Mesons From String Theory

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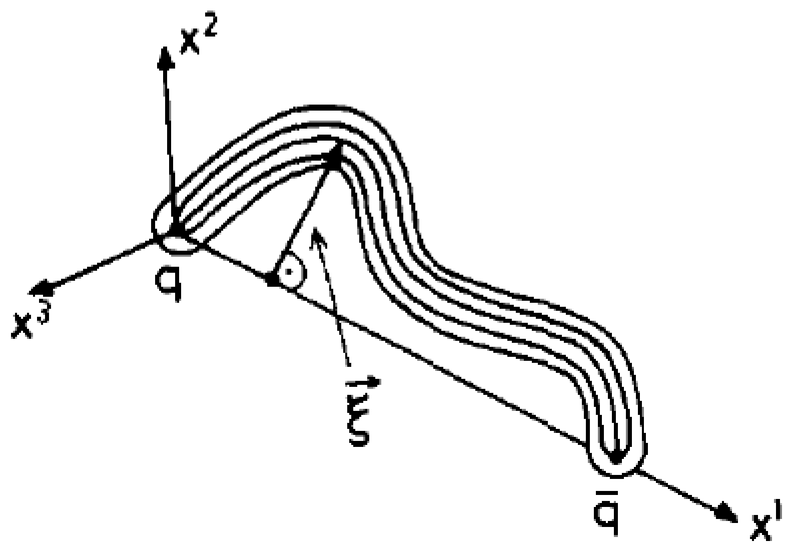
Outline

- History of strings and quarks: Lüschers fundamental string: Tensions and the Lüscher term
- The *k*-string: combinations of fundamental strings "glued together"
- t'Hooft large N limit: confining gauge theories can be described by string theory.
- *k*-strings as dual descriptions of D-branes in supergravity backgrounds (SUGRA)
- Energy Fluctuations: the Lüscher term from supergravity duals

History of Strings and Quarks (Lüscher, 1981)

- Lüscher created a simple model of a quark – antiquark pair *tied* together by a string, or color flux tube.



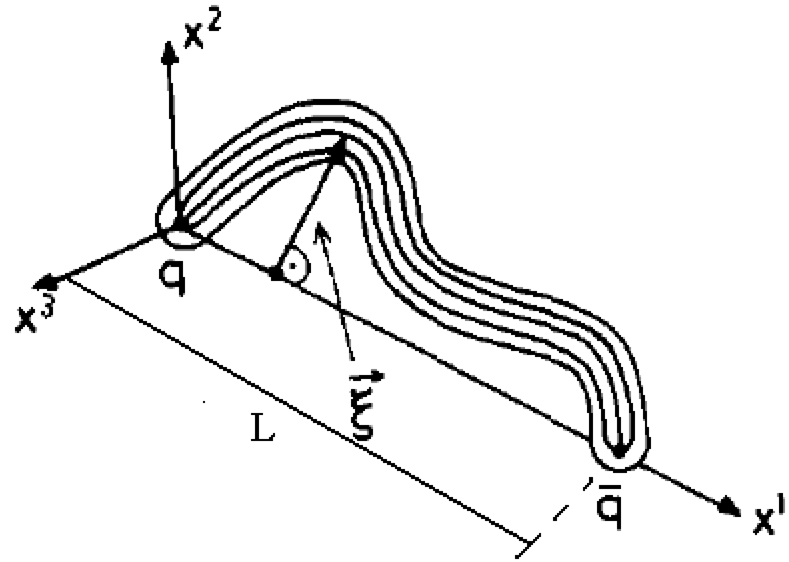
Thin flux tube at a fixed time x^0. The position vector ξ is parallel to the (x^2, x^3) plane.

