Study of collective effect in muon ionization cooling

Dazhang Huang, *Illinois Institute of Technology*
King Y. Ng, *Fermilab*
Thomas J. Roberts, *Muons Inc.*

DPF09, Detroit
July 27, 2009

Outline

- **Basic ideas**
- **Derivations of the wakefield introduced by a incident beam particle and stopping power perturbation due to collective effect**
- **Estimation of collective perturbation in muon ionization cooling (MIC) with LH2 as the absorber**
- **Estimation of collective perturbation in plasma medium with density comparable to beam density**
- **Conclusions**
- **References**

Coordinate system & units

- **Cylindrical coordinate system with $\hat{z}$ the direction of motion of the incident particle.**

- **All units Gaussian**

Basic ideas

- As a charged particle beam passes through matter, it polarizes and ionizes the matter. The polarized electrons in matter then build up wakefields that interact back on the incident beam particles (density effect of matter [3]).

- The wakefields of all the beam particles then superpose together, bring collective perturbation to the stopping power of beam particles.

Derivation of wake electric field

- In lab frame, the polarized electrons and the incident particle set up the wake potential at $\vec{r}(z,\rho,\theta)$ which can be expressed as [1,2]:

$$\phi(\vec{r},t) = \frac{Ze}{2\pi^2 v}\int_0^\infty \kappa d\kappa \int_{-\pi}^{\pi} d\theta \int_{-\infty}^{\infty} d\omega \frac{e^{i[\kappa\rho\cos\theta + (\frac{z}{v}-t)\omega]}}{(\kappa^2 + \frac{\omega^2}{v^2})}\frac{1}{\epsilon(k,\omega)} \qquad (1.1)$$

  where $k^2 = \kappa^2 + \omega^2/v^2$ is the wave number, and κ is the transverse wave number , z is along the direction of particle momentum, $\epsilon(k,\omega)$ is the complex dielectric constant in matter which is a function of wave number and frequency. Note that if replacing $1/\epsilon$ by $1/\epsilon - 1$, one obtains the potential purely generated by the polarized electrons.

- The electric wakefield produced by the wake potential above in the center of mass frame can be computed by:

$$E_z = \frac{\partial\phi}{\partial z} = \frac{2Zci}{\pi v^2}\int_0^\infty \kappa d\kappa \int_{-\infty}^{\infty}\frac{\omega d\omega}{\kappa^2+\omega^2/v^2}\frac{1}{\epsilon(k,\omega)}J_0(\kappa\rho)e^{i(z/v-t)\omega} \qquad (1.2)$$

Derivation (cont.)

$$E_\rho = \frac{\partial \phi}{\partial \rho} = -\frac{2Ze}{\pi v} \int_0^\infty \kappa^2 d\kappa \int_{-\infty}^{\infty} \frac{d\omega e^{i(z/v-t)\omega}}{\kappa^2 + \omega^2/v^2} \frac{1}{\epsilon(k,\omega)} J_1(\kappa\rho) \quad (1.3)$$

$$E_\theta = \frac{1}{\rho}\frac{\partial \phi}{\partial \theta} = 0 \quad (1.4)$$

- **In dispersive media, the dielectric constant is a complex number, In the high-velocity case that we are discussing, the wakefield frequency is high and the inverse of dielectric constant takes the form [3]:**

Derivation (cont.)

$$\frac{1}{\epsilon} \approx \frac{\omega^2}{\omega^2 - \omega_p^2 + i\omega\Gamma} \approx \frac{\omega^2}{(\omega + \omega_p + \frac{i}{2}\Gamma)(\omega - \omega_p + \frac{i}{2}\Gamma)} \quad (1.5)$$

where $\omega_p = (4\pi n_e e^2/m)^{1/2}$ **is the plasma frequency,** Γ **is the damping constant and** $\Gamma \longrightarrow 0$

- **After path integration around the poles in complex plane we have (approximately):**

Derivation (cont.)

$$E_z = \frac{2Ze\omega_p^2}{v^2} K_0\left(\frac{\omega_p}{v}\rho\right)\cos\left[\left(\frac{z}{v} - t\right)\omega_p\right] - \frac{Ze}{c^2}\frac{\frac{z}{v} - t}{\left[\left(\frac{z}{v} - t\right)^2 + \frac{\rho^2}{v^2}\right]^{\frac{3}{2}}} \quad (1.6)$$

$$E_\rho = \frac{2Ze\omega_p^2}{v^2} K_1\left(\frac{\omega_p}{v}\rho\right)\sin\left[\left(\frac{z}{v} - t\right)\omega_p\right] + \text{small term by vector potential} \quad (1.7)$$

- **The 2nd terms in Eq. (1.6) & (1.7) come from vector potential. They are in fact ignorable because they are much smaller than the contribution of electric potential (the 1st terms)**

Collective perturbation

- Based on Eq. (1.6) and (1.7) , one can conclude that the wakefield induced by an incident charged particle oscillates with the plasma frequency ω_p . In other words, only the wave with $\omega = \omega_p$ can propagate in the media.

