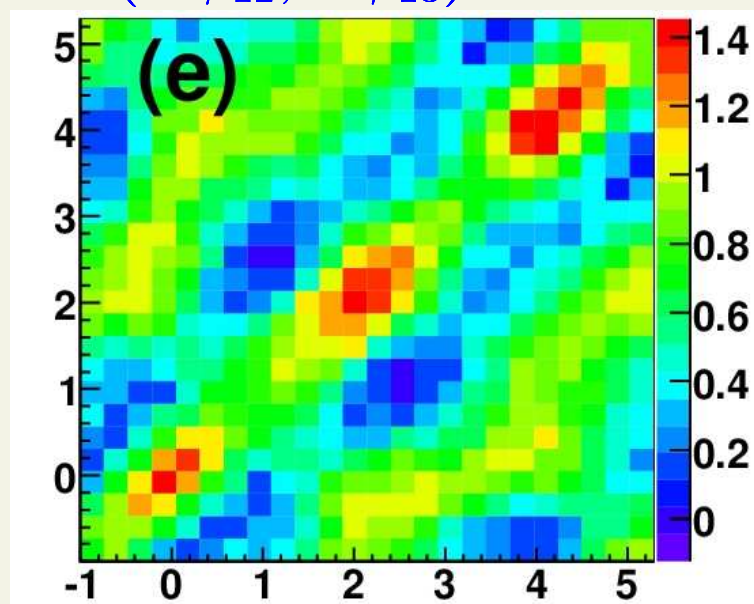


STAR, PRL102 ('09)

$C(\Delta\phi)$



$C(\Delta\phi_{12}, \Delta\phi_{13})$



double peaks **on away-side in correlation with soft $p_T \sim 1-3$ GeV partners**

further corroboration by three-particle correlation data