History of Strings and Quarks (Lüscher, 1981)

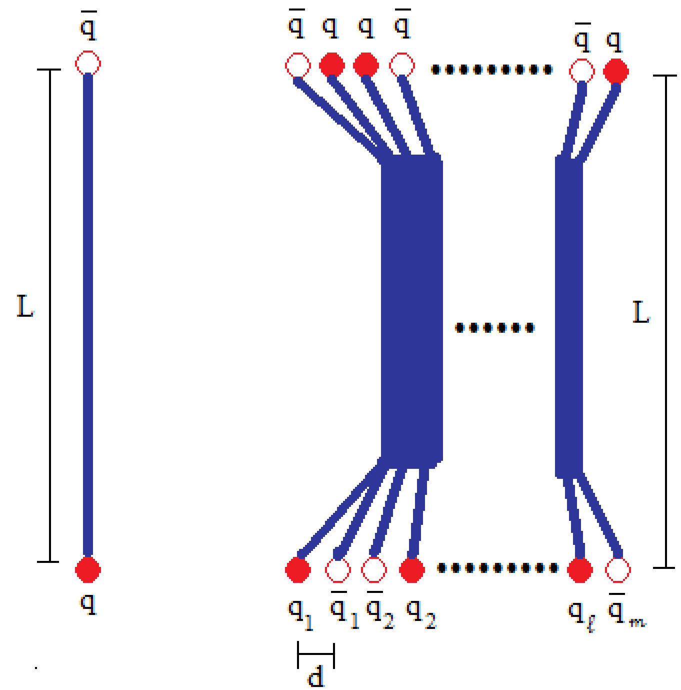
- Lüscher calculated the energy of this configuration

$$E = T\,L - \frac{\pi(d-2)}{24L} + \beta + O\left(\frac{1}{L^2}\right)$$

- *T* is tension, *d* is the dimension of space-time



SU(N) k-strings (Shifman, 2005)



Lüschers fundamental string (left) and *k*-string (right)

- *SU(N) k*-strings are composed of quark – antiquark pairs pulled a distance L apart
- $k = |\ell - m|$
- For large L, *k*-strings exhibit *k*-ality: tension vanishes when *k = N*
- *k*-ality exhibited by models in lattice gauge theory, analytic Hamiltonian methods, and string theory using gauge/gravity correspondence

Energy of k-strings

Both lattice gauge theory* and Hamiltonian methods† find:

$$E_k = T_k L$$

Two possible forms for T_k:

$$T_k \propto \begin{cases} N \sin \dfrac{k\pi}{N} & \text{sine law} \\ k \dfrac{N-k}{N} & \text{casimir law} \end{cases}$$

*Teper, 1999, † Nair et. al., 1998

k-string Tensions in d=2+1

T_k/T_f from Various Methods

S=symmetric, A=antisymmetric, M=mixed, *=antisymmetric

$Group$	k	CGLP	MNa(Sine)	Casimir	lattice	Karabali-Nair
$SU(4)$	2	1.310	1.414	1.333	1.353(A) 2.139(S)	1.332(A) 2.400(S)
$SU(5)$	2	1.466	1.618	1.5	1.528*	1.529*
$SU(6)$	2	1.562	1.732	1.6	1.617(A)	1.601(A)
					2.190(S)	2.286(S)
	3	1.744	2.0	1.8	1.808(A)	1.800(A)
					3.721(S)	3.859(S)
					2.710(M)	2.830(M)
$SU(8)$	2	1.674	1.848	1.714	1.752*	1.741*
	3	2.060	2.414	2.143	2.174*	2.177*
	4	2.194	2.613	2.286	2.366*	2.322*

A, S,M data from from Nair et. al., 2008
* data from Teper et. al., 2008

k-strings from Supergravity Duals

- t'Hoofts large *N* argument: basically a weakly coupled string theory is dual to a strongly coupled gauge theory at large *N* (t' Hooft, circa 1974)
- Low energy effective action of string theory <=> classical SUGRA (Green and Schwarz, circa 1985)
- Strings and/or D-branes (objects strings end on) embedded in SUGRA are dual descriptions of configurations of large *N* gauge theories (Maldacena, 1998)
- One such *SU(N)* gauge theory configuration: the *k*-string, dual to a D-brane (Herzog and Klebanov, 2002)

Lüscher Terms for *k*-strings

- *Assumed* in lattice calculations to be (Teper, 1999):

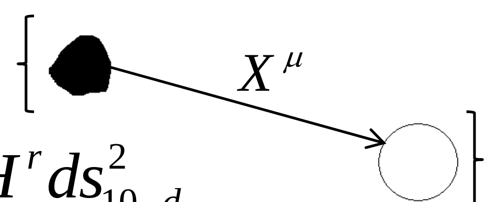
$$\alpha = -\frac{\pi(d-2)}{24L}$$

d is the dimension of space-time.