- Because the wakefield is a function of time, source location and target location, if the wakefields of incident particles happen to coherently interfere at the target location, it might have non-ignorable impulse on the beam particle at that location. Then the stopping power of that beam particle may be perturbed.

- For a two-particles-system (#1, #2) with small velocity deviation, the total stopping power can be described by:

Collective perturbation (cont.)

$$S_c = \frac{2c^2 i}{\pi}\int_0^\infty \kappa d\kappa \int_{-\infty}^\infty \omega d\omega \left\{ \frac{1}{(\kappa^2 v_1^2+\omega^2)}\frac{1}{\epsilon(k,\omega)}\left\{ [Z_1^2+Z_2^2+ \right.\right.$$
$$\left.\left. 2Z_1Z_2 J_0(\kappa b)\cos\frac{d\omega}{v_1}] - \frac{2\kappa^2 v_1 \Delta v}{(\kappa^2 v_1^2+\omega^2)^2}[Z_2^2 + Z_1 Z_2 J_0(\kappa b)\cos\frac{d\omega}{v_1}]\right\}\right\} \quad (1.8)$$

where $v_2 = v_1 + \Delta v$ is the velocity of incident particle #2, Z_1 & Z_2 are the charges of particle #1 & #2, respectively. The two particles are separated by vector $\vec{R} = \vec{d} + \vec{b}$ where $\vec{d}$ in $\hat{z}$ and $\vec{b}$ in $\hat{r}$ From Eq. (1.8) one can tell that the cross-term with $Z_1 Z_2$ is the perturbation due to the two-particle-interference. (Fig. 1) shows its strength as a function of separation normalized to the shielding length $R = v/\omega_p$, w/ $\Delta v = 0$, averaged over all directions in 3-D space:

Collective perturbation (cont.)

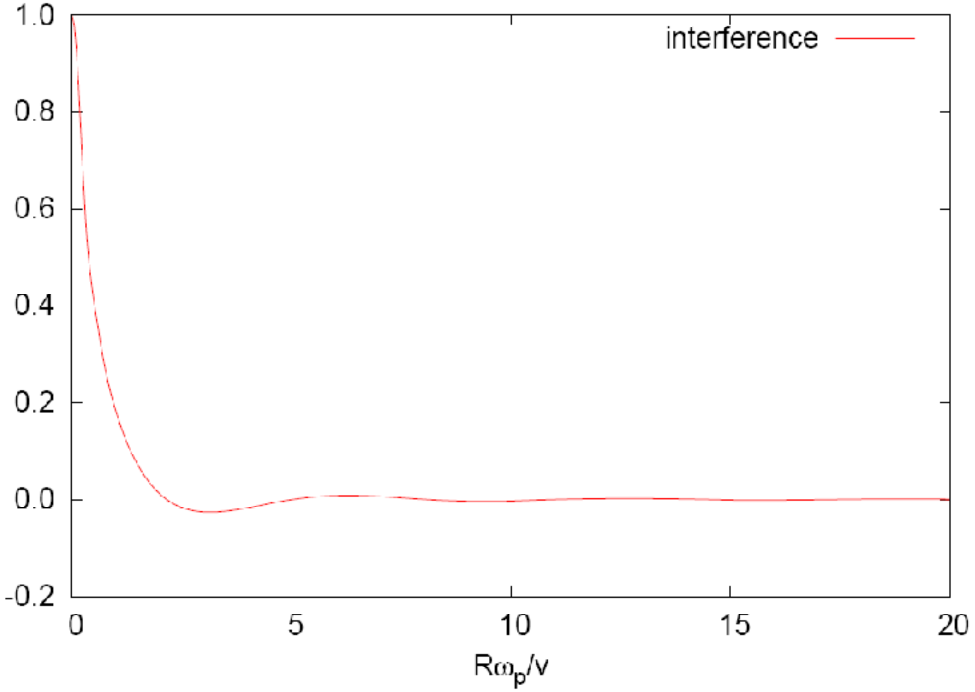

Fig. 1

- **We can see that the interference strength drops very fast to almost zero within a few shielding lengths and has small oscillations in distance.**

