# On the absence of spin-orbit inversion in heavy-light mesons

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# Introduction

- In potential model of heavy-light mesons $D_s(2317)$ and $D(2308)$ are thought to be the missing P-wave states $(n = 1, j = 1/2, l = 1)$. But the observed masses are substantially smaller than predictions.

Di Pierro and Eichten (2001)

| $H(n^jL_J)$ | $m_{expt}$ | $E^0$ | $E^{phys}$ | $\phi$ (%) |
|---|---|---|---|---|
| $D(1^{1/2}S_0)$ | 1.865 | 1.895 | 1.868 | |
| $D(1^{1/2}S_1)$ | 2.007 | 1.895 | 2.005 | |
| $D(1^{1/2}P_0)$ | | 2.282 | 2.377 | |
| $D(1^{3/2}P_1)$ | 2.422 | 2.253 | 2.417 | −10.92 |
| $D(1^{3/2}P_2)$ | 2.459 | 2.253 | 2.460 | |
| $D(1^{1/2}P_1)$ | | 2.282 | 2.490 | 10.92 |
| $D(2^{1/2}S_0)$ | | 2.447 | 2.589 | |
| $D(2^{1/2}S_1)$ | | 2.447 | 2.692 | 2.17 |
| $D(1^{5/2}D_2)$ | | 2.504 | 2.775 | −5.41 |
| $D(1^{3/2}D_1)$ | | 2.553 | 2.795 | −2.17 |
| $D(1^{5/2}D_3)$ | | 2.504 | 2.799 | |
| $D(1^{3/2}D_2)$ | | 2.553 | 2.833 | 5.41 |
| $D(2^{1/2}P_0)$ | | 2.683 | 2.949 | |
| $D(2^{3/2}P_1)$ | | 2.679 | 2.995 | −10.70 |
| $D(2^{3/2}P_2)$ | | 2.679 | 3.035 | 1.79 |
| $D(2^{1/2}P_1$ | | 2.683 | 3.045 | 10.70 |
| $D(1^{7/2}F_3)$ | | 2.709 | 3.074 | −3.17 |
| $D(1^{7/2}F_4)$ | | 2.709 | 3.091 | |
| $D(1^{5/2}F_2)$ | | 2.760 | 3.101 | −1.79 |
| $D(1^{5/2}F_3)$ | | 2.760 | 3.123 | 3.17 |
| $D(3^{1/2}S_0)$ | | 2.823 | 3.141 | |
| $D(3^{1/2}S_1)$ | | 2.823 | 3.226 | |

| $H(n^jL_J)$ | $m_{expt}$ | $E^0$ | $E^{phys}$ | $\phi$ (%) |
|---|---|---|---|---|
| $D_s(1^{1/2}S_0)$ | 1.969 | 1.988 | 1.965 | |
| $D_s(1^{1/2}S_1)$ | 2.112 | 1.988 | 2.113 | |
| $D_s(1^{1/2}P_0)$ | | 2.374 | 2.487 | |
| $D_s(1^{3/2}P_1)$ | 2.535 | 2.353 | 2.535 | −11.62 |
| $D_s(1^{3/2}P_2)$ | 2.573 | 2.353 | 2.581 | |
| $D_s(1^{1/2}P_1)$ | | 2.374 | 2.605 | 11.62 |
| $D_s(2^{1/2}S_0)$ | | 2.540 | 2.700 | |
| $D_s(2^{1/2}S_1)$ | | 2.540 | 2.806 | 1.97 |
| $D_s(1^{5/2}D_2)$ | | 2.606 | 2.900 | −6.11 |
| $D_s(1^{3/2}D_1)$ | | 2.648 | 2.913 | −1.97 |
| $D_s(1^{5/2}D_3)$ | | 2.606 | 2.925 | |
| $D_s(1^{3/2}D_2)$ | | 2.648 | 2.953 | 6.11 |
| $D_s(2^{1/2}P_0)$ | | 2.777 | 3.067 | |
| $D_s(2^{3/2}P_1)$ | | 2.775 | 3.114 | −10.58 |
| $D_s(2^{3/2}P_2)$ | | 2.775 | 3.157 | 1.81 |
| $D_s(2^{1/2}P_1)$ | | 2.777 | 3.165 | 10.58 |
| $D_s(1^{7/2}F_3)$ | | 2.812 | 3.203 | −3.60 |
| $D_s(1^{7/2}F_4)$ | | 2.812 | 3.220 | |
| $D_s(1^{5/2}F_2)$ | | 2.857 | 3.224 | −1.81 |
| $D_s(1^{5/2}F_3)$ | | 2.857 | 3.247 | 3.60 |
| $D_s(3^{1/2}S_0)$ | | 2.917 | 3.259 | |
| $D_s(3^{1/2}S_1)$ | | 2.917 | 3.345 | |