- *Calculated* in string theory models to be (Stiffler et al. 2008):

$$\alpha = -\frac{\pi(d+p-3)}{24L}$$

Multi-dimensional gravitational source:



Dp-brane: p spatial dimensions

$$ds_{10}^2 = H^q dx^\mu dx^\nu \underbrace{\eta_{\mu\nu}}_{d \text{ dimensional Minkowski space}} + H^r ds_{10-d}^2$$

k-strings from Supergravity Duals

(Herzog and Klebanov, 2002)

- k-string is a Dp-brane with U(1) gauge flux

$$F = dA = F_{tx} dt \wedge dx + F_{\theta\phi} d\theta \wedge d\phi,$$

embedded in a classical SUGRA sourced by

$$F_n = dC_n, \quad H_3 = dB_2, \ \Phi.$$

- Sources are functions of Bosonic SUGRA coordinates, X^μ
- Action for this Dp-brane: $S_p = S_p\left[X(\varsigma), \partial X(\varsigma), A(\varsigma), \partial A(\varsigma), \partial\Theta(\varsigma), \overline{\Theta}(\varsigma)\right]$

$$S_p = -\mu_p \int d^{p+1}\varsigma\, e^{-\Phi} \sqrt{-\det\left(g_{ab} + \mathcal{F}_{ab}\right)} + \mu_p \int \sum_q C_q \wedge \mathcal{F}_2 + S_f$$

$$\mathcal{F} = B_2 + 2\pi\alpha' F, \qquad \mu_p^{-1} = (2\pi)^p \alpha'^{(p+1)/2}, \qquad S_f = S_f\left[\partial\Theta(\varsigma), \overline{\Theta}(\varsigma)\right] = 0 \text{ classically},$$

g_{ab} is pullback of SUGRA onto Dp-brane

k-string Tension and Lüscher Term

(Stiffler et. al. 2008)

- Tension is the minimum classical energy of these D4-branes

$$S_p^{(0)} = \int dt \mathcal{L}(A_0, \dot{A}_0, X_0), \qquad X_0, A_0 \text{ are classical SUGRA bosons}$$

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} \dot{A}_0 - \mathcal{L}, \quad \mathcal{H}_{\min} = T_k L$$

- Lüscher term is the first quantum correction:

$$X = X_0 + \delta X, \ \ A = A_0 + \delta A, \ \ \Theta = 0 + \delta\Theta$$

$$S_p = S_p^{(0)} + S_p^{(1)} + S_p^{(2)} + \ldots.$$

$$e^{iE_1 T} = Z_2 = \int DX\, DA\, D\overline{\Theta}\, D\Theta\, e^{iS_2}, \qquad E_1 = \frac{\alpha}{L} + \beta$$

CGLP Supergravity Background

(Cvetic, Gibbons, Lu, and Pope, 2001)

$$ds^2 = H^{-1/4} dx^\mu dx^\nu \eta_{\mu\nu} + H^{1/4} l^2 [h^2 dr^2 + a^2 (D\mu^i)^2 + b^2 d\Omega_4^2]$$

$$X = \left(x^0, x^1, x^2, r, \mu^1, \mu^2, \mu^3, \psi, \chi, \theta, \phi\right)$$

- H, h, a, b are functions of r, l is a constant

$$D\mu^i = d\mu^i + \epsilon^{ijk} A^j \mu^k, \quad J^i = dA^i + \frac{1}{2}\epsilon^{ijk} A^j \wedge A^k$$

