
Entangled B_s - B_s States at The Upsilon ($5s$)

Outline

- Motivation
- Upsilon(5s): Bs menu
- Experimental Scenario
- Categories of B_s Decays
- Oscillation Formalism
- Basic (qualitative) Idea
- Correlations
- Asymmetries
- Conclusions

Motivation

- The SM predicts that CP violating phases in the c will be small. We should check that.

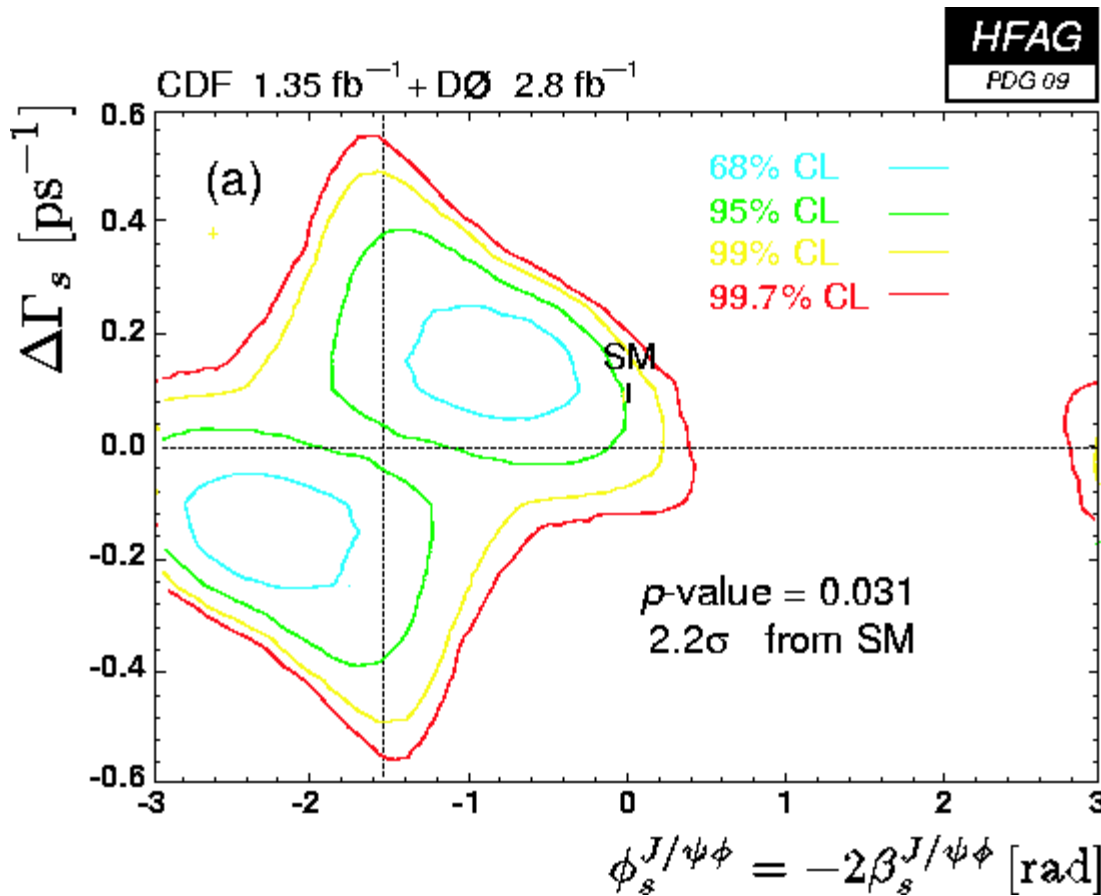
- Theory now gives a better prediction for $\Delta\Gamma_s$, We therefore want a better measurement of this.

Nierst and Lenz JHEP 0706:072 (2007)

Badin, Gabbiani and Petrov Phys.Lett.B653:230-240 (2007)

- In contrast to hadron colliders, electron-positron B-factories produce B_s in a unique entangled state.
- In this talk, I will consider how to make use of this correlated initial state to address these two issues.

