Renormalon subtraction from the average plaquette and the gluon condensate

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Outline

- Brief introduction:dim-2 condensate from plaquette?
- Problem with the existing renormalon subtraction
- New scheme
- Consistency with dim-4 condensate
- Conclusion

Introduction

- Extraction of the gluon condensate $\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle$ from a lattice simulation of the average plaquette is an old problem
- The average plaquette can be expanded as

$$P \equiv \langle 1 - \frac{1}{3} \mathrm{TrU}_{\Box} \rangle = \sum_{n=1} \frac{c_n}{\beta^n} + \frac{\pi^2}{36} Z(\beta) \langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle a^4 + O(a^6)$$

- The difficulty lies in the accurate subtraction of the perturbative contribution: need to know better than one part in 10^4
- The pertubative coefficients were computed to 10 loop order in stochastic perturbation theory (F. Di Renzo, et al,1995,2000), and grow rapidly as expected from the presence of renormalons

- Burgio et. al. (1998) subtracted the renormalon contribution in a continuum scheme, by matching the large order behavior in the lattice scheme to that of the continuum scheme. Specifically,
- The average plaquette was expressed as

$$P = P^{\mathrm{ren}}(\beta_c) + \delta P(\beta_c) + P_{\mathrm{NP}} \,,$$

$$P^{\mathrm{ren}}(\beta_c) = \int_0^{b_{\max}} e^{-\beta_c b} \frac{\mathcal{N}}{(1 - b/z_0)^{1+\nu}} db$$

where $$\beta_c = \beta - r_1 - \frac{r_2}{\beta}$$

- The divergent contribution is contained in $P^{\rm ren}$ and δP represents the remaining perturbative contribution: It is renormalon-free, so can be expanded in a convergent series

- Now, define $P_{\rm NP}^{(N)} \equiv P(\beta) - P^{\rm ren}(\beta_c) - \sum_{n=1}^{N}(c_n - C_n^{\rm ren})\beta^{-n}$ where $C_n^{\rm ren}$ are the coefficients of $P^{\rm ren}$ in lattice scheme
- $P_{\rm NP}^{(\rm N)}$ is free of perturbative coefficients to $O(N)$ and regarded as a power correction.
- $P_{\rm NP}^{(8)}$ was shown to scale as a dim-2 object

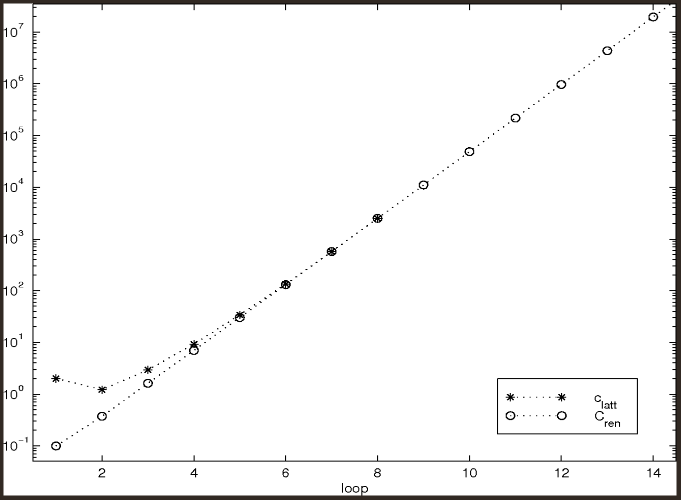
Dim-2 Condensate

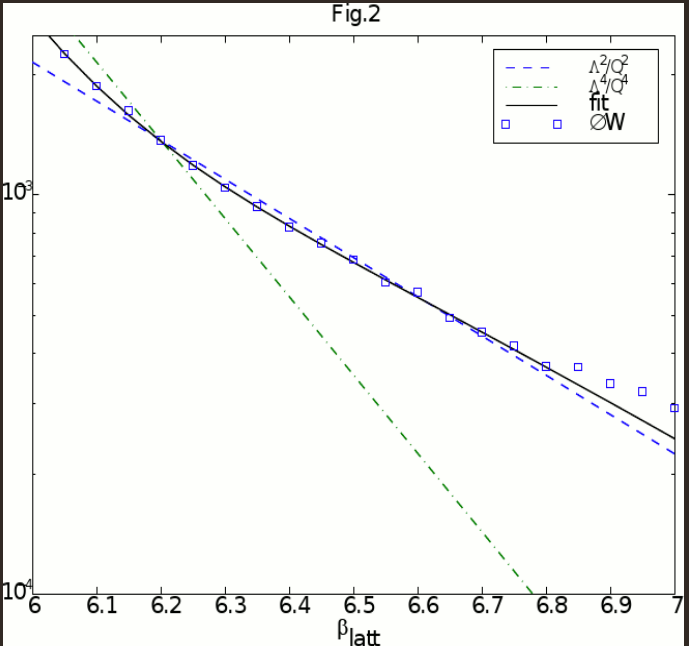
- $r_1, \quad r_2$ and $\mathcal{N}$ were fitted so that $C_n^{\text{ren}} \rightarrow c_n$ at large orders

- $r_1 = 3.1 \quad r_2 = 2.0$

- Lo and behold, a dim−2 condensate

- A dim−2 condensate is puzzling considering the OPE





Critical comments

- The purported continuum scheme is far from being a renormalon-dominant scheme
- When mapping the pertubative coefficients in the lattice scheme to the continuum scheme we find

$c_1^{\rm cont}$	$c_2^{\rm cont}$	$c_3^{\rm cont}$	$c_4^{\rm cont}$	$c_5^{\rm cont}$	$c_6^{\rm cont}$	$c_7^{\rm cont}$	$c_8^{\rm cont}$
2.0	-4.9792	10.613	-10.200	-44.218	316.34	-1096.	1947.