CGLP Supergravity Background

(Cvetic, Gibbons, Lu, and Pope, 2001)

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$$H_3 = dB_2 = \frac{m}{l}\Big(f_1 dr \wedge X_2 + f_2 dr \wedge J_2 + f_3 X_3 \Big), \quad e^\Phi = g_s H^{1/4}$$

$$\widetilde{F_4} = F_4 = g_s^{-1} d^3x \wedge dH^{-1} + m\Big(f_4 \epsilon_{ijk} \mu^i dr \wedge D\mu^j \wedge J^k + f_5 X_2 \wedge J_2 + f_6 J_2 \wedge J_2 \Big)$$

$$X_2 = \frac{1}{2}\epsilon_{ijk}\mu^i D\mu^j \wedge D\mu^k, \;\; J_2 = \mu^i J^i, \;\; X_3 = dX_2$$

$$m = 8\pi\alpha'^{3/2} g_s N$$

N is number of D-branes sourcing the F_4 and H_3 flux on the gravity side
N is number of colors on the gauge theory side *(SU(N))*

SU(N) k-strings in CGLP Background

- Place a "Probe" D4-brane in the CGLP Background
- Calculate dynamics of this brane using a Born-Infeld type action
- Gauge/Gravity correspondence says this gravitational theory should be a dual description of some configuration in strongly coupled gauge theories
- Calculate the Energy: we calculate up to one loop quantum corrections, it is dual to *SU(N)* k-strings

k-string Tension from Classical D4-brane Action in CGLP (Herzog, 2002)

$$S^{(0)} = -\mu_4 \int d^5 x\, e^{-\Phi} \sqrt{-\det\left(g_{ab} + 2\pi\alpha' F_{ab}\right)} + \mu_4 \int C_3 \wedge F$$

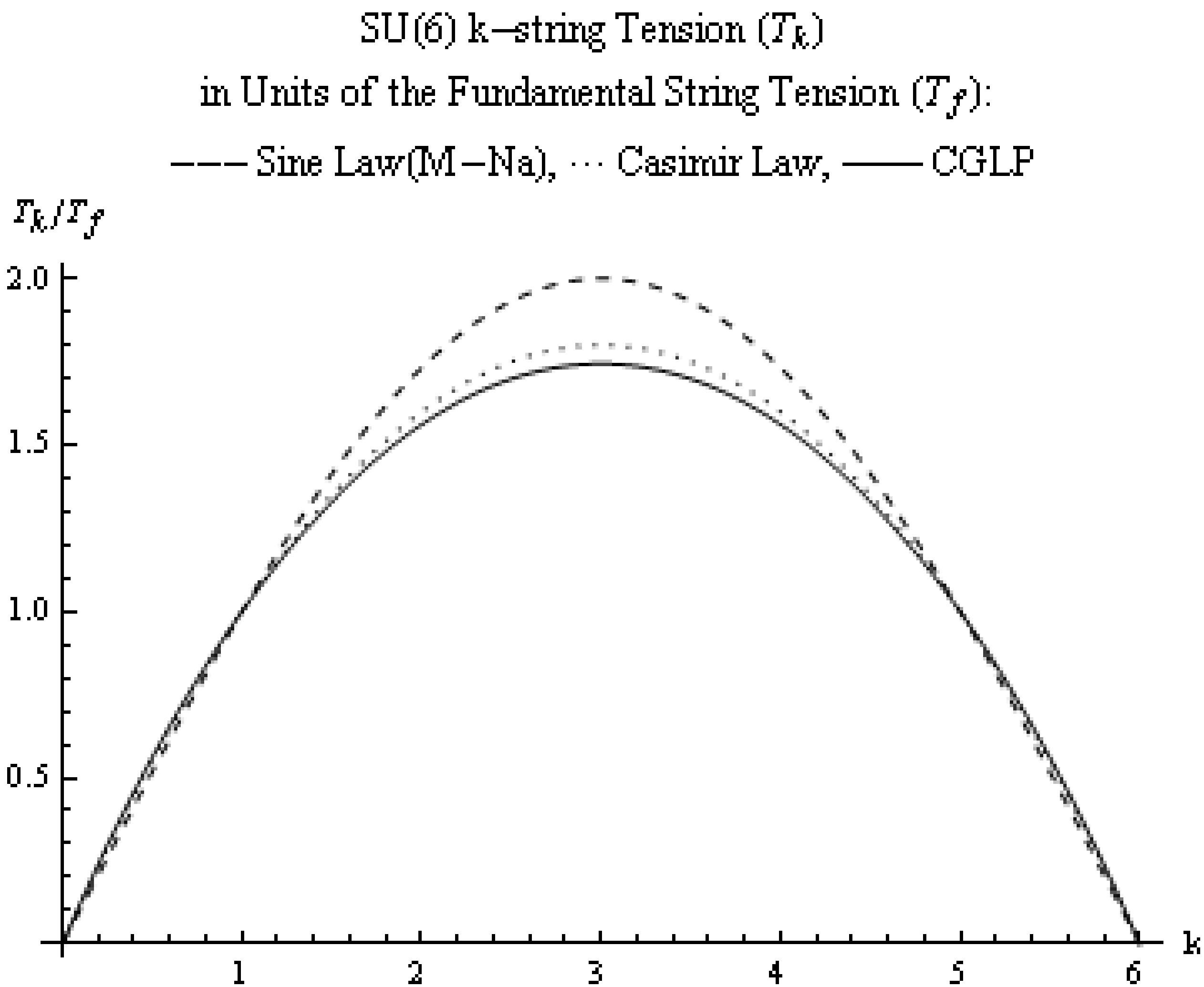
Yields the Hamiltonian, minimized w.r.t. bosonic SUGRA coordinate ψ :