- Moreover, abnormally small mass difference between strange and non-strange states. Where has the strange mass gone?

- Experimentally,

$$Gap = [m(D(0^+)) - m(D(0^-))] - [m(D_s(0^+)) - m(D_s(0^-))] = 95 MeV$$

whereas potential model predicts vanishing Gap.

- Spin Orbit Problem. In potential model strong scalar confining potential leads to spin-orbit inversion in P-wave mesons (Schnitzer (1978), Isgur (1998) and Ebert, et.al (1998))

- Data show no spin-orbit inversion.

| | $D_s(1, \frac{1}{2}, 0)$ | $D_s(1, \frac{1}{2}, 1)$ | $D_s(1, \frac{3}{2}, 1)$ | $D_s(1, \frac{3}{2}, 2)$ |
|---|---|---|---|---|
| $m_{expt.}$ | 2317 | 2460 | 2535 | 2573 |
| $E$ | 2487 | 2605 | 2535 | 2581 |

- Can the mass puzzle and spin-orbit problem be resolved within potential model?

# Potential model

- The potential model of heavy-light system is based on the chiral quark model (Goity and Roberts ('99), Di Pierro and Eichten('01)).
- Lagrangian:

$$\mathcal{L} = \Psi^\dagger (i\partial_0 - H)\Psi$$

with Hamiltonian given by

$$H = H_0 + \frac{1}{m_h} H_1 + \cdots$$

where $m_h$ denotes the heavy quark mass and the leading Hamiltonian $H_0$ reads

$$H_0 = \gamma^0(-i\not\nabla + m) + V(r)$$

with the potential given in the form

$$V(r) = M_h + \gamma^0 V_s(r) + V_v(r)\,,$$

and the scalar and vector potentials $V_s$ and $V_v$ read

$$V_s = br + c\,, \quad V_v = -\frac{4}{3}\frac{\alpha_s}{r}$$

and $m = m_i \delta_{ij}$ denotes the constituent quark masses.

- Nine free parameters to be fixed by fitting: $\{\alpha_s, \lambda, b, m_u, m_d, m_s, M_c, m_b, M_b\}$
- Problem: Chiral symmetry breaking is encoded only in the constituent light-quark masses.
- The light quarks carry chiral charges, so chiral radiative corrections must be taken into account.
- Decay widths $(-2Im(\Sigma))$ in potential model are of a few hundred MeVs $\Longrightarrow$ Chiral loop corrections expected to be $\sim 100 MeVs$

# Chiral radiative corrections

EPJC 49, 737(2007), (with I. Lee, D. Min, B. Park); PRD 76, 014017(2007), (with I. Lee)

- Chiral quark model (Georgi-Manohar) contains an infinite tower of derivative expansions for constituent light-quark—Goldstone boson coupling.