- The coefficients do not display a renormalon behavior: The low order coefficients must be taken into account when mapping coefficients at the computed orders
- Let's also check the consistency of the program

Consistency check

- The nonperturbative term can be written as

$$P_{\mathrm{NP}} = P_{\mathrm{NP}}^{(N)} - \{\delta P(\beta_c) - \sum_{n=1}^{N} (c_n - C_n^{\mathrm{ren}})\beta^{-n}\}$$

- For $P_{\mathrm{NP}}^{(\mathrm{N})}$ to represent the nonperturbative term

$$\left| \delta P(\beta_c) - \sum_{n=1}^{N} (c_n - C_n^{\mathrm{ren}})\beta^{-n} \right| \ll P_{\mathrm{NP}}^{(N)}$$

must be satisfied

- Since δP is a convergent quantity it can be computed to O(N), so

$$\frac{|\sum_{n=1}^{N} D_n \beta_c^{-n} - \sum_{n=1}^{N} (c_n - C_n^{\mathrm{ren}})\beta^{-n}|}{P_{\mathrm{NP}}^{(N)}} \ll 1$$

- It is severely violated, for instance, at N=8,beta=6 the ratio is 70

Lesson

- At the computed orders, the low order coefficients must be included when mapping perturbative coefficients between schemes.
- A renormalon-dominant scheme may be found by mapping the pertubative coefficients in the lattice scheme to a continuum scheme, not the other way around.

Bilocal expansion of Borel transform

- The renormalon (perturbative) contribution should be subtracted in a continuum scheme
- The bilocal expansion of Borel transform (Lee,2003) provides the necessary framework for this
- Expand the Borel transform of the average plaquette at two points, the origin and the singularity
- $$\tilde{\mathcal{P}}(b) = \sum_{n=0} \frac{h_n}{n!}\left(\frac{b}{\beta_0}\right)^n + \frac{\mathcal{N}}{(1-b/z0)^{1+\nu}}\left[1+\sum_{i=1} c_i(1-b/z_0)^i\right]$$
- Demanding the Borel transform reproduce the perturbative coefficients in continuum scheme

  the average plaquette can be written as

$$P = P_{\rm BR}(\beta_c) + P_{\rm NP}$$

where

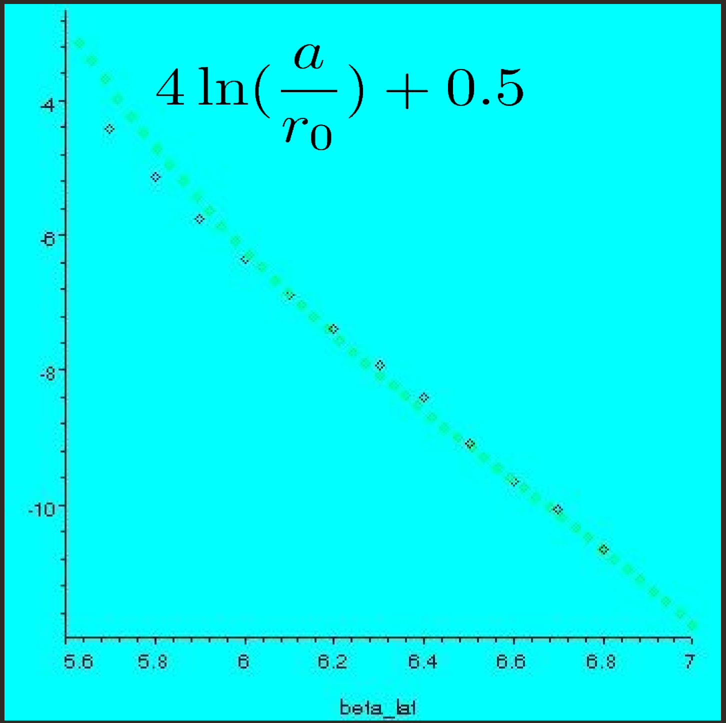
$$P_{\rm BR}(\beta_c) = \int_0^\infty e^{-\beta_c b}\tilde{P}(b)db$$

- The Borel resummed $P_{\rm BR}(\beta_c)$, by definition, reproduces the lattice coefficients exactly when expanded in β

Power correction: a dim-4 condensate

- The simulated average plaquette subtracted of perturbative contribution agrees well with a dim−4 condensate
- $r_1 = 1.61, r_2 = 0.24, \mathcal{N} = 170$
- $$\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle = \frac{36}{\pi^2} e^{0.5} \frac{1}{r_0^4} \approx 0.14 \text{ GeV}^4$$

$\ln(\mathrm{P_{simul.}} - \mathrm{P_{BR}})$ vs. β_{lat}



- Note: Condensate depends on the prescription of pertubrative contribution

Conclusions

- The existing scheme for renormalon subtraction from the average plaquette has a consistency problem
- No evidence of dim−2 condensate
- The bilocal expansion of Borel transform provides a cosistent framework for renormalon subtraction, and its result shows a dim−4 condensate
- $$\langle \frac{\alpha_s}{\pi} G^2_{\mu\nu} \rangle = \frac{36}{\pi^2} e^{0.5} \frac{1}{r_0^4}$$