Current knowledge (from HFAG 2009)



Results from CDF and DØ:
 CDF, Phys.Rev.Lett.100:121803 (2008)
 DØ, Phys.Rev.Lett.95:171801 (2005)
 Phys.Rev.Lett.98:121801 (2007)

$$|\Delta\Gamma_s| = .154^{+0.054}_{-0.070} \text{ ps}^{-1}$$

$$\tau_s = 1.412 \pm 0.042 \text{ ps}$$

$$y = \frac{1}{2} \tau_s \Delta\Gamma_s = 0.109^{+0.038}_{-0.050}$$

$\phi_s = -2\beta_s$ (90%
 CL range)

$[-1.47; -0.29] \cup [-2.85; -1.65]$

Upsilon(5s): B_s Menu

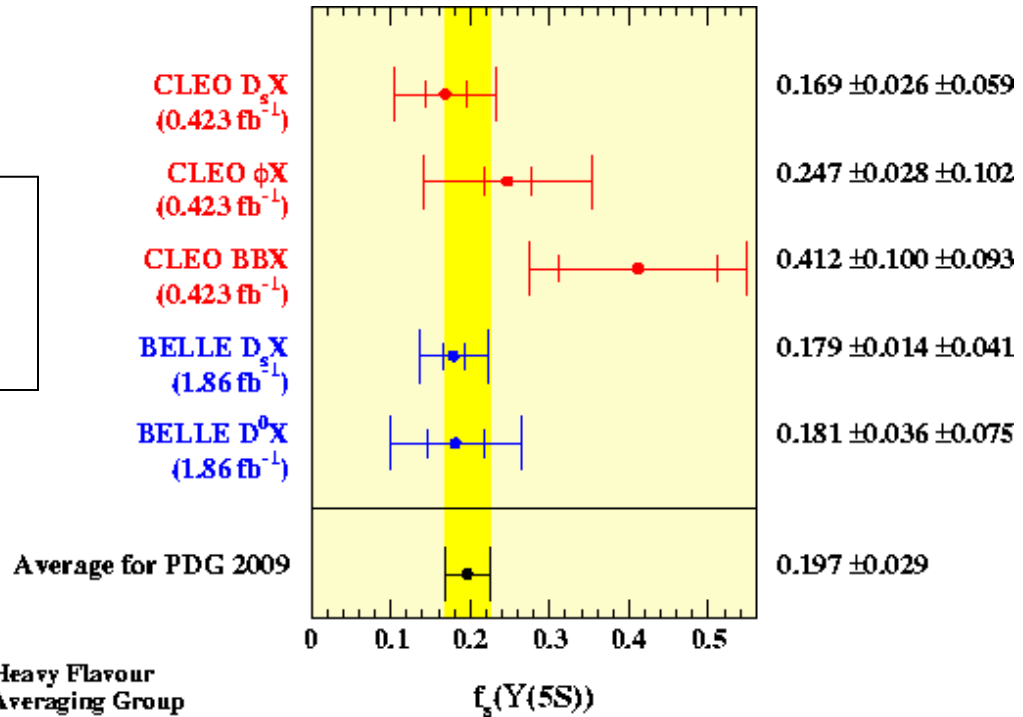
- Constraints we will try to live with:
 - 10^7 - 10^8 $b\bar{b}$ events ($\sigma_{b\bar{b}}=0.3\text{nb}\rightarrow L=30\text{-}300\text{ fb}^{-1}$)
 - Limited time resolution (cannot see Δm_s oscillations)
- Using this, what will we do?
 - Measure $\Delta\Gamma_s$ using time independent correlations of inclusive states
 - Measure mixing/decay phases correlating inclusive with exclusive decays
 - Measure $\Delta\Gamma_s$ using time asymmetry of inclusive states (much better)
 - Constrain $\tan(\phi_s)$ (i.e. mixing phase) using time asymmetry of inclusive states

B factories at Upsilon(5s)

- CLEO has run at the Upsilon(5s) for 0.4fb^{-1}
- More recently BELLE has run at Upsilon(5s)
BELLE Collab.. PRL 102, 021801 (2009)
- BELLE has now gathered $\sim 100\text{fb}^{-1}$ [= 3×10^7 bb events]

Basics of $B_s B_s$ Production at Upsilon(5s)

Fractions of Upsilon(5s) events going to each $B_s B_s$ mode (from HFAG 2009)



b hadron species	fraction in $Y(5S)$ decay	ratio
$B_s^{(*)} \text{ anti-}B_s^{(*)}$	$f_s^{(*)} = f_s(Y(5S)) = 0.197 \pm 0.029$	
$B_s^* \text{ anti-}B_s^*$	f_s^{**}	$f_s^{**}/f_s^{(*)} = 0.901 +0.038 -0.040$