- One loop corrections arise from

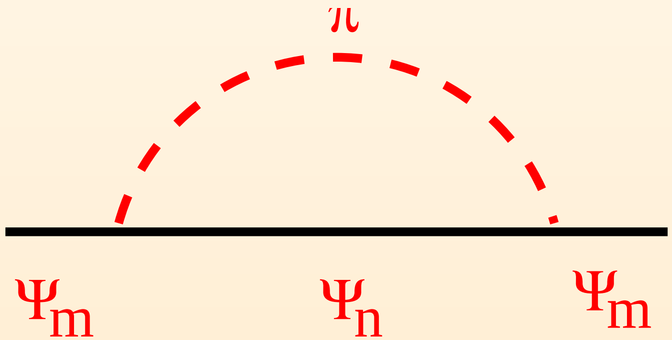
$$H_{\bar{\psi}\psi\pi} = -g_A \bar{\Psi} \not{A} \gamma_5 \Psi = \frac{g_A}{2f_\pi} \bar{\Psi}_i \gamma^\mu \gamma_5 \Psi_j \partial_\mu \Pi_{ij} + O(\Pi^2)$$

where $g_A$ is an axial coupling constant, and

$$A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$$

with $\xi = e^{i\Pi/2f_\pi}$, where $f_\pi = 93$ MeV, and

$$\Pi = \begin{pmatrix} \pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2\eta}{\sqrt{3}} \end{pmatrix}$$




- One loop induced Hamiltonian:

$$\delta H = \Psi^\dagger \Sigma \Psi$$

where

$$\Sigma \equiv -i(\frac{g_A}{2f_\pi})^2\gamma^0\gamma^\mu\gamma_5\langle\Psi\Psi^\dagger\rangle\gamma^0\gamma^\nu\gamma_5\langle\partial_\mu\pi\partial_\nu\pi\rangle$$

with the quark propagator

$$\langle\Psi\Psi^\dagger\rangle = \frac{i}{i\partial_0 - H_0 + i\epsilon}$$

- Energy correction for the state $\Psi_m$, where $m = \{n, l, j, m_j, q\}$:

$$\Delta E_m = \frac{ig_A^2}{4f_\pi^2}\sum_n\sum_\pi\zeta_\pi\int\frac{d^4k}{(2\pi)^4}\frac{|j_{mn}(k)|^2}{(E_m - k_0 - E_n + i\epsilon)(k^2 - m_\pi^2 + i\epsilon)}$$

where

$$j_{mn}(k) = \left(\int d^3\vec{x}\ \Psi_m^\dagger(\vec{x})\gamma^0\gamma^\mu\gamma_5\Psi_n(\vec{x})e^{i\vec{k}\cdot\vec{x}}\right)k_\mu$$

- Using the Dirac equation for $\Psi_m$ and performing the $k_0$ and angular integration, with $e^{i\vec{k}\cdot\vec{x}}$ expanded in spherical harmonics, we obtain

$$\Delta E_m^{loop} = \sum_n \sum_{\pi, l_\pi} \zeta_\pi J(m,n,l_\pi) C(l_\pi, l_n, j_n),$$

where

$$J(m,n,l_\pi) = -\frac{g_A^2}{8f_\pi^2}\int_0^\infty \frac{k^2 dk}{(2\pi)^3 E_\pi} e^{-\vec{k}^2/\Lambda_{UV}^2}\Big[(E_n^0 - E_m^0)|\rho_{mn}^{(1)}(|\vec{k}|, l_\pi)|^2 + 2Re[\rho_{mn}^{(1)}(|\vec{k}|, l_\pi)\rho_{mn}^{(2)*}(|\vec{k}|, l_\pi)] + \frac{|\rho_{mn}^{(2)}(|\vec{k}|, l_\pi)|^2}{E_\pi - E_m^0 + E_n^0 - i\epsilon}\Big]$$

with

$$\rho^{(1)}_{m,n}(|\vec{k}|, l_\pi) = \sqrt{4\pi}\int_0^\infty r^2 dr (f_m(r) g_n(r) - f_n(r) g_m(r)) j_{l_\pi}(kr),$$

$$\rho^{(2)}_{m,m}(|\vec{k}|, l_\pi) = \sqrt{4\pi}\int_0^\infty r^2 dr (f_m(r) g_n(r) + f_n(r) g_m(r)) \times (m_m + m_n + 2V_s) j_{l_\pi}(kr) \,.$$