$$\mathcal{H}_{\min} = TL = \alpha N L \sin^2\psi_0 \sqrt{\sin^2\psi_0 + \left(\frac{3\alpha}{q}\right)^2 \cos^2\psi_0}$$

Subject to the constraint

$$\frac{4k}{3N} = \xi(\psi) + 3\frac{\alpha^2}{q^2}\sin^2\psi_0 \cos\psi_0$$

Tension for CGLP *k*-strings (Herzog, 2002)



Lüscher Term from Supergravity Duals

(Stiffler et. al., 2008)

- Fluctuate about classical solution:

$$X = X_0 + \delta X,\ \ A = A_0 + \delta A,\ \ \Theta = 0 + \delta\Theta$$

$$S = S[X, \partial X, A, \partial A, \overline{\Theta}, \partial\Theta] = S^{(0)} + S^{(1)} + S^{(2)} + \ldots.$$

$$= S^{(0)} + 0 + S^{(2)} + \ldots.$$

$$e^{iE_1T} = Z_2 = \int DX\ DA\, D\overline{\Theta}\, D\Theta\, e^{iS_2}, \qquad E_1 = \frac{\alpha}{L} + \beta$$

$$\alpha = -\frac{\pi(d+p-3)}{24L}$$

Summary: Lüscher Term from Supergravity Duals

- Two backgrounds (CGLP and KS) have been shown through this rigorous method to exhibit this formula (Stiffler et. al., 2008)

$$\alpha = -\frac{\pi(d+p-3)}{24L}$$

- 2+1 *k*-string dual (CGLP background) D4-brane
d = 3, *p = 4:* $\alpha = -\frac{\pi}{6L}$
- 3+1 *k*-string dual (KS background) D3-brane:
d = 4, *p = 3:* $\alpha = -\frac{\pi}{6L}$

k-string Tensions in d=2+1

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Conclusion

- Lüscher calculated the energy for the fundamental string from 1st principles in 1981
- *k*-string tension calculated from lattice gauge theory as well as with Hamiltonian methods
- *k*-string tension calculated from supergravity duals, as well as a Lüscher term, though have no calculations that we have found to compare this to in the lattice and Hamiltionian community

Thank You

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- Organizers of DPF conference
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