Experimental Scenario

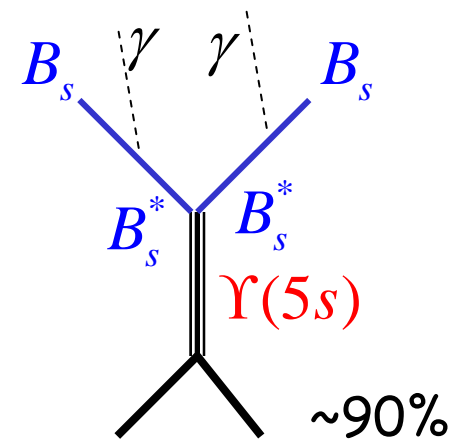
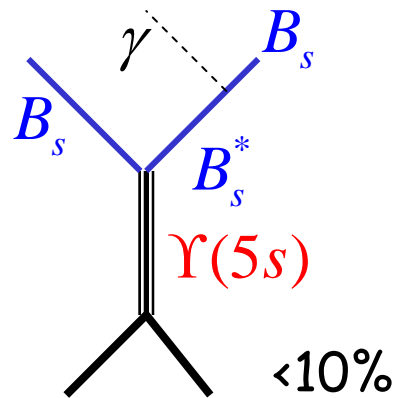
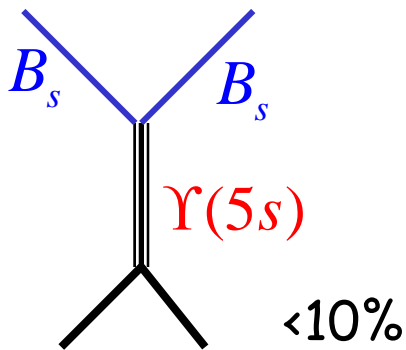
- A $B_s \bar{B}_s$ pair is produced in about 20% of the $e^+e^- \rightarrow b\bar{b}$ events at the Upsilon(5s).

- $B_s \bar{B}_s$
- $B_s^* \bar{B}_s$
- $B_s^* \bar{B}_s^* \rightarrow 90\%$ of the time!

Define

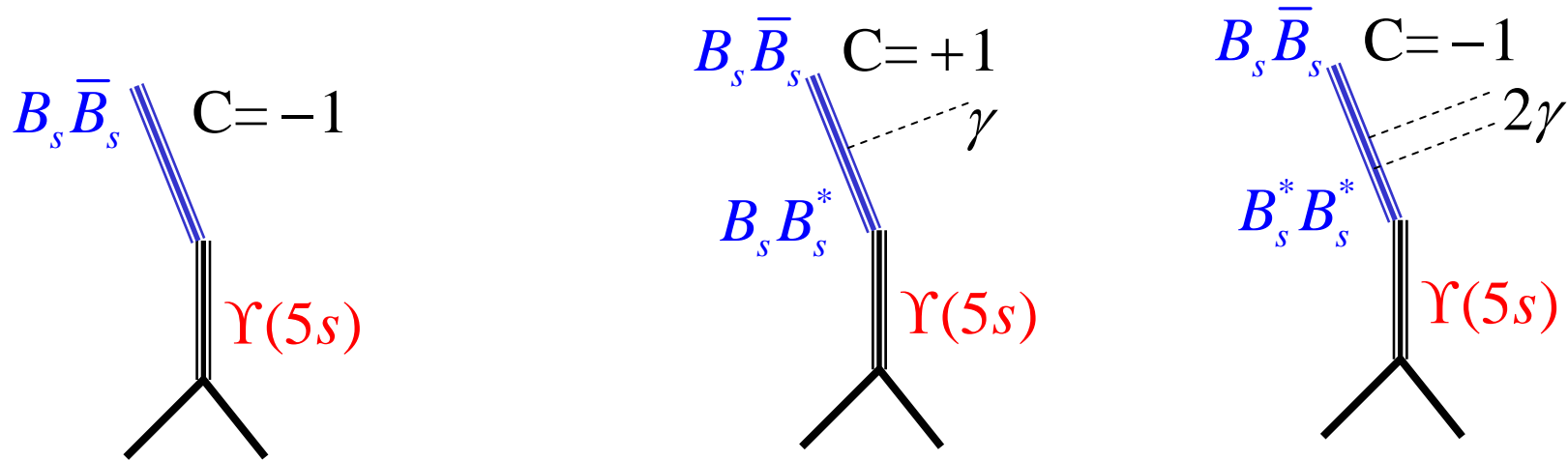
$$r^* = \frac{B_s^{(*)} \bar{B}_s + B_s \bar{B}_s^{(*)}}{B_s^{(*)} \bar{B}_s^{(*)}} \leq 10\%$$

- In all cases the system decays radiatively to a $B_s \bar{B}_s$ pair.