and

$$C(l_\pi, l_n, j_n) = j_n + \frac{1}{2}, \quad for \quad j_m = 1/2$$

and

$$C(l_\pi, l_n, j_n) = \left(\frac{3l_\pi(l_\pi - 1)}{2(2l_\pi - 1)}\delta_{l_n, l_\pi - 2} + \frac{l_\pi(l_\pi + 1)}{2(2l_\pi + 3)}\delta_{l_n, l_\pi}\right)\delta_{j_n, l_n + \frac{1}{2}} + \left(\frac{l_\pi(l_\pi + 1)}{2(2l_\pi - 1)}\delta_{l_n, l_\pi} + \frac{3(l_\pi + 1)(l_\pi + 2)}{2(2l_\pi + 3)}\delta_{l_n, l_\pi + 2}\right)\delta_{j_n, l_n - \frac{1}{2}}$$

for $l_m = 1, j_m = 3/2$, and

$$C(l_\pi, l_n, j_n) \;=\; \left(\frac{l_\pi(l_\pi+1)}{2(2l_\pi-1)}\delta_{l_n,l_\pi-1} + \frac{3(l_\pi+1)(l_\pi+2)}{2(2l_\pi+3)}\delta_{l_n,l_\pi+1}\right)\delta_{j_n,l_n+\frac{1}{2}}$$
$$\left(\frac{l_\pi(l_\pi+1)}{2(2l_\pi+3)}\delta_{l_n,l_\pi+1} + \frac{3l_\pi(l_\pi-1)}{2(2l_\pi-1)}\delta_{l_n,l_\pi-1}\right)\delta_{j_n,l_n-\frac{1}{2}}$$

for $l_m = 2, j_m = 3/2$, and

$$C(l_\pi, l_n, j_n) = \left(\frac{l_\pi(l_\pi-1)(l_\pi+1)}{(2l_\pi-1)(2l_\pi+3)}\delta_{l_n,l_\pi-1} + \frac{l_\pi(l_\pi+1)(l_\pi+2)}{2(2l_\pi+3)(2l_\pi+5)}\delta_{l_n,l_\pi+1}\right.$$
$$\left. + \frac{5l_\pi(l_\pi-1)(l_\pi-2)}{2(2l_\pi-1)(2l_\pi-3)}\delta_{l_n,l_\pi-3}\right)\delta_{j_n,l_n+\frac{1}{2}} +$$
$$\left(\frac{l_\pi(l_\pi-1)(l_\pi+1)}{2(2l_\pi-1)(2l_\pi-3)}\delta_{l_n,l_\pi-1} + \frac{l_\pi(l_\pi+1)(l_\pi+2)}{(2l_\pi-1)(2l_\pi+3)}\delta_{l_n,l_\pi+1}\right.$$

$$+\frac{5(l_\pi+1)(l_\pi+2)(l_\pi+3)}{2(2l_\pi+3)(2l_\pi+5)}\delta_{l_n,l_\pi+3}\Bigg)\,\delta_{j_n,l_n-\frac{1}{2}}$$

for $l_m = 2, j_m = 5/2$.

# Mass Gap

- $$Gap \equiv [m(D(0^+)) - m(D(0^-))] - [m(D_s(0^+)) - m(D_s(0^-))]$$

- Modified energy levels:

$$\bar{E}_m = E_m + \delta E_m^0 + \delta E_m^{loop} \,,$$

where $E_m$ denotes the conventional energy levels. This is valid to the leading order of loop corrections.

$$\overline{Gap} \;\; = \;\; Gap + \delta Gap^0 + \delta Gap^{loop}$$

- $$Gap \;\; \approx \;\; Gap^0 \approx 0 \Longrightarrow \overline{Gap} \approx \delta Gap^{loop}$$

TABLE I: Energy corrections from the first five radial excitations at $l_\pi = 0$. Gap $\equiv (\Delta E1_d - \Delta E0_d) - (\Delta E1_s - \Delta E0_s)$. Values are at $\Lambda_{UV} = 700$ MeV and $g_A = 1$. Units are in MeV.