Estimation for MIC

• As an example, assuming 200 MeV/c muon beam, which will be commonly used in muon ionization cooling channel; LH2 absorber and 1.5e11 muons/bunch. In the most aggressive cooling scheme proposed by David Neuffer [4], $\epsilon_n = 25\ mm \cdot mrad$ with 0.05 mm in radius and 30 cm (~1 nsec) length of beam; targeting a muon particle in the bunch, considering the contributions of the wakefields of other particles in a sphere of radius *r*, by analytical computation, compared to the stopping power of a single particle w/o collective effect by Bethe formula[5], and assuming uniform beam, we have:

- At $r = 10 \times v/\omega_p$, perturbation fraction: 0.003%
- At $r = 100 \times v/\omega_p$, perturbation fraction: 4.8%
- At $r = 200 \times v/\omega_p$, perturbation fraction: 2.3%
- At $r = 500 \times v/\omega_p$, perturbation fraction: 3.4%

In this case the shielding length is in the order of ***1.e-5 mm, much smaller than the average separation of beam particles.*** Also it is not surprising that the perturbation is oscillating because of Fig. 1

Estimation for MIC (cont.)

- OOPICPro [6] developed by Tech-X Corporation is able to simulate the charged particle beam passing through matter. However, in ionization cooling, because the electron density in LH_2 is much larger than that of the incident muon beam, the polarization is almost invisible (Fig. 2), and results in small collective perturbation. It is consistent with our numerical computation.

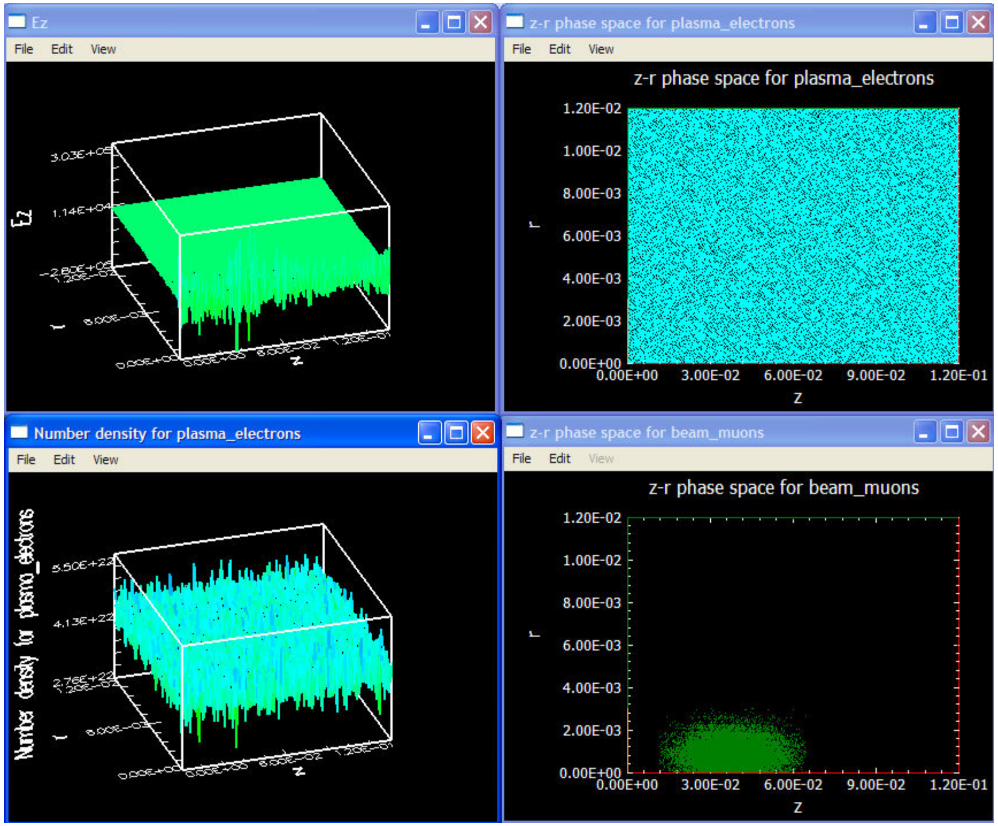

Fig. 2

Estimation for plasma medium

• As the incident beam density is comparable to the pre-ionized plasma density, this collective effect could be important. The longitudinal on-axis wake electric field computed by OOPICPro (Fig. 3) is on the same order of our analytical solution. Assuming a 200 MeV/c Gaussian muon beam, with density 5e13 cm^{-3}, standard derivation 1 mm in all three directions; and plasma density 4.28e12 cm^{-3}, we obtain the peak longitudinal on-axis wake electric field to be ~ 1.e8 V/m by simulation; and analytical solution (Fig. 4) has ~ 4.e8 V/m. The reason why there is a factor of 4 is being investigated.