Properties of Meson Pairs

- In asymmetric B-factories, the system is in motion wrt the lab frame
- This allows us to get time information from the position of the decay (as in the Upsilon(4s)).
- I will assume time resolution not fine enough to see Δm_s oscillations.
- Unlike the Upsilon(4s), there are a variety of initial states



Initial State

- $C=-1$ implies $L=\text{odd}$ and $P=-1$ (>90% of the time)

$$\Psi^{C=-1} = \frac{1}{\sqrt{2}} \left(|B_s\rangle |\bar{B}_s\rangle - |\bar{B}_s\rangle |B_s\rangle \right)$$

- $C=+1$ implies $L=\text{even}$ and $P=+1$

$$\Psi^{C=+1} = \frac{1}{\sqrt{2}} \left(|B_s\rangle |\bar{B}_s\rangle + |\bar{B}_s\rangle |B_s\rangle \right)$$

Categories of B_s Decays

- Given limited # of mesons, we want to consider final states with as much BR as possible.
- Use inclusive states composed of many exclusive states.
- Taggable Decays (t)
 - This includes semileptonic decays as well as hadronic decays which give the flavor of the B_s meson.
 - Usually we won't actually care whether it tags a B_s meson ($t+$) or a B_s meson ($t-$) [i.e. due to rapid oscillations].
- Hadronic Decays (h)
 - This includes all decays that are not taggable
 - Decays with quark content $c\bar{c}s\bar{s}$ should be included in this sample.
 - It is advantageous to cut away as many non-ccss states as possible.
 - The width difference is entirely due to a difference in the decay rate of the eigenstates to hadronic states

- Exclusive final state

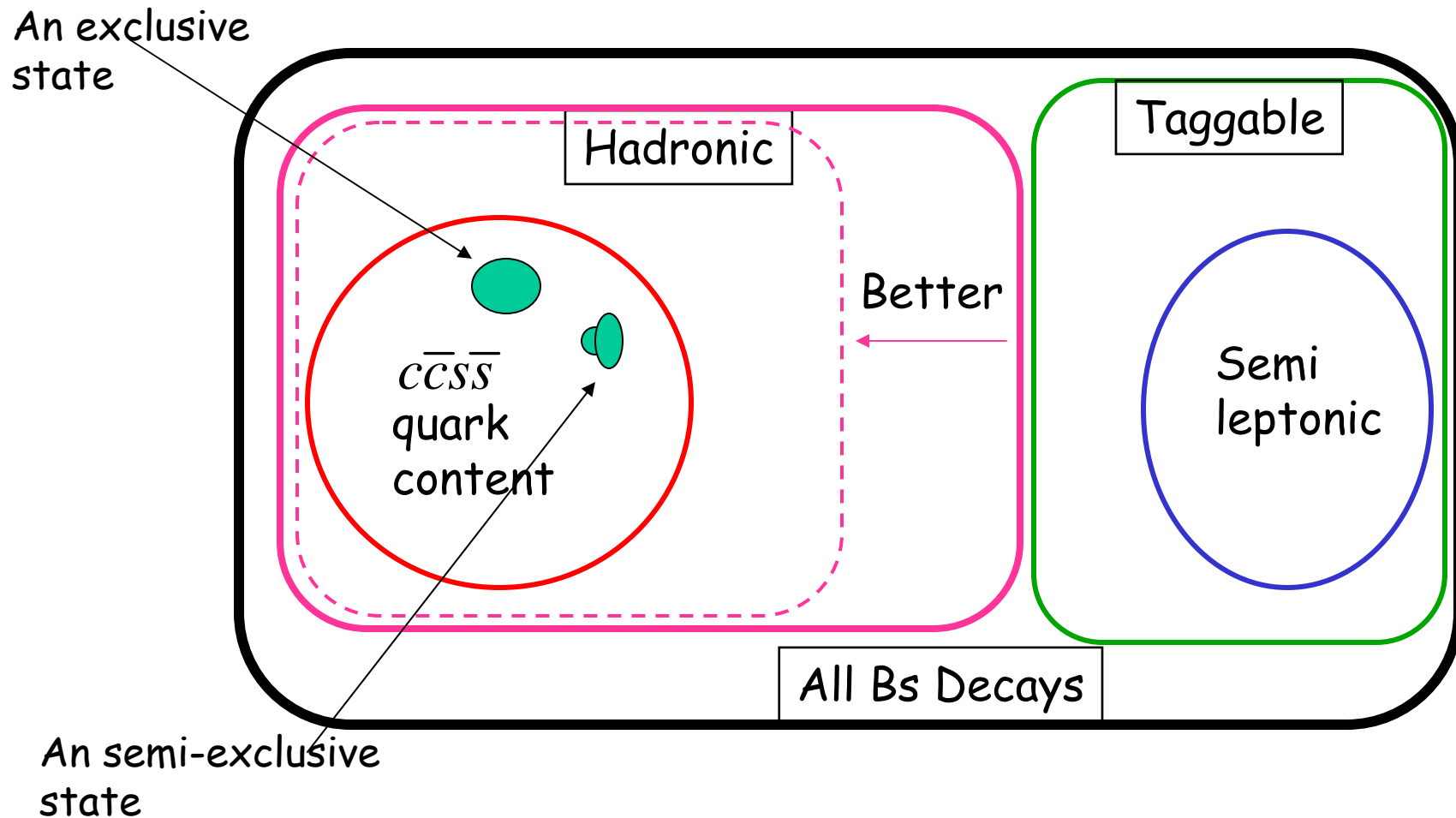
- A single quantum amplitude (e.g. $D_s^+ D_S^-$)

- Semi-exclusive state

- A set of related amplitudes (e.g. $D_s^{+*} D_S^{-*}$: three polarization amplitudes; $D_s^+ D_S^- \eta'$: s-wave vs p-wave)

- It is generally possible to separate out the components from the decay distribution.