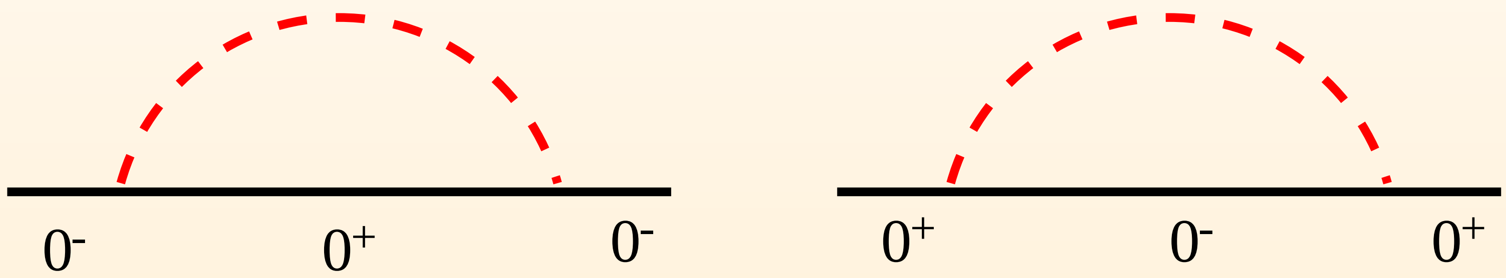
| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\Delta E0_d$ | -23 | -3 | 0 | 0 | 0 |
| $\Delta E1_d$ | 29 | -49 | -8 | -1 | 0 |
| $\Delta E0_s$ | -33 | -3 | 0 | 0 | 0 |
| $\Delta E1_s$ | -108 | -49 | -9 | -1 | 0 |
| Gap | 127 | 0 | 1 | 0 | 0 |

TABLE II: Energy corrections ($g_A = 1$) vs $l_\pi$. Gap $\equiv (\Delta E1_d - \Delta E0_d) - (\Delta E1_s - \Delta E0_s)$. Units are in MeV.

| | $l_\pi$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Lambda_{UV} = 700$ | -26 | -142 | -50 | -16 | -5 | -1 | 0 | 0 | 0 | 0 |
| $\Delta E0_d$ | $\Lambda_{UV} = 1000$ | -116 | -361 | -196 | -93 | -41 | -17 | -7 | -3 | -1 | 0 |
| | $\Lambda_{UV} = 1200$ | -201 | -561 | -362 | -202 | -105 | -52 | -25 | -12 | -6 | -3 |
| | $\Lambda_{UV} = 700$ | -29 | -192 | -101 | -42 | -15 | -5 | -2 | 0 | 0 | 0 |
| $\Delta E1_d$ | $\Lambda_{UV} = 1000$ | -64 | -400 | -310 | -185 | -98 | -49 | -23 | -11 | -5 | -2 |
| | $\Lambda_{UV} = 1200$ | -93 | -558 | -502 | -349 | -215 | -124 | -69 | -37 | -19 | -10 |
| | $\Lambda_{UV} = 700$ | -37 | -136 | -47 | -14 | -4 | 0 | 0 | 0 | 0 | 0 |
| $\Delta E0_s$ | $\Lambda_{UV} = 1000$ | -141 | -376 | -197 | -88 | -37 | -15 | -6 | -2 | 0 | 0 |
| | $\Lambda_{UV} = 1200$ | -239 | -599 | -372 | -197 | -97 | -46 | -21 | -10 | -4 | -2 |
| | $\Lambda_{UV} = 700$ | -168 | -186 | -98 | -39 | -14 | -4 | -1 | 0 | 0 | 0 |
| $\Delta E1_s$ | $\Lambda_{UV} = 1000$ | -217 | -413 | -317 | -183 | -94 | -45 | -20 | -9 | -4 | -2 |
| | $\Lambda_{UV} = 1200$ | -253 | -589 | -524 | -354 | -210 | -117 | -63 | -32 | -16 | -8 |
| | $\Lambda_{UV} = 700$ | 128 | 0 | 0 | -1 | 0 | 0 | -1 | 0 | 0 | 0 |
| Gap | $\Lambda_{UV} = 1000$ | 128 | -2 | 6 | 3 | 0 | -2 | -2 | -1 | 0 | 0 |
| | $\Lambda_{UV} = 1200$ | 122 | -7 | 12 | 10 | 3 | -1 | -2 | -3 | -1 | -1 |

- Contributions from the radially excited states drop rapidly.