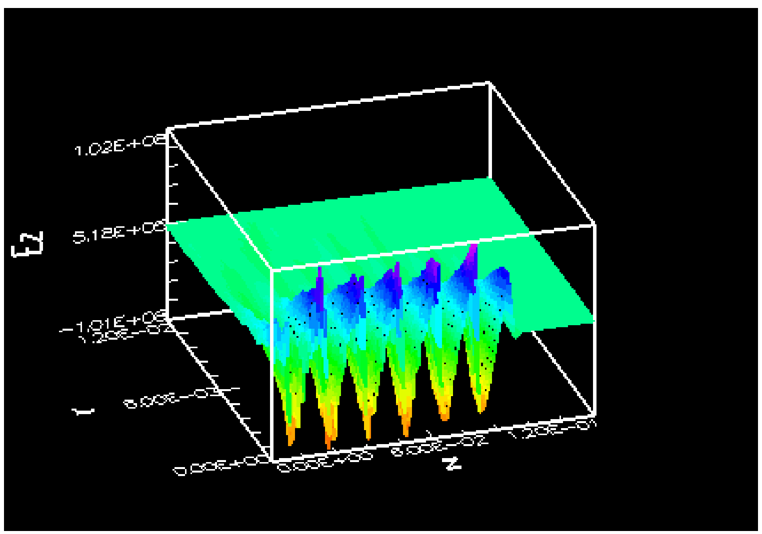

Fig. 3

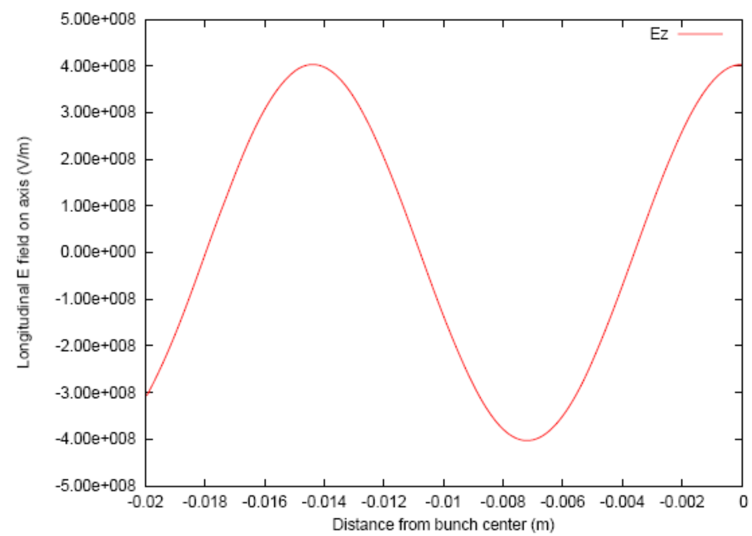

Fig. 4

Conclusions

- Cooling happens when there exists a resistive force along the direction of motion of each incident particle. In ionization cooling, the wake force due to matter polarization can serve as the required resistive force, other than the single particle cooling scheme which has been well-established, It is a collective effect and needs to be studied.

- Both analytical computation and simulation indicate that in muon ionization cooling, the collective effect of multiple beam particles due to matter polarization is not important.

- In wakefield acceleration, because the beam density is comparable to the pre-ionized plasma (matter) density, this effect could be significant. More investigation needs to be done to figure out its importance.

References

- [1] M. G. Calkin & P. J. Nicholson, Reviews of Modern Physics, v.39 no.2 (361), 1967
- [2] W. Brandt et al, Physical Review Letters, v.33 no.22 (1325), 1974
- [3] J. D. Jackson, Classical Electrodynamics, 3rd edition, 1998
- [4] D. Neuffer, LEMC09 presentation
- :
- [5] Particle Data Group (PDG): Passage of Particles through matter
- [6] OOPICPro, Tech-X Corporation
  http://www.txcorp.com/products/OOPIC_Pro/