Map of States



Oscillation Formalism: Decay Matrix for a General State

- For a single state, let
 - A =amplitude($B_s \rightarrow f$)
 - \bar{A} =amplitude($\bar{B}_s \rightarrow f$)

$$R_f = \begin{bmatrix} A^* A & A^* \bar{A} \\ \bar{A}^* A & \bar{A}^* \bar{A} \end{bmatrix} = \begin{bmatrix} u_f + v_f & w_f e^{i\theta_f} \\ w_f e^{-i\theta_f} & u_f - v_f \end{bmatrix} \quad \text{for a single quantum state } u^2 = v^2 + w^2$$

- For a set of states $F=\{f\}$

$$R_F = \sum_f R_f = \begin{bmatrix} u_F + v_F & w_F e^{i\theta_F} \\ w_F e^{-i\theta_F} & u_F - v_F \end{bmatrix} \quad \text{for a set of quantum states } u^2 \geq v^2 + w^2$$

Oscillation of a Single Meson

- At time t , $|\psi(t)\rangle = a(t)|B_s\rangle + \bar{a}(t)|\bar{B}_s\rangle \equiv \begin{bmatrix} a(t) \\ \bar{a}(t) \end{bmatrix}$

- The time evolution is given by

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle$$

$$g_{\pm}(t) = \frac{1}{2} \left(e^{-im_1 t} e^{-\frac{1}{2}\Gamma_1 t} \pm e^{-im_2 t} e^{-\frac{1}{2}\Gamma_2 t} \right)$$

$$U(t) = \begin{bmatrix} g_+(t) & e^{-i\phi} g_-(t) \\ e^{+i\phi} g_-(t) & g_+(t) \end{bmatrix}$$

- Thus the time dependent decay rate is

$$\Gamma(F, t) = \text{Tr} \left[U^\dagger(t) R_F U(t) \rho_0 \right]$$

ρ_0 = Initial state density Matrix

General Result Single Meson

- Define

$$x = \frac{\Delta m}{\Gamma} \quad S_x = \sin(x\Gamma t) \quad C_x = \cos(x\Gamma t)$$

$$y = \frac{\Delta\Gamma}{2\Gamma} \quad S_y = \sinh(y\Gamma t) \quad C_y = \cosh(y\Gamma t)$$

$$C_\theta^\phi = \cos(\phi + \theta) \quad S_\theta^\phi = \sin(\phi + \theta)$$

- Result $\Gamma(B_s \rightarrow F, t) = e^{-\Gamma t} [u_F C_y + v_F C_x - w_F S_y C_\theta^\phi + w_F S_x S_\theta^\phi] \leftrightarrow \rho_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Gamma(\bar{B}_s \rightarrow F, t) = e^{-\Gamma t} [u_F C_y - v_F C_x - w_F S_y C_\theta^\phi - w_F S_x S_\theta^\phi] \leftrightarrow \rho_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Gamma(\{B_s \bar{B}_s\} \rightarrow F, t) = e^{-\Gamma t} [u_F C_y - w_F S_y C_\theta^\phi] \leftrightarrow \rho_0 = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Average Branching Ratio (as measured e.g. at a collider)

$$\hat{\Gamma}(F) = \int \Gamma(\{B_s \bar{B}_s\} \rightarrow F, t) dt = \frac{1}{\Gamma} \left[\frac{u_F - y w_F C_\theta^\phi}{1 - y^2} \right]$$

Oscillation of Entangled Mesons

- For the $C=\pm 1$ state, the time dependent decay to a final state $F_1 F_2$ is:

$$\Gamma^\pm(F_1 F_2, t_1 t_2) = s_{12} \text{Tr} \left(\left[U^\dagger(t_1) R_1 U(t_1) \right] T_\pm \left[U^\dagger(t_2) R_2 U(t_2) \right]^T T_\pm^\dagger \right)$$

$$T_\pm = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ \pm 1 & 0 \end{bmatrix}$$

- s_{12} =statistical factor=1/2 for $F_1 = F_2$ or 1 otherwise.