- $\delta Gap^{loop} = (\Delta E1_d - \Delta E0_d) - (\Delta E1_s - \Delta E0_s)$ is dominated by the n=1, $\ell_\pi = 0$ modes. Contributions from the higher angular and radial modes are not so sensitive to the quark flavors because the larger energy differences between internal and external states offset the light-meson masses. $\Longrightarrow$ Light meson mass effects are significant only for $0^-$ and $0^+$ exchanges for $0^+$ and $0^-$, respectively.



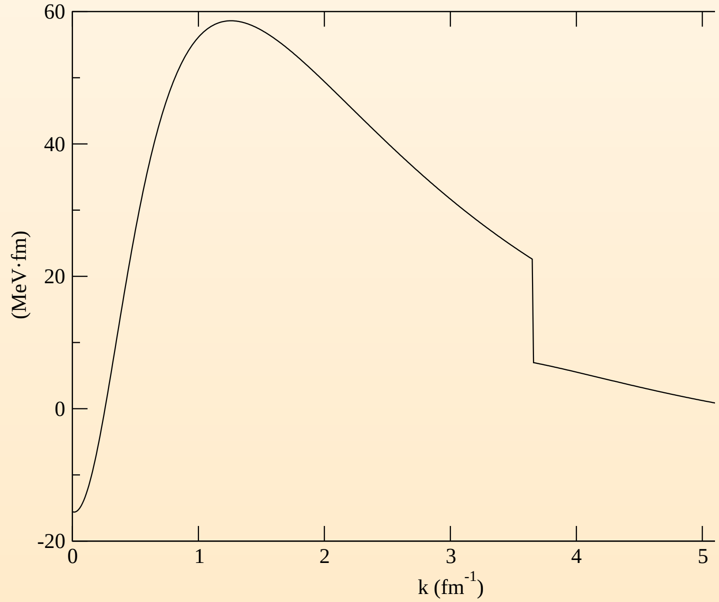

- The total corrections are sensitive to the cutoff, but the differences in energy corrections are less sensitive.

$$\overline{Gap} = (\Delta E1_d - \Delta E0_d) - (\Delta E1_s - \Delta E0_s) = 84, 87, 89 \quad MeV$$

at $\Lambda_{UV} = 700, 1000, 1200$ MeV, respectively, and $g_A = 0.82$.

- The Gap is not sensitive to the cutoff. Consider

$$Gap = \int_0^\infty \mathcal{G}(k)dk$$



- Main contribution to the Gap comes from the low energy region, $k \sim 250$MeV, far down the cutoff.
- Good agreement with the experimental value $Gap = 95$ MeV.

## Spin-orbit inversion

- $$\bar{E}_j - \bar{E}_{j'} = E_j - E_{j'} + \delta E_j^{loop} - \delta E_{j'}^{loop} + \delta(E_j^0 - E_{j'}^0)\,,$$

- $\delta(E_j^0 - E_{j'}^0)$ are small, so
$$\bar{E}_j - \bar{E}_{j'} = E_j - E_{j'} + \delta E_j^{loop} - \delta E_{j'}^{loop}\,.$$

- $\Lambda_{UV} = 700$

| $\delta E^{loop}_{1,\frac{1}{2},d}$ | $\delta E^{loop}_{1,\frac{3}{2},d}$ | $\delta E^{loop}_{1,\frac{1}{2},s}$ | $\delta E^{loop}_{1,\frac{3}{2},s}$ | $\delta E^{loop}_{2,\frac{3}{2},d}$ | $\delta E^{loop}_{2,\frac{5}{2},d}$ | $\delta E^{loop}_{2,\frac{3}{2},s}$ | $\delta E^{loop}_{2,\frac{5}{2},s}$ |
|---|---|---|---|---|---|---|---|
| -261 | -183 | -344 | -181 | -257 | -184 | -275 | -184 |

- Loop corrections are negative and larger for smaller j, crucial for spin-orbit inversion.