General Result: Entangled Mesons

$$S_y^\pm = \sinh(y\Gamma(t_1 \pm t_2)) \quad C_y^\pm = \cosh(y\Gamma(t_1 \pm t_2))$$

$$S_x^\pm = \sin(x\Gamma(t_1 \pm t_2)) \quad C_x^\pm = \cos(x\Gamma(t_1 \pm t_2))$$

$$\Gamma^-(F_1 F_2, t_1 t_2) = 2s_{12} e^{-\Gamma(t_1+t_2)} \left[(u_1 u_2 - w_1 w_2 C_1^\phi C_2^\phi) C_y^- + (u_1 w_2 C_2^\phi - u_2 w_1 C_1^\phi) S_y^- \right. \\ \left. - (v_1 w_2 S_2^\phi - v_2 w_1 S_1^\phi) S_x^- - (v_1 v_2 + w_1 w_2 S_1^\phi S_2^\phi) C_x^- \right]$$

Only depends
on time
difference

$$\Gamma^+(F_1 F_2, t_1 t_2) = 2s_{12} e^{-\Gamma(t_1+t_2)} \left[(u_1 u_2 + w_1 w_2 C_1^\phi C_2^\phi) C_y^+ - (u_1 w_2 C_2^\phi + u_2 w_1 C_1^\phi) S_y^+ \right. \\ \left. + (v_1 w_2 S_2^\phi + v_2 w_1 S_1^\phi) S_x^+ - (v_1 v_2 - w_1 w_2 S_1^\phi S_2^\phi) C_x^+ \right]$$

$$B^- = \int \Gamma^-(F_1 F_2, t_1 t_2) dt_1 dt_2 = \frac{2s_{12}}{\Gamma^2} \left[(u_1 u_2 - w_1 w_2 C_1^\phi C_2^\phi) \frac{1}{1-y^2} - (v_1 v_2 + w_1 w_2 S_1^\phi S_2^\phi) \frac{1}{1-x^2} \right]$$

$$B^+ = \int \Gamma^+(F_1 F_2, t_1 t_2) dt_1 dt_2 = \frac{2s_{12}}{\Gamma^2} \left[(u_1 u_2 + w_1 w_2 C_1^\phi C_2^\phi) C_y^+ \frac{1+y^2}{(1-y^2)^2} - (u_1 w_2 C_2^\phi + u_2 w_1 C_1^\phi) S_y^+ \frac{2y}{(1-y^2)^2} \right. \\ \left. + (v_1 w_2 S_2^\phi + v_2 w_1 S_1^\phi) \frac{2x}{(1+x^2)^2} - (v_1 v_2 - w_1 w_2 S_1^\phi S_2^\phi) \frac{1-x^2}{(1+x^2)^2} \right]$$

Basic (qualitative) Idea: What happens if CP is conserved?

- The eigenstates are CP eigenstates

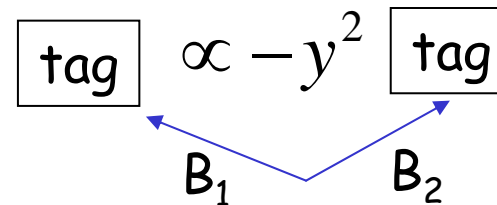
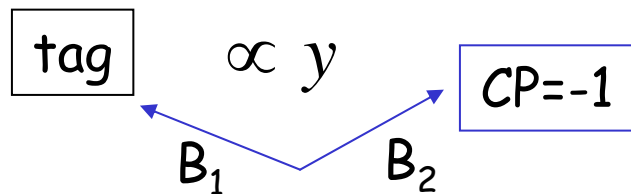
$$|B_1\rangle = \frac{1}{\sqrt{2}} \left(|B_s\rangle + |\bar{B}_s\rangle \right) \quad |B_2\rangle = \frac{1}{\sqrt{2}} \left(|B_s\rangle - |\bar{B}_s\rangle \right)$$

- The decay rate of B_1 and B_2 to taggable states is the same.
- The branching ratio of B_1 to taggable states will therefore be smaller because it has a larger total width.
- Just one catch:
 - there is no easy way to get a pure B_i sample.

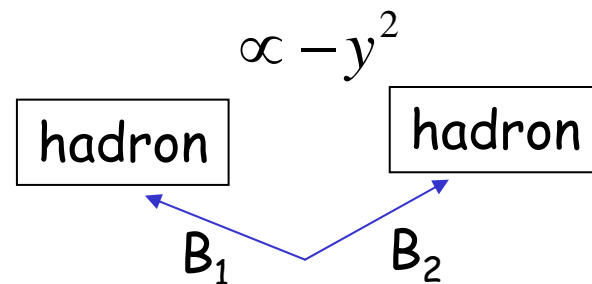
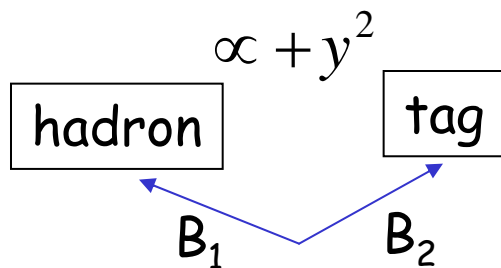
e.g. DA and Petrov PRD 71 054032 (2005)

Measuring y If CP is conserved

- This leads to a couple of ways to find y at B-factories (e.g. starting from a $C=-1$ pair):
 - You can find the BR of B_1 to taggable states by correlating with a $CP=-1$ eigenstate; this in effect gives a sample of B_2 states
 - If you observe one of the mesons decaying to a taggable state, it is more likely to be a B_2 .
 - The other meson is thus more likely to be a B_1 and thus less likely to decay itself to taggable states.
 - There will be an effect proportional to y^2 .



- By the same argument, B_1 has a larger branching ratio to hadronic states since they account for the width difference.
- Thus $t\bar{t}$ $t\bar{h}$ and $h\bar{h}$ correlations will all show effects proportional to y^2 .
- The correlation of t or h states with a CP eigenstate will show an effect proportional to y



Correlations of Inclusive States

- Define $a = \left[\frac{e^+e^- \rightarrow B_s^{(*)} \bar{B}_s^{(*)}}{e^+e^- \rightarrow b\bar{b}} \right]_{\Upsilon(5s)} \approx 20\%$

$$B_{5s}(F_1 F_2) = \left[\frac{e^+e^- \rightarrow F_1 F_2}{e^+e^- \rightarrow b\bar{b}} \right]_{\Upsilon(5s)}$$

- The correlated branching ratios are

$$B_{5s}(tt) = a \hat{B}_t^2 \left[1 - (1 - 2r^*) y^2 \right]$$

$$B_{5s}(hh) = a \hat{B}_h^2 \left[1 - (1 - 2r^*) \left[\frac{1 - \hat{B}_h}{\hat{B}_h} \right]^2 y^2 \right]$$

$$B_{5s}(ht) = 2a \hat{B}_h \hat{B}_t \left[1 + (1 - 2r^*) \left[\frac{1 - \hat{B}_h}{\hat{B}_h} \right] y^2 \right]$$

These are all sensitive to y^2

How Well Can y be measured?

- Assume that
 - $B_{\tau}=0.3$
 - $B_h=0.7$ (loose cuts) or 0.5 (tight cuts)
 - $y=0.1$
 - $r^*=0.1$
 - I assume that taggable states are not contaminated by ccss states
 - Likewise all ccss states are designated hadronic
- How many $ee \rightarrow bb$ events do you need to get a 5σ effect just including statistical errors?
 - tt correlation: $N=21 \times 10^6$.
 - hh correlation: $N=120 \times 10^6$ (loose cuts) $N=8 \times 10^6$ (tight cuts)
 - ht correlation $N=25 \times 10^6$ (loose cuts) $N=7 \times 10^6$ (tight cuts)
- Note, the number increases like y^{-4} as y decreases.

Inclusive / Semi-exclusive Correlations

- For a state Y , define

$$P_Y = \frac{w_Y C_Y^\phi}{u_Y}$$

$$\hat{P}_Y = \frac{P_Y - y}{1 - yP_Y} = \frac{Br(B_1 \rightarrow Y) - Br(B_2 \rightarrow Y)}{Br(B_1 \rightarrow Y) + Br(B_2 \rightarrow Y)}$$

- Measuring this will give us information about $\theta + \phi$,
 - if Y is a single quantum state $w = u$ so $\cos(\theta + \phi)$ is determined by P_Y .

$$B_{5s}(tY) = 2a\hat{B}_t \hat{B}_Y \left[1 + (1 - 2r^*)y\hat{P}_Y \right]$$

$$B_{5s}(hY) = 2a\hat{B}_h \hat{B}_Y \left[1 - (1 - 2r^*) \left[\frac{1 - \hat{B}_h}{\hat{B}_h} \right] y\hat{P}_Y \right]$$

How Well can P_Y be Measured?

- Consider a final state like $Y=DsDs$ with $Br \sim 1\%$.
- We expect $P \sim 1$ since this a CP eigenstate.
- If the acceptance is about 10%, in effect the Br is 0.1%
- How many $ee \rightarrow bb$ events do you need to get a 5σ effect?
 - tY correlation $N=33 \times 10^6$.
 - hY correlation $N=76 \times 10^6$ (loose cuts); $N=20 \times 10^6$ (tight cuts)

Problem with This Approach

- For hh, ht and tt correlations, the deviation from the product branching ratios is $O(\gamma^2) \sim 1\%$.
- To use this method the input branching ratios, \hat{B}_h and \hat{B}_t must be known to better than 1%
- In the tY and hY correlations the deviations are $O(\gamma) \sim 10\%$
- Likewise the input branching ratios must be known to this precision.
- How do we get around this?