- Charmed Mesons

| | $D(1,\frac{1}{2},0)$ | $D(1,\frac{1}{2},1)$ | $D(1,\frac{3}{2},1)$ | $D(1,\frac{3}{2},2)$ |
|---|---|---|---|---|
| $m_{expt.}$ | 2308 | 2427 | 2422 | 2459 |
| $\bar{E}$ | 2308 | 2421 | 2425 | 2468 |
| $E$ | 2377 | 2490 | 2417 | 2460 |

| | $D_s(1,\frac{1}{2},0)$ | $D_s(1,\frac{1}{2},1)$ | $D_s(1,\frac{3}{2},1)$ | $D_s(1,\frac{3}{2},2)$ |
|---|---|---|---|---|
| $m_{expt.}$ | 2317 | 2460 | 2535 | 2573 |
| $\bar{E}$ | 2317 | 2435 | 2527 | 2573 |
| $E$ | 2487 | 2605 | 2535 | 2581 |

- Bottom Mesons: Loop corrections are heavy-quark mass independent, so

$$\bar{E}_{B_s(l,j,J)} - E_{B_s(l,j,J)} = \bar{E}_{D_s(l,j,J)} - E_{D_s(l,j,J)} \,.$$

| | $B(1,\frac{1}{2},0)$ | $B(1,\frac{1}{2},1)$ | $B(1,\frac{3}{2},1)$ | $B(1,\frac{3}{2},2)$ |
|---|---|---|---|---|
| $\bar{E}$ | 5637 | 5673 | 5709 | 5723 |
| $E$ | 5706 | 5742 | 5700 | 5714 |

| | $B_s(1,\frac{1}{2},0)$ | $B_s(1,\frac{1}{2},1)$ | $B_s(1,\frac{3}{2},1)$ | $B_s(1,\frac{3}{2},2)$ |
|---|---|---|---|---|
| $\bar{E}$ | 5634 | 5672 | 5798 | 5813 |
| $E$ | 5804 | 5842 | 5805 | 5820 |

- For D-wave states, define $\Delta \equiv \bar{E}_{H(2,j,J)} - \bar{E}_{H(2,\frac{3}{2},1)}$

| | $D(2,\frac{3}{2},2)$ | $D(2,\frac{5}{2},2)$ | $D(2,\frac{5}{2},3)$ | $D_s(2,\frac{3}{2},2)$ | $D_s(2,\frac{5}{2},2)$ | $D_s(2,\frac{5}{2},3)$ |
|---|---|---|---|---|---|---|
| $\Delta$ | 38 | 53 | 77 | 40 | 78 | 103 |

Table 1: Level differences in D-wave charmed mesons. (Units are in MeV.)

| | $B(2,\frac{3}{2},2)$ | $B(2,\frac{5}{2},2)$ | $B(2,\frac{5}{2},3)$ | $B_s(2,\frac{3}{2},2)$ | $B_s(2,\frac{5}{2},2)$ | $B_s(2,\frac{5}{2},3)$ |
|---|---|---|---|---|---|---|
| $\Delta$ | 12 | 33 | 41 | 13 | 59 | 67 |

Table 2: Level differences in D-wave bottom mesons. (Units are in MeV.)

# Summary

- One loop chiral corrections are sizable and must be incorporated in potential model.

- $[m(D(0^+)) - m(D(0^-))] - [m(D_s(0^+)) - m(D_s(0^-))]$ is in good agreement with data. Expect to hold on B system as well.

- No spin-orbit inversion once loop corrections are taken into account.

- Some of P-wave bottom meson masses are predicted: smaller mass for $B_s(0^+)$