Better yet: Time Asymmetry

- Recall the time dependent rate

$$\Gamma^-(F_1 F_2, t_1 t_2) = 2s_{12} e^{-\Gamma(t_1+t_2)} \left[(u_1 u_2 - w_1 w_2 C_1^\phi C_2^\phi) C_y^- + (u_1 w_2 C_2^\phi - u_2 w_1 C_1^\phi) S_y^- \right. \\ \left. - (v_1 w_2 S_2^\phi - v_2 w_1 S_1^\phi) S_x^- - (v_1 v_2 + w_1 w_2 S_1^\phi S_2^\phi) C_x^- \right]$$

- The colored terms are the only ones anti-symmetric under $t_1 \leftrightarrow t_2$
- Consider the asymmetry

$$A(F_1 F_2) = \frac{(F_2 \text{ decays first}) - (F_1 \text{ decays first})}{\text{Total } F_1 F_2}$$

- In general this will be sensitive to the red term

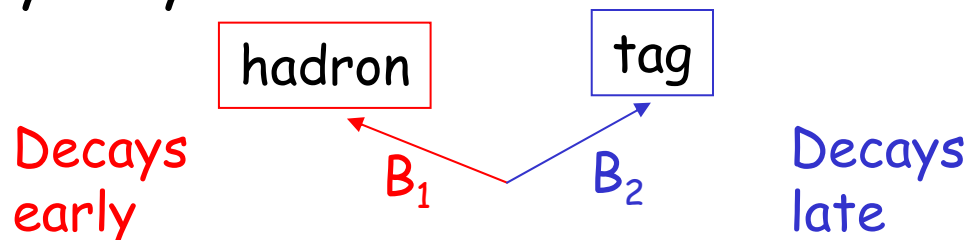
$$A(F_1 F_2) = (1 - r^*) (\hat{P}_2 - \hat{P}_1) y + O(1/x)$$

Time Asymmetry for ht Correlations

- For the ht correlation

$$A(ht) = (1 - r^*) \frac{y^2}{\hat{B}_h} \approx 1.5\%$$

- How many $ee \rightarrow bb$ events do you need to get a 5σ effect?
 - $A(ht)$ correlation $N = 1.8 \times 10^6$ (loose cuts)
 - $A(ht)$ correlation $N = 1.3 \times 10^6$ (tight cuts)
- Why do you do so well???



- Further advantage of an asymmetry: null experiment, don't need to subtract another measurement to find the signal

Charge/Time Asymmetry

$$\Gamma^-(F_1 F_2, t_1 t_2) = 2s_{12} e^{-\Gamma(t_1+t_2)} \left[\left(u_1 u_2 - w_1 w_2 C_1^\phi C_2^\phi \right) C_y^- + \left(u_1 w_2 C_2^\phi - u_2 w_1 C_1^\phi \right) S_y^- \right. \\ \left. - \left(v_1 w_2 S_2^\phi - v_2 w_1 S_1^\phi \right) S_x^- - \left(v_1 v_2 + w_1 w_2 S_1^\phi S_2^\phi \right) C_x^- \right]$$

$\sim 1/x$
 not \leftarrow
 $\sim 1/x^2$

- We can look for the blue term by considering the asymmetry taking into account the sign of the taggable decay.

$$A' = A(ht^+) - A(ht^-) = \frac{(1-r^*)}{\hat{B}_h} \left[\frac{x^2(1-y^2)}{(1+x^2)} \right] \frac{y}{x} \tan(\phi + \theta_h)$$

- For $\tan=1$, $A' \sim 0.5\%$
- This asymmetry therefore gives us $\tan(\text{phase})$.

Charge/Time Asymmetry

- How many $ee \rightarrow bb$ events do you need to get a 5σ effect for $A'(ht)$ if $\tan(\phi+\theta)=1$?
 - $A'(ht)$ correlation
 $N=7.0 \times 10^6$ (loose cuts) $N=6.0 \times 10^6$ (tight cuts)

Summary

- At the Upsilon(5s), 90% of the time, B_s pairs are produced in the $C=-1$ correlated state.
- The correlation between the product branching ratio and the correlated branching ratio tells us about mixing
 - Two inclusive states: $|y|$
 - Inclusive/exclusive $y \cos(\phi+\theta)$
- Time taggable/hadronic time asymmetry gives us y^2 and $\tan(\phi+\theta)$.
- This method is better because it is a null experiment
- All this may be done with existing or soon to